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Abstract

Purpose of this paper is to interpret the multiplication of neutrosophic cubic set. Here we define the notation of ɤ-multiplication of neutrosophic cubic set and study it with the help of neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal and neutrosophic cubic closed normal ideal. We also study ɤ-multiplication under homomorphism and cartesian product through significant characteristics.

Keywords: B-algebra, Neutrosophic cubic set, ɤ-Multiplication, Cartesian product, Homomorphism.

1.Introduction

Theory of existing and non-existing value was first introduced by Zadeh [1,2]. Cubic set was defined by Jun et al. [3] in 2012, which was the modern form of interval-valued fuzzy set. Cubic set with the help of subalgebras, ideals and closed ideals of *B*-algebra was studied by Senapati et al. [4]. After the defing of *BCK*-algebra and *BCI*-algebra by Imai et al. [5] and Iseki [6], cubic set through subalgebras and q-ideals in BCK/BCI-algebra was investigated by Jun et al. [7, 8]. Notion of M-subalgebra on G-algebra is introduced and analyzed by Khalid et al. [9]. Interval-valued fuzzy set on *B*-algebra was studied by Senapati et al. [10,11]. Intuitionistic fuzzy translation and multiplication of G-algebra were deeply studied by by Khalid et al. [19]. Neutrosophic cubic set is the extended form of interval valued intuitionistic fuzzy theory with indeterminacy was introduced by Smarandache [12]. Neutrosophic logics and neutrosophic probability gave the new idea of research were interpret by Smarandache [13]. Neutrosophic cubic was introduced by Jun et al. [14]. Neutrosophic cubic point, (α, β)-fuzzy ideals and neutrosophic cubic (α, β)-ideals were analyzed by Gulistan et al. [15]. A new idea of normal ideal and closed normal ideal under neutrosophic cubic set was given and investigated by Khalid et al. [16]. Neutrosophic cubic set was investigated by Jun et al. [17]. PS fuzzy ideals were studied by Priya et al. [18]. Rosenfeld's fuzzy subgroup was studied by Biswas [20]. B-homomorphism was deeply studied by Neggers et al. [21]. Neutrosophic soft cubic subalgebra was extensively studied by Khalid et al. [22]. A B-algebra is an important logical class of algebra was defined by Neggers et al. [23]. T-Neutrosophic Cubic Set was defined and deeply investigated by Khalid et al. [24].

In this paper, we define ɤ-multiplication of neutrosophic cubic set and investigate the neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal (NCNID) and neutrosophic cubic closed normal ideal (NCCNID) under ɤ-multiplication with the help of P-intersection, P-union etc. We also study the cartesian product and homomorphism of ɤ-multiplication of neutrosophic cubic normal ideal (ɤMNCNID) and ɤ-multiplication of neutrosophic cubic closed normal ideal (ɤMNCCNID) with important results.

2. Preliminaries

Definition 2.1 [19] A nonempty set X with a constant 0 and $*$ is said to be B-algebra if it fulfills these conditions:

1: $ʈ * ʈ = 0$,

2: $ʈ * 0 = 0$, for all $ʈ \in X$.

3: $(ʈ * ȶ) * t = ʈ * (t * (0 * ȶ))$ $\forall$ $ʈ, ȶ, t \in X$.

Definition 2.2 [21] A nonempty subset K of B-algebra X is called a subalgebra of Y if $ʈ * ȶ \in K$ $\forall$ $ʈ, ȶ \in K$, a mapping $f : X \rightarrow Y$ of B-algebra is called B-homomorphism if $f(ʈ * ȶ) = f(ʈ) * f(ȶ)$ $\forall$ $ʈ, ȶ \in X$.

Definition 2.3 [1] Let X be a collection of elements like ʈ. Then a FS J in X is defined as $J = \{< ʈ, \nu_J(ʈ) > | ʈ \in X\}$, where $\mu_J(ʈ)$ is called the existenceship value of ʈ in J and $\nu_J(ʈ) \in [0,1]$.

For a family $J_i = \{< ʈ, \nu_{J_i}(ʈ) > | ʈ \in X\}$ of FSs in X, where $i \in k$ and k is index set, Then join ($\vee$) and meet ($\wedge$) are as follows:

$$\bigvee_{i\in k} J_i = (\bigvee_{i\in k} \nu_{J_i})(ʈ) = \sup\{\nu_{J_i} | i \in k\}$$

and

$$\bigwedge_{i\in k} J_i = (\bigwedge_{i\in k} \nu_{J_i})(ʈ) = \inf\{\nu_{J_i} | i \in k\},$$

respectively, $\forall$ $ʈ \in X$.

Definition 2.4 [2] An IVFS B is of the form $B = \{< ʈ, \tilde{\nu}_B(ʈ) > | ʈ \in X\}$, where $\tilde{\nu}_B | X \rightarrow D[0,1]$, here $D[0,1]$ is the collection of all subintervals of $[0,1]$. The intervals $\tilde{\nu}_B(ʈ) = [\nu_B^-(ʈ), \nu_B^+(ʈ)]$ $\forall$ $ʈ \in X$ denote the degree of existence of ʈ to the set B, also $\tilde{\nu}_B^c = [1 - \nu_B^-(ʈ), 1 - \nu_B^+(ʈ)]$ shows the complement of $\tilde{\nu}_B$.

For a family $B_i = \{< ʈ, \tilde{\nu}_B(ʈ) > | ʈ \in X\}$ of IVFSs in X where k is an index set and $i \in k$, the union $G = \bigcup_{i\in k} \tilde{\nu}_{B_i}(ʈ)$ and the intersection $F = \bigcap_{i\in k} \tilde{\nu}_{B_i}(ʈ)$ are defined below:

$$G(ʈ) = r\sup\{\tilde{\nu}_{B_i}(ʈ) i \in k\}$$

and

$$F(ʈ) = r\inf\{\tilde{\nu}_{B_i}(ʈ) | i \in k\},$$

respectively, $\forall$ $ʈ \in X$.

Definition 2.5 [20] Consider two elements $D_1, D_2 \in D[0,1]$. If $D_1 = [ʈ_1^-, ʈ_1^+]$ and $D_2 = [ʈ_2^-, ʈ_2^+]$, then $r\max(D_1, D_2) = [\max(ʈ_1^-, ʈ_2^-), \max(ʈ_1^+, ʈ_2^+)]$ which is denoted by $D_1 \vee^r D_2$ and $r\min(D_1, D_2) = [\min(ʈ_1^-, ʈ_2^-), \min(ʈ_1^+, ʈ_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus, if $D_i = [ʈ_i^-, ʈ_2^+] \in D[0,1]$ for $i = 1,2,3, \ldots$, then we define $r\sup_i(D_i) = [\sup_i(ʈ_i^-), \sup_i(ʈ_i^+)]$, i.e., $\vee_i^r D_i = [\vee_i ʈ_i^-, \vee_i ʈ_i^+]$. Similarly we define $r\inf_i(D_i) = [\inf_i(ʈ_i^-), \inf_i(ʈ_i^+)]$, i.e., $\wedge_i^r D_i = [\wedge_i ʈ_i^-, \wedge_i ʈ_i^+]$. Now we call $D_1 \geq D_2 \Leftarrow ʈ_1^- \geq ʈ_2^-$ and $ʈ_1^+ \geq ʈ_2^+$. Similarly the relations $D_1 \leq D_2$ and $D_1 = D_2$ are defined.

Definition 2.6 [19] A fuzzy set $B = \{< ṭ, \nu_B(ṭ) > | ṭ \in X\}$ is called a fuzzy subalgebra of X if $\nu_B(ṭ * ŧ) \geq \min\{\nu_B(ṭ), \nu_B(ŧ)\}\ \forall\ ṭ, ŧ \in X$.

Definition 2.7 [14] Let X be a nonempty set. A NCS is $P_k = (B, \Lambda)$, where $B = \{\langle ṭ; B_T(ṭ), B_I(ṭ), B_F(ṭ)\rangle | ṭ \in X\}$ is an interval neutrosophic set in X and $\Lambda = \{\langle ṭ; \lambda_T(ṭ), \lambda_I(ṭ), \lambda_F(ṭ)\rangle | ṭ \in X\}$ is a neutrosophic set in X.

Definition 2.8 [3] Let U be a universe and cubic set in U, we mean a structure $\{ṭ, \overline{\nu}_A(ṭ), \lambda_A(ṭ) | ṭ \in U\}$ in which $\overline{\nu}_A$ is an IVF set in U and λ_A is a fuzzy set in U. A cubic set $A = \{ṭ, \overline{\nu}_A(ṭ), \lambda_A(ṭ) | ṭ \in U\}$ is simply denoted by C(U), which is the set of all cubic sets in U.

Definition 2.9 [3] Let $C = \{\langle ṭ, C(ṭ), \lambda(ṭ)\rangle\}$ be a cubic set, where $C(ṭ)$ is an IVFS in Y, $\lambda(ṭ)$ is a fuzzy set in Y. Then A is cubic subalgebra under $*$ if it fulfills these axioms:

C1: $C(ṭ * ŧ) \geq \text{rmin}\{C(ṭ), C(ŧ)\}$,

C2: $\lambda(ṭ * ŧ) \leq \max\{\lambda(ṭ), \lambda(ŧ)\}\ \forall\ ṭ, ŧ \in X$.

Definition 2.10 [18] A fuzzy set $B = \{< ṭ, \nu_B(ṭ) > | ṭ \in X\}$ is called a fuzzy ideal of X if

(i) $\nu_B(0) \geq \nu_B(ṭ)$,

(ii) $\nu_B(ṭ) \geq \min\{\nu_B(ṭ * ŧ), \nu_B(ŧ)\}\ \forall\ ṭ, ŧ \in X$.

Definition 2.11 [14] For any $C_i = (A_I, F_I)$, where $A_i = \{\langle t_1; A_{iT}(ṭ), A_{iI}(ṭ), A_{iF}(ṭ)\rangle | ṭ \in Y\}$, $F_i = \{\langle t_1; F_{iT}(ṭ), F_{iI}(ṭ), F_{iF}(ṭ)\rangle | ṭ \in Y\}$ for $i \in k$, then

P-union: $\bigcup_{P\, i\in k} C_i = (\bigcup_{i\in k} A_I, \bigvee_{i\in k} F_i)$,

P-intersection: $\bigcap_{P\, i\in k} C_i = (\bigcap_{i\in k} A_i, \bigwedge_{i\in k} F_i)$,

R-union: $\bigcup_{R\, i\in k} C_i = (\bigcup_{i\in k} A_I, \bigwedge_{i\in k} F_i)$,

R-intersection: $\bigcap_{R\, i\in k} C_i = (\bigcap_{i\in k} A_i, \bigvee_{i\in k} F_i)$.

Definition 2.12 [16] A NCS $Ɽ = (R_{T,I,F}, \lambda_{T,I,F})$ of X is called a NCNID of X if it fulfills following axioms:

N3. $R_{T,I,F}(0) \geq R_{T,I,F}(ṭ * \alpha)$ and $\lambda_{T,I,F}(0) \leq \lambda_{T,I,F}(ṭ * \alpha)$,

N4. $R_{T,I,F}(ṭ * \alpha) \geq \text{rmin}\{R_{T,I,F}((ṭ * \alpha) * (ŧ * \beta)), R_{T,I,F}(ŧ * \beta)\}$,

N5. $\lambda_{T,I,F}(ṭ * \alpha) \leq \max\{\lambda_{T,I,F}(ṭ * \alpha) * (ŧ * \beta)), \lambda_{T,I,F}(ŧ * \beta)\}, \forall\ ṭ, ŧ$ X and $\alpha, \beta \in [0,1]$.

Let $Ɽ = \{R_{T,I,F}, \lambda_{T,I,F}\}$ be a NCS X then it is called NCCNID of X if it fulfills N4, N5 and N6: $R_{T,I,F}(0 * (ṭ * \alpha)) \geq R_{T,I,F}(ṭ * \alpha)$ and $\lambda_{T,I,F}(0 * (ṭ * \alpha)) \leq \lambda_{T,I,F}(ṭ * \alpha), \forall\ ṭ \in X$ and $\alpha \in [0,1]$.

Definition 2.13 [16] Let $Ɽ = (R_{T,I,F}, \lambda_{T,I,F})$ and $\mathcal{B} = (B_{T,I,F}, \upsilon_{T,I,F})$ are two NCSs of X and Y respectively. The Cartesian product $Ɽ \times \mathcal{B} = (X \times Y, R_{T,I,F} \times B_{T,I,F}, \lambda_{T,I,F} \times \upsilon_{T,I,F})$ is defined by $(R_{T,I,F} \times B_{T,I,F})(ṭ * \alpha, ŧ * \beta) = \text{rmin}\{R_{T,I,F}(ṭ * \alpha), B_{T,I,F}(ŧ * \beta)\}$ and $(\lambda_{T,I,F} \times \upsilon_{T,I,F})(ṭ * \alpha, ŧ * \beta) = \max\{\lambda_{T,I,F}(ṭ * \alpha), \upsilon_{T,I,F}(ŧ * \beta))\}$, where $R_{T,I,F} \times B_{T,I,F} | X \times Y \to D[0,1]$ and $\lambda_{T,I,F} \times \upsilon_{T,I,F} | X \times Y \to [0,1]\ \forall\ (ṭ, ŧ) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

Definition 2.14 [16] A neutrosophic cubic subset $Ŗ \times Ƒ = (X \times Y, R_{T,I,F} \times F_{T,I,F}, \lambda_{T,I,F} \times \mu_{T,I,F})$ is called a NCNID if satisfies these conditions:

1. $(R_{T,I,F} \times F_{T,I,F})(0,0) \geq (R_{T,I,F} \times F_{T,I,F})((ţ * \alpha), (ƀ * \beta))$ and $(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})((ţ * \alpha), (ƀ * \beta)) \ \forall\ (ţ, ƀ) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

2. $(R_{T,I,F} \times F_{T,I,F})(ţ_1 * \alpha, ƀ_1 * \beta) \geq \mathrm{rmin}\{(R_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha, ƀ_1 * \beta) * (ţ_2 * \alpha, ƀ_2 * \beta)), (R_{T,I,F} \times F_{T,I,F})(ţ_2 * \alpha, ƀ_2 * \beta)\}$.

3.$(\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_1 * \alpha, ƀ_1 * \beta) \leq \max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha, ƀ_1 * \beta)(ţ_2 * \alpha, ƀ_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_2 * \alpha, ƀ_2 * \beta)\}$ and $R \times Ƒ$ is closed normal ideal if it satisfies 2, 3, and 4. $(R_{T,I,F} \times F_{T,I,F})((0,0) * (ţ_1 * \alpha, ƀ_1 * \beta)) \geq (R_{T,I,F} \times F_{T,I,F})(ţ * \alpha, ƀ * \beta)$ and $(\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (ţ * \alpha, ƀ * \beta)) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ * \alpha, ƀ * \beta)\ \ \forall\ \ (ţ_1, ƀ_1)$ and $(ţ_2, ƀ_2) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

Definition 2.15 [9] Let $\tilde{\mathcal{F}}_k = (A_{e_i}, \Lambda_{e_i})$ be a neutrosophic soft cubic set, where Y is subalgebra. Then $\tilde{\mathcal{F}}_k$ is NSCMSU under binary operation $*$ where $t_1, t_2 \in Y$ and $\alpha, \beta \in [0,1]$ if it fulfills these conditions:

$A^{\varrho}_{e_i}((t_1 * \alpha) * (t_2 * \beta)) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1 * \alpha), A^{\varrho}_{e_i}(t_2 * \beta)\}$ and $\lambda^{\varrho}_{e_i}((t_1 * \alpha) * (t_2 * \beta)) \leq \max\{\lambda^{\varrho}_{e_i}(t_1 * \alpha), \lambda^{\varrho}_{e_i}(t_2 * \beta)\}$.

3. ɤ-Multiplication of Neutrosophic Cubic Normal Ideal and Closed Normal Ideal

Definition 3.1. Let $Ƕ = (H_{T,I,F}, \lambda_{T,I,F})$ be a NCS of X and $ɤ \in [0,1]$. An object of the form $Ƕ^{M}_{ɤ} = ({}^{M}_{ɤ}H^{Ƕ}_{T,I,F}, {}^{M}_{ɤ}\lambda^{Ƕ}_{T,I,F})$ is called neutrosophic cubic ɤ multiplication of Ƕ X if it fulfills following axioms:

$${}^{M}_{ɤ}H^{Ƕ}_{T}(x) = ɤ.\, H^{Ƕ}_{T}(x), \qquad {}^{M}_{ɤ}\lambda^{Ƕ}_{T}(x) = ɤ.\, \lambda^{Ƕ}_{T}(x),$$

$${}^{M}_{ɤ}H^{Ƕ}_{I}(x) = ɤ.\, H^{Ƕ}_{I}(x), \qquad {}^{M}_{ɤ}\lambda^{Ƕ}_{I}(x) = ɤ.\, \lambda^{Ƕ}_{I}(x),$$

$${}^{M}_{ɤ}H^{Ƕ}_{F}(x) = ɤ.\, H^{Ƕ}_{F}(x), \qquad {}^{M}_{ɤ}\lambda^{Ƕ}_{F}(x) = ɤ.\, \lambda^{Ƕ}_{F}(x).$$

For convinience we use ${}^{M}_{ɤ}H^{Ƕ}_{T,I,F} = ɤ.\, H^{Ƕ}_{T,I,F}(x)$ and ${}^{M}_{ɤ}\lambda^{Ƕ}_{T,I,F} = ɤ.\, \lambda^{Ƕ}_{T,I,F}(x)$.

Theorem 3.1 A ɤ-multilplication of NCCNID of B-algebra X is also a ɤ-multilplication of NCMSU of X.

Proof. Suppose $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ be a NCCNID of X, then for any $ţ \in X$, we have ${}^{M}_{ɤ}H_{T,I,F}(0 * (ţ * \alpha)) = ɤ.\, H_{T,I,F}(0 * (ţ * \alpha)) \geq ɤ.\, H_{T,I,F}(ţ * \alpha)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(0 * (ţ * \alpha)) = ɤ.\, \lambda_{T,I,F}(0 * (ţ * \alpha)) \leq ɤ.\, \lambda_{T,I,F}(ţ * \alpha)$. Now by N4, N6, and through proposition 3.3 of article M subalgebra, we know that ${}^{M}_{ɤ}H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) = ɤ.\, H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) \geq ɤ.\, \mathrm{rmin}\{H_{T,I,F}(((ţ * \alpha) * (ƀ * \beta)) * (0 * (ƀ * \beta))), H_{T,I,F}(0 * (ƀ * \beta))\} = ɤ.\, \mathrm{rmin}\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(0 * (ƀ * \beta))\} \geq ɤ.\, \mathrm{rmin}\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(ƀ * \beta)\} = \mathrm{rmin}\{ɤ.\, H_{T,I,F}(ţ * \alpha), ɤ.\, H_{T,I,F}(ƀ * \beta)\} = \mathrm{rmin}\{{}^{M}_{ɤ}H_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}H_{T,I,F}(ƀ * \beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) = ɤ.\, \lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) \leq ɤ.\, \max\{\lambda_{T,I,F}(((ţ * \alpha) * (ƀ * \beta)) * (0 * (ƀ * \beta))), \lambda_{T,I,F}(0 * (ƀ * \beta))\} = ɤ.\, \max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(0 * (ƀ * \beta))\} \leq ɤ.\, \max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(ƀ * \beta)\} = \max\{ɤ.\, \lambda_{T,I,F}(ţ * \alpha), ɤ.\, \lambda_{T,I,F}(ƀ * \beta)\} = \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}\lambda_{T,I,F}(ƀ * \beta)\}$. Hence, ɤMNCCNID is ɤMNCMSU of X.

Proposition 3.1 Every ɤ-multiplication of NCCNID is a ɤ-multiplication NCNID but the converse is not true in general.

Theorem 3.2 The R-intersection of any set of ɤMNCNIDs of X is also a ɤMNCNID of X.

Proof. Let $\text{Ƕ}_i = \{H^{i}_{T,I,F}, \lambda^{i}_{T,I,F}\}$, where $i \in k$, be a ɤMNCNID of X and $\text{ţ}, \text{ƀ} \in X$. Then

$$(\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(0) = \text{rinf}\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F}(0) = \text{rinf}\, H^{i}_{T,I,F}(0).\text{ɤ}$$

$$\geq \text{rinf}H^{i}_{T,I,F}(\text{ţ} * \alpha).\text{ɤ} = \text{rinf}\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F}(\text{ţ} * \alpha)$$

$$= (\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(\text{ţ} * \alpha)$$

$$\Rightarrow (\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(0) \geq (\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(\text{ţ} * \alpha)$$

and

$$(\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(0) = \text{sup}\,{}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F}(0) = \text{sup}\lambda^{i}_{T,I,F}(0)\,.\text{ɤ}$$

$$\leq \text{sup}\lambda^{i}_{T,I,F}(\text{ţ} * \alpha)\,.\text{ɤ} = \text{sup}\,{}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F}(\text{ţ} * \alpha)$$

$$= (\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(\text{ţ} * \alpha)$$

$$\Rightarrow (\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(0) \leq (\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(\text{ţ} * \alpha),$$

now

$$(\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(\text{ţ} * \alpha) = \text{rinf}\,{}^{M}_{\text{ɤ}}H^{i}_{T,I,F}(\text{ţ} * \alpha) = \text{rinf}\, H^{i}_{T,I,F}(\text{ţ} * \alpha)\,.\text{ɤ}$$

$$\geq \text{rinf}\{\text{rmin}\{H^{i}_{T,I,F}((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), H^{i}_{T,I,F}(\text{ƀ} * \beta)\}\}\,.\text{ɤ}$$

$$= \text{rmin}\{\text{rinf}H^{i}_{T,I,F}((\text{ţ} * \alpha) * (\text{ƀ} * \beta)).\text{ɤ}, \text{rinf}H^{i}_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\}$$

$$= \text{rmin}\{\text{rinf}\,{}^{M}_{\text{ɤ}}H^{i}_{T,I,F}((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), \text{rinf}\,{}^{M}_{\text{ɤ}}H^{i}_{T,I,F}(\text{ƀ} * \beta)\}$$

$$= \text{rmin}\{(\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), (\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})((\text{ƀ} * \beta))\} \Rightarrow (\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(\text{ţ} * \alpha) \geq \text{rmin}\{(\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), (\cap\, {}^{M}_{\text{ɤ}}H^{i}_{T,I,F})(\text{ƀ} * \beta)\}$$

and

$$(\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(\text{ţ} * \alpha) = \text{sup}\,{}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F}(\text{ţ} * \alpha) = \text{sup}\lambda^{i}_{T,I,F}((\text{ţ} * \alpha)\,.\text{ɤ}$$

$$\leq \text{sup}\{\max\{\lambda^{i}_{T,I,F}((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), \lambda^{i}_{T,I,F}(\text{ƀ} * \beta)\}\}\,.\text{ɤ}$$

$$= \max\{\text{sup}\lambda^{i}_{T,I,F}((\text{ţ} * \alpha) * (\text{ƀ} * \beta)).\text{ɤ}, \text{sup}\lambda^{i}_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\}$$

$$= \max\{\text{sup}\,{}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F}((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), \text{sup}\,{}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F}(\text{ƀ} * \beta)\}$$

$$= \max\{(\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), (\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(\text{ƀ} * \beta)\}$$

$$\Rightarrow (\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(\text{ţ} * \alpha) \leq \max\{(\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})((\text{ţ} * \alpha) * (\text{ƀ} * \beta)), (\vee {}^{M}_{\text{ɤ}}\lambda^{i}_{T,I,F})(\text{ƀ} * \beta)\},$$

which show that R-intersection is a ɤMNCNID of X.

Theorem 3.3. The R-intersection of any set of ɤMNCCNIDs of X is also a ɤ-multiplication of NCCNID of X.

Proof**.** We can prove this theorem as Theorem 3.2.

Theorem 3.4. Let $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ be a NCS of X. Then ɤMNCNID of Ƕ is a NCNID of X iff ${}_{ɤ}^{M}H_{T,I,F}^{-}$, ${}_{ɤ}^{M}H_{T,I,F}^{+}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}$ are fuzzy ideals of X.

Proof. Suppose that $ţ, ƀ \in X$. Since ${}_{ɤ}^{M}H_{T,I,F}^{-}(0) = H_{T,I,F}^{-}(0).ɤ \geq H_{T,I,F}^{-}(ţ * \alpha).ɤ = {}_{ɤ}^{M}H_{T,I,F}^{-}(ţ * \alpha)$, ${}_{ɤ}^{M}H_{T,I,F}^{+}(0) = H_{T,I,F}^{+}(0).ɤ \geq H_{T,I,F}^{+}(ţ * \alpha).ɤ = {}_{ɤ}^{M}H_{T,I,F}^{-}(ţ * \alpha)$, therefore, $H_{T,I,F}(0) \geq H_{T,I,F}(ţ * \alpha)$, also ${}_{ɤ}^{M}\lambda_{T,I,F}(0) = \lambda_{T,I,F}(0).ɤ \leq \lambda_{T,I,F}(ţ * \alpha).ɤ = {}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha)$. Suppose that ${}_{ɤ}^{M}H_{T,I,F}^{-}$, ${}_{ɤ}^{M}H_{T,I,F}^{+}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}$ are ɤ-multiplication of fuzzy ideals of X. Then ${}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) = H_{T,I,F}(ţ * \alpha).ɤ = \{H_{T,I,F}^{-}(ţ * \alpha), H_{T,I,F}^{+}(ţ * \alpha)\}.ɤ \geq [\min\{H_{T,I,F}^{-}((ţ * \alpha) * (ƀ * \beta)), H_{T,I,F}^{-}((ƀ * \beta))\}, \min\{H_{T,I,F}^{+}((ţ * \alpha) * (ƀ * \beta)), H_{T,I,F}^{+}(ƀ * \beta)\}.ɤ = r\min\{[H_{T,I,F}^{-}((ţ * \alpha) * (ƀ * \beta)), H_{T,I,F}^{+}((ţ * \alpha) * (ƀ * \beta))], [H_{T,I,F}^{-}((ƀ * \beta)), H_{T,I,F}^{+}(ƀ * \beta)]\}.ɤ = r\min\{H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), H_{T,I,F}(ƀ * \beta)\}.ɤ = r\min\{H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)).ɤ, H_{T,I,F}(ƀ * \beta).ɤ\} = r\min\{{}_{ɤ}^{M}H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta)\}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha) \leq \max\{{}_{ɤ}^{M}\lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}\lambda_{T,I,F}(ƀ * \beta)\}$. Therefore ɤMNCNID of Ƕ is a NCNID of X.

Conversely, assume that ɤMNCNID Ƕ is a NCNID of X. For any $ţ, ƀ \in X$, we have $\{{}_{ɤ}^{M}H_{T,I,F}^{-}(ţ * \alpha), {}_{ɤ}^{M}H_{T,I,F}^{+}(ţ * \alpha)\} = \{H_{T,I,F}^{-}(ţ * \alpha).ɤ, H_{T,I,F}^{+}(ţ * \alpha).ɤ\} = \{H_{T,I,F}^{-}(ţ * \alpha), H_{T,I,F}^{+}(ţ * \alpha)\}.ɤ = H_{T,I,F}(ţ * \alpha).ɤ = {}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) = r\min\{{}_{ɤ}^{M}H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta)\} = r\min\{[{}_{ɤ}^{M}H_{T,I,F}^{-}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}^{+}((ţ * \alpha) * (ƀ * \beta))], [{}_{ɤ}^{M}H_{T,I,F}^{-}(ƀ * \beta), {}_{ɤ}^{M}H_{T,I,F}^{+}(ƀ * \beta)]\} = [\min\{H_{T,I,F}^{-}((ţ * \alpha) * (ƀ * \beta)).ɤ, H_{T,I,F}^{-}(ƀ * \beta).ɤ\}, \min\{H_{T,I,F}^{+}((ţ * \alpha) * (ƀ * \beta)).ɤ, H_{T,I,F}^{+}(ƀ * \beta).ɤ\} = [\min\{{}_{ɤ}^{M}H_{T,I,F}^{-}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}^{-}(ƀ * \beta)\}, \min\{{}_{ɤ}^{M}H_{T,I,F}^{+}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}^{+}(ƀ * \beta)\}]$. Thus, ${}_{ɤ}^{M}H_{T,I,F}^{-}(ţ * \alpha) \geq \min\{{}_{ɤ}^{M}H_{T,I,F}^{-}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}^{-}(ƀ * \beta)\}$, ${}_{ɤ}^{M}H_{T,I,F}^{+}(ţ * \alpha) \geq \min\{{}_{ɤ}^{M}H_{T,I,F}^{+}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}H_{T,I,F}^{+}(ƀ * \beta)\}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha) \leq \max\{{}_{ɤ}^{M}\lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), {}_{ɤ}^{M}\lambda_{T,I,F}(ƀ * \beta)\}$. Hence, ${}_{ɤ}^{M}H_{T,I,F}^{-}$, ${}_{ɤ}^{M}H_{T,I,F}^{+}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}$ are fuzzy ideals of X.

Theorem 3.5. For a NCNID $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ of X, the following statements are valid:

1. If $(ţ * \alpha) * (ƀ * \beta) \leq z * \gamma$, then ${}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) \geq r\min\{{}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta), {}_{ɤ}^{M}H_{T,I,F}(z * \gamma)\}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha) \leq \max\{{}_{ɤ}^{M}\lambda_{T,I,F}(ƀ * \beta), {}_{ɤ}^{M}\lambda_{T,I,F}(z * \gamma)\}$,

2. If $(ţ * \alpha) \leq (ƀ * \beta)$, then ${}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) \geq {}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta)$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha) \leq {}_{ɤ}^{M}\lambda_{T,I,F}(ƀ * \beta)$ $\forall$ $ţ, ƀ, z \in X$ and $\alpha, \beta, \gamma \in [0,1]$.

Proof. 1. Assume that $ţ, ƀ, z \in X$ such that $(ţ * \alpha) * (ƀ * \beta) \leq (z * \gamma)$. Then $((ţ * \alpha) * (ƀ * \beta)) * (z * \gamma) = 0$ and thus ${}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) = H_{T,I,F}(ţ * \alpha).ɤ \geq r\min\{H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), H_{T,I,F}(ƀ * \beta)\}.ɤ \geq r\min\{r\min\{H_{T,I,F}(((ţ * \alpha) * (ƀ * \beta)) * (z * \gamma)), H_{T,I,F}(z * \gamma)\}, H_{T,I,F}(ƀ * \beta)\}.ɤ = r\min\{r\min\{H_{T,I,F}(0), H_{T,I,F}(z * \gamma)\}, H_{T,I,F}(ƀ * \beta)\}.ɤ = r\min\{H_{T,I,F}(ƀ * \beta).ɤ, H_{T,I,F}(z * \gamma).ɤ\} = r\min\{{}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta), {}_{ɤ}^{M}H_{T,I,F}(z * \gamma)\}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha) = \lambda_{T,I,F}(ţ * \alpha).ɤ \leq \max\{\lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), \lambda_{T,I,F}(ƀ * \beta)\}.ɤ \leq \max\{\max\{\lambda_{T,I,F}(((ţ * \alpha) * (ƀ * \beta)) * (z * \gamma)), \lambda_{T,I,F}(z * \gamma)\}, \lambda_{T,I,F}(ƀ * \beta)\}.ɤ = \max\{\max\{\lambda_{T,I,F}(0), \lambda_{T,I,F}(z * \gamma)\}, \lambda_{T,I,F}(ƀ * \beta)\}.ɤ = \max\{\lambda_{T,I,F}(ƀ * \beta).ɤ, \lambda_{T,I,F}(z * \gamma).ɤ\} = \max\{{}_{ɤ}^{M}\lambda_{T,I,F}(ƀ * \beta), {}_{ɤ}^{M}\lambda_{T,I,F}(z * \gamma)\}$.

2. Again, take $ţ, ƀ \in X$ and $\alpha, \beta \in [0,1]$, such that $(ţ * \alpha) \leq (ƀ * \beta)$. Then $(ţ * \alpha) * (ƀ * \beta) = 0$ and thus ${}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) = H_{T,I,F}(ţ * \alpha).ɤ \geq rmin\{H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), H_{T,I,F}(ƀ * \beta)\}.ɤ = rmin\{H_{T,I,F}(0), H_{T,I,F}(ƀ * \beta)\}.ɤ = H_{T,I,F}(ƀ * \beta).ɤ = {}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta)$, *so* ${}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha) \geq {}_{ɤ}^{M}H_{T,I,F}(ƀ * \beta)$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha) = \lambda_{T,I,F}(ţ * \alpha).ɤ \leq$

$max\{\lambda_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)),\lambda_{T,I,F}(ȶ*\beta)\}.ɤ = \max\{\lambda_{T,I,F}(0),\lambda_{T,I,F}(ȶ*\beta)\}.ɤ = \lambda_{T,I,F}(ȶ*\beta).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta), so\ {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\alpha) \le {}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)$.

Theorem 3.6. Let ${}^{M}_{ɤ}Ƕ\ of\ Ƕ = \{H_{T,I,F},\lambda_{T,I,F}\}$ is a NCNID of X. $\forall\ ƫ,ȶ \in X\ and\ \alpha,\beta \in [0,1]$, then Ƕ is a NCMSU of X.

Proof. Assume that ${}^{M}_{ɤ}Ƕ$ is a NCNID of X, $\forall\ ƫ,ȶ \in X\ and\ \alpha,\beta \in [0,1]$. Then $ɤ.H_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)) = {}^{M}_{ɤ}H_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)) \ge rmin\{{}^{M}_{ɤ}H_{T,I,F}((ȶ*\beta)*((ƫ*\alpha)*(ȶ*\beta))),{}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)\} = rmin\{{}^{M}_{ɤ}H_{T,I,F}(0),{}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)\} \ge rmin\{{}^{M}_{ɤ}H_{T,I,F}(ƫ*\alpha),{}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)\} = rmin\{H_{T,I,F}(ƫ*\alpha).ɤ,H_{T,I,F}(ȶ*\beta).ɤ\} = rmin\{H_{T,I,F}(ƫ*\alpha),H_{T,I,F}(ȶ*\beta)\}.ɤ \Rightarrow H_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)) \ge rmin\{H_{T,I,F}(ƫ*\alpha),H_{T,I,F}(ȶ*\beta)\}$ and $ɤ.\lambda_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)) = {}^{M}_{ɤ}\lambda_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)) \le max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ȶ*\beta)*((ƫ*\alpha)*(ȶ*\beta))),{}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)\} = max\{{}^{M}_{ɤ}\lambda_{T,I,F}(0),{}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)\} \le max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\alpha),{}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)\} = max\{\lambda_{T,I,F}(ƫ*\alpha).ɤ,\lambda_{T,I,F}(ȶ*\beta).ɤ\} = max\{\lambda_{T,I,F}(ƫ*\alpha),\lambda_{T,I,F}(ȶ*\beta)\}.ɤ \Rightarrow \lambda_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)) \le max\{\lambda_{T,I,F}(ƫ*\alpha),\lambda_{T,I,F}((ȶ*\beta))\}$. Hence, $Ƕ\{H_{T,I,F},\lambda_{T,I,F}\}$ is a NCMSU of X.

4. ɤ-Multiplication under Homomorphism

Theorem 4.1. Suppose that $\Gamma|X \to Y$ is a homomorphic mapping of PS-algebra. If ${}^{M}_{ɤ}Ƕ\ of\ Ƕ = (H_{T,I,F},\lambda_{T,I,F})$ is a NCNID of Y, then pre-image $\Gamma^{-1}({}^{M}_{ɤ}Ƕ) = (\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}),\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}))$ of ${}^{M}_{ɤ}Ƕ$ under Γ of X is a NCNID of X.

Proof. For all $ƫ \in X\ and\ \alpha \in [0,1]$, $\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ƫ*\alpha) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ƫ*\alpha)) = H_{T,I,F}(\Gamma(ƫ*\alpha)).ɤ \le H_{T,I,F}(\Gamma(0)).ɤ = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(0)) = \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(0)$ and $\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ƫ*\alpha) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ƫ*\alpha)) = \lambda_{T,I,F}(\Gamma(ƫ*\alpha)).ɤ \ge \lambda_{T,I,F}(\Gamma(0)).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(0)) = \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(0)$.

Let $ƫ,ȶ \in X$, $\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ƫ*\alpha) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ƫ*\alpha)) = H_{T,I,F}(\Gamma(ƫ*\alpha)).ɤ \ge rmin\{H_{T,I,F}(\Gamma(ƫ*\alpha)*\Gamma(ȶ*\beta)),H_{T,I,F}(\Gamma(ȶ*\beta))\}.ɤ = rmin\{H_{T,I,F}(\Gamma((ƫ*\alpha)*(ȶ*\beta))),H_{T,I,F}(\Gamma(ȶ*\beta))\}.ɤ = rmin\{\Gamma^{-1}(H_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)).ɤ),\Gamma^{-1}(H_{T,I,F}(ȶ*\beta).ɤ)\} = rmin\{\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}((ƫ*\alpha)*(ȶ*\beta))),\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta))\}$ and $\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ƫ*\alpha) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ƫ*\alpha)) = \lambda_{T,I,F}(\Gamma(ƫ*\alpha)).ɤ \le max\{\lambda_{T,I,F}(\Gamma(ƫ*\alpha)*\Gamma(ȶ*\beta)),\lambda_{T,I,F}(\Gamma(ȶ*\beta))\}.ɤ = max\{\lambda_{T,I,F}(\Gamma((ƫ*\alpha)*(ȶ*\beta))),\lambda_{T,I,F}(\Gamma(ȶ*\beta))\}.ɤ = max\{\Gamma^{-1}(\lambda_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)).ɤ),\Gamma^{-1}(\lambda_{T,I,F}(ȶ*\beta).ɤ)\} = max\{\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}((ƫ*\alpha)*(ȶ*\beta)),\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta))\}$. Hence, $\Gamma^{-1}({}^{M}_{ɤ}Ƕ) = (\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}),\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}))$ is a NCNID of X.

Theorem 4.2. Let $\Gamma|X \to Y$ be a homomorphic mapping of B-algebra. If ${}^{M}_{ɤ}Ƕ_i\ of\ Ƕ_i = (H^{i}_{T,I,F},\lambda^{i}_{T,I,F})$ is a NCNID of Y where $i \in k$, then the pre-image $\Gamma^{-1}(\bigcap_{i\in k}{}^{M}_{ɤ}Ƕ^{i}_{T,I,F}) = (\Gamma^{-1}(\bigcap_{i\in k}{}^{M}_{ɤ}H^{i}_{T,I,F}),\Gamma^{-1}(\bigcap_{i\in k}{}^{M}_{ɤ}\lambda^{i}_{T,I,F}))$ is a NCNID of X.

Proof. We can prove this theorem through Theorem 3.2 and Theorem 4.1.

Theorem 4.3. Let $\Gamma|X \to Y$ is an epimorphic mapping of B-algebra.Then ${}^{M}_{ɤ}Ƕ = ({}^{M}_{ɤ}H_{T,I,F},{}^{M}_{ɤ}\lambda_{T,I,F})$ is a NCNID of Y, if pre-image $\Gamma^{-1}({}^{M}_{ɤ}Ƕ) = (\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}),\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}))$ of ${}^{M}_{ɤ}Ƕ$ under Γ of X is a NCNID of X

Proof. For any $ȶ \in Y$, $ƫ \in X\ and\ \alpha,\beta \in [0,1]$ such that $(ȶ*\beta) = \Gamma(ƫ*\alpha)$. Then ${}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ƫ*\alpha)) = \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ƫ*\alpha) = \Gamma^{-1}(H_{T,I,F})(ƫ*\alpha).ɤ \ge \Gamma^{-1}(H_{T,I,F})(0).ɤ = H_{T,I,F}(\Gamma(0)).ɤ = H_{T,I,F}(0).ɤ = {}^{M}_{ɤ}H_{T,I,F}(0)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ƫ*\alpha)) = \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ƫ*\alpha) = \Gamma^{-1}(\lambda_{T,I,F})(ƫ*\alpha).ɤ \le \Gamma^{-1}(\lambda_{T,I,F})(0).ɤ = \lambda_{T,I,F}(\Gamma(0)).ɤ = \lambda_{T,I,F}(0).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(0)$.

Assume $ȶ_1,ȶ_2 \in Y$. Then $\Gamma(ƫ_1*\alpha) = ȶ_1*\beta$ and $\Gamma(ƫ_2*\alpha) = ȶ_2*\beta$ for some $ƫ_1,ƫ_2 \in X\ and\ \alpha,\beta \in [0,1]$. Thus ${}^{M}_{ɤ}H_{T,I,F}(ȶ_1*\beta) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ƫ_1*\alpha)) = \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ƫ_1*\alpha) = \Gamma^{-1}(H_{T,I,F})(ƫ_1*\alpha).ɤ \ge rmin\{\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})((ƫ_1*$

$\alpha) * (\text{ţ}_2 * \alpha)), \Gamma^{-1}({}^{M}_{\text{ɤ}}H_{T,I,F})(\text{ţ}_2 * \alpha)\}.\text{ɤ} = rmin\{{}^{M}_{\text{ɤ}}H_{T,I,F}(\Gamma((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha))), H_{T,I,F}(\Gamma(\text{ţ}_2 * \alpha))\}.\text{ɤ} = rmin\{H_{T,I,F}(\Gamma(\text{ţ}_1 * \alpha) * \Gamma(\text{ţ}_2 * \alpha)), H_{T,I,F}(\Gamma(\text{ţ}_2 * \alpha))\} = rmin\{H_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), H_{T,I,F}(\text{ƀ}_2 * \beta)\}.\text{ɤ} = rmin\{H_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)).\text{ɤ}, H_{T,I,F}(\text{ƀ}_2 * \beta).\text{ɤ}\} = rmin\{{}^{M}_{\text{ɤ}}H_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), {}^{M}_{\text{ɤ}}H_{T,I,F}(\text{ƀ}_2 * \beta)\}$ and ${}^{M}_{\text{ɤ}}\lambda_{T,I,F}(\text{ƀ}_1 * \beta) = {}^{M}_{\text{ɤ}}\lambda_{T,I,F}(\Gamma(\text{ţ}_1 * \alpha)) = \Gamma^{-1}({}^{M}_{\text{ɤ}}\lambda_{T,I,F})(\text{ţ}_1 * \alpha) = \Gamma^{-1}(\lambda_{T,I,F})(\text{ţ}_1 * \alpha).\text{ɤ} \leq max\{\Gamma^{-1}({}^{M}_{\text{ɤ}}\lambda_{T,I,F})((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha)), \Gamma^{-1}({}^{M}_{\text{ɤ}}\lambda_{T,I,F})(\text{ţ}_2 * \alpha)\}.\text{ɤ} = max\{{}^{M}_{\text{ɤ}}\lambda_{T,I,F}(\Gamma((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha))), \lambda_{T,I,F}(\Gamma(\text{ţ}_2 * \alpha))\}.\text{ɤ} = max\{\lambda_{T,I,F}(\Gamma(\text{ţ}_1 * \alpha) * \Gamma(\text{ţ}_2 * \alpha)), \lambda_{T,I,F}(\Gamma(\text{ţ}_2 * \alpha))\} = max\{\lambda_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), \lambda_{T,I,F}(\text{ƀ}_2 * \beta)\}.\text{ɤ} = max\{\lambda_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)).\text{ɤ}, \lambda_{T,I,F}(\text{ƀ}_2 * \beta).\text{ɤ}\} = max\{{}^{M}_{\text{ɤ}}\lambda_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), {}^{M}_{\text{ɤ}}\lambda_{T,I,F}(\text{ƀ}_2 * \beta)\}$.
Hence, ${}^{M}_{\text{ɤ}}\text{Ƕ} = ({}^{M}_{\text{ɤ}}H_{T,I,F}, {}^{M}_{\text{ɤ}}\lambda_{T,I,F})$ is a NCNID of Y.

5. ɤ-MULTIPLICATION OF CARTESIAN PRODUCT

Theorem 5.1. Let ${}^{M}_{\text{ɤ}}\text{Ƕ} = ({}^{M}_{\text{ɤ}}H_{T,I,F}, {}^{M}_{\text{ɤ}}\lambda_{T,I,F})$ and ${}^{M}_{\text{ɤ}}\text{Ƒ} = ({}^{M}_{\text{ɤ}}F_{T,I,F}, {}^{M}_{\text{ɤ}}\mu_{T,I,F})$ are NCNIDs of X and Y respectively. Then ${}^{M}_{\text{ɤ}}\text{Ƕ} \times {}^{M}_{\text{ɤ}}\text{Ƒ}$ is a neutrosophic cubic normal ideal of $X \times Y$.

Proof. For any $(\text{ţ}, \text{ƀ}) \in X \times Y$ and $\alpha, \beta \in [0,1]$. We have $({}^{M}_{\text{ɤ}}H_{T,I,F} \times {}^{M}_{\text{ɤ}}F_{T,I,F})(0,0) = \text{ɤ}.(H_{T,I,F} \times F_{T,I,F})(0,0) = \text{ɤ}.rmin\{H_{T,I,F}(0), F_{T,I,F}(0)\} \geq \text{ɤ}.rmin\{H_{T,I,F}(\text{ţ} * \alpha), F_{T,I,F}(\text{ƀ} * \beta)\} = rmin\{H_{T,I,F}(\text{ţ} * \alpha).\text{ɤ}, F_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\} = rmin\{{}^{M}_{\text{ɤ}}H_{T,I,F}(\text{ţ} * \alpha), {}^{M}_{\text{ɤ}}F_{T,I,F}(\text{ƀ} * \beta)\} = ({}^{M}_{\text{ɤ}}H_{T,I,F} \times {}^{M}_{\text{ɤ}}F_{T,I,F})(\text{ţ} * \alpha, \text{ƀ} * \beta)$ and $({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})(0,0) = \text{ɤ}.(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) = \text{ɤ}.max\{\lambda_{T,I,F}(0), \mu_{T,I,F}(0)\} \leq \text{ɤ}.max\{\lambda_{T,I,F}(\text{ţ} * \alpha), \mu_{T,I,F}(\text{ƀ} * \beta)\} = max\{\lambda_{T,I,F}(\text{ţ} * \alpha).\text{ɤ}, \mu_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\} = max\{{}^{M}_{\text{ɤ}}\lambda_{T,I,F}(\text{ţ} * \alpha), {}^{M}_{\text{ɤ}}\mu_{T,I,F}(\text{ƀ} * \beta)\} = ({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})(\text{ţ} * \alpha, \text{ƀ} * \beta)$.

Let $(\text{ţ}_1, \text{ƀ}_1), (\text{ţ}_2, \text{ƀ}_2) \in X \times Y$ and $\alpha, \beta \in [0,1]$. Then $({}^{M}_{\text{ɤ}}H_{T,I,F} \times {}^{M}_{\text{ɤ}}F_{T,I,F})(\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) = \text{ɤ}.(H_{T,I,F} \times F_{T,I,F})(\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) = \text{ɤ}.rmin\{H_{T,I,F}(\text{ţ}_1 * \alpha), F_{T,I,F}(\text{ƀ}_1 * \beta)\} \geq \text{ɤ}.rmin\{rmin\{H_{T,I,F}((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha)), H_{T,I,F}(\text{ţ}_2 * \alpha)\}, rmin\{F_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), F_{T,I,F}(\text{ƀ}_2 * \beta)\}\} = \text{ɤ}.rmin\{rmin\{H_{T,I,F}((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha)), F_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta))\}, rmin\{H_{T,I,F}(\text{ţ}_2 * \alpha), F_{T,I,F}(\text{ƀ}_2 * \beta)\}\} = \text{ɤ}.rmin\{(H_{T,I,F} \times F_{T,I,F})((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha), (\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), (H_{T,I,F} \times F_{T,I,F})((\text{ţ}_2 * \alpha), (\text{ƀ}_2 * \beta))\} = rmin\{(H_{T,I,F} \times F_{T,I,F})((\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) * (\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta)).\text{ɤ}, (H_{T,I,F} \times F_{T,I,F})(\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta).\text{ɤ}\} = rmin\{({}^{M}_{\text{ɤ}}R_{T,I,F} \times {}^{M}_{\text{ɤ}}F_{T,I,F})((\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) * (\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta)), ({}^{M}_{\text{ɤ}}R_{T,I,F} \times {}^{M}_{\text{ɤ}}F_{T,I,F})(\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta)\}$ and $({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})(\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) = \text{ɤ}.(\lambda_{T,I,F} \times \mu_{T,I,F})(\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) = \text{ɤ}.max\{\lambda_{T,I,F}(\text{ţ}_1 * \alpha), \mu_{T,I,F}(\text{ƀ}_1 * \beta)\} \leq \text{ɤ}.max\{max\{\lambda_{T,I,F}((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha)), \lambda_{T,I,F}(\text{ţ}_2 * \alpha)\}, max\{\mu_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), F_{T,I,F}(\text{ƀ}_2 * \beta)\}\} = \text{ɤ}.max\{max\{\lambda_{T,I,F}((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha)), \mu_{T,I,F}((\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta))\}, max\{\lambda_{T,I,F}(\text{ţ}_2 * \alpha), \mu_{T,I,F}(\text{ƀ}_2 * \beta)\}\} = \text{ɤ}.max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((\text{ţ}_1 * \alpha) * (\text{ţ}_2 * \alpha), (\text{ƀ}_1 * \beta) * (\text{ƀ}_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})((\text{ţ}_2 * \alpha), (\text{ƀ}_2 * \beta))\} = max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) * (\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta)).\text{ɤ}, (\lambda_{T,I,F} \times \mu_{T,I,F})(\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta).\text{ɤ}\} = max\{({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})((\text{ţ}_1 * \alpha, \text{ƀ}_1 * \beta) * (\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta)), ({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})(\text{ţ}_2 * \alpha, \text{ƀ}_2 * \beta)\}$. Hence, ${}^{M}_{\text{ɤ}}\text{Ƕ} \times {}^{M}_{\text{ɤ}}\text{Ƒ}$ is a neutrosophic cubic normal ideal of $X \times Y$.

Theorem 5.2. Let ${}^{M}_{\text{ɤ}}\text{Ƕ} = ({}^{M}_{\text{ɤ}}H_{T,I,F}, {}^{M}_{\text{ɤ}}\lambda_{T,I,F})$ and ${}^{M}_{\text{ɤ}}\text{Ƒ} = ({}^{M}_{\text{ɤ}}F_{T,I,F}, {}^{M}_{\text{ɤ}}\mu_{T,I,F})$ are two ɤ-multiplications of neutrosophic cubic closed normal ideals of X and Y respectively. Then ${}^{M}_{\text{ɤ}}\text{Ƕ} \times {}^{M}_{\text{ɤ}}\text{Ƒ}$ is a NCCNID of $X \times Y$.

Proof. By Proposition 3.1 and Theorem 5.1, ${}^{M}_{\text{ɤ}}\text{Ƕ} \times {}^{M}_{\text{ɤ}}\text{Ƒ}$ is NCNID. Now, $({}^{M}_{\text{ɤ}}H_{T,I,F} \times {}^{M}_{\text{ɤ}}F_{T,I,F})((0,0) * (\text{ţ} * \alpha, \text{ƀ} * \beta)) = (H_{T,I,F} \times F_{T,I,F})((0,0) * (\text{ţ} * \alpha, \text{ƀ} * \beta)).\text{ɤ} = (H_{T,I,F} \times F_{T,I,F})(0 * (\text{ţ} * \alpha), 0 * (\text{ƀ} * \beta)).\text{ɤ} = \text{ɤ}.\mathrm{rmin}\{\mathrm{H}_{T,I,F}(0 * (\text{ţ} * \alpha)), \mathrm{F}_{T,I,F}(0 * (\text{ƀ} * \beta))\} \geq \text{ɤ}.\mathrm{rmin}\{\mathrm{H}_{T,I,F}(\text{ţ} * \alpha), \mathrm{F}_{T,I,F}(\text{ƀ} * \beta)\} = \mathrm{rmin}\{\mathrm{H}_{T,I,F}(\text{ţ} * \alpha).\text{ɤ}, \mathrm{F}_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\} = \mathrm{rmin}\{{}^{M}_{\text{ɤ}}\mathrm{H}_{T,I,F}(\text{ţ} * \alpha), {}^{M}_{\text{ɤ}}\mathrm{F}_{T,I,F}(\text{ƀ} * \beta)\} = ({}^{M}_{\text{ɤ}}\mathrm{H}_{T,I,F} \times {}^{M}_{\text{ɤ}}\mathrm{F}_{T,I,F})(\text{ţ} * \alpha, \text{ƀ} * \beta)$ and $({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})((0,0) * (\text{ţ} * \alpha, \text{ƀ} * \beta)) = (\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (\text{ţ} * \alpha, \text{ƀ} * \beta)).\text{ɤ} = (\lambda_{T,I,F} \times \mu_{T,I,F})(0 * (\text{ţ} * \alpha), 0 * (\text{ƀ} * \beta)).\text{ɤ} = \text{ɤ}.\max\{\lambda_{T,I,F}(0 * (\text{ţ} * \alpha)), \mu_{T,I,F}(0 * (\text{ƀ} * \beta))\} \leq \text{ɤ}.\max\{\lambda_{T,I,F}(\text{ţ} * \alpha), \mu_{T,I,F}(\text{ƀ} * \beta)\} = \max\{\lambda_{T,I,F}(\text{ţ} * \alpha).\text{ɤ}, \mu_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\} = \max\{{}^{M}_{\text{ɤ}}\lambda_{T,I,F}(\text{ţ} * \alpha), {}^{M}_{\text{ɤ}}\mu_{T,I,F}(\text{ƀ} * \beta)\} = ({}^{M}_{\text{ɤ}}\lambda_{T,I,F} \times {}^{M}_{\text{ɤ}}\mu_{T,I,F})(\text{ţ} * \alpha, \text{ƀ} * \beta)$. Hence, ${}^{M}_{\text{ɤ}}\text{Ƕ} \times {}^{M}_{\text{ɤ}}\text{Ƒ}$ is a neutrosophic cubic closed normal ideal of $X \times Y$.

6. Conclusion

In this paper, the notion of ϒ-multiplication of neutrosophic cubic set was introduced and ϒ-multiplication was studies by several useful results. This study will provide the base for further work like t-neutrosophic soft cubic and intuitionistic soft cubic set etc

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