\textbf{Nβ-Closed Sets In Neutrosophic Topological Spaces}

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\textbf{Abstract}——The aim of this paper to introduced the new concept of β-closed sets in Neutrosophic topological spaces. We also analyze the properties and characterize the Neutrosophic β-closed sets.

\textbf{Keywords}—Neutrosophic Closed Sets,Neutrosophic Topological Spaces,Neutrosophic β-closed sets,N-semi open,N-preopen.

I. INTRODUCTION

In 1965, Zadeh[9] introduced fuzzy set theory which deals with uncertainties where each element has a degree of membership. In 1983, Atanassov[1] introduced the Intuitionistic fuzzy set where each element has a degree of membership and degree of non-membership values.In 2005, Florentin Smarandache[7] introduced the Neutrosophic set and explained that the generalization of intuitionistic fuzzy set is the Neutrosophic set. In 2012, A.A.Salama and S.A.Alblowi[6] introduced the concept of Neutrosophic topological spaces besides the degree of membership, degree of indeterminacy and the degree of non-membership for each element. In 2014 A.A.Salama, Smarandache and Veleri[5] introduced the concept of Neutrosophic closed sets and continuous functions. In this paper, we introduce the concept N\textsubscript{β} closed set and characterized some of its properties in Neutrosophic topological spaces.

II. PRELIMINARIES

In this paper the Neutrosophic topological space is denoted by (X,τ).Alsoneutrosophic interior of A is denoted by N\text{Int}(A) andneutrosophic closure of A is denoted by N\text{Cl}(A).The complement of neutrosophic A is denoted by A\textsuperscript{c}.

\textbf{Definition 2.1:}
Let X be a nonempty fixed set A is an object having the form A={⟨x,μ\textsubscript{A}(x),σ\textsubscript{A}(x),ν\textsubscript{A}(x):x∈X⟩} where μ\textsubscript{A}(x),σ\textsubscript{A}(x),ν\textsubscript{A}(x) represents the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x∈X to the set A.

\textbf{Definition 2.2:}
Let A={⟨x,μ\textsubscript{A}(x),σ\textsubscript{A}(x),ν\textsubscript{A}(x):x∈X⟩} and B={⟨x,μ\textsubscript{B}(x),σ\textsubscript{B}(x),ν\textsubscript{B}(x):x∈X⟩} are the two neutrosophic sets on X, then the complements become,

\text{C(A)}=\{x,1−μ\textsubscript{A}(x),1−σ\textsubscript{A}(x),1−ν\textsubscript{A}(x):x∈X\}
\text{C(A)}=\{x,ν\textsubscript{A}(x),σ\textsubscript{A}(x),μ\textsubscript{A}(x):x∈X\}
\text{C(A)}=\{x,ν\textsubscript{A}(x),1−σ\textsubscript{A}(x),μ\textsubscript{A}(x):x∈X\}
\text{C(A∪B)}=\text{C(A)}\cap\text{C(B)}
\text{C(A∩B)}=\text{C(A)}∪\text{C(B)}.
**Definition 2.3:**
Let \( A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X\} \) and \( B = \{ (x, \mu_B(x), \sigma_B(x), \nu_B(x)) : x \in X\} \) are the two neutrosophic sets on \( X \), then \( A \subseteq B \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \)
\[ A = B \iff A \subseteq B \text{ and } B \subseteq A \]
\[ A \cup B = \{ (x, \mu_A(x) \lor \mu_B(x), \sigma_A(x) \lor \sigma_B(x), \nu_A(x) \land \nu_B(x)) \} \]
\[ A \cap B = \{ (x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \land \sigma_B(x), \nu_A(x) \lor \nu_B(x)) \} \]

**Definition 2.4**
A neutrosophic topological space on a nonempty set \( X \) is a family \( \tau \) of neutrosophic subsets in \( X \) satisfies the following axioms:

i) \( 0_N, 1_N \in \tau \),

ii) \( G_1 \cap G_2 \in \tau \), for every \( G_1 \) and \( G_2 \in \tau \),

iii) \( \cup G_i \in \tau \) for every \( G_i : i \in J \subseteq \tau \)

The pair \( (X, \tau) \) is a neutrosophic topological space (NTS) and the element \( \tau \) is called neutrosophic open sets (NOS) in \( X \). A neutrosophic set \( A \) is called the neutrosophic closed set \( A \) if and only if its complement \( C(A) \) is a neutrosophic open set in \( X \).

The empty set \( (0_N) \) and the whole set \( (1_N) \) may be defined as follows:

\[ (0_1) \quad 0_N = \{ (x, 0, 0, 1) : x \in X \} \]
\[ (0_2) \quad 0_N = \{ (x, 0, 1, 1) : x \in X \} \]
\[ (0_3) \quad 0_N = \{ (x, 0, 1, 0) : x \in X \} \]
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\[ (1_3) \quad 1_N = \{ (x, 1, 1, 0) : x \in X \} \]
\[ (1_4) \quad 1_N = \{ (x, 1, 1, 1) : x \in X \} \]

**Definition 2.5:** Let \( (X, \tau) \) be the neutrosophic topological spaces and \( A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \} \) be the neutrosophic set in \( X \). Then the neutrosophic interior and closure becomes,
\[ \text{NInt}(A) = \bigcup \{ G : G \text{ is an NOS in } X \text{ and } G \subseteq A \} \]
\[ \text{NCl}(A) = \bigcap \{ K : K \text{ is an NCS in } X \text{ and } A \subseteq K \} \]

**Definition 2.6:**
A neutrosophic set of a neutrosophic topological space \( X \) is said to be
i) A neutrosophic pre-open set (NPOS) if \( A \subseteq \text{NInt}(\text{NCl}(A)) \)
ii) A neutrosophic semi-open set (NSOS) if \( A \subseteq \text{NCl}(\text{NInt}(A)) \)
iii) A
neutrosophic $\alpha$-open set (N$\alpha$-OS) if $A \subseteq \text{NInt(}\text{NCl(}\text{NInt(}A))\text{)}$
iv) A neutrosophic regular open set (NR-OS) if $A = \text{NInt(}\text{NCl(}A)\text{)}$

**Definition 2.7:** A neutrosophic set of a neutrosophic topological space $X$ is said to be
i) A neutrosophic pre-closed set (NPCS) if $\text{NCl(}\text{NInt(}A)\text{)} \subseteq A$
ii) A neutrosophic semi-closed set (NSCS) if $\text{NInt(}\text{NCl(}A)\text{)} \subseteq A$
iii) A neutrosophic $\alpha$-closed set (N$\alpha$-CS) if $\text{NCl(}\text{NInt(}\text{NCl(}A)\text{)}) \subseteq A$
v) A neutrosophic regular closed set (NRCS) if $A = \text{NCl(}\text{NInt(}A)\text{)}$

## III. NEUTROSOPHIC $\beta$-CLOSED SETS

**Definition 3.1:**
Let $(X, \tau)$ be a neutrosophic topological space. Then $A$ is said to be a neutrosophic $\beta$ closed set (N$\beta$-CS) if $\text{NCl(}\text{NβCl(}A)\text{)} \subseteq U$ whenever $A \subseteq U$ and $U$ is neutrosophic pre-open set in $X$ (NPOS). The complement $C(A)$ of a N$\beta$CS $A$ is a N$\beta$OS in $X$.

**Theorem 3.2:** Every Neutrosophic closed set $A$ is N$\beta$ closed set.
*Proof:* Let $A \subseteq U$ where $U$ is pre-open set in $X$.

Let $\text{NInt(A)} \subseteq A$. Then $\text{NCl(}\text{NInt(A)}\text{)} \subseteq \text{NCl(}A\text{)}$.

By the definition of NCS, $\text{NCl(}\text{NInt(A)}\text{)} \subseteq A$.

Which implies $\text{NInt(NCl(NInt(A)))} \subseteq \text{NInt(A)}$.

Therefore $A$ is N$\beta$ closed set in $X$.

**Remark 3.3:** The converse of the above theorem need not be true which can be shown by the following example.

**Example 3.4:**
Let $X = \{a,b,c\}$ and $\tau = \{0_N, 1_N, G_1, G_2, G_3, G_4\}$ where $G_1 = \{x, (0.5,0.6), (0.4,0.7), (0.3,0.7)\}$
$G_2 = \{x, (0.7,0.5), (0.6,0.5), (0.7,0.4)\}$ $G_3 = \{x, (0.7,0.5), (0.6,0.7), (0.7,0.7)\}$
$G_4 = \{x, (0.5,0.5), (0.4,0.5), (0.3,0.4)\}$ Let $M = \{x, (0.7,0.6), (0.3,0.2), (0.8,0.9)\}$ Then the set $M$ is N$\beta$ closed set but $M$ is not Neutrosophic closed set.

**Theorem 3.5:**
Every Neutrosophic pre-closed set is N$\beta$ closed set in $(X, \tau_N)$.
*Proof:* Let $A \subseteq U$ where $U$ is pre-open set in $X$.

Given, Let $A$ be neutrosophic pre-closed set.

Let $\text{NCl(}\text{NInt(A)}\text{)} \subseteq A$. Then $\text{NInt(NCl(NInt(A)))} \subseteq \text{NInt(A)}$.

Which implies $\text{NInt(NCl(NInt(A)))} \subseteq A$.

Therefore $A$ is N$\beta$ closed set in $X$. 

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Remark 3.6: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.7:
Let $X=\{a,b,c\}$ and $\tau=\{0_N,1_N,G_1,G_2\}$ where
$G_1=(x, (0.7,0.6), (0.5,0.5), (0.5,0.7))$ \hspace{1cm} $G_2=(x, (0.3,0.4), (0.5,0.3), (0.6,0.8))$
Let $M=(x, (0.4,0.5), (0.5,0.5), (0.5,0.8))$ Then the set $M$ is $N_0$ closed set but $M$ is not Neutrosophic pre-closed set.

Theorem 3.8:
Every Neutrosophic semi-closed set is $N_0$ closed set in $(X,\tau_N)$.

Proof:
Let $A\subseteq U$ where $U$ is preopen set in $X$.
Given, Let $A$ be neutrosophic semi-closed set.
That is $N\text{Int}(N\text{Cl}(A))\subseteq A$.
Consider $N\text{Int}(A)\subseteq A$, \hspace{1cm} Then

$N\text{Cl}(N\text{Int}(A))\subseteq N\text{Cl}(A)$ implies $N\text{Int}(N\text{Cl}(N\text{Int}(A)))\subseteq N\text{Int}(N\text{Cl}(A))$
which implies $N\text{Int}(N\text{Cl}(N\text{Int}(A)))\subseteq A$.
Therefore $A$ is $N_0$ closed set in $X$.

Remark 3.9: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.10:
Let $X=\{a,b,c\}$ and $\tau=\{0_N,1_N,G_1,G_2\}$ where
$G_1=(x, (0.2,0.3), (0.4,0.3), (0.5,0.6))$
$G_2=(x, (0.6,0.8), (0.5,0.4), (0.4,0.2))$ Let $M=(x, (0.6,0.5), (0.4,0.4), (0.7,0.8))$ Then the set $M$ is $N_0$ closed set but $M$ is not Neutrosophic semi-closed set.

Theorem 3.11:
Every Neutrosophic $\alpha$-closed set is $N_0$ closed set in $(X,\tau_N)$.

Proof:
Let $A\subseteq U$ where $U$ is preopen set in $X$.
Given, Let $A$ be neutrosophic $\alpha$-closed set.
That is $N\text{Cl}(N\text{Int}(N\text{Cl}(A)))\subseteq A$.
Consider $N\text{Cl}(N\text{Int}(N\text{Cl}(A)))\subseteq A$.

$A\subseteq N\text{Cl}(A)$. Then $N\text{Int}(A)\subseteq N\text{Int}(N\text{Cl}(A))$. \hspace{1cm} Then

$N\text{Cl}(N\text{Int}(A))\subseteq N\text{Cl}(N\text{Int}(N\text{Cl}(A)))\subseteq A$.
Which implies $N\text{Cl}(N\text{Int}(A))\subseteq A$ \hspace{1cm} Then

$N\text{Int}(N\text{Int}(N\text{Cl}(A)))\subseteq N\text{Int}(A)\subseteq A$. \hspace{1cm} Hence
NInt(Ncl(NInt(A)))⊆A. Therefore A is $\mathbb{N}_\beta$ closed set in X.

Remark 3.12: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.13:
Let X={a,b,c} and $\tau=\{0_\mathbb{N},1_\mathbb{N},G_1,G_2,G_3,G_4\}$ where $G_1=(x, (0.4,0.3), (0.5,0.8), (0.4,0.3))$ $G_2=(x, (0.2,0.5), (0.6,0.3), (0.5,0.7))$ $G_3=(x, (0.4,0.5), (0.6,0.8), (0.4,0.3))$ $G_4=(x, (0.2,0.3), (0.5,0.3), (0.5,0.7))$
Let M=(x, (0.2,0.4), (0.6,0.8), (0.6,0.7))Then the set M is $\mathbb{N}_\beta$ closed set but M is not Neutrosophic $\alpha$-closed set.

Theorem 3.14:
Every Neutrosophic regular closed set is $\mathbb{N}_\beta$ closed set in $(X,\tau_\mathbb{N})$.

Proof:
Let A $\subseteq$ U where U is preopen set in X.

Given, Let A be neutrosophic regular closed set.

That is NCl(NInt(A))=A. which implies NInt(NCl(NInt(A)))$\subseteq$NInt(A)$\subseteq$A.

Which implies NInt(NCl(NInt(A)))$\subseteq$A.

Therefore A is $\mathbb{N}_\beta$ closed set in X.

Remark 3.15: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.16:
Let X={a,b,c} and $\tau=\{0_\mathbb{N},1_\mathbb{N},G_1,G_2\}$ where $G_1=(x, (0.2,0.3), (0.4,0.3), (0.6,0.7))$ $G_2=(x, (0.6,0.5), (0.5,0.5), (0.4,0.3))$
Let M=(x, (0.4,0.4), (0.5,0.4), (0.5,0.5))Then the set M is $\mathbb{N}_\beta$ closed set but M is not Neutrosophic regular closed set.

Theorem 3.17:
The Union of two $\mathbb{N}_\beta$ closed set in $(X,\tau_\mathbb{N})$ is a $\mathbb{N}_\beta$ closed set in $(X,\tau_\mathbb{N})$.

Proof:
Let A and B are $\mathbb{N}_\beta$ closed sets in $(X,\tau_\mathbb{N})$. From the definition of $\mathbb{N}_\beta$ closed set,

$\subseteq$ U whenever A $\subseteq$ U and U is Preopen in $(X,\tau_\mathbb{N})$.

Similarly, $\mathbb{N}_\beta$cl(B) $\subseteq$ U whenever B $\subseteq$ U and U is Preopen in $(X,\tau_\mathbb{N})$.

Since A and B are the subsets of U then AUB also the subsets of U and U is the neutrosophic Preopen set, which implies $\mathbb{N}_\beta$cl(AUB) $\subseteq$ U.

Therefore AUB is $\mathbb{N}_\beta$ closed set in $(X,\tau_\mathbb{N})$. 
Theorem 3.18: Suppose A and B are $\mathbb{N}_0$ closed set in $(X,\tau)$ then $N_0\text{cl}(A\cap B) \subseteq N_0\text{cl}(A)\cap N_0\text{cl}(B)$. 

Proof: 
Let A be $\mathbb{N}_0$ closed set in $(X,\tau)$, Then $N_0\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is neutrosophic preopen in $(X,\tau)$. Moreover, Since A and B are subsets of U, then $A\cap B$ is a subset of U where U is a neutrosophic preopen set. From the result, If $A \subseteq B$, then $N_0\text{cl}(A) \subseteq N_0\text{cl}(B)$, which implies $N_0\text{cl}(A\cap B) \subseteq N_0\text{cl}(A)$ and $N_0\text{cl}(A\cap B) \subseteq N_0\text{cl}(B)$. Therefore $N_0\text{cl}(A\cap B) \subseteq N_0\text{cl}(A)\cap N_0\text{cl}(B)$.

Example 3.19: The intersection of two $\mathbb{N}_0$ closed sets are need not be $\mathbb{N}_0$ closed set. 

Let $X=\{a,b,c\}$ and $\tau=\{0_0,1_0,G_1,G_2,G_3,G_4\}$ where $G_1=(x,(0.8,0.6),(0.5,0.4),(0.5,0.7))$ $G_2=(x,(0.7,0.3),(0.7,0.1),(0.4,0.9))$ $G_3=(x,(0.8,0.6),(0.7,0.4),(0.4,0.7))$ $G_4=(x,(0.7,0.3),(0.5,0.1),(0.5,0.9))$ Let $M=(x,(0.7,0.9),(0.6,0.3),(0.5,0.5))$ and $N=(x,(0.7,0.6),(0.7,0.4),(0.4,0.9))$. Then M and N are $\mathbb{N}_0$ closed set, and the union $M\cup N=(x,(0.7,0.9),(0.7,0.4),(0.4,0.5))$ is also an $\mathbb{N}_0$ closed set. But the intersection $M\cap N=(x,(0.7,0.6),(0.6,0.3),(0.5,0.9))$ is not an $\mathbb{N}_0$ closed set.

Theorem 3.20: Suppose M is $\mathbb{N}_0$ closed set in $(X,\tau)$ and $M \subseteq N \subseteq N_0\text{cl}(A)$, then N is also $\mathbb{N}_0$ closed set in $(X,\tau)$. 

Proof: Let $N \subseteq U$ and U is neutrosophic preopen in $(X,\tau)$. Then $M \subseteq N$ which implies $M \subseteq U$, and $M \subseteq N_0\text{cl}(N) \subseteq U$ and $N_0\text{cl}(N) \subseteq N_0\text{cl}(M)$. Which implies $M \subseteq N_0\text{cl}(N) \subseteq N_0\text{cl}(M) \subseteq U$. Therefore $N_0\text{cl}(N) \subseteq U$. and so N is also $\mathbb{N}_0$ closed set in $(X,\tau)$.

Theorem 3.21: If M be a $\mathbb{N}_0$ closed subset of $(X,\tau)$, then $N_0\text{cl}(M)-M$ does not contain any nonempty $\mathbb{N}_0$ closed set. 

Proof: Assume that M is $\mathbb{N}_0$ closed set. Let $F$ be a nonempty $\mathbb{N}_0$ closed set, such that $F \subseteq N_0\text{cl}(M)-M = N_0\text{cl}(M) \cap \overline{M}$. 

That is, $F \subseteq N_0\text{cl}(M)$ and $F \subseteq \overline{M}$. Therefore $M \subseteq \overline{F}$. 

Since $\overline{F}$ is $\mathbb{N}_0$ open set, $N_0\text{cl}(M) \subseteq \overline{F}$ implies $F \subseteq (N_0\text{cl}(M)-M) \cap (\overline{N_0\text{cl}(M)}) \subseteq N_0\text{cl}(M) \cap (\overline{N_0\text{cl}(M)})$. 

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That is $F \subseteq \emptyset \Rightarrow F$ is empty.

Therefore $N_\emptyset cl(M) - M$ does not contain any nonempty $N_\emptyset$ closed set.

**Theorem 3.22:**

If $M$ is neutrosophic open also $N_\emptyset$ closed set, then $M$ is neutrosophic closed set.

**Proof:**

Since $M$ is both neutrosophic open and $N_\emptyset$ closed set in $(X, \tau_N)$.

Then $N_\emptyset cl(M) \subseteq M$ and $M \subseteq N_\emptyset cl(M)$

$\Rightarrow N_\emptyset cl(M) = M$.

Therefore $M$ is neutrosophic closed set in $(X, \tau_N)$.

**Theorem 3.23:**

If $A$ is both $N_\emptyset$ closed set and neutrosophic closed set if and only if $N cl(A) - A$ is neutrosophic closed.

**Proof:**

From the hypothesis, Let $A$ be $N_\emptyset$ closed set.

If $A$ is neutrosophic closed set then $N\ cl(A) = A$.

$\Rightarrow N\ cl(A) - A = \emptyset$.

Therefore $N\ cl(A) - A$ is neutrosophic closed set.

Conversely,

Assume that $N\ cl(A) - A$ is neutrosophic closed,

But $A$ is $N_\emptyset$ closed set and $N\ cl(A) - A \subseteq$ neutrosophic closed.

$\Rightarrow N\ cl(A) - A = \emptyset$, which implies $N\ cl(A) = A$.

Therefore $A$ is Neutrosophic set.

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