International Journal for Uncertainty Quantification, 8(6):511–526 (2018)

NEUTROSOPHIC FILTERS IN PSEUDO-BCI ALGEBRAS

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Original Manuscript Submitted: 8/21/2017; Final Draft Received: 9/9/2018

The concept of the neutrosophic set was introduced by Smarandache; it is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The notion of pseudo-BCI algebra was introduced by Dudek and Jun; it is a kind of nonclassical logic algebra and has a close connection with various noncommutative fuzzy logics. In this paper, neutrosophic set theory is applied to pseudo-BCI algebras. The new concepts of neutrosophic filter, neutrosophic normal filter, antigrouped neutrosophic filter, and neutrosophic p-filter in pseudo-BCI algebras are proposed, and their basic properties are presented. Moreover, by using the concept of (alpha, beta, gamma)-level set in neutrosophic sets, the relationships between fuzzy filters and neutrosophic filters are discussed.

KEY WORDS: *neutrosophic set, fuzzy set, pseudo-BCI algebra, neutrosophic filter, fuzzy filter*

1. INTRODUCTION

To represent uncertain, imprecise, incomplete, and inconsistent information, Smarandache introduced the concept of a neutrosophic set from a philosophical point of view (see [1–3]). The neutrosophic set is a powerful general formal framework that generalizes the concept of the fuzzy set and intuitionistic fuzzy set. In the neutrosophic set, truth-membership, indeterminacy-membership, and falsity-membership are represented independently. If U is a set, a neutrosophic set defined on the universe U assigns to each element $x \in U$, a triple $(T(x), I(x), F(x))$, where $T(x)$, $I(x)$, and $F(x)$ are standard or nonstandard elements of a nonstandard unit interval $]0^{-}, 1^{+}[= 0^{-} \cup [0, 1] \cup 1^{+}$. T is the degree of truth-membership in the set U, I is the degree of indeterminacy-membership in the set U, and F is the degree of nonmembership in the set U. In this paper we work with special neutrosophic sets where their neutrosophic elements are standard real numbers in [0,1]; they are called single valued neutrosophic sets (see [4]). The neutrosophic set theory is applied to many scientific fields (see [2,5–7]). In recent years neutrosophic set theory has been applied to algebraic structures (see [8,9]); it is similar to the applications of fuzzy set (soft set, rough set) theory in algebraic structures (see [10–13]).

Iséki introduced the concept of BCI-algebra as an algebraic counterpart of the BCI-logic (see [14,15]). As a generalization of BCI-algebra, Dudek and Jun [16] introduced the notion of pseudo-BCI algebras. Moreover, pseudo-BCI algebra is also as a generalization of pseudo-BCK algebra (which has a close connection with various noncommutative fuzzy logic formal systems; see [17–24]). For nonclassical logic algebra systems, the theory of filters (ideals) plays an important role (see [25–33]). In 2006, the notion of a pseudo-BCI filter (ideal) of pseudo-BCI algebras was introduced in [34]. In 2009, some special pseudo-BCI filters (ideals) were discussed in [35]. Recently, some articles related to filter theory of pseudo-BCI algebras have been published (see [13,36–39]).

2152–5080/18/$35.00

In this paper, we study the applications of neutrosophic sets to pseudo-BCI algebras. We introduce the new concepts of neutrosophic filter, neutrosophic normal filter, antigrouped neutrosophic filter, and neutrosophic p-filter in pseudo-BCI algebras, and investigate their basic properties and present relationships between neutrosophic filters and fuzzy filters in [33]. It is worth noting that the notion of pseudo-BCI algebra in this paper is a dual of the original definition in [16], so the notion of filter is a dual of ideal. Moreover, the notion of filter of pseudo-BCI algebra is a simple name of the notion of pseudo-BCI filter (or pseudo-filter) in the original and other articles (see [34,35]).

2. PRELIMINARIES

At first, we recall some basic concepts and properties of neutrosophic sets and pseudo-BCI algebras.

Definition 2.1 ([1–3])**.** Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$. That is, $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. Thus, there is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2 ([4])**.** Let X be a space of points (objects) with generic elements in X denoted by x. A simple valued neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. Then, a simple valued neutrosophic set A can be denoted by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\},$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each point x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3 ([4])**.** The complement of a simple valued neutrosophic set A is denoted by A^c and is defined as

$$T_{A^c}(x) = F_A(x),\ I_{A^c}(x) = 1 - I_A(x),\ F_{A^c}(x) = T_A(x),\ \forall x \in X.$$

Then

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}.$$

Definition 2.4 ([4])**.** A simple valued neutrosophic set A is contained in the other simple valued neutrosophic set B, denoted $A \subseteq B$, if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X.

Definition 2.5 ([4])**.** Two simple valued neutrosophic sets A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

For convenience, "simple valued neutrosophic set" is abbreviated to "neutrosophic set" later.

Definition 2.6 ([4])**.** The union of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership, and falsity-membership functions are related to those of A and B by

$$T_C(x) = \max(T_A(x), T_B(x)),\ I_C(x) = \max(I_A(x), I_B(x)),\ F_C(x) = \min(F_A(x), F_B(x)),\ \forall x \in X.$$

Definition 2.7 ([4])**.** The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership, and falsity-membership functions are related to those of A and B by

$$T_C(x) = \min(T_A(x), T_B(x)),\ I_C(x) = \min(I_A(x), I_B(x)),\ F_C(x) = \max(F_A(x), F_B(x)),\ \forall x \in X.$$

Definition 2.8 ([5])**.** Let A be a neutrosophic set in X and $\alpha, \beta, \gamma \in [0, 1]$ with $0 \leq \alpha + \beta + \gamma \leq 3$ and (α, β, γ)-level set of A denoted by $A^{(\alpha,\beta,\gamma)}$ is defined as

$$A^{(\alpha,\beta,\gamma)} = \{x \in X | T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}.$$

Definition 2.9 ([16])**.** A pseudo-BCI algebra is a structure (X; $\leq$, $\rightarrow$, $\rightsquigarrow$, 1), where "$\leq$" is a binary relation on X, "$\rightarrow$" and "$\rightsquigarrow$" are binary operations on X, and "1" is an element of X, verifying the axioms: For all x, y, $z \in X$,

(1) $y \rightarrow z \leq (z \rightarrow x) \rightsquigarrow (y \rightarrow x)$, $y \rightsquigarrow z \leq (z \rightsquigarrow x) \rightarrow (y \rightsquigarrow x)$;
(2) $x \leq (x \rightarrow y) \rightsquigarrow y$, $x \leq (x \rightsquigarrow y) \rightarrow y$;
(3) $x \leq x$;
(4) $x \leq y$, $y \leq x \Rightarrow x = y$;
(5) $x \leq y \Leftrightarrow x \rightarrow y = 1 \Leftrightarrow x \rightsquigarrow y = 1$.

If (X; $\leq$, $\rightarrow$, $\rightsquigarrow$, 1) is a pseudo-BCI algebra satisfying $x \rightarrow y = x \rightsquigarrow y$ for all x, $y \in X$, then (X; $\rightarrow$, 1) is a BCI-algebra.

Proposition 2.1 ([16,34,35])**.** *Let (X; $\leq$, $\rightarrow$, $\rightsquigarrow$, 1) be a pseudo-BCI algebra, then X satisfies the following properties ($\forall x$, y, $z \in X$):*

(1) $1 \leq x \Rightarrow x = 1$;
(2) $x \leq y \Rightarrow y \rightarrow z \leq x \rightarrow z$, $y \rightsquigarrow z \leq x \rightsquigarrow z$;
(3) $x \leq y$, $y \leq z \Rightarrow x \leq z$;
(4) $x \rightsquigarrow (y \rightarrow z) = y \rightarrow (x \rightsquigarrow z)$;
(5) $x \leq y \rightarrow z \Leftrightarrow y \leq x \rightsquigarrow z$;
(6) $x \rightarrow y \leq (z \rightarrow x) \rightarrow (z \rightarrow y)$, $x \rightsquigarrow y \leq (z \rightsquigarrow x) \rightsquigarrow (z \rightsquigarrow y)$;
(7) $x \leq y \Rightarrow z \rightarrow x \leq z \rightarrow y$, $z \rightsquigarrow x \leq z \rightsquigarrow y$;
(8) $1 \rightarrow x = x$, $1 \rightsquigarrow x = x$;
(9) $((y \rightarrow x) \rightsquigarrow x) \rightarrow x = y \rightarrow x$, $((y \rightsquigarrow x) \rightarrow x) \rightsquigarrow x = y \rightsquigarrow x$;
(10) $x \rightarrow y \leq (y \rightarrow x) \rightsquigarrow 1$, $x \rightsquigarrow y \leq (y \rightsquigarrow x) \rightarrow 1$;
(11) $(x \rightarrow y) \rightarrow 1 = (x \rightarrow 1) \rightsquigarrow (y \rightsquigarrow 1)$, $(x \rightsquigarrow y) \rightsquigarrow 1 = (x \rightsquigarrow 1) \rightarrow (y \rightarrow 1)$;
(12) $x \rightarrow 1 = x \rightsquigarrow 1$.

Definition 2.10 ([34])**.** A nonempty subset F of pseudo-BCI algebra X is called a pseudo-BCI filter (briefly, filter) of X if it satisfies

(F1) $1 \in F$;
(F2) $x \in F$, $x \rightarrow y \in F \Rightarrow y \in F$;
(F3) $x \in F$, $x \rightsquigarrow y \in F \Rightarrow y \in F$.

Definition 2.11 ([36])**.** A pseudo-BCI algebra X is said to be antigrouped pseudo-BCI algebra if it satisfies the following identity:

(G1) $\forall x, y, z \in X$, $(x \rightarrow y) \rightarrow (x \rightarrow z) = y \rightarrow z$,
(G2) $\forall x, y, z \in X$, $(x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z) = y \rightsquigarrow z$.

Proposition 2.2 ([36])**.** *A pseudo-BCI algebra X is an antigrouped pseudo-BCI algebra if and only if it satisfies*

$$\forall x \in X,\ (x \rightarrow 1) \rightarrow 1 = x \text{ or } (x \rightsquigarrow 1) \rightsquigarrow 1 = x.$$

Definition 2.12 ([36])**.** A filter F of a pseudo-BCI algebra X is called an antigrouped filter of X if it satisfies

(GF) $\forall x \in X$, $(x \rightarrow 1) \rightarrow 1 \in F$ or $(x \rightsquigarrow 1) \rightsquigarrow 1 \in F \Rightarrow x \in F$.

Definition 2.13 ([35,36]). A subset F of a pseudo-BCI algebra X is called a p-filter of X if it satisfies

(P1) $1 \in F$,

(P2) $(x \to y) \rightsquigarrow (x \to z) \in F$ and $y \in F$ imply $z \in F$,

(P3) $(x \rightsquigarrow y) \to (x \rightsquigarrow z) \in F$ and $y \in F$ imply $z \in F$.

Definition 2.14 ([37,38]). A fuzzy set A in pseudo-BCI algebra X is called a fuzzy filter of X if it satisfies

(FF1) $\forall x \in X$, $\mu_A(x) \leq \mu_A(1)$;

(FF2) $\forall x, y \in X$, min$\{\mu_A(x), \mu_A(x \to y)\} \leq \mu_A(y)$;

(FF3) $\forall x, y \in X$, min$\{\mu_A(x), \mu_A(x \rightsquigarrow y)\} \leq \mu_A(y)$.

Definition 2.15 ([38]). A fuzzy set A in pseudo-BCI algebra X is called fuzzy antigrouped filter if it satisfies

(1) $\forall x \in X$, $\mu_A(x) \leq \mu_A(1)$;

(2) $\forall x, y, z \in X$, min$\{\mu_A(y), \mu_A((x \to y) \to (x \to z))\}z))\} \leq \mu_A(z)$;

(3) $\forall x, y, z \in X$, min$\{\mu_A(y), \mu_A((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z))\}z))\} \leq \mu_A(z)$.

Proposition 2.3 ([38]). *Let A be a fuzzy filter of pseudo-BCI algebra X. Then A is a fuzzy antigrouped filter of X if and only if it satisfies:*

$$\forall x \in X,\ \mu_A(x) \geq \mu_A((x \to 1) \to 1),\ \mu_A(x) \geq \mu_A((x \rightsquigarrow 1) \rightsquigarrow 1).$$

Definition 2.16 ([35,38]). A fuzzy set A: $X \to$ [0, 1] is called a fuzzy p-filter of pseudo-BCI algebra X if it satisfies (FF1) and

(FpF1) $\forall x, y, z \in X$, $\mu_A(z) \geq$ min$\{\mu_A((x \to y) \rightsquigarrow (x \to z)), \mu_A(y)\}$;

(FpF2) $\forall x, y, z \in X$, $\mu_A(z) \geq$ min$\{\mu_A((x \rightsquigarrow y) \to (x \rightsquigarrow z)), \mu_A(y)\}$.

Definition 2.17 ([38]). A fuzzy set A: $X \to [0, 1]$ is called a fuzzy normal filter of pseudo-BCI algebra X if it is a fuzzy filter such that

(FNF) $\mu_A(x \to y) = \mu_A(x \rightsquigarrow y), \forall x, y \in X$.

Proposition 2.4 ([38]). *Let A be a fuzzy filter of a pseudo-BCI algebra X. Then A is a fuzzy normal filter of X if and only if it satisfies*

(1) $\forall x, y \in X$, $\mu_A((x \to y) \to y) \geq \mu_A(x)$;

(2) $\forall x, y \in X$, $\mu_A((x \rightsquigarrow y) \rightsquigarrow y) \geq \mu_A(x)$.

Proposition 2.5 ([38]). *Let A be a fuzzy filter of a pseudo-BCI algebra X. Then the following conditions are equivalent:*

(1) A is a fuzzy p-filter of X;

(2) A is both a fuzzy antigrouped filter and a fuzzy normal filter of X.

3. NEUTROSOPHIC FILTERS IN PSEUDO-BCI ALGEBRAS

Definition 3.1. A neutrosophic set A in pseudo-BCI algebra X is called a neutrosophic filter in X if it satisfies

(NSF1) $\forall x \in X$, $T_A(x) \leq T_A(1)$, $I_A(x) \leq I_A(1)$ and $F_A(x) \geq F_A(1)$;

(NSF2) $\forall x, y \in X$, $\min\{T_A(x), T_A(x \to y)\} \leq T_A(y)$, $\min\{I_A(x), I_A(x \to y)\} \leq I_A(y)$, and $\max\{F_A(x), F_A(x \to y)\} \geq F_A(y)$;

(NSF3) $\forall x, y \in X$, $\min\{T_A(x), T_A(x \rightsquigarrow y)\} \leq T_A(y)$, $\min\{I_A(x), I_A(x \rightsquigarrow y)\} \leq I_A(y)$, and $\max\{F_A(x), F_A(x \rightsquigarrow y)\} \geq F_A(y)$.

Proposition 3.1. *Let A be a neutrosophic filter in pseudo-BCI algebra X, then*

(NSF4) $\forall x, y \in X$, $x \leq y \Rightarrow T_A(x) \leq T_A(y)$, $I_A(x) \leq I_A(y)$ *and* $F_A(x) \geq F_A(y)$.

Proof. Suppose $x \leq y$, then $x \to y = 1$ [by Definition 2.9 (5)]. It follows that $T_A(x \to y) = T_A(1)$. From this and using Definition 3.1 (NSF1) and (NSF2) we get

$$T_A(x) = \min\{T_A(x), T_A(1)\} = \min\{T_A(x), T_A(x \to y)\} \leq T_A(y).$$

That is, $x \leq y \Rightarrow T_A(x) \leq T_A(y)$.

Similarly, we can get that $x \leq y \Rightarrow I_A(x) \leq I_A(y)$ and $F_A(x) \geq F_A(y)$. □

Proposition 3.2. *If A and B are two neutrosophic filters in pseudo-BCI algebra X, then $A \cap B$ is also a neutrosophic filter in X.*

Proof. Suppose that A and B are two neutrosophic filters in pseudo-BCI algebra X. By Definition 3.1 (NSF1), we have

$$\forall x \in X,\ T_A(x) \leq T_A(1),\ I_A(x) \leq I_A(1),\ \text{and } F_A(x) \geq F_A(1);$$

$$T_B(x) \leq T_B(1),\ I_B(x) \leq I_B(1),\ \text{and } F_B(x) \geq F_B(1).$$

It follows that

$$\forall x \in X,\ \min(T_A(x), T_B(x)) \leq \min(T_A(1), T_B(1)),\ \min(I_A(x), I_B(x)) \leq \min(I_A(1), I_B(1)),$$

$$\text{and } \max(F_A(x), F_B(x)) \geq \max(F_A(1), F_B(1)).$$

From this, using Definition 2.7, we get that

$$\forall x \in X,\ T_{A\cap B}(x) \leq T_{A\cap B}(1),\ I_{A\cap B}(x) \leq I_{A\cap B}(1),\ \text{and } F_{A\cap B}(x) \geq F_{A\cap B}(1).$$

That is, $A \cap B$ satisfies (NSF1).

Moreover, by Definition 3.1 (NSF2), we have

$$\forall x, y \in X,\ \min\{T_A(x), T_A(x \to y)\} \leq T_A(y),\ \min\ \{I_A(x), I_A(x \to y)\} \leq I_A(y),$$

$$\text{and } \max\{F_A(x), F_A(x \to y)\} \geq F_A(y);$$

$$\forall x, y \in X,\ \min\{T_B(x), T_B(x \to y)\} \leq T_B(y),\ \min\{I_B(x), I_B(x \to y)\} \leq I_B(y),$$

$$\text{and } \max\{F_B(x), F_B(x \to y)\} \geq F_B(y).$$

Then, $\forall x, y \in X$,

$$\min\{T_{A\cap B}(x), T_{A\cap B}(x \to y)\} = \min\{\min(T_A(x), T_B(x)),$$

$$\min(T_A(x \to y), T_B(x \to y))\} \leq \min\{T_A(x), T_A(x \to y)\} \leq T_A(y),$$

$$\min\{T_{A\cap B}(x), T_{A\cap B}(x \to y)\} = \min\{\min(T_A(x), T_B(x)),$$

$$\min(T_A(x \to y), T_B(x \to y))\} \le \min\{T_B(x), T_B(x \to y)\} \le T_B(y);$$

$$\min\{I_{A\cap B}(x), I_{A\cap B}(x \to y)\} = \min\{\min(I_A(x), I_B(x)),$$

$$\min(I_A(x \to y), I_B(x \to y))\} \le \min\{I_A(x), I_A(x \to y)\} \le I_A(y),$$

$$\min\{I_{A\cap B}(x), I_{A\cap B}(x \to y)\} = \min\{\min(I_A(x), I_B(x)),$$

$$\min(I_A(x \to y), I_B(x \to y))\} \le \min\{I_B(x), I_B(x \to y)\} \le I_B(y);$$

$$\max\{F_{A\cap B}(x), F_{A\cap B}(x \to y)\} = \max\{\max(F_A(x), F_B(x)),$$

$$\max(F_A(x \to y), F_B(x \to y))\} \ge \max\{F_A(x), F_A(x \to y)\} \ge F_A(y),$$

$$\max\{F_{A\cap B}(x), F_{A\cap B}(x \to y)\} = \max\{\max(F_A(x), F_B(x)),$$

$$\max(F_A(x \to y), F_B(x \to y))\} \ge \max\{F_B(x), F_B(x \to y)\} \ge F_B(y).$$

It follows that

$$\min\{T_{A\cap B}(x), T_{A\cap B}(x \to y)\} \le \min\{T_A(y), T_B(y)\} = T_{A\cap B}(y);$$

$$\min\{I_{A\cap B}(x), I_{A\cap B}(xv \to y)\} \le \min\{I_A(y), I_B(y)\} = I_{A\cap B}(y);$$

$$\max\{F_{A\cap B}(x), F_{A\cap B}(x \to y)\} \ge \max\{F_A(y), F_B(y)\} = F_{A\cap B}(y).$$

That is, $A \cap B$ satisfies (NSF2).

Similarly, we can prove that $A \cap B$ satisfies (NSF3). Therefore, $A \cap B$ is a neutrosophic filter in X. □

Proposition 3.3. *Let A be a neutrosophic filter in pseudo-BCI algebra X; denote that*

(1) $A_T = \{x \in X | T_A(x) = T_A(1)\}$*;*

(2) $A_I = \{x \in X | I_A(x) = I_A(1)\}$*;*

(3) $A_F = \{x \in X | F_A(x) = F_A(1)\}$.

Then A_T, A_I, and A_F are filters of X.

Proof. Obviously, $1 \in A_T$. Assume that $x \in A_T$ and $x \to y \in A_T$, then $T_A(x) = T_A(1)$, $T_A(x \to y) = T_A(1)$. From this, using Definition 3.1, we have

$$T_A(1) = \min\{T_A(x), T_A(x \to y)\} \le T_A(y) \le T_A(1).$$

It follows that $T_A(y) = T_A(1)$, that is, $y \in A_T$. In the same way, we can get $x \in A_T$, $x \rightsquigarrow y \in A_T \Rightarrow y \in A_T$. By Definition 2.10 we know that A_T is a filter of X.

Similarly, we can get that AI is a filter of X.

Moreover, $1 \in A_F$. Assume that $x \in A_F$ and $x \to y \in A_F$, then $F_A(x) = F_A(1)$, $F_A(x \to y) = F_A(1)$. From this, using Definition 3.1, we have

$$F_A(1) = \max\{F_A(x), F_A(x \to y)\} \ge F_A(y) \ge F_A(1).$$

It follows that $F_A(y) = F_A(1)$; that is, $y \in A_F$. In the same way, we can get $x \in A_F$, $x \rightsquigarrow y \in A_F \Rightarrow y \in A_F$. By Definition 2.10 we know that A_F is a filter of X. □

The following example shows that the union of two neutrosophic filters may be not a neutrosophic filter.

Example 1. Let $X = \{a, b, c, d, e, 1\}$ with two binary operations given in Tables 1 and 2. Then $(X; \leq, \rightarrow, \rightsquigarrow, 1)$ is a pseudo-BCI algebra, where $x \leq y$ if and only if $x \rightarrow y = 1$.

Define neutrosophic sets A and B in X as follows:

$$T_A(a) = T_A(b) = T_A(e) = T_A(d) = 0,\ T_A(c) = T_A(1) = 0.7,$$

$$I_A(a) = I_A(b) = I_A(e) = I_A(d) = 0,\ I_A(c) = I_A(1) = 0.1,$$

$$F_A(a) = F_A(b) = F_A(e) = F_A(d) = 0.8,\ F_A(c) = F_A(1) = 0.15;$$

$$T_B(a) = T_B(b) = T_B(c) = T_B(d) = 0.1,\ T_B(e) = 0.6,\ T_B(1) = 0.8,$$

$$I_B(a) = I_B(b) = I_B(c) = I_B(d) = 0.05,\ I_B(e) = 0.1,\ I_B(1) = 0.15,$$

$$F_B(a) = F_B(b) = F_B(c) = F_B(d) = 0.78,\ F_B(e) = 0.2,\ F_B(1) = 0.05.$$

Then A, B are neutrosophic filters in X. Let $C = A \cap B$ and $D = A \cup B$, then

$$T_{A\cap B}(a) = T_{A\cap B}(b) = T_{A\cap B}(d) = T_{A\cap B}(e) = 0,\ T_{A\cap B}(c) = 0.1,\ T_{A\cap B}(1) = 0.7,$$

$$I_{A\cap B}(a) = I_{A\cap B}(b) = I_{A\cap B}(d) = I_{A\cap B}(e) = 0,\ I_{A\cap B}(c) = 0.05,\ I_{A\cap B}(1) = 0.1,$$

$$F_{A\cap B}(a) = F_{A\cap B}(b) = F_{A\cap B}(d) = F_{A\cap B}(e) = 0.8,\ F_{A\cap B}(c) = 0.78,\ F_{A\cap B}(1) = 0.15;$$

$$T_{A\cup B}(a) = T_{A\cup B}(b) = T_{A\cup B}(d) = 0.1,\ T_{A\cup B}(c) = 0.7,\ T_{A\cup B}(e) = 0.6,\ T_{A\cup B}(1) = 0.8,$$

$$I_{A\cup B}(a) = I_{A\cup B}(b) = I_{A\cup B}(d) = 0.05,\ I_{A\cup B}(c) = I_{A\cup B}(e) = 0.1,\ I_{A\cup B}(1) = 0.15,$$

$$F_{A\cup B}(a) = F_{A\cup B}(b) = F_{A\cup B}(d) = 0.78,\ F_{A\cup B}(c) = 0.15,\ F_{A\cup B}(e) = 0.2,\ F_{A\cup B}(1) = 0.05.$$

We can verify that $C = A \cap B$ is a neutrosophic filter in X, but $D = A \cup B$ is not a neutrosophic filter in X, since

$$\min\{T_{A\cup B}(c), T_{A\cup B}(c \rightarrow b)\} = \min\{T_{A\cup B}(c), T_{A\cup B}(e)\} = \min\{0.7, 0.6\} = 0.6 \nleq 0.1 = T_{A\cup B}(b),$$

$$\max\{F_{A\cup B}(c), F_{A\cup B}(c \rightarrow b)\} = \max\{F_{A\cup B}(c), F_{A\cup B}(e)\} = \max\{0.15, 0.2\} = 0.2 \ngeq 0.78 = F_{A\cup B}(b).$$

TABLE 1: First set of binary operations

$\rightarrow$	a	b	c	d	e	1
a	1	a	d	e	c	b
b	b	1	e	c	d	a
c	d	e	1	a	b	c
d	e	c	b	1	a	d
e	c	d	a	b	1	e
1	a	b	c	d	e	1

TABLE 2: Second set of binary operations

$\rightsquigarrow$	a	b	c	d	e	1
a	1	a	e	c	d	b
b	b	1	d	e	c	a
c	e	d	1	b	a	c
d	c	e	a	1	b	d
e	d	c	b	a	1	e
1	a	b	c	d	e	1

4. ANTIGROUPED NEUTROSOPHIC FILTERS AND NEUTROSOPHIC P-FILTERS

Definition 4.1. A neutrosophic set A in pseudo-BCI algebra X is called an antigrouped neutrosophic filter if it satisfies $\forall x, y, z \in X$,

(1) $T_A(x) \leq T_A(1)$, $I_A(x) \leq I_A(1)$, and $F_A(x) \geq F_A(1)$;

(2) $\min\{T_A(y), T_A((x \to y) \to (x \to z))\} \leq T_A(z)$, $\min\{I_A(y), I_A((x \to y) \to (x \to z))\} \leq I_A(z)$, and $\max\{F_A(x), F_A((x \to y) \to (x \to z))\} \geq F_A(z)$;

(3) $\min\{T_A(y), T_A((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z))\} \leq T_A(z)$, $\min\{I_A(y), I_A((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z))\} \leq I_A(z)$, and $\max\{F_A(x), F_A((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z))\} \geq F_A(z)$.

When $x = y$ in Definition 4.1 (2) and (3), we can get (NSF2) and (NSF3) in Definition 3.1; this means that the following proposition is true.

Proposition 4.1. *Let A be an antigrouped neutrosophic filter in pseudo-BCI algebra X. Then A is a neutrosophic filter in X.*

Proposition 4.2. *Let A be an antigrouped neutrosophic filter in pseudo-BCI algebra X. Then A satisfies the following conditions:*

(i) $\forall x \in X, T_A(x) \geq T_A((x \to 1) \to 1), T_A(x) \geq T_A((x \rightsquigarrow 1) \rightsquigarrow 1)$;

(ii) $\forall x \in X, I_A(x) \geq I_A((x \to 1) \to 1), I_A(x) \geq I_A((x \rightsquigarrow 1) \rightsquigarrow 1)$;

(iii) $\forall x \in X, F_A(x) \leq F_A((x \to 1) \to 1), F_A(x) \leq F_A((x \rightsquigarrow 1) \rightsquigarrow 1)$.

Proof. Putting $z = x$ and y = 1 in Definition 4.1 (2) and (3), we can get the results. □

Lemma 4.1 ([38]). *Let X be a pseudo-BCI algebra X. Then X satisfies the following properties:*

(1) $\forall x, y, z \in X, ((x \to y) \to (x \to z)) \to 1 = (y \to z) \to 1$;

(2) $\forall x, y, z \in X, ((x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow z)) \rightsquigarrow 1 = (y \rightsquigarrow z) \rightsquigarrow 1$;

(3) $\forall x, y \in X, (x \to y) \to (x \to 1) = y \to 1$;

(4) $\forall x, y \rightsquigarrow X, (x \rightsquigarrow y) \rightsquigarrow (x \rightsquigarrow 1) = y \rightsquigarrow 1$.

Theorem 4.1. *Let A be a neutrosophic filter in pseudo-BCI algebra X. Then A is an antigrouped neutrosophic filter in X if and only if it satisfies*

(i) $\forall x \in X, T_A(x) \geq T_A((x \to 1) \to 1), T_A(x) \geq T_A((x \rightsquigarrow 1) \rightsquigarrow 1)$;

(ii) $\forall x \in X, I_A(x) \geq I_A((x \to 1) \to 1), I_A(x) \geq I_A((x \rightsquigarrow 1) \rightsquigarrow 1)$;

(iii) $\forall x \in X, F_A(x) \leq F_A((x \to 1) \to 1), F_A(x) \leq F_A((x \rightsquigarrow 1) \rightsquigarrow 1)$.

Proof. If A is an antigrouped neutrosophic filter in X, then by Proposition 4.2 we know that the conditions (i), (ii), and (iii) hold.

Conversely, suppose that A satisfies the conditions (i), (ii), and (iii). For any $x, y, z \in X$, by Definition 2.9 (2) and Lemma 4.1 (1) we have

$$(x \to y) \to (x \to z) \leq (((x \to y) \to (x \to z)) \to 1) \to 1 = ((y \to z) \to 1) \to 1.$$

From this, using Proposition 3.1 (NSF4) and conditions (i), (ii), and (iii) we have

$$T_A((x \to y) \to (x \to z)) \leq T_A(((y \to z) \to 1) \to 1) \leq T_A(y \to z);$$

$$I_A((x \to y) \to (x \to z)) \leq I_A(((y \to z) \to 1) \to 1) \leq I_A(y \to z);$$

$$F_A((x \to y) \to (x \to z)) \geq F_A(((y \to z) \to 1) \to 1) \geq F_A(y \to z).$$

From this, using Definition 3.1 (NSF2) we get

$$\min\{T_A(y), T_A((x \to y) \to (x \to z))\} \leq \min\{T_A(y), T_A(y \to z)\} \leq T_A(z);$$

$$\min\{I_A(y), I_A((x \to y) \to (x \to z))\} \leq \min\{I_A(y), I_A(y \to z)\} \leq I_A(z);$$

$$\max\{F_A(y), F_A((x \to y) \to (x \to z))\} \geq \max\{F_A(y), F_A(y \to z)\} \geq F_A(z).$$

This means that Definition 4.1 (2) holds. By the same way, we can prove that Definition 4.1 (3) holds. Therefore, A is an antigrouped neutrosophic filter in X. □

Definition 4.2. A neutrosophic filter A in pseudo-BCI algebra X is called a neutrosophic normal filter in X if it satisfies

(NSNF) $T_A(x \to y) = T_A(x \rightsquigarrow y)$, $I_A(x \to y) = I_A(x \rightsquigarrow y)$, $F_A(x \to y) = F_A(x \rightsquigarrow y)$, $\forall x, y \in X$.

Theorem 4.2. *Let A be a neutrosophic filter in pseudo-BCI algebra X. Then A is a neutrosophic normal filter in X if and only if it satisfies*

(1) $\forall x, y \in X, T_A((x \to y) \to y) \geq T_A(x)$, $I_A((x \to y) \to y) \geq I_A(x)$, $F_A((x \to y) \to y) \leq F_A(x)$;

(2) $\forall x, y \in X, T_A((x \rightsquigarrow y) \rightsquigarrow y) \geq T_A(x)$, $I_A((x \rightsquigarrow y) \rightsquigarrow y) \geq I_A(x)$, $F_A((x \rightsquigarrow y) \rightsquigarrow y) \leq F_A(x)$.

Proof. Suppose that A is a neutrosophic normal filter in X. For any $x, y \in X$, by Definition 2.9 (2), $x \leq (x \to y) \rightsquigarrow y$. Applying Proposition 3.1, $T_A(x) \leq T_A((x \to y) \rightsquigarrow y)$, $I_A(x) \leq I_A((x \to y) \rightsquigarrow y)$, $F_A(x) \geq F_A((x \to y) \rightsquigarrow y)$. On the other hand, by Definition 4.2 (NSNF),

$$T_A((x \to y) \rightsquigarrow y) = T_A((x \to y) \to y),\ I_A((x \to y) \rightsquigarrow y) = I_A((x \to y) \to y),$$

$$F_A((x \to y) \rightsquigarrow y) = F_A((x \to y) \to y).$$

Thus, $T_A(x) \leq T_A((x \to y) \to y)$, $I_A(x) \leq I_A((x \to y) \to y)$, $F_A(x) \geq F_A((x \to y) \to y)$. That is, (1) holds. Similarly, we can get (2).

Conversely, suppose that A satisfies the conditions (1) and (2). For any $x, y \in X$, by Definition 2.9 (1),

$$x \to ((x \rightsquigarrow y) \to y) \leq (((x \rightsquigarrow y) \to y) \to y) \rightsquigarrow (x \to y).$$

Moreover, by Definition 2.9 (2) and (5), $x \leq (x \rightsquigarrow y) \to y$ and $x \to ((x \rightsquigarrow y) \to y)$ = 1. Thus, $1 \leq (((x \rightsquigarrow y) \to y) \to y) \rightsquigarrow (x \to y)$. From this, by Proposition 2.1 (1), $((x \rightsquigarrow y) \to y) \to y \leq x \to y$. Applying Proposition 3.1 we get

$$T_A(((x \rightsquigarrow y) \to y) \to y) \leq T_A(x \to y),\ I_A(((x \rightsquigarrow y) \to y) \to y) \leq I_A(x \to y),$$

$$F_A(((x \rightsquigarrow y) \to y) \to y) \geq F_A(x \to y).$$

On the other hand, by (1), $T_A(((x \rightsquigarrow y) \to y) \to y) \geq T_A(x \rightsquigarrow y)$, $I_A(((x \rightsquigarrow y) \to y) \to y) \geq I_A(x \rightsquigarrow y)$, $F_A(((x \rightsquigarrow y) \to y) \to y) \leq F_A(x \rightsquigarrow y)$. Therefore,

$$T_A(x \rightsquigarrow y) \leq T_A(x \to y),\ I_A(x \rightsquigarrow y) \leq I_A(x \to y),\ F_A(x \rightsquigarrow y) \geq F_A(x \to y).$$

Similarly, by (2) we can get

$$T_A(x \to y) \leq T_A(x \rightsquigarrow y),\ I_A(x \to y) \leq I_A(x \rightsquigarrow y),\ F_A(x \to y) \geq F_A(x \rightsquigarrow y).$$

It follows that $T_A(x \to y) = T_A(x \rightsquigarrow y)$, $I_A(x \to y) = I_A(x \rightsquigarrow y)$, $F_A(x \to y) = F_A(x \rightsquigarrow y)$, $\forall x, y \rightsquigarrow X$. By Definition 4.2, A is a neutrosophic normal filter in X. □

Definition 4.3. A neutrosophic set A in pseudo-BCI algebra X is called a neutrosophic p-filter in X if it satisfies (NSF1) and $\forall x, y, z \in X$,

(NSpF1) $T_A(z) \geq \min\{T_A((x \to y) \rightsquigarrow (x \to z)), T_A(y)\}$, $I_A(z) \geq \min\{I_A((x \to y) \rightsquigarrow (x \to z)), I_A(y)\}$, $F_A(z) \leq \max\{F_A((x \to y) \rightsquigarrow (x \to z)), F_A(y)\}$;

(NSpF2) $T_A(z) \geq \min\{T_A((x \rightsquigarrow y) \to (x \rightsquigarrow z)), T_A(y)\}$, $I_A(z) \geq \min\{I_A((x \rightsquigarrow y) \to (x \rightsquigarrow z)), I_A(y)\}$, $F_A(z) \leq \max\{F_A((x \rightsquigarrow y) \to (x \rightsquigarrow z)), F_A(y)\}$.

When $x = y$ in Definition 4.3 (NSpF1) and (NSpF2), we can get (NSF2) and (NSF3) in Definition 3.1; this means that the following proposition is true.

Proposition 4.3. *Let A be a neutrosophic p-filter in pseudo-BCI algebra X. Then A is a neutrosophic filter in X.*

Theorem 4.3. *Let A be a neutrosophic filter in pseudo-BCI algebra X. Then the following conditions are equivalent:*

(i) A is a neutrosophic p-filter in X;

(ii) A is both a neutrosophic antigrouped filter and a neutrosophic normal filter in X.

Proof. (i) $\Rightarrow$ (ii)

Suppose that A is a neutrosophic p-filter in X. When $x = z$ and y = 1 in Definition 4.3 (NSpF1) and (NSpF2), we can get

$$T_A(x) \geq \min\{T_A((x \to 1) \rightsquigarrow 1), T_A(1)\},\ I_A(x) \geq \min\{I_A((x \to 1) \rightsquigarrow 1), I_A(1)\},$$
$$F_A(x) \leq \max\{F_A((x \to 1) \rightsquigarrow 1), F_A(1)\};$$
$$T_A(x) \geq \min\{T_A((x \rightsquigarrow 1) \to 1)), T_A(1)\},\ I_A(x) \geq \min\{I_A((x \rightsquigarrow 1) \to 1), I_A(1)\},$$
$$F_A(x) \leq \max\{F_A((x \rightsquigarrow 1) \to 1), F_A(1)\}.$$

From this, applying (NSF1) and Proposition 2.1 (12) we have

$$T_A(x) \geq T_A((x \to 1) \to 1),\ I_A(x) \geq I_A((x \to 1) \to 1),\ F_A(x) \leq F_A((x \to 1) \to 1);$$
$$T_A(x) \geq T_A((x \rightsquigarrow 1) \rightsquigarrow 1),\ I_A(x) \geq I_A((x \rightsquigarrow 1) \rightsquigarrow 1),\ F_A(x) \leq F_A((x \rightsquigarrow 1) \rightsquigarrow 1).$$

Using Theorem 4.1 and Proposition 4.3, we know that A is a neutrosophic antigrouped filter in X.

Moreover, for any $x, y \in X$, by Definition 2.9 (1), Proposition 2.1 (4), and (12), we have

$$x \to y \leq (y \to 1) \rightsquigarrow (x \to 1) = (y \to 1) \rightsquigarrow (x \rightsquigarrow 1) = (y \to 1) \rightsquigarrow (x \rightsquigarrow (y \to y)) = (y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y)).$$

From this, using Proposition 3.1 we have

$$T_A(x \to y) \leq T_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))),\ I_A(x \to y) \leq I_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))),$$
$$F_A(x \to y) \geq F_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))).$$

On the other hand, by (NSF1) and Definition 4.3 (NSpF1) we get

$$T_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))) = \min\{T_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))),\ T_A(1)\} \leq T_A(x \rightsquigarrow y),$$
$$I_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))) = \min\{I_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))),\ I_A(1)\} \leq I_A(x \rightsquigarrow y),$$
$$F_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))) = \max\{F_A((y \to 1) \rightsquigarrow (y \to (x \rightsquigarrow y))),\ F_A(1)\} \geq F_A(x \rightsquigarrow y).$$

Combining above results, we have

$$T_A(x \to y) \leq T_A(x \rightsquigarrow y),\ I_A(x \to y) \leq I_A(x \rightsquigarrow y),\ F_A(x \to y) \geq F_A(x \rightsquigarrow y).$$

Similarly, we can get

$$T_A(x \rightsquigarrow y) \leq T_A(x \to y),\ I_A(x \rightsquigarrow y) \leq I_A(x \to y),\ F_A(x \rightsquigarrow y) \geq F_A(x \to y).$$

Hence, $T_A(x \to y) = T_A(x \rightsquigarrow y)$, $I_A(x \to y) = I_A(x \rightsquigarrow y)$, $F_A(x \to y) = F_A(x \rightsquigarrow y)$. By Definition 4.2 we know that A is a neutrosophic normal filter in X.

(ii) ⇒ (i)

Conversely, suppose that A is both a neutrosophic antigrouped filter and a neutrosophic normal filter in X. For any x, y, $z \in X$, by Definition 4.1 (2),

$$\min\{T_A(y), T_A((x \to y) \to (x \to z))\} \leq T_A(z),\ \min\{I_A(y), I_A((x \to y) \to (x \to z))\} \leq I_A(z),$$
$$\max\{F_A(x), F_A((x \to y) \to (x \to z))\} \geq F_A(z).$$

On the other hand, using Definition 4.2 (NSNF),

$$T_A((x \to y) \to (x \to z)) = T_A((x \to y) \rightsquigarrow (x \to z)),\ I_A((x \to y) \to (x \to z)) = I_A((x \to y) \rightsquigarrow (x \to z)),$$
$$F_A((x \to y) \to (x \to z)) = F_A((x \to y) \rightsquigarrow (x \to z)).$$

Therefore,

$$\min\{T_A(y), T_A((x \to y) \rightsquigarrow (x \to z))\} \leq T_A(z),\ \min\{I_A(y), I_A((x \to y) \rightsquigarrow (x \to z))\} \leq I_A(z),$$
$$\max\{F_A(x), F_A((x \to y) \rightsquigarrow (x \to z))\} \geq F_A(z).$$

This means that Definition 4.3 (NSpF1) holds. Similarly, we can get (NSpF2). Hence, A is a neutrosophic p-filter in X. □

Example 2. Let $X = \{a, b, c, d, e, 1\}$ with two binary operations given in Tables 1 and 2 (see Example 3.1). Then $(X; \leq, \to, \rightsquigarrow, 1)$ is a pseudo-BCI algebra, where $x \leq y$ if and only if $x \to y = 1$. Define neutrosophic sets A and B in X as follows:

$$T_A(a) = T_A(b) = T_A(c) = T_A(d) = 0,\ T_A(e) = T_A(1) = 0.9,$$
$$I_A(a) = I_A(b) = I_A(c) = I_A(d) = 0,\ I_A(e) = I_A(1) = 0.05,$$
$$F_A(a) = F_A(b) = F_A(c) = F_A(d) = 0.9,\ F_A(e) = F_A(1) = 0;$$
$$T_B(a) = T_B(b) = T_B(c) = T_B(d) = T_B(e) = 0.75,\ T_B(1) = 0.95,$$
$$I_B(a) = I_B(b) = I_B(c) = I_B(d) = I_B(e) = 0.15,\ I_B(1) = 0.05,$$
$$F_B(a) = F_B(b) = F_B(c) = F_B(d) = F_B(e) = 0.1,\ F_B(1) = 0.$$

Then A, B are neutrosophic filters in X. We can verify that A is a neutrosophic antigrouped filter in X. But A is not a neutrosophic p-filter in X, since

$$T_A(a) = 0 \ngeq 0.9 = \min\{T_A((b \to e) \in (b \to a)), T_A(e)\},$$
$$I_A(a) = 0 \ngeq 0.05 = \min\{I_A((b \to e) \in (b \to a)), I_A(e)\},$$
$$F_A(a) = 0.9 \nleq 0 = \max\{F_A((b \to e) \in (b \to a)), F_A(e)\}.$$

Moreover, we can verify that B is a neutrosophic p-filter in X.

Example 3. Let $X = \{a, b, c, d, 1\}$ with two binary operations given in Tables 3 and 4. Then $(X; \leq, \to, \rightsquigarrow, 1)$ is a pseudo-BCI algebra, where $x \leq y$ if and only if $x \to y = 1$.

Define neutrosophic set A in X as follows:

$$T_A(a) = T_A(b) = T_A(c) = T_A(1) = 0.65,\ T_A(d) = 0.3,$$
$$I_A(a) = I_A(b) = I_A(c) = I_A(1) = 0.2,\ I_A(d) = 0.15,$$
$$F_A(a) = F_A(b) = F_A(c) = F_A(1) = 0.1,\ F_A(d) = 0.55.$$

We can verify that A is both a neutrosophic antigrouped filter and a neutrosophic normal filter in X, so it is a neutrosophic p-filter in X.

TABLE 3: Third set of binary operations

$\to$	a	b	c	d	1
a	1	1	1	d	1
b	b	1	1	d	1
c	b	b	1	d	1
d	d	d	d	1	d
1	a	b	c	d	1

TABLE 4: Fourth set of binary operations

$\rightsquigarrow$	a	b	c	d	1
a	1	1	1	d	1
b	c	1	1	d	1
c	a	b	1	d	1
d	d	d	d	1	d
1	a	b	c	d	1

5. THE RELATIONSHIPS BETWEEN NEUTROSOPHIC FILTERS AND FUZZY FILTERS

Theorem 5.1. *Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is a neutrosophic filter in X if and only if A satisfies*

(i) T_A is a fuzzy filter of X;

(ii) I_A is a fuzzy filter of X;

(iii) $1 - F_A$ is a fuzzy filter of X, where $(1 - F_A)(x) = 1 - F_A(x), \forall x \in X$.

Proof. Suppose that A is a neutrosophic filter in X. Then T_A is a fuzzy set on X; and using Definition 3.1 we have

$$\forall x, y \in X,\ T_A(x) \leq T_A(1),\ \min\{T_A(x), T_A(x \to y)\} \leq T_A(y),\ \min\{T_A(x), T_A(x \rightsquigarrow y)\} \leq T_A(y).$$

By Definition 2.14 we know that T_A is a fuzzy filter of X. Similarly, we can get that I_A is a fuzzy filter of X. Moreover, it is easy to verify that $1 - F_A$ is a fuzzy set on X; and using Definition 3.1 we have $\forall x, y \in X$,

$$(1 - F_A)(x) = 1 - F_A(x) \leq 1 - F_A(1) = (1 - F_A)(1);$$

$$\begin{aligned}
&\min\{(1 - F_A)(x), (1 - F_A)(x \to y)\} = \min\{1 - F_A(x), 1 - F_A(x \to y)\} \\
&= 1 - \max\{F_A(x), F_A(x \to y)\} \leq 1 - F_A(y) = (1 - F_A)(y); \\
&\min\{(1 - F_A)(x), (1 - F_A)(x \rightsquigarrow y)\} = \min\{1 - F_A(x), 1 - F_A(x \rightsquigarrow y)\} \\
&= 1 - \max\{F_A(x), F_A(x \rightsquigarrow y)\} \leq 1 - F_A(y) = (1 - F_A)(y).
\end{aligned}$$

By Definition 2.14 we know that $1 - F_A$ is a fuzzy filter of X.

Conversely, suppose that neutrosophic set A satisfies the conditions (i), (ii), and (iii). Then, by Definition 2.14 we have

$$\forall x, y \in X,\ T_A(x) \leq T_A(1),\ \min\{T_A(x), T_A(x \to y)\} \leq T_A(y),\ \min\{T_A(x), T_A(x \rightsquigarrow y)\} \leq T_A(y);$$

$$\forall x, y \in X,\ I_A(x) \leq T_A(1),\ \min\{I_A(x), I_A(x \to y)\} \leq I_A(y),\ \min\{I_A(x), I_A(x \rightsquigarrow y)\} \leq I_A(y);$$

$$\forall x, y \in X,\ (1 - F_A)(x) \leq (1 - F_A)(1),\ \min\{(1 - F_A)(x), (1 - F_A)(x \to y)\} \leq (1 - F_A)(y),$$
$$\min\{(1 - F_A)(x), (1 - F_A)(x \rightsquigarrow y)\} \leq (1 - F_A)(y).$$

Thus $\forall x, y \in X$,

$$T_A(x) \le T_A(1),\ I_A(x) \le I_A(1),\ \text{and } F_A(x) = 1 - (1-F_A)(x) \ge 1 - (1-F_A)(1) = F_A(1);$$

$$\min\{T_A(x), T_A(x \to y)\} \le T_A(y),\ \min\{I_A(x), I_A(x \to y)\} \le I_A(y)$$

and

$$\max\{F_A(x), F_A(x \to y)\} = 1 - \min\{(1-F_A)(x), (1-F_A)(x \to y)\} \ge 1 - (1-F_A)(y) = F_A(y);$$

$$\min\{T_A(x), T_A(x \rightsquigarrow y)\} \le T_A(y),\ \min\{I_A(x), I_A(x \rightsquigarrow y)\} \le I_A(y)$$

and

$$\max\{F_A(x), F_A(x \rightsquigarrow y)\} = 1 - \min\{(1-F_A)(x), (1-F_A)(x \rightsquigarrow y)\} \ge 1 - (1-F_A)(y) = F_A(y).$$

From this, by Definition 3.1 we get that A is a neutrosophic filter in X. □

Theorem 5.2. *Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is an antigrouped neutrosophic filter in X if and only if A satisfies*

(i) T_A is a fuzzy antigrouped filter of X;

(ii) I_A is a fuzzy antigrouped filter of X;

(iii) $1 - F_A$ is a fuzzy antigrouped filter of X, where $(1-F_A)(x) = 1 - F_A(x), \forall x \in X$.

Proof. By Theorem 5.1, we only prove the following fact:

For any neutrosophic filter A in X, A is antigrouped if and only if T_A, I_A, and $1 - F_A$ are fuzzy antigrouped filters of X.

Assume that A is antigrouped neutrosophic filter in X. By Theorem 4.1 we have ($\forall x \in X$)

$$T_A(x) \ge T_A((x \to 1) \to 1),\ T_A(x) \ge T_A((x \rightsquigarrow 1) \rightsquigarrow 1);\ I_A(x) \ge I_A((x \to 1) \to 1),\ I_A(x) \ge I_A((x \rightsquigarrow 1) \rightsquigarrow 1);$$

$$F_A(x) \le F_A((x \to 1) \to 1),\ F_A(x) \le F_A((x \rightsquigarrow 1) \rightsquigarrow 1).$$

Thus,

$$(1-F_A)(x) = 1 - F_A(x) \ge 1 - F_A((x \to 1) \to 1) = (1-F_A)((x \to 1) \to 1),$$
$$(1-F_A)(x) = 1 - F_A(x) \ge 1 - F_A((x \in 1) \in 1) = (1-F_A)((x \rightsquigarrow 1) \rightsquigarrow 1).$$

Therefore, using Proposition 2.3, we get that T_A, I_A, and $1 - F_A$ are fuzzy antigrouped filters of X.

Conversely, assume that T_A, I_A, and $1 - F_A$ are fuzzy antigrouped filters of X. Then, by Proposition 2.3,

$$T_A(x) \ge T_A((x \to 1) \to 1),\ T_A(x) \ge T_A((x \rightsquigarrow 1) \rightsquigarrow 1);\ I_A(x) \ge I_A((x \to 1) \to 1),\ I_A(x) \ge I_A((x \rightsquigarrow 1) \rightsquigarrow 1);$$

$$(1-F_A)(x) \ge (1-F_A)((x \to 1) \to 1),\ (1-F_A)(x) \ge (1-F_A)((x \rightsquigarrow 1) \rightsquigarrow 1).$$

Therefore,

$$F_A(x) = 1 - (1-F_A)(x) \le 1 - (1-F_A)((x \to 1) \to 1) = F_A((x \to 1) \to 1),$$
$$F_A(x) = 1 - (1-F_A)(x) \le 1 - (1-F_A)((x \rightsquigarrow 1) \rightsquigarrow 1) = F_A((x \rightsquigarrow 1) \rightsquigarrow 1).$$

Hence, applying Theorem 4.1 we get that A is antigrouped neutrosophic filter A in X. □

Similar to Theorem 5.2 we can get the following results (the proofs are omitted).

Theorem 5.3. *Let A be a neutrosophic set in pseudo-BCI algebra X. Then A is a neutrosophic normal filter in X if and only if A satisfies* *(i)* T_A *is a fuzzy normal filter of* X*;*

(ii) I_A *is a fuzzy normal filter of* X*;*

(iii) $1 - F_A$ *is a fuzzy normal filter of* X*, where* $(1 - F_A)(x) = 1 - F_A(x), \forall x \in X$.

Theorem 5.4. *Let* A *be a neutrosophic set in pseudo-BCI algebra* X*. Then* A *is a neutrosophic p-filter in* X *if and only if* A *satisfies*

(i) T_A *is a fuzzy p-filter of* X*;*

(ii) I_A *is a fuzzy p-filter of* X*;*

(iii) $1 - F_A$ *is a fuzzy p-filter of* X*, where* $(1 - F_A)(x) = 1 - F_A(x), \forall x \in X$.

Lemma 5.1 ([10,38])**.** *Let* X *be a pseudo-BCI algebra. Then a fuzzy set* $\mu : X \to [0, 1]$ *is a fuzzy filter of* X *if and only if the level set* $\mu_t = \{x \in X | \mu(x) \geq t\}$ *is a filter of* X *for all* $t \in Im(\mu)$.

Theorem 5.5. *Let* X *be a pseudo-BCI algebra, and* A *be a neutrosophic set in* X *such that* $T_A(x) \geq \alpha_0$, $I_A(x) \geq \beta_0$, *and* $F_A(x) \leq \gamma_0, \forall x \in X$*, where* $\alpha_0 \in Im(T_A)$, $\beta_0 \in Im(I_A)$*, and* $\gamma_0 \in Im(F_A)$*. Then* A *is a neutrosophic filter in* X *if and only if* (α, β, γ)*-level set* $A^{(\alpha,\beta,\gamma)}$ *is a filter of* X *for all* $\alpha \in Im(T_A)$, $\beta \in Im(I_A)$*, and* $\gamma \in Im(F_A)$.

Proof. Assume that A is a neutrosophic filter in X. By Theorem 5.1 and Lemma 5.1, for any $\alpha \in \text{Im}(T_A)$, $\beta \in \text{Im}(I_A)$, and $\gamma \in \text{Im}(F_A)$, we have

$$(T_A)_\alpha = \{x \in X | T_A(x) \geq \alpha\},\ (I_A)_\beta = \{x \in X | I_A(x) \geq \beta\},$$

and

$$(1 - F_A)_{1-\gamma} = \{x \in X | (1 - F_A)(x) \geq 1 - \gamma\} = \{x \in X | F_A(x) \leq \gamma\} \text{ are filters of } X.$$

Thus $(T_A)_\alpha \cap (I_A)_\beta \cap (1 - F_A)_{1-\gamma}$ is a filters of X. Moreover, by Definition 2.8, it is easy to verify that $A^{(\alpha,\beta,\gamma)} = (T_A)_\alpha \cap (I_A)_\beta \cap (1 - F_A)_{1-\gamma}$. Therefore, $A^{(\alpha,\beta,\gamma)}$ is filter of X for all $\alpha \in \text{Im}(T_A)$, $\beta \in \text{Im}(I_A)$, and $\gamma \in \text{Im}(F_A)$.

Conversely, assume that $A^{(\alpha,\beta,\gamma)}$ is a filter of X for all $\alpha \in \text{Im}(T_A)$, $\beta \in \text{Im}(I_A)$, and $\gamma \in \text{Im}(F_A)$. Since $T_A(x) \geq \alpha_0$, $I_A(x) \geq \beta_0$, and $F_A(x) \leq \gamma_0$, $\forall x \in X$, then

$$(T_A)_\alpha = \{x \in X | T_A(x) \geq \alpha\} = (T_A)_\alpha \cap X \cap X = (T_A)_\alpha \cap (I_A)_{\beta_0} \cap (1 - F_A)_{1-\gamma_0} = A^{(\alpha,\beta_0,\gamma_0)};$$

$$(I_A)_\beta = \{x \in X | I_A(x) \geq \beta\} = X \cap (I_A)_\beta \cap X = (T_A)_{\alpha_0} \cap (I_A)_\beta \cap (1 - F_A)_{1-\gamma_0} = A^{(\alpha_0,\beta,\gamma_0)};$$

$$\begin{aligned}(1 - F_A)_{1-\gamma} &= \{x \in X | (1 - F_A)(x) \geq 1 - \gamma\} = X \cap X \cap \{x \in X | F_A(x) \leq \gamma\} \\ &= (T_A)_{\alpha_0} \cap (I_A)_{\beta_0} \cap \{x \in X | F_A(x) \leq \gamma\} = A^{(\alpha_0,\beta_0,\gamma)}.\end{aligned}$$

Thus,

$$(T_A)_\alpha = \{x \in X | T_A(x) \geq \alpha\}, (I_A)_\beta = \{x \in X | I_A(x) \geq \beta\},$$

and

$$(1 - F_A)_{1-\gamma} = \{x \in X | (1 - F_A)(x) \geq 1 - \gamma\} = \{x \in X | F_A(x) \leq \gamma\} \text{ are filters of } X.$$

From this, applying Lemma 5.1, we know that T_A, I_A, and $1 - F_A$ are fuzzy filters of X. By Theorem 5.1 we get that A is neutrosophic filter in X. □

Similar to Theorem 5.5 we can get the following results (the proofs are omitted).

Theorem 5.6. *Let* X *be a pseudo-BCI algebra, and* A *be a neutrosophic set in* X *such that* $T_A(x) \geq \alpha_0$, $I_A(x) \geq \beta_0$, *and* $F_A(x) \leq \gamma_0, \forall x \in X$*, where* $\alpha_0 \in Im(T_A)$, $\beta_0 \in Im(I_A)$*, and* $\gamma_0 \in Im(F_A)$*. Then* A *is a antigrouped neutrosophic filter in* X *if and only if* (α, β, γ)*-level set* $A^{(\alpha,\beta,\gamma)}$ *is an antigrouped filter of* X *for all* $\alpha \in Im(T_A)$, $\beta \in Im(I_A)$*, and* $\gamma \in Im(F_A)$.

Theorem 5.7. *Let X be a pseudo-BCI algebra, and A be a neutrosophic set in X such that $T_A(x) \geq \alpha_0$, $I_A(x) \geq \beta_0$, and $F_A(x) \leq \gamma_0$, $\forall x \in X$, where $\alpha_0 \in Im(T_A)$, $\beta_0 \in Im(I_A)$, and $\gamma_0 \in Im(F_A)$. Then A is a neutrosophic normal filter in X if and only if (α, β, γ)-level set $A^{(\alpha,\beta,\gamma)}$ is a normal filter of X for all $\alpha \in Im(T_A)$, $\beta \in Im(I_A)$, and $\gamma \in Im(F_A)$.*

Theorem 5.8. *Let X be a pseudo-BCI algebra, and A be a neutrosophic set in X such that $T_A(x) \geq \alpha_0$, $I_A(x) \geq \beta_0$, and $F_A(x) \leq \gamma_0$, $\forall x \in X$, where $\alpha_0 \in Im(T_A)$, $\beta_0 \in Im(I_A)$, and $\gamma_0 \in Im(F_A)$. Then A is a neutrosophic p-filter in X if and only if (α, β, γ)-level set $A^{(\alpha,\beta,\gamma)}$ is a p-filter of X for all $\alpha \in Im(T_A)$, $\beta \in Im(I_A)$, and $\gamma \in Im(F_A)$.*

Now, some new research results on neutrosophic sets and related algebraic structures have been published (see [40–42]), and we will further expand the research content of this paper on the basis of these studies.

ACKNOWLEDGMENT

This work was supported by National Natural Science Foundation of China (Grant No. 61573240).

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