



Neutrosophic EOQ Model with Price Break

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Abstract—Inventory control of an ideal resource is the most important one which fulfils various activities (functions) of an organisation. The supplier gives the discount for an item in the cost of units inorder to motivate the buyers (or) customers to purchase the large quantity of that item. These discounts take the form of price breaks where purchase cost is assumed to be constant. In this paper an EOQ model with price break in inventory model is developed to obtain its optimum solution by assuming neutrosophic demand and neutrosophic purchasing cost as triangular neutrosophic numbers. A numerical example is provided to illustrate the proposed model.

Keywords: Price break, neutrosophic demand, neutrosophic purchase cost, neutrosophic sets, triangular neutrosophic number.

1 INTRODUCTION

Bai and Li[1] have discussed triangular and trapezoidal fuzzy numbers in inventory model for determining the optimal order quantity and the optimal cost. The quantity discount problem has been analyzed from a buyers perspective. Hadley and Whintin[2], Peterson and Silver[3], and Starr and Miller[6] considered various discount polices and demand assumptions.

Yang and Wee[7] developed an economic ordering policy in the view of both the supplier and the buyer. Prabjot Kaur and Mahuya Deb[5] developed an intuitionistic approach for price breaks in EOQ from buyer's perspective. Smarandache[5] introduced neutrosophic set and neutrosophic logic by considering the non-standard analysis. Also, neutrosophic inventory model without shortages is introduced by M. Mullai and S. Broumi[3].

In this paper, we introduce the neutrosophic inventory models with neutrosophic price break to find the optimal solution of the model for the optimal order quantity. Also the neutrosophic inventory model under neutrosophic demand and neutrosophic purchasing cost at which the quantity discount are offered to be triangular neutrosophic number. Also the optimal order quantity for the neutrosophic total cost is determined by defining the accuracy function of triangular neutrosophic numbers.

2 NOTATIONS:

 Q^N = Number of pieces per order

 C_0^N = Neutrosophic Ordering cost for each order

 C_h^N = Neutrosophic Holding cost per unit per year

 D^N = Neutrosophic Annual demand in units

3 NEUTROSOPHIC EOQ MODEL WITH PRICE BREAK:

The Neutrosophic inventory model with neutrosophic price break is introduced to find the optimal solutions for the optimal neutrosophic order quantity. Here we assume that there is no stock outs, no backlogs, replenishment is instantaneous, the neutrosophic ordering cost involved to receive an order are known and constant and purchasing values at which discounts are offered as triangular neutrosophic numbers.

Consider the following variables: $\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1$

D^{*N*}: Neutrosophic yearly demand,

P^{*N*}: Neutrosophic purchasing cost

Let
$$D^{N} = (D_{1}^{N}, D_{2}^{N}, D_{3}^{N}) (D_{1}^{\prime N}, D_{2}^{N}, D_{3}^{\prime N}) (D_{1}^{\prime \prime N}, D_{2}^{N}, D_{3}^{\prime \prime N})^{T}$$

 $P^{N} = (P_{1}^{N}, P_{2}^{N}, P_{3}^{N}) (P_{1}^{\prime N}, P_{2}^{N}, P_{3}^{\prime N}) (P_{1}^{\prime \prime N}, P_{2}^{N}, P_{3}^{\prime \prime N})$
 $P_{1}^{N} = (P_{11}^{N}, P_{12}^{N}, P_{13}^{N}) (P_{11}^{\prime N}, P_{12}^{N}, P_{13}^{\prime N}) (P_{11}^{\prime \prime N}, P_{12}^{N}, P_{13}^{\prime \prime N})$
 $P_{2}^{N} = (P_{21}^{N}, P_{22}^{N}, P_{23}^{N}) (P_{21}^{\prime N}, P_{22}^{N}, P_{23}^{\prime N}) (P_{21}^{\prime \prime N}, P_{22}^{N}, P_{23}^{\prime \prime N})$

are non negative triangular neutrosophic numbers.

Now, we introduce the neutrosophic inventory model under neutrosophic demand and neutrosophic purchasing cost at which the quantity discounts are offered. Total neutrosophic inventory cost is given by

$$(\mathsf{TC})^N = D^N \otimes P^N \oplus \frac{D^N C_0^N}{Q^N} \oplus \frac{Q^N P^N \otimes I^N}{2}$$

Then the total neutrosophic inventory cost is

 $(\mathbf{TC})^{N} = (D_{1}^{N}P_{1}^{N} + \frac{D_{1}^{N}C_{0}^{N}}{Q^{N}} + \frac{Q^{N}P_{1}^{N}I^{N}}{2}, D_{2}^{N}P_{2}^{N} + \frac{D_{2}^{N}C_{0}^{N}}{Q^{N}} + \frac{Q^{N}P_{2}^{N}I^{N}}{2}, D_{3}^{N}P_{3}^{N} + \frac{D_{3}^{N}C_{0}^{N}}{Q^{N}} +$

$$\frac{\frac{Q^{N}P_{3}^{N}I^{N}}{2}}{\frac{Q^{N}P_{3}^{N}I^{N}}{Q^{N}}} + \frac{\frac{D_{1}^{\prime N}C_{0}^{N}}{Q^{N}}}{\frac{Q^{N}P_{2}^{\prime N}I^{N}}{2}}, D_{2}^{N}P_{2}^{N} + \frac{\frac{D_{2}^{\prime N}C_{0}^{N}}{2}}{\frac{Q^{N}P_{3}^{\prime N}I^{N}}{2}} + \frac{\frac{Q^{N}P_{2}^{\prime N}I^{N}}{2}}{\frac{Q^{N}P_{3}^{\prime N}I^{N}}{2}} + \frac{\frac{D_{3}^{\prime N}C_{0}^{N}}{Q^{N}}}{\frac{Q^{N}P_{3}^{\prime N}I^{N}}{2}} + \frac{\frac{D_{3}^{\prime N}C_{0}^{N}}{Q^{N}}}{\frac{Q^{N}P_{2}^{\prime N}I^{N}}{2}}, D_{2}^{N}P_{2}^{N} + \frac{\frac{D_{2}^{\prime N}C_{0}^{N}}{2}}{\frac{D_{2}^{\prime N}C_{0}^{N}}{2}} + \frac{\frac{Q^{N}P_{2}^{\prime N}I^{N}}{2}}{\frac{D_{2}^{\prime N}C_{0}^{N}}{2}} + \frac{\frac{Q^{N}P_{2}^{\prime N}I^{N}}{2}}{\frac{Q^{N}P_{2}^{\prime N}I^{N}}{2}} + \frac{\frac{Q^{N}P_{2}^{\prime N$$

The defuzzified total neutrosophic cost using accuracy function is given by

$D(TC)^{N} = \frac{1}{8} \left[\left(D_{1}^{N} P_{1}^{N} + \frac{D_{1}^{N} C_{0}^{N}}{Q^{N}} + \frac{Q^{N} P_{1}^{N} I^{N}}{2} \right) + \right]$
$2(D_2^N P_2^N + \frac{D_2^N C_0^N}{O^N} + \frac{Q^N P_2^N I^N}{2}) + (D_3^N P_3^N + \frac{D_3^N C_0^N}{O^N} +$
$\frac{Q^{N}P_{3}^{N}I^{N}}{2}) + \left(D_{1}^{\prime\prime N}P_{1}^{\prime\prime N} + \frac{D_{1}^{\prime\prime N}C_{0}^{N}}{Q^{N}} + \frac{Q^{N}P_{1}^{\prime\prime N}I^{N}}{2}\right) +$
$2(D_2^N P_2^N + \frac{D_2^N C_0^N}{Q^N} + \frac{Q^N P_2^N T^N}{2}) + (D_3^{\prime\prime N} P_3^{\prime\prime N} + $
$\frac{D_{3}^{\prime\prime N}C_{0}^{N}}{Q^{N}} + \frac{Q^{N}P_{3}^{\prime\prime N}I^{N}}{2})]$

To find the minimum of $D(TC)^N$ by taking the derivative $D(TC)^N$ and equating it to zero,

(i.e)
$$\frac{1}{8Q^{2N}} [(D_1^N C_0^N + 2D_2^N C_0^N + D_3^N C_0^N) + (D_1''^N C_0^N + 2D_2^N C_0^N + D_3''^N C_0^N)] + \frac{1}{16} [(P_1^N I^N + 2P_2^N I^N + P_3^N I^N) + (P_1''^N I^N + 2P_2^N I^N + P_3''^N I^N)] = 0, \text{ we get}$$

$$Q^{N} = \sqrt{\frac{2[(D_{1}^{N}C_{0}^{N} + 2D_{2}^{N}C_{0}^{N} + D_{3}^{N}C_{0}^{N}) + (D_{1}^{\prime\prime}N_{0}^{N} + 2D_{2}^{N}C_{0}^{N} + D_{3}^{\prime\prime}N_{0}^{N})]}{[(P_{1}^{N}I^{N} + 2P_{2}^{N}I^{N} + P_{3}^{\prime\prime}I^{N}) + (P_{1}^{\prime\prime}N_{1}I^{N} + 2P_{2}^{N}I^{N} + P_{3}^{\prime\prime\prime}I^{N})]}}$$

Neutrosophic Price Break:

S.No.	Quantity	Price Per Unit (Rs)
1	$0 \le Q_1^N \le b$	P_1^N
2	$b \leq Q_2^N$	$P_2^N (< P_1^N)$

4 ALGORITHM FOR FINDING NEUTRO-SOPHIC OPTIMAL QUANTITY AND NEU-TROSOPHIC OPTIMAL COST:

Step I:

Consider the lowest price P_2^N and determine Q_2^N by using the economic order quantity (EOQ) formula:

$$Q^{N} = \sqrt{\frac{2[(D_{1}^{N}C_{0}^{N}+2D_{2}^{N}C_{0}^{N}+D_{3}^{N}C_{0}^{N})+(D_{1}^{\prime\prime N}C_{0}^{N}+2D_{2}^{N}C_{0}^{N}+D_{3}^{\prime\prime N}C_{0}^{N})}{[(P_{1}^{N}I^{N}+2P_{2}^{N}I^{N}+P_{3}^{\prime\prime N}I^{N})+(P_{1}^{\prime\prime N}I^{N}+2P_{2}^{N}I^{N}+P_{3}^{\prime\prime N}I^{N})]}}$$

If Q_2^N lies in the range specified, $b \ge Q_2^N$ then Q_2^N is the EOQ .The defuzzified optimal total cost $(TC)^N$ associated with Q^N is calculated as follows:

$$(\text{TC})^{N} = D^{N} * P_{2}^{N} + \frac{D^{N}C_{0}^{N}}{b} + \frac{bP_{2}^{N}*I^{N}}{2},$$

by using the accuracy function
$$A^{N} = \frac{(a_{1}+2a_{2}+a_{3})+(a_{1}''+2a_{2}+a_{3}'')}{8}$$

Step 2:

(i) If $Q_2^N < b$, we cannot place an order at the lowest price P_2^N . (ii) We calculate Q_1^N with price P_1^N and the

corresponding total cost TC at Q^N . (iii) If $(TC)^N b > (TC)^N Q_1^N$, then EOQ is $Q^{*N} = Q_1^N$, Otherwise $Q^{*N} = b$ is the required EOO.

The EOQ in crisp, fuzzy and intuitionistic fuzzy sets are discussed detail in [5]. They are (i) Crisp:

 $Q_2^* = \sqrt{\frac{2DC_0}{P_2I}}$

(ii) Fuzzy: $\widetilde{Q}_2^* = \sqrt{\frac{2(D_1C_0 + 2D_2C_0 + D_3C_0)}{P_1I + 2P_2I + P_3I}}$

(iii) Intuitionistic fuzzy: $\overline{\overline{Q}}_{2}^{*} = \sqrt{\frac{2(D_{1}C_{0}+4D_{2}C_{0}+D_{3}C_{0}+D_{1}'C_{0}+D_{3}'C_{0})}{P_{1}I+4P_{2}I+P_{3}I+P_{1}'I+P_{3}'I}}$

Using these formula, the numerical example for neutrosophic set is illustrated as follows.

5 NUMERICAL EXAMPLE:

A manufacturing company issues the supply of a special component which has the following price schedule:

0 to 99 items: Rs.800 per unit

100 items and above: Rs.600 per unit

The inventory holding costs are estimated to be Rs.30/- of the value of the inventory. The procurement ordering costs are estimated to be Rs.1500 per order. If the annual requirement of the special component is 350, then compute the economic order quantity for the procurement of these items.

Solution:

(i) **Crisp Case**:

Given D = 350, $P_1 = Rs.800$, $P_2 = Rs.600$, $C_0 = Rs.1500, I = 0.3$

 $Q_2^* = 76$

 $TC(P_1 = 800) = Rs.296039$

TC(b=100) = Rs.224250, which is lower than the total cost corresponding to Q_2

(ii) Fuzzy Case:

Given $\widetilde{D} = (300, 350, 400), \widetilde{P}_1 = (750, 800,$

$$\widetilde{P}_2$$
 = (550, 600, 650), C_0 = Rs.1500,I = 0.3

 $\widetilde{Q}_{2}^{*} = 88.192$

 $\widetilde{TC}(P_1 = 800) = \text{Rs.297785.95}$

 \widetilde{TC} (b=100) = Rs.225500, which is lower than the total cost corresponding to Q_2 .

(iii) Intuitionistic Fuzzy Case:

Given $\overline{D} = (300, 350, 400) (250, 350, 450)$

 $\overline{\overline{P}}_1 = (750, 800, 850) (700, 800, 900)$

$$\overline{\overline{P}}_2 = (550, 600, 650) (500, 600, 700),$$

 $C_0 = \text{Rs.}1500, \text{I} = 0.3$

 $\overline{\overline{Q}}_{2}^{*} = 88.19$

 $\overline{\overline{TC}}(P_1 = 800) = \text{Rs.299660.85}$

 $\overline{TC}(b = 100) = \text{Rs.}227375$, which is lower than the total cost corresponding to Q_2 .

(iv) Neutrosophic Case:

Given $D^N = (300, 350, 400) (250, 350, 450)$ (150, 350, 550)

 $P_1^N = (750, 800, 850) (700, 800, 900)(600,$ 800, 1000)

 $P_2^N = (550, 600, 650) (500, 600, 700)(400,$ 600, 800)

$$C_0^N = \text{Rs.1500}, I^N = 0.3$$

We calculate $Q_2^{*^N}$ corresponding to the lowest price 600,

$$Q_2^{*^N} = \sqrt{\frac{2[(D_1^N C_0^N + 2D_2^N C_0^N + D_3^N C_0^N) + (D_1^{\prime\prime N} C_0^N + 2D_2^N C_0^N + D_3^{\prime\prime N} C_0^N)]}{[(P_1^N I^N + 2P_2^N I^N + P_3^N I^N) + (P_1^{\prime\prime N} I^N + 2P_2^N I^N + P_3^{\prime\prime N} I^N)]}}$$

= 76.376, which is less than the price break point.

Therefore, we have to determine the optimal total cost for the first price and the total cost at the price- break corresponding to the second price and compare the two.

The defuzzified optimal total cost $(TC)^N$ associated with P_1^N is calculated as follows:

$$(TC)^{N}(P_{1}^{N} = 800) = D^{N} * P_{1}^{N} + \frac{D^{N}C_{0}^{N}}{Q_{2}^{N}} + \frac{Q_{2}^{N}P_{1}^{N} * I^{N}}{2}$$

= Rs.306664.13

$$(TC)^{N}(b = 100) = D^{N} * P_{2}^{N} + \frac{D^{N}C_{0}^{N}}{b} + \frac{bP_{2}^{N} * I^{N}}{2}$$

= Rs.173812.5

which is lower than the total cost corresponding to Q_2^N .

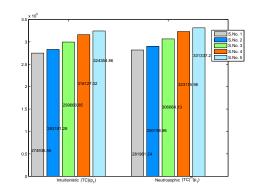


Figure 2. Analysis of first price between intuitionistic fuzzy set and neutrosophic set

6 SENSITIVITY ANALYSIS

In this section, the analysis between intuitionistic set and neutrosophic set is tabulated and the results are compared graphically.

S.No.	Intuitionistic Demand	Neutrosophic Demand
1	(270,320,370) (220,320,420)	(270,320,370) (220,320,420) (120,320,520)
2	(280,330,380) (230,330,430)	(280,330,380) (230,330,430) (130,330,530)
3	(300,350,400) (250,350,450)	(300,350,400) (250,350,450) (150,350,550)
4	(320,370,420) (270,370,470)	(320,370,420) (270,370,470) (170,370,570)
5	(330,380,430) (280,380,480)	(330,380,430) (280,380,480) (180,380,580)

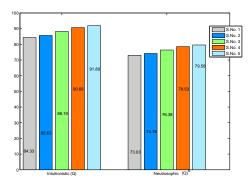


Figure 1. Analysis of economic order quantity (EOQ) between intuitionistic fuzzy set and neutrosophic set

Figure 3. Analysis of price break corresponding to second price between intuitionistic fuzzy set and neutrosophic set

Conclusion

In this paper, EOQ model with price break in neutrosophic environment is introduced. An inventory model is developed for price breaks and its optimum solution is obtained by using triangular neutrosophic number. An algorithm for solving neutrosophic optimal quantity and neutrosophic optimal cost is also developed. This will be an advantage for the buyer who can easily decrease the bad cases and increase the better ones. Hence, the neutrosophic set gives the better solutions to the real world problems than fuzzy and intuitionistic fuzzy sets. In future, the various neutrosophic inventory models will be developed with various limitations such as lead time, backlogging, back order and deteriorating items, etc.

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