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Multiple Attribute Group Decision-Making Method Based on Linguistic Neutrosophic Numbers

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Academic Editors: Florentin Smarandache and Sergei D. Odintsov
Received: 2 June 2017; Accepted: 3 July 2017; Published: 7 July 2017

Abstract: Existing intuitionistic linguistic variables can describe the linguistic information of both the truth/membership and falsity/non-membership degrees, but it cannot represent the indeterminate and inconsistent linguistic information. To deal with the issue, this paper originally proposes the concept of a linguistic neutrosophic number (LNN), which is characterized independently by the truth, indeterminacy, and falsity linguistic variables. Then, we define the basic operational laws of LNNs and the score and accuracy functions of LNN for comparing LNNs. Next, we develop an LNN-weighted arithmetic averaging (LNNWAA) operator and an LNN-weighted geometric averaging (LNNWGA) operator to aggregate LNN information and investigate their properties. Further, a multiple attribute group decision-making method based on the proposed LNNWAA or LNNWGA operator is established under LNN environment. Finally, an illustrative example about selecting problems of investment alternatives is presented to demonstrate the application and effectiveness of the developed approach.

Keywords: linguistic neutrosophic number; score function; accuracy function; linguistic neutrosophic number weighted arithmetic averaging (LNNWAA) operator; linguistic neutrosophic number weighted geometric averaging (LNNWGA) operator; multiple attribute group decision-making

1. Introduction

In complex decision-making problems, human judgments, including preference information, may be difficultly stated in numerical values due to the ambiguity of human thinking about the complex objective things in the real world, and then may be easily expressed in linguistic terms, especially for some qualitative attributes. Thus, decision-making problems under linguistic environments are interesting research topics, which have received more and more attentions from researchers in past decades. Zadeh [1] firstly introduced the concept of linguistic variables and the application in fuzzy reasoning. Later, Herrera et al. [2] and Herrera and Herrera-Viedma [3] presented linguistic decision analyses to deal with decision-making problems with linguistic information. Next, Xu [4] put forward a linguistic hybrid arithmetic averaging operator for multiple attribute group decision-making (MAGDM) problems with linguistic information. Further, Xu [5] developed goal programming models for multiple attribute decision-making (MADM) problems with linguistic information. Some scholars [6–8] also proposed two-dimension uncertain linguistic operations and aggregation operators and applied them to decision-making. By combining intuitionistic fuzzy numbers (IFNs) (basic elements in intuitionistic fuzzy sets) introduced in [9] and linguistic variables introduced in [1], Chen et al. [10] proposed the linguistic intuitionistic fuzzy number (LIFN) denoted by the form of $s = (l_p, l_q)$, where l_p and l_q stand for the linguistic variables of the truth/membership and falsity/non-membership degrees, respectively, and developed a MAGDM method with LIFNs. Then, Liu and Wang [11] presented some improved LIFN aggregation operators for MADM. It is obvious that the LIFN consists of two linguistic variables l_p and l_q and describes the linguistic information of both the truth/membership and falsity/non-membership degrees, which are expressed by linguistic values rather than exact values like IFNs. However, LIFNs cannot describe indeterminate and inconsistent linguistic information. Then, a single-valued neutrosophic number (SVNN), which is a basic element in a single-valued neutrosophic set (SVNS) [12,13], can only express the truth, indeterminacy, and falsity degrees independently, and describe the incomplete, indeterminate, and inconsistent information in SVNN rather than linguistic information; then, it cannot express linguistic information in linguistic decision-making problems, while linguistic variables can represent the qualitative information for attributes in complex MADM problems. Hence, Ye [13] proposed the single-valued neutrosophic linguistic number (SVNLN), which is composed of a linguistic variable and an SVNN, where the linguistic variable is represented as the decision-maker's judgment to an evaluated object and the SVNN is expressed as the reliability of the given linguistic variable, and developed an extended TOPSIS method for MAGDM problems with SVNLNs. However, SVNLN cannot also describe the truth, indeterminacy, and falsity linguistic information according to a linguistic term set. Tian et al. [14] put forward a simplified neutrosophic linguistic MAGDM approach for green product development. Liu and Tang [15] presented an interval neutrosophic uncertain linguistic Choquet integral method for MAGDM. Liu and Shi [16] introduced some neutrosophic uncertain linguistic number Heronian mean operators for MAGDM. However, all existing linguistic decision-making methods cannot express and deal with decision-making problems with indeterminate and inconsistent linguistic information.

To overcome the aforementioned insufficiency for SVNNs, LIFNs, and SVNLNs, a feasible solution is to represent the truth, indeterminacy, and falsity degrees independently by three linguistic variables to an evaluated object. On the other hand, human judgments under a linguistic decision-making environment should also contain the linguistic information of truth/determinacy, indeterminacy, and falsity degrees since SVNN contains the information of the truth/determinacy, indeterminacy, and falsity degrees. Based on this idea, it is necessary to propose the concept of a linguistic neutrosophic number (LNN) by combining SVNN and linguistic variables, where its truth, indeterminacy, and falsity degrees can be described by three linguistic variables rather than three exact values, like an SVNN, or both a linguistic value and an SVNN, like an SVNLN. For example, a company wants to select a supplier. Suppose that a decision-maker evaluates it based on a linguistic term set $L = \{l_0$ = extremely low, l_1 = very low, l_2 = low, l_3 = slightly low, l_4 = medium, l_5 = slightly high, l_6 = high, l_7 = very high, l_8 = extremely high$\}$. If the evaluation of a supplier with respect to its service performance is given as l_6 for the truth/membership degree, l_2 for the indeterminacy degree, and l_3 for the falsity/non-membership degree, respectively, by the decision-maker corresponding to the linguistic term set L then, for the concept of an LNN, it can be expressed as the form of an LNN $e = <l_6, l_2, l_3>$. Obviously, LIFN and SVNLN cannot express such kinds of linguistic evaluation values; while LNN can easily describe them in a linguistic setting by the extension of SVNN and LIFN to LNN. Therefore, it is necessary to introduce LNN for expressing indeterminate and inconsistent linguistic information corresponding to human fuzzy thinking about complex problems, especially for some qualitative evaluations for attributes, and solving linguistic decision-making problems with indeterminate and inconsistent linguistic information. However, LNNs are very suitable for describing more complex linguistic information of human judgments under linguistic decision-making environment since LNNs contain the advantages of both SVNNs and linguistic variables, which imply the truth, falsity, and indeterminate linguistic information. To aggregate LNN information in MAGDM problems, we have to develop some weighted aggregation operators, including an LNN-weighted arithmetic averaging (LNNWAA) operator and an LNN-weighted geometric averaging (LNNWGA) operator, which are usually used for MADM/MAGDM problems, score, and accuracy functions for the comparison of LNNs, and their decision-making method. Thus, the purposes of this paper are (1) to propose LNNs and their basic operational laws; (2) to introduce the score and accuracy functions of the LNN for comparing LNNs; (3) to present the LNNWAA and LNNWGA operators, their properties, and special cases; (4) to develop a MAGDM method based on the LNNWAA or LNNWGA operator under an LNN environment; and (5) to explain the advantages of the proposed method.

The rest of this paper is organized as follows: Section 2 briefly reviews the basic concept of LIFNs, the basic operational laws of LIFNs, and the score and accuracy functions of LIFN for the comparison of LIFNs. In Section 3, LNNs and their basic operational laws are presented as the extension of LIFNs, and then the score and accuracy functions for an LNN are defined to compare LNNs. Section 4 develops the LNNWAA and LNNWGA operators for aggregating LNNs and discusses their properties and some special cases. In Section 5, a MAGDM method is developed by using the LNNWAA or LNNWGA operator under LNN environment. In Section 6, an illustrative example about selecting problem of investment alternatives demonstrates the application of the presented method. Section 7 gives conclusions and future research directions.

2. Linguistic Intuitionistic Fuzzy Numbers

Under a linguistic intuitionistic fuzzy environment, Chen et al. [10] introduced the concept of LIFNs and gave the following definition:

Definition 1. *[10] Assume that $L = \{l_0, l_1, \ldots, l_t\}$ is a linguistic term set with odd cardinality $t + 1$, where l_j ($j = 0, 1, \ldots, t$) is a possible value for a linguistic variable. If there is $s = (l_p, l_q)$ for $l_p, l_q \in L$ and $p, q \in [0, t]$, then s is called LIFN.*

Definition 2. *[10] Let $s = (l_p, l_q)$, $s_1 = (l_{p_1}, l_{q_1})$, and $s_2 = (l_{p_2}, l_{q_2})$ be three LIFNs in L and $\rho > 0$, then there are the following operational laws of the LIFNs:*

$$s_1 \oplus s_2 = (l_{p_1}, l_{q_1}) \oplus (l_{p_2}, l_{q_2}) = \left(l_{p_1+p_2-\frac{p_1p_2}{t}}, l_{\frac{q_1q_2}{t}}\right); \tag{1}$$

$$s_1 \otimes s_2 = (l_{p_1}, l_{q_1}) \otimes (l_{p_2}, l_{q_2}) = \left(l_{\frac{p_1p_2}{t}}, l_{q_1+q_2-\frac{q_1q_2}{t}}\right); \tag{2}$$

$$\rho s = \rho(l_p, l_q) = \left(l_{t-t(1-\frac{p}{t})^\rho}, l_{t(\frac{q}{t})^\rho}\right); \tag{3}$$

$$s^\rho = (l_p, l_q)^\rho = \left(l_{t(\frac{p}{t})^\rho}, l_{t-t(1-\frac{q}{t})^\rho}\right). \tag{4}$$

Then, Chen et al. [10] defined the score and accuracy functions for the comparison of LIFNs.

Definition 3. *[10] Let $s = (l_p, l_q)$ be a LIFN in L, then the score and accuracy functions are defined as follows:*

$$S(s) = p - q; \tag{5}$$

$$H(s) = p + q. \tag{6}$$

Definition 4. *[10] Let $s_1 = (l_{p_1}, l_{q_1})$ and $s_2 = (l_{p_2}, l_{q_2})$ be two LIFNs in L, then there are the following comparative relations:*

(1) If $S(s_1) < S(s_2)$, then $s_1 \prec s_2$;
(2) If $S(s_1) > S(s_2)$, then $s_1 \succ s_2$;
(3) If $S(s_1) = S(s_2)$ and $H(s_1) < H(s_2)$, then $s_1 \prec s_2$;
(4) If $S(s_1) = S(s_2)$ and $H(s_1) > H(s_2)$, then $s_1 \succ s_2$;
(5) If $S(s_1) = S(s_2)$ and $H(s_1) = H(s_2)$, then $s_1 = s_2$.

3. Linguistic Neutrosophic Numbers

An SVNS is described independently by the truth, indeterminacy, and falsity membership functions, which is a subclass of a neutrosophic set [12]. Then, an SVNN (a basic element in an SVNS) consists of the truth T, indeterminacy I, and falsity F, which is denoted by N = <T, I, F> for T, I, $F \in$ [0, 1] and $0 \leq T + I + F \leq 3$. Then in some complex decision situations (especially for some qualitative arguments), it is difficult for decision-makers to give the truth, indeterminacy, and falsity degrees with crisp numbers. A feasible solution is to express them by linguistic arguments. Based on this idea, we can introduce a linguistic neutrosophic concept to express incomplete, indeterminate, inconsistent linguistic information. In this section, we propose an LNN, which consists of the truth, indeterminacy, and falsity linguistic variables. Intuitively, LNNs can more easily deal with fuzzy linguistic information because the three linguistic variables in an LNN can be expressed independently by three linguistic values rather than exact values, like a SVNN.

Definition 5. *Assume that* $L = \{l_0, l_1, \ldots, l_t\}$ *is a linguistic term set with odd cardinality* $t + 1$*. If* $e = \langle l_p, l_q, l_r \rangle$ *is defined for* $l_p, l_q, l_r \in L$ *and* $p, q, r \in [0, t]$*, where* l_p*,* l_q*, and* l_r *express independently the truth degree, indeterminacy degree, and falsity degree by linguistic terms, respectively, then e is called an LNN.*

Definition 6. *Let* $e = \langle l_p, l_q, l_r \rangle$*,* $e_1 = \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle$*, and* $e_2 = \langle l_{p_2}, l_{q_2},, l_{r_2} \rangle$ *be three LNNs in L and* $\rho > 0$*, then there are the following operational laws of the LNNs:*

$$e_1 \oplus e_2 = \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle \oplus \langle l_{p_2}, l_{q_2}, l_{r_2} \rangle = \left(l_{p_1+p_2-\frac{p_1p_2}{t}}, l_{\frac{q_1q_2}{t}}, l_{\frac{r_1r_2}{t}} \right); \tag{7}$$

$$e_1 \otimes e_2 = \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle \otimes \langle l_{p_2}, l_{q_2}, l_{r_2} \rangle = \left\langle l_{\frac{p_1p_2}{t}}, l_{q_1+q_2-\frac{q_1q_2}{t}}, l_{r_1+r_2-\frac{r_1r_2}{t}} \right\rangle; \tag{8}$$

$$\rho e = \rho \langle l_p, l_q, l_r \rangle = \left\langle l_{t-t(1-\frac{p}{t})^\rho}, l_{t(\frac{q}{t})^\rho}, l_{t(\frac{r}{t})^\rho} \right\rangle; \tag{9}$$

$$e^\rho = \langle l_p, l_q, l_r \rangle^\rho = \left\langle l_{t(\frac{p}{t})^\rho}, l_{t-t(1-\frac{q}{t})^\rho}, l_{t-t(1-\frac{r}{t})^\rho} \right\rangle. \tag{10}$$

It is obvious that the above operational results are still LNNs.

Example 1. *Assume that* $e_1 = \langle l_6, l_2, l_3 \rangle$ *and* $e_2 = \langle l_5, l_1, l_2 \rangle$ *be two LNNs in L and* $\rho = 0.5$*, then there are the following operational results:*

(1)
$$\begin{aligned} e_1 \oplus e_2 &= \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle \oplus \langle l_{p_2}, l_{q_2}, l_{r_2} \rangle = \left\langle l_{p_1+p_2-\frac{p_1p_2}{t}}, l_{\frac{q_1q_2}{t}}, l_{\frac{r_1r_2}{t}} \right\rangle \\ &= \langle l_{6+5-6\times5/8}, l_{2\times1/8}, l_{3\times2/8} \rangle = \langle l_{7.25}, l_{0.25}, l_{0.75} \rangle, \end{aligned}$$

(2)
$$\begin{aligned} e_1 \otimes e_2 &= \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle \otimes \langle l_{p_2}, l_{q_2}, l_{r_2} \rangle = \left\langle l_{\frac{p_1p_2}{t}}, l_{q_1+q_2-\frac{q_1q_2}{t}}, l_{r_1+r_2-\frac{r_1r_2}{t}} \right\rangle \\ &= \left\langle l_{\frac{6\times5}{8}}, l_{2+1-\frac{2\times1}{8}}, l_{3+2-\frac{3\times2}{8}} \right\rangle = \langle l_{3.75}, l_{2.75}, l_{4.25} \rangle, \end{aligned}$$

(3)
$$\begin{aligned} \rho e_1 &= \rho \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle = \left\langle l_{t-t(1-\frac{p_1}{t})^\rho}, l_{t(\frac{q_1}{t})^\rho}, l_{t(\frac{r_1}{t})^\rho} \right\rangle = \left\langle l_{8-8(1-\frac{6}{8})^{0.5}}, l_{8(\frac{2}{8})^{0.5}}, l_{8(\frac{3}{8})^{0.5}} \right\rangle \\ &= \langle l_4, l_4, l_{4.899} \rangle, \end{aligned}$$

(4) $e_1^{\rho} = \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle^{\rho} = \left\langle l_{t(\frac{p_1}{t})^{\rho}}, l_{t-t(1-\frac{q_1}{t})^{\rho}}, l_{t-t(1-\frac{r_1}{t})^{\rho}} \right\rangle = \left\langle l_{8(\frac{6}{8})^{0.5}}, l_{8-8(1-\frac{2}{8})^{0.5}}, l_{8-8(1-\frac{3}{8})^{0.5}} \right\rangle = \langle l_{6.9282}, l_{1.0718}, l_{1.6754} \rangle$.

Then, we can define the score function and accuracy function for the comparison of LNNs.

Definition 7. *Let e = <l_p, l_q, l_r> be an LNN in L. Then the score and accuracy functions of e are defined as follows:*

$$Q(e) = (2t + p - q - r)/(3t) \text{ for } Q(e) \in [0, 1]; \quad (11)$$

$$T(e) = (p - r)/t \text{ for } T(e) \in [-1, 1]. \quad (12)$$

Definition 8. *Let* $e_1 = \langle l_{p_1}, l_{q_1}, l_{r_1} \rangle$ *and* $e_2 = \langle l_{p_2}, l_{q_2}, , l_{r_2} \rangle$ *be two LNNs in L, then their comparative relations are as follows:*

(1) *If $Q(e_1) < Q(e_2)$, then $e_1 \prec e_2$;*
(2) *If $Q(e_1) > Q(e_2)$, then $e_1 \succ e_2$;*
(3) *If $Q(e_1) = Q(e_2)$ and $T(e_1) < T(e_2)$, then $e_1 \prec e_2$;*
(4) *If $Q(e_1) = Q(e_2)$ and $T(e_1) > T(e_2)$, then $e_1 \succ e_2$;*
(5) *If $Q(e_1) = Q(e_2)$ and $T(e_1) = T(e_2)$, then $e_1 = e_2$.*

Example 2. *Assume that e_1 = <l_6, l_3, l_4>, e_2 = <l_5, l_1, l_3>, and e_3 = <l_6, l_4, l_3> be three LNNs in L, then the values of their score and accuracy functions are as follows:*

$Q(e_1) = (2 \times 8 + 6 - 3 - 4)/24 = 0.625$, $Q(e_2) = (2 \times 8 + 5 - 1 - 3)/24 = 0.7083$, and $Q(e_3) = (2 \times 8 + 6 - 4 - 3)/24 = 0.625$;
$T(e_1) = (6 - 4)/8 = 0.25$ and $T(e_3) = (6 - 3)/8 = 0.375$.
According to Definition 8, their ranking order is $e_2 \succ e_3 \succ e_1$.

4. Weighted Aggregation Operators of LNNs

4.1. LNNWAA Operator

Definition 9. *Let* $e_j = \left\langle l_{p_j}, l_{q_j}, l_{r_j} \right\rangle$ *(j =1, 2, . . . , n) be a collection of LNNs in L, then we can define LNNWAA operator as follows:*

$$LNNWAA(e_1, e_2, ..., e_n) = \sum_{j=1}^{n} w_j e_j, \quad (13)$$

where $w_j \in [0, 1]$ is the weight of e_j (j = 1, 2, . . . , n), satisfying $\sum_{j=1}^{n} w_j = 1$.

According to Definitions 6 and 9, we can present the following theorem:

Theorem 1. *Let* $e_j = \left\langle l_{p_j}, l_{q_j}, l_{r_j} \right\rangle$ *(j = 1, 2, . . . , n) be a collection of LNNs in L, then the aggregation result obtained by Equation (13) is still an LNN, and has the following aggregation formula:*

$$LNNWAA(e_1, e_2, ..., e_n) = \sum_{j=1}^{n} w_j e_j = \left\langle l_{t - t\prod_{j=1}^{n}(1-\frac{p_j}{t})^{w_j}}, l_{t\prod_{j=1}^{n}(\frac{q_j}{t})^{w_j}}, l_{t\prod_{j=1}^{n}(\frac{r_j}{t})^{w_j}} \right\rangle, \quad (14)$$

where $w_j \in [0, 1]$ is the weight of e_j (j = 1, 2, . . . , n), satisfying $\sum_{j=1}^{n} w_j = 1$.

Theorem 1 can be proved by mathematical induction.

Proof.

(1) When $n = 2$, by Equation (9), we obtain:

$$w_1e_1 = \left\langle l_{t-t(1-\frac{p_1}{t})^{w_1}}, l_{t(\frac{q_1}{t})^{w_1}}, l_{t(\frac{r_1}{t})^{w_1}} \right\rangle,$$

$$w_2e_2 = \left\langle l_{t-t(1-\frac{p_2}{t})^{w_2}}, l_{t(\frac{q_2}{t})^{w_2}}, l_{t(\frac{r_2}{t})^{w_2}} \right\rangle.$$

By Equation (7), there is the following result:

$$\begin{aligned} LNNWAA(e_1, e_2) = w_1e_1 \oplus w_2e_2 &= \left\langle l_{t-t(1-\frac{p_1}{t})^{w_1}+t-t(1-\frac{p_2}{t})^{w_2}-\frac{(t-t(1-\frac{p_1}{t})^{w_1})(t-t(1-\frac{p_2}{t})^{w_2})}{t}}, l_{t(\frac{q_1}{t})^{w_1}(\frac{q_2}{t})^{w_2}}, l_{t(\frac{r_1}{t})^{w_1}(\frac{r_2}{t})^{w_2}} \right\rangle \\ &= \left\langle l_{t-t(1-\frac{p_1}{t})^{w_1}+t-t(1-\frac{p_2}{t})^{w_2}-(t-t(1-\frac{p_1}{t})^{w_1}-t(1-\frac{p_2}{t})^{w_2}+t(1-\frac{p_1}{t})^{w_1}(1-\frac{p_2}{t})^{w_2})}, l_{t(\frac{q_1}{t})^{w_1}(\frac{q_2}{t})^{w_2}}, l_{t(\frac{r_1}{t})^{w_1}(\frac{r_2}{t})^{w_2}} \right\rangle \\ &= \left\langle l_{t-t(1-\frac{p_1}{t})^{w_1}(1-\frac{p_2}{t})^{w_2}}, l_{t(\frac{q_1}{t})^{w_1}(\frac{q_2}{t})^{w_2}}, l_{t(\frac{r_1}{t})^{w_1}(\frac{r_2}{t})^{w_2}} \right\rangle = \left\langle l_{t-t\prod_{j=1}^{2}(1-\frac{p_j}{t})^{w_j}}, l_{t\prod_{j=1}^{2}(\frac{q_j}{t})^{w_j}}, l_{t\prod_{j=1}^{2}(\frac{r_j}{t})^{w_j}} \right\rangle. \end{aligned} \tag{15}$$

(2) When $n = k$, by applying Equation (14), we obtain:

$$LNNWAA(e_1, e_2, \ldots, e_k) = \sum_{j=1}^{k} w_je_j = \left\langle l_{t-t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j}}, l_{t\prod_{j=1}^{k}(\frac{q_j}{t})^{w_j}}, l_{t\prod_{j=1}^{k}(\frac{r_j}{t})^{w_j}} \right\rangle, \tag{16}$$

(3) When $n = k + 1$, by applying Equations (15) and (16), which yields:

$$\begin{aligned} LNNWAA(e_1, e_2, \ldots, e_{k+1}) &= \sum_{j=1}^{k+1} w_je_j \\ &= \left\langle l_{t-t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j}+t-t(1-\frac{p_{k+1}}{t})^{w_{k+1}}-\frac{(t-t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j})(t-t(1-\frac{p_{k+1}}{t})^{w_{k+1}})}{t}}, l_{t\prod_{j=1}^{k}(\frac{q_j}{t})^{w_j}(\frac{q_{k+1}}{t})^{w_{k+1}}}, l_{t\prod_{j=1}^{k}(\frac{r_j}{t})^{w_j}(\frac{r_{k+1}}{t})^{w_{k+1}}} \right\rangle \\ &= \left\langle l_{t-t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j}+t-t(1-\frac{p_{k+1}}{t})^{w_{k+1}}-(t-t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j}-t(1-\frac{p_{k+1}}{t})^{w_{k+1}}+t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j}(1-\frac{p_{k+1}}{t})^{w_{k+1}})}, l_{t\prod_{j=1}^{k}(\frac{q_j}{t})^{w_j}(\frac{q_{k+1}}{t})^{w_{k+1}}}, l_{t\prod_{j=1}^{k}(\frac{r_j}{t})^{w_j}(\frac{r_{k+1}}{t})^{w_{k+1}}} \right\rangle \\ &= \left\langle l_{t-t\prod_{j=1}^{k}(1-\frac{p_j}{t})^{w_j}(1-\frac{p_{k+1}}{t})^{w_{k+1}}}, l_{t\prod_{j=1}^{k}(\frac{q_j}{t})^{w_j}(\frac{q_{k+1}}{t})^{w_{k+1}}}, l_{t\prod_{j=1}^{k}(\frac{r_j}{t})^{w_j}(\frac{r_{k+1}}{t})^{w_{k+1}}} \right\rangle = \left\langle l_{t-t\prod_{j=1}^{k+1}(1-\frac{p_j}{t})^{w_j}}, l_{t\prod_{j=1}^{k+1}(\frac{q_j}{t})^{w_j}}, l_{t\prod_{j=1}^{k+1}(\frac{r_j}{t})^{w_j}} \right\rangle. \end{aligned}$$

Corresponding to the above results, we have Equation (14) for any n. This finishes the proof. □

It is obvious that the LNNWAA operator satisfies the following properties:

(1) Idempotency: Let e_j (j = 1, 2, ... , n) be a collection of LNNs in L. If e_j (j = 1, 2, ... , n) is equal, i.e., $e_j = e$ for j = 1, 2, ... , n, then $LNNWAA(e_1, e_2, \cdots, e_n) = e$.

(2) Boundedness: Let e_j (j = 1, 2, ... , n) be a collection of LNNs in L and let $e^- = \left\langle \min_j(l_{p_j}), \max_j(l_{q_j}), \max_j(l_{r_j}) \right\rangle$ and $e^+ = \left\langle \max_j(l_{p_j}), \min_j(l_{q_j}), \min_j(l_{r_j}) \right\rangle$. Then $e^- \leq LNNWAA(e_1, e_2, \cdots, e_n) \leq e^+$.

(3) Monotonicity: Let e_j (j = 1, 2, ... , n) be a collection of LNNs in L. If $e_j \leq e_j^*$ for j = 1, 2, ... , n, then $LNNWAA(e_1, e_2, \cdots, e_n) \leq LNNWAA(e_1^*, e_2^*, \cdots, e_n^*)$.

Proof.

(1) Since $e_j = e$, i.e., $p_j = p$; $q_j = q$; $t_j = r$ for j = 1, 2, ..., n, we have:

$$LNNWAA(e_1, e_2, ..., e_n) = \sum_{j=1}^{n} w_j e_j = \left\langle l_{t-t\prod_{j=1}^{n}(1-\frac{p_j}{t})^{w_j}}, l_{t\prod_{j=1}^{n}(\frac{q_j}{t})^{w_j}}, l_{t\prod_{j=1}^{n}(\frac{r_j}{t})^{w_j}} \right\rangle$$
$$= \left\langle l_{t-t(1-\frac{p}{t})^{\sum_{j=1}^{n} w_j}}, l_{t(\frac{q}{t})^{\sum_{j=1}^{n} w_j}}, l_{t(\frac{r}{t})^{\sum_{j=1}^{n} w_j}} \right\rangle = \left\langle l_{t-t(1-\frac{p}{t})}, l_{t(\frac{q}{t})}, l_{t(\frac{r}{t})} \right\rangle$$
$$= \langle l_p, l_q, l_r \rangle = e.$$

(2) Since the minimum LNN is e^- and the maximum LNN is e^+, $e^- \le e_j \le e^+$. Thus, $\sum_{j=1}^{n} w_j e^- \le \sum_{j=1}^{n} w_j e_j \le \sum_{j=1}^{n} w_j e^+$. According to the above property (1), $e^- \le \sum_{j=1}^{n} w_j e_j \le e^+$, i.e., $e^- \le LNNWAA(e_1, e_2, \cdots, e_n) \le e^+$.

(3) Since $e_j \le e_j^*$ for j = 1, 2, ..., n, $\sum_{j=1}^{n} w_j e_j \le \sum_{j=1}^{n} w_j e_j^*$, i.e., $LNNWAA(e_1, e_2, \cdots, e_n) \le LNNWAA(e_1^*, e_2^*, \cdots, e_n^*)$.

Thus, the proofs of these properties are completed. □

Especially when $w_j = 1/n$ for j = 1, 2, ..., n, the LNNWAA operator is reduced to the LNN arithmetic averaging operator.

4.2. LNNWGA Operator

Definition 10. *Let* $e_j = \left\langle l_{p_j}, l_{q_j}, l_{r_j} \right\rangle$ *(j = 1, 2, ..., n) be a collection of LNNs in L, then we can define LNNWGA operator as follows:*

$$LNNWGA(e_1, e_2, \cdots, e_n) = \prod_{j=1}^{n} e_j^{w_j}, \quad (17)$$

where $w_j \in$ *[0, 1] is the weight of* e_j *(j = 1, 2, ..., n), satisfying* $\sum_{j=1}^{n} w_j = 1$.

According to Definitions 6 and 10, we can present the following theorem:

Theorem 2. *Let* $e_j = \left\langle l_{p_j}, l_{q_j}, l_{r_j} \right\rangle$ *(j = 1,2, ..., n) be a collection of LNNs in L, then the aggregation result obtained by Equation (17) is still an LNN, and has the following aggregation formula:*

$$LNNWGA(e_1, e_2, ..., e_n) = \prod_{j=1}^{n} e_j^{w_j} = \left\langle l_{t\prod_{j=1}^{n}(\frac{p_j}{t})^{w_j}}, l_{t-t\prod_{j=1}^{n}(1-\frac{q_j}{t})^{w_j}}, l_{t-t\prod_{j=1}^{n}(1-\frac{r_j}{t})^{w_j}} \right\rangle, \quad (18)$$

where $w_j \in$ *[0, 1] is the weight of* e_j *(j =1, 2, ..., n), satisfying* $\sum_{j=1}^{n} w_j = 1$. *Especially when* w_j *= 1/n for j = 1, 2, ..., n, the LNNWGA operator is reduced to the LNN geometric averaging operator.*

Since the proof manner of Theorem 2 is similar to that of Theorem 1, it is not repeated here.
It is obvious that the LNNWGA operator implies the following properties:

(1) Idempotency: Let e_j (j = 1, 2, ..., n) be a collection of LNNs in L. If e_j (j = 1, 2, ..., n) is equal, i.e., $e_j = e$ for j = 1, 2, ..., n, then $LNNWGA(e_1, e_2, \cdots, e_n) = e$.

(2) Boundedness: Let e_j (j = 1, 2, . . . , n) be a collection of LNNs in L and let $e^- = \left\langle \min_j(l_{p_j}), \max_j(l_{q_j}), \max_j(l_{r_j}) \right\rangle$ and $e^+ = \left\langle \max_j(l_{p_j}), \min_j(l_{q_j}), \min_j(l_{r_j}) \right\rangle$. Then $e^- \le LNNWGA(e_1, e_2, \cdots, e_n) \le e^+$.

(3) Monotonicity: Let e_j (j = 1, 2, . . . , n) be a collection of LNNs in L. If $e_j \le e_j^*$ for j = 1, 2, . . . , n, then $LNNWGA(e_1, e_2, \cdots, e_n) \le LNNWGA(e_1^*, e_2^*, \cdots, e_n^*)$.

Due to the similar proof manner of the properties of the LNNWAA operator we can prove these properties, which are omitted here.

5. MAGDM Method Based on the LNNWAA or LNNWGA Operator

In this section, the LNNWAA and LNNWGA operators and the score and accuracy functions are applied to MAGDM problems with LNN information.

In a MAGDM problem, let $Y = \{Y_1, Y_2, \ldots, Y_m\}$ be a set of alternatives and $Z = \{Z_1, Z_2, \ldots, Z_n\}$ be a set of attributes. The weigh vector of the attributes Z_j (j = 1, 2, . . . , n) is $W = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$. Then, a group of decision-makers $D = \{D_1, D_2, \ldots, D_d\}$ can be assigned with a corresponding weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_d)^{\mathrm{T}}$ to evaluate the alternatives Y_i (i = 1, 2, . . . , m) on the attributes Z_j (j = 1, 2, . . . , n) by LNNs from the linguistic term set $L = \{l_0$ = extremely low, l_1 = very low, l_2 = low, l_3 = slightly low, l_4 = medium, l_5 = slightly high, l_6 = high, l_7 = very high, l_8 = extremely high$\}$. In the evaluation process, the decision-makers can assign the three linguistic values of the truth, falsity, and indeterminacy degrees, composed of an LNN, to each attribute Z_j on an alternative Y_i according to the linguistic terms. Thus, the LNN evaluation information of the attributes Z_j (j = 1, 2, . . . , n) on the alternatives Y_i (i = 1, 2, . . . , m) provided by each decision maker D_k (k = 1, 2, . . . , d) can be established as an LNN decision matrix $M^k = (e_{ij}^k)_{m\times n}$, where $e_{ij}^k = \left\langle l_{p_{ij}^k}, l_{q_{ij}^k}, l_{r_{ij}^k} \right\rangle$ (k = 1, 2, . . . , d; i = 1,2, . . . , m; j = 1, 2, . . . , n) is an LNN.

Then, we apply the LNNWAA or LNNWGA operator and the score function (accuracy function if necessary) to the MAGDM problem with LNN information to rank the alternatives and to select the best one. The decision-making steps are introduced as follows:

Step 1: Obtain the integrated matrix $R = (e_{ij})_{m\times n}$, where $e_{ij} = \left\langle l_{p_{ij}}, l_{q_{ij}}, l_{r_{ij}} \right\rangle$ (i = 1, 2, . . . , m; j = 1, 2, . . . , n) is an integrated LNN, by using the following LNNWAA operator:

$$e_{ij} = LNNWAA(e_{ij}^1, e_{ij}^2, \ldots, e_{ij}^d) = \sum_{k=1}^{d} \omega_k e_{ij}^k = \left\langle l_{t - t\prod_{k=1}^{d}(1-\frac{p_{ij}^k}{t})^{\omega_k}}, l_{t\prod_{k=1}^{d}(\frac{q_{ij}^k}{t})^{\omega_k}}, l_{t\prod_{k=1}^{d}(\frac{r_{ij}^k}{t})^{\omega_k}} \right\rangle. \quad (19)$$

Step 2: Obtain the collective overall LNN e_i for Y_{i} (i = 1, 2, . . . , m) by using the following LNNWAA operator or LNNWGA operator:

$$e_i = LNNWAA(e_{i1}, e_{i2}, \ldots, e_{in}) = \sum_{j=1}^{n} w_j e_{ij} = \left\langle l_{t - t\prod_{j=1}^{n}(1-\frac{p_{ij}}{t})^{w_j}}, l_{t\prod_{j=1}^{n}(\frac{q_{ij}}{t})^{w_j}}, l_{t\prod_{j=1}^{n}(\frac{r_{ij}}{t})^{w_j}} \right\rangle, \quad (20)$$

or

$$e_i = LNNWGA(e_{i1}, e_{i2}, \ldots, e_{in}) = \prod_{j=1}^{n} e_{ij}^{w_j} = \left\langle l_{t\prod_{j=1}^{n}(\frac{p_{ij}}{t})^{w_j}}, l_{t - t\prod_{j=1}^{n}(1-\frac{q_{ij}}{t})^{w_j}}, l_{t - t\prod_{j=1}^{n}(1-\frac{r_{ij}}{t})^{w_j}} \right\rangle. \quad (21)$$

Step 3: Calculate the score function $Q(e_i)$ (accuracy function $T(e_i)$ if necessary) (i = 1, 2, . . . , m) of the collective overall LNN e_i (i = 1, 2, . . . , m) by Equation (11) (Equation (12) if necessary).

Step 4: Rank the alternatives corresponding to the score (accuracy if necessary) values, and then select the best one.

Step 5: End.

6. An Illustrative Example

This section considers the selection problem of investment alternatives in an investment company as an illustrative example, which is adapted from [13], in order to demonstrate the application of the proposed method.

Some investment company needs to invest a sum of money to an industry. A panel provides a set of four possible investment alternatives $Y = \{Y_1, Y_2, Y_3, Y_4\}$, where Y_1 is a car company; Y_2 is a food company; Y_3 is a computer company; Y_4 is an arms company. The evaluation of the four alternatives must satisfy the requirements of three attributes: (1) Z_1 is the risk; (2) Z_2 is the growth; (3) Z_3 is the environmental impact. The importance of the three attributes is provided by the weigh vector $\boldsymbol{W} = (0.35, 0.25, 0.4)^{\mathrm{T}}$. Then, three decision-makers are invited and denoted as a set of the decision-makers $D = \{D_1, D_2, D_3\}$ and the importance of the three decision-makers is given as a weight vector $\boldsymbol{\omega} = (0.37, 0.33, 0.3)^{\mathrm{T}}$. The three decision-makers are required to give the suitability evaluation of the four possible alternatives Y_i ($i = 1, 2, 3, 4$) with respect to the three attributes Z_j ($j = 1, 2, 3$) by the expression of the linguistic values of LNNs from the linguistic term set $L = \{l_0$ = extremely low, l_1 = very low, l_2 = low, l_3 = slightly low, l_4 = medium, l_5 = slightly high, l_6 = high, l_7 = very high, l_8 = extremely high$\}$ with the odd cardinality $t + 1 = 9$. Thus, the linguistic evaluation information given by each decision-maker D_k ($k = 1, 2, 3$) can be established as the following the LNN decision matrix M^k:

$$M^1 = \begin{bmatrix} \langle l_6, l_1, l_2\rangle & \langle l_7, l_2, l_1\rangle & \langle l_6, l_2, l_2\rangle \\ \langle l_7, l_1, l_1\rangle & \langle l_7, l_3, l_2\rangle & \langle l_7, l_2, l_1\rangle \\ \langle l_6, l_2, l_2\rangle & \langle l_7, l_1, l_1\rangle & \langle l_6, l_2, l_2\rangle \\ \langle l_7, l_1, l_2\rangle & \langle l_7, l_2, l_3\rangle & \langle l_7, l_2, l_1\rangle \end{bmatrix},$$

$$M^2 = \begin{bmatrix} \langle l_6, l_1, l_2\rangle & \langle l_6, l_1, l_1\rangle & \langle l_4, l_2, l_3\rangle \\ \langle l_7, l_2, l_3\rangle & \langle l_6, l_1, l_1\rangle & \langle l_4, l_2, l_3\rangle \\ \langle l_5, l_1, l_2\rangle & \langle l_5, l_1, l_2\rangle & \langle l_5, l_4, l_2\rangle \\ \langle l_6, l_1, l_1\rangle & \langle l_5, l_1, l_1\rangle & \langle l_5, l_2, l_3\rangle \end{bmatrix},$$

$$M^3 = \begin{bmatrix} \langle l_7, l_3, l_4\rangle & \langle l_7, l_3, l_3\rangle & \langle l_5, l_2, l_5\rangle \\ \langle l_6, l_3, l_4\rangle & \langle l_5, l_1, l_2\rangle & \langle l_6, l_2, l_3\rangle \\ \langle l_7, l_2, l_4\rangle & \langle l_6, l_1, l_2\rangle & \langle l_7, l_2, l_4\rangle \\ \langle l_7, l_2, l_3\rangle & \langle l_5, l_2, l_1\rangle & \langle l_6, l_1, l_1\rangle \end{bmatrix}.$$

Hence, the proposed method can be applied to this decision-making problem and the computational procedures are given as follows:

Step 1: Get the following integrated matrix $R = (e_{ij})_{m\times n}$ by using Equation (19):

$$R = \begin{bmatrix} \langle l_{6.3755}, l_{1.3904}, l_{2.4623}\rangle & \langle l_{6.7430}, l_{1.7969}, l_{1.3904}\rangle & \langle l_{5.1608}, l_{2.0000}, l_{3.0097}\rangle \\ \langle l_{6.7689}, l_{1.7477}, l_{2.1781}\rangle & \langle l_{6.2523}, l_{1.5015}, l_{1.5911}\rangle & \langle l_{6.0547}, l_{2.0000}, l_{1.9980}\rangle \\ \langle l_{6.1429}, l_{1.5911}, l_{2.4623}\rangle & \langle l_{6.2309}, l_{1.0000}, l_{1.5476}\rangle & \langle l_{6.1429}, l_{2.5140}, l_{2.4623}\rangle \\ \langle l_{6.7430}, l_{1.2311}, l_{1.7969}\rangle & \langle l_{6.0020}, l_{1.5911}, l_{1.5015}\rangle & \langle l_{6.2309}, l_{1.6245}, l_{1.4370}\rangle \end{bmatrix}.$$

Step 2: By using Equation (20), the collective overall LNNs of e_i for Y_i ($i = 1, 2, 3, 4$) can be obtained as follows:

$$e_1 = \langle l_{6.0951}, l_{1.7145}, l_{2.3129}\rangle,\ e_2 = \langle l_{6.3863}, l_{1.7759}, l_{1.9453}\rangle,\ e_3 = \langle l_{6.1653}, l_{1.7011}, l_{2.1924}\rangle, \text{and } e_4 = \langle l_{6.3818}, l_{1.4666}, l_{1.5711}\rangle.$$

Step 3: Calculate the score values of $Q(e_i)$ (i = 1, 2, 3, 4) of the collective overall LNNs of e_i (i = 1, 2, 3, 4) by Equation (11):

$$Q(e_1) = 0.7528,\ Q(e_2) = 0.7777,\ Q(e_3) = 0.7613, \text{ and } Q(e_4) = 0.8060.$$

Step 4: Ranking order of the four alternatives is $Y_4 \succ Y_2 \succ Y_3 \succ Y_1$ corresponding to the score values. Thus, the alternative Y_4 is the best choice among the four alternatives.

Or by using Equation (21), the computational procedures are given as follows:

Step 1': The same as Step 1.

Step 2': By using Equation (21), the collective overall LNNs of e_i for Y_i (i = 1, 2, 3, 4) are obtained as follows:

$$e_1 = \langle l_{5.9413}, l_{1.7414}, l_{2.4479}\rangle,\ e_2 = \langle l_{6.3464}, l_{1.7902}, l_{1.9634}\rangle, e_3 = \langle l_{6.1648}, l_{1.8433}, l_{2.2465}\rangle, \text{ and } e_4 = \langle l_{6.3459}, l_{1.4810}, l_{1.5811}\rangle.$$

Step 3': By using Equation (11), we calculate the score values of $Q(e_i)$ (i = 1, 2, 3, 4) of the collective overall LNNs of e_i (i = 1, 2, 3, 4) as follows:

$$Q(e_1) = 0.7397,\ Q(e_2) = 0.7747,\ Q(e_3) = 0.7531, \text{ and } Q(e_4) = 0.8035.$$

Step 4': The ranking order of the four alternatives is $Y_4 \succ Y_2 \succ Y_3 \succ Y_1$. Thus, the alternative Y_4 is still the best choice among the four alternatives.

Clearly, the above two ranking orders and the best alternative based on the LNNWAA and LNNWGA operators are the same, which are in agreement with Ye's results [13].

Compared with the relevant papers [10,11] which proposed the decision-making approaches with LIFNs, the decision information used in [10,11] is LIFNs, whereas the decision information in this paper are LNNs. As mentioned above, the LNN is a further generalization of the LIFN and contains more information than the LIFN. Thus, the decision-making method proposed in this paper is more typical and more general in application since the decision-making method proposed in [10,11] cannot handle indeterminate and inconsistent linguistic information and the MAGDM problem with LNN information in this paper. Furthermore, compared with the relevant papers [6–8,13–16], the decision-making approach proposed in this study can be used to solve decision-making problems with LNN information, while the MADM/MAGDM methods with various linguistic information presented in [6–8,13–16] are not suitable for handling the decision-making problems with LNN information in this paper since existing various linguistic numbers in [6–8,13–16] cannot express indeterminate and inconsistent linguistic information.

In fact, all decision-making methods based on various linguistic variables in existing literature not only cannot express indeterminate and inconsistent linguistic information, but also lose the useful information in linguistic evaluation process, and then they cannot also deal with decision-making problems with indeterminate and inconsistent linguistic information; while the linguistic method proposed in the study is a generalization of existing linguistic methods and can represent and handle linguistic decision-making problems with LNN information. Obviously, the main contribution in this study is that our new method can express indeterminate and inconsistent linguistic information corresponding to human fuzzy thinking about complex problems, especially for some qualitative evaluations of attributes, and solve linguistic decision-making problems with indeterminate and inconsistent linguistic information.

From above comparative analyses with relevant papers, one can see that main advantages of the developed new method are summarized as follows:

(1) The developed new method is more suitable for expressing and handling indeterminate and inconsistent linguistic information in linguistic decision-making problems to overcome the insufficiency of various linguistic decision-making methods in the existing literature.

(2) The developed new method contains much more information (the three linguistic variables of truth, indeterminate, and falsity degrees contained in an LNN) than the existing method in [10,11] (the two linguistic variables of truth and falsity degrees contained in a LIFN) and can better describe people's linguistic expression to objective things evaluated in detail.

(3) The developed new method enriches the neutrosophic theory and decision-making method under a linguistic environment and provides a new way for solving linguistic MAGDM problems with indeterminate and inconsistent linguistic information.

7. Conclusions

This paper originally presented LNNs, the operational laws of LNNs, and the score and accuracy functions of LNNs. Then, we proposed the LNNWAA and LNNWGA operators to aggregate LNNs and investigated their properties and special cases. Further, we developed a MAGDM method based on the LNNWAA or LNNWGA operator and the score and accuracy functions to solve MAGDM problems with LNN information. Finally, an illustrative example was provided to demonstrate the application of the developed MAGDM method under LNN environment. The developed MAGDM method with LNNs enriches fuzzy decision-making theory and provides a new way for decision-makers under LNN environment. In the future research directions, we shall further develop new aggregation operators of LNNs and apply them to decision-making, pattern recognition, medical diagnosis, and so on.

Acknowledgments: This paper was supported by the National Natural Science Foundation of China (71471172, 51272159) and the Natural Science Foundation of Zhejiang province (LY15A040001).

Author Contributions: Jun Ye originally proposed LNNs and the LNNWAA and LNNWGA operators and investigated their properties, and Zebo Fang provided the calculation and comparative analysis of examples. We wrote the paper together.

Conflicts of Interest: The authors declare that we have no conflicts of interest regarding the publication of this paper.

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