

Neutrosophic Fuzzy Ideals of Near-Rings

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Abstract

In this paper, the notion of Neutrosophic fuzzy ideals of near-rings are introduced and discussed some algebraic properties like union ,intersection , homomorphic image and preimage of neutrosophic fuzzy ideals of near-rings. Further we discuss about the direct product of Neutrosophic fuzzy ideals of near-rings .

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1 Introduction

The concept of fuzzy set was introduced by Zadeh [12] in 1965. The notion fuzzy sub near-rings and fuzzy ideals of near-rings was introduced by Abou zaid[1]. This concept is further discussed by Kim [6] and Dutta[5]. V.Chinnadurai and S.Kadalarasi[4] discussed the direct product of $n(n=1,2,\dots,k)$ fuzzy subnearring, fuzzy ideal and fuzzy R-subgroups. The concept of neutrosophy was introduced by Florentin Smarandache[10] as a new branch of philosophy. Neutrosophy is a base of Neutrosophic logic which is an extension of fuzzy logic in which indeterminacy is included. In Neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. The theory of neutrosophic set have achieved great success in various fields like medical diagnosis, image processing, decision making problem ,robotics and so on. I.Arockiarani[3] consider the neutrosophic set with value from the subset of $[0,1]$ and extended the research in fuzzy neutrosophic set. J.Martina Jency and I.Arockia rani [8] initiate the concept of subgroupoids in fuzzy neutrosophic set. In this paper we introduce the notion of neutrosophic fuzzy sub near-ring, neutrosophic fuzzy ideals of near-rings and discussed some algebraic properties. Also we discussed about the direct product of neutrosophic neutrosophic fuzzy ideals of near-rings.

2 Preliminaries

- Definition 1.** 1. $(N, +)$ is a group
 2. (N, \cdot) is a semigroup
 3. $(x + y) \cdot z = x \cdot z + y \cdot z$ for $x, y, z \in N$

Definition 2. An ideal of a near-ring N is a subgroup I of N such that

1. $(I, +)$ is a normal subgroup of $(N, +)$
 2. $IN \subseteq I$
 3. $y(x + i) - -yx \in I$, for $x, y \in N$ and $i \in I$.

I is a left ideal of N if I satisfies (i) and (ii) and right ideal of N if I satisfies (i) and (iii).

Definition 3. A fuzzy set μ in a near-ring N is said to be fuzzy sub near-ring of N if

1. $\mu(x - y) \geq \min(\mu(x), \mu(y))$, for all $x, y \in N$
 2. $\mu(xy) \geq \min(\mu(x), \mu(y))$, for all $x, y \in N$

Definition 4. A fuzzy set μ in a near-ring N is said to be fuzzy ideal of N if

- (i) $\mu(x - y) \geq \min(\mu(x), \mu(y))$, for all $x, y \in N$
 (ii) $\mu(y + x - y) \geq \mu(x)$, for all $x, y \in N$
 (iii) $\mu(xy) \geq \mu(y)$, for all $x, y \in N$
 (iv) $\mu((x + z)y - xy) \geq \mu(z)$, for all $x, y \in N$

A fuzzy set with (i)-(iii) is called fuzzy left ideal of N where as fuzzy set with (i),(ii) and (iv) is called a fuzzy right ideal of N .

Definition 5. A Neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, a indeterminacy function $I_A(x)$ and a falsity membership function $F_A(x)$ is defined as

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

where $T_A, I_A, F_A : X \rightarrow [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Definition 6. Let A and B be neutrosophic fuzzy sets of X . Then ,

1. $A \cup B = \{ \langle x, T_{A \cup B}(x), I_{A \cup B}(x), F_{A \cup B}(x) \rangle \}$, where
 $T_{A \cup B}(x) = \max(T_A(x), T_B(x)), I_{A \cup B}(x) = \min(I_A(x), I_B(x)), F_{A \cup B}(x) = \min(F_A(x), F_B(x))$
 ,for all $x \in X$.
 2. $A \cap B = \{ \langle x, T_{A \cap B}(x), I_{A \cap B}(x), F_{A \cap B}(x) \rangle \}$, where
 $T_{A \cap B}(x) = \min(T_A(x), T_B(x)), I_{A \cap B}(x) = \max(I_A(x), I_B(x)), F_{A \cap B}(x) = \max(F_A(x), F_B(x))$

Definition 7. A Neutrosophic fuzzy set A in a near-ring N is called neutrosophic fuzzy sub near-ring of N if

1. $T_A(x - y) \geq \min(T_A(x), T_A(y)), I_A(x - y) \leq \max(I_A(x), I_A(y)), F_A(x - y) \leq \max(F_A(x), F_A(y))$
 2. $T_A(xy) \geq \min(T_A(x), T_A(y)), I_A(xy) \leq \max(I_A(x), I_A(y)), F_A(xy) \leq \max(F_A(x), F_A(y))$

3 Neutrosophic fuzzy ideals of near-rings

Definition 8. Let N be a near-ring. A Neutrosophic fuzzy set A in a near-ring N is called neutrosophic fuzzy ideal of N if

- (i) $T_A(x-y) \geq \min(T_A(x), T_A(y)), I_A(x-y) \leq \max(I_A(x), I_A(y)), F_A(x-y) \leq \max(F_A(x), F_A(y))$
- (ii) $T_A(y+x-y) \geq T_A(x), I_A(y+x-y) \leq I_A(x), F_A(y+x-y) \leq F_A(x)$
- (iii) $T_A(xy) \geq T_A(y), I_A(xy) \leq I_A(y), F_A(xy) \leq F_A(y)$
- (iv) $T_A((x+z)y-xy) \geq T_A(z), I_A((x+z)y-xy) \leq I_A(z), F_A((x+z)y-xy) \leq F_A(z).$

A Neutrosophic fuzzy subset with the above condition (i)-(iii) is called a neutrosophic fuzzy left ideal of N where as with (i),(ii) and (iv) is called neutrosophic fuzzy right ideal of N .

Theorem 9. Let A and B be neutrosophic fuzzy ideal of N . If $A \subset B$, then $A \cup B$ is a neutrosophic fuzzy ideal of N .

Proof. Let A and B are neutrosophic fuzzy ideals of N . Let $x,y,z \in N$.

(i)

$$\begin{aligned}
 T_{A \cup B}(x-y) &= \max(T_A(x-y), T_B(x-y)) \\
 &\geq \max(\min(T_A(x), T_A(y)), \min(T_B(x), T_B(y))) \\
 &= \min\{\max(T_A(x), T_B(x)), \max(T_A(y), T_B(y))\} \\
 &= \min(T_{A \cup B}(x), T_{A \cup B}(y)) \\
 I_{A \cup B}(x-y) &= \min(I_A(x-y), I_B(x-y)) \\
 &\leq \min(\max(I_A(x), I_A(y)), \max(I_B(x), I_B(y))) \\
 &= \max\{\min(I_A(x), I_B(x)), \max(I_A(y), I_B(y))\} \\
 &= \max(I_{A \cup B}(x), I_{A \cup B}(y)) \\
 F_{A \cup B}(x-y) &= \min(F_A(x-y), F_B(x-y)) \\
 &\leq \min(\max(F_A(x), F_A(y)), \max(F_B(x), F_B(y))) \\
 &= \max\{\min(F_A(x), F_B(x)), \max(F_A(y), F_B(y))\} \\
 &= \max(F_{A \cup B}(x), F_{A \cup B}(y))
 \end{aligned}$$

(ii)

$$\begin{aligned}
 T_{A \cup B}(y+x-y) &= \max(T_A(y+x-y), T_B(y+x-y)) \\
 &\geq \max(T_A(x), T_B(x)) \\
 &= T_{A \cup B}(x) \\
 I_{A \cup B}(y+x-y) &= \min(I_A(y+x-y), I_B(y+x-y)) \\
 &\leq \min(I_A(x), I_B(x)) \\
 &= I_{A \cup B}(x) \\
 F_{A \cup B}(y+x-y) &= \min(F_A(y+x-y), F_B(y+x-y)) \\
 &\leq \min(F_A(x), F_B(x)) \\
 &= F_{A \cup B}(x)
 \end{aligned}$$

(iii)

$$\begin{aligned}
 T_{A \cup B}(xy) &= \max(T_A(xy), T_B(xy)) \geq \max(T_A(y), T_B(y)) = T_{A \cup B}(y) \\
 I_{A \cup B}(xy) &= \min(I_A(xy), I_B(xy)) \leq \min(I_A(y), I_B(y)) = I_{A \cup B}(y) \\
 F_{A \cup B}(xy) &= \min(F_A(xy), F_B(xy)) \leq \min(F_A(y), F_B(y)) = F_{A \cup B}(y)
 \end{aligned}$$

(iv)

$$\begin{aligned}
 T_{A \cup B}((x+z)y - xy) &= \max\{T_A((x+z)y - xy), T_B((x+z)y - xy)\} \\
 &\geq \max(T_A(z), T_B(z)) = T_{A \cup B}(z) \\
 I_{A \cup B}((x+z)y - xy) &= \min\{I_A((x+z)y - xy), I_B((x+z)y - xy)\} \\
 &\leq \min(I_A(z), I_B(z)) = I_{A \cup B}(z) \\
 T_{A \cup B}((x+z)y - xy) &= \min\{F_A((x+z)y - xy), F_B((x+z)y - xy)\} \\
 &\leq \min(F_A(z), F_B(z)) = F_{A \cup B}(z).
 \end{aligned}$$

Therefore, $A \cup B$ is a neutrosophic fuzzy ideal of N . □

Theorem 10. *Let A and B be neutrosophic fuzzy ideal of N . Then $A \cap B$ is a neutrosophic fuzzy ideal of N .*

Proof. Let A and B are neutrosophic fuzzy ideals of N . Let $x, y, z \in N$

(i)

$$\begin{aligned}
 T_{A \cap B}(x - y) &= \min(T_A(x - y), T_B(x - y)) \\
 &\geq \min(\min(T_A(x), T_A(y)), \min(T_B(x), T_B(y))) \\
 &= \min\{\min(T_A(x), T_B(x)), \min(T_A(y), T_B(y))\} \\
 &= \min(T_{A \cap B}(x), T_{A \cap B}(y)) \\
 I_{A \cap B}(x - y) &= \max(I_A(x - y), I_B(x - y)) \\
 \leq \max(\max(I_A(x), I_A(y)), \max(I_B(x), I_B(y))) \\
 &= \max\{\max(I_A(x), I_B(x)), \max(I_A(y), I_B(y))\} \\
 &= \max(I_{A \cap B}(x), I_{A \cap B}(y)) \\
 F_{A \cap B}(x - y) &= \max(F_A(x - y), F_B(x - y)) \\
 &\leq \max(\max(F_A(x), F_A(y)), \max(F_B(x), F_B(y))) \\
 &= \max\{\max(F_A(x), F_B(x)), \max(F_A(y), F_B(y))\} \\
 &= \max(F_{A \cap B}(x), F_{A \cap B}(y))
 \end{aligned}$$

(ii)

$$\begin{aligned}
 T_{A \cap B}(y + x - y) &= \min(T_A(y + x - y), T_B(y + x - y)) \\
 &\geq \min(T_A(x), T_B(x)) \\
 &= T_{A \cap B}(x) \\
 I_{A \cap B}(y + x - y) &= \max(I_A(y + x - y), I_B(y + x - y)) \\
 &\leq \max(I_A(x), I_B(x)) \\
 &= I_{A \cap B}(x) \\
 F_{A \cap B}(y + x - y) &= \max(F_A(y + x - y), F_B(y + x - y)) \\
 &\leq \max(F_A(x), F_B(x)) \\
 &= F_{A \cap B}(x)
 \end{aligned}$$

(iii)

$$\begin{aligned}
 T_{A \cap B}(xy) &= \min(T_A(xy), T_B(xy)) \geq \min(T_A(y), T_B(y)) = F_{A \cap B}(y) \\
 I_{A \cap B}(xy) &= \max(I_A(xy), I_B(xy)) \leq \max(I_A(y), I_B(y)) = I_{A \cap B}(y) \\
 F_{A \cap B}(xy) &= \max(F_A(xy), F_B(xy)) \leq \max(F_A(y), F_B(y)) = F_{A \cap B}(y)
 \end{aligned}$$

(iv)

$$\begin{aligned}
 T_{A \cap B}((x+z)y - xy) &= \min\{T_A((x+z)y - xy), T_B((x+z)y - xy)\} \\
 &\geq \min(T_A(z), T_B(z)) = T_{A \cap B}(z) \\
 I_{A \cap B}((x+z)y - xy) &= \max\{I_A((x+z)y - xy), I_B((x+z)y - xy)\} \\
 &\leq \max(I_A(z), I_B(z)) = I_{A \cap B}(z) \\
 F_{A \cap B}((x+z)y - xy) &= \max\{F_A((x+z)y - xy), F_B((x+z)y - xy)\} \\
 &\leq \max(F_A(z), F_B(z)) = F_{A \cap B}(z)
 \end{aligned}$$

Therefore, $A \cap B$ is a neutrosophic fuzzy ideal of N . □

Corollary 11. *If A_1, A_2, \dots, A_n are neutrosophic fuzzy ideals of N , then $A = \bigcap_{i=1}^n A_i$ is a neutrosophic fuzzy ideal of N .*

Lemma 12. *For all $a, b \in I$ and i is any positive integer, if $a = b$, then*

- (i) $a^i \leq b^i$
- (ii) $[\min(a, b)]^i = \min(a^i, b^i)$
- (iii) $[\max(a, b)]^i = \max(a^i, b^i)$

Theorem 13. *Let A be a Neutrosophic fuzzy ideal of N .*

Then $A^m = \{ \langle x, T_{A^m}(x), I_{A^m}(x), F_{A^m}(x) \rangle : x \in N \}$

is a Neutrosophic fuzzy ideal of N^m , where m is a positive integer and

$$T_{A^m}(x) = (T_A(x))^m, I_{A^m}(x) = (I_A(x))^m, F_{A^m}(x) = (F_A(x))^m.$$

Proof. Let A be a Neutrosophic fuzzy ideal of N . Let $x, y, z \in N$.

(i)

$$\begin{aligned}
 T_{A^m}(x - y) &= (T_A(x - y))^m \\
 &\geq [\min(T_A(x), T_A(y))]^m = \min((T_A(x))^m, (T_A(y))^m) \\
 &= \min(T_{A^m}(x), T_{A^m}(y)) \\
 I_{A^m}(x - y) &= (I_A(x - y))^m \\
 &\leq [\max(I_A(x), I_A(y))]^m = \max((I_A(x))^m, (I_A(y))^m) \\
 &= \max(I_{A^m}(x), I_{A^m}(y)) \\
 F_{A^m}(x - y) &= (F_A(x - y))^m \\
 &\leq [\max(F_A(x), F_A(y))]^m = \max((F_A(x))^m, (F_A(y))^m) \\
 &= \max(F_{A^m}(x), F_{A^m}(y))
 \end{aligned}$$

(ii)

$$\begin{aligned} T_{A^m}(y + x - y) &= (T_A(y + x - y))^m \geq (T_A(x))^m = T_{A^m}(x) \\ I_{A^m}(y + x - y) &= (I_A(y + x - y))^m \leq (I_A(x))^m = I_{A^m}(x) \\ F_{A^m}(y + x - y) &= (F_A(y + x - y))^m \leq (F_A(x))^m = F_{A^m}(x) \end{aligned}$$

(iii)

$$\begin{aligned} T_A^m(xy) &= (T_A(xy))^m \geq (T_A(y))^m = T_A^m(y), \\ I_A^m(xy) &= (I_A(xy))^m \leq (I_A(y))^m = I_A^m(y), \\ F_A^m(xy) &= (F_A(xy))^m \leq (F_A(y))^m = F_A^m(y). \end{aligned}$$

(iv)

$$\begin{aligned} T_{A^m}((x + z)y - xy) &= (T_A((x + z)y - xy))^m \geq (T_A(z))^m = T_{A^m}(z) \\ I_{A^m}((x + z)y - xy) &= (I_A((x + z)y - xy))^m \leq (I_A(z))^m = I_{A^m}(z) \\ F_{A^m}((x + z)y - xy) &= (F_A((x + z)y - xy))^m \leq (F_A(z))^m = F_{A^m}(z) \end{aligned}$$

Therefore, A^m is a Neutrosophic fuzzy ideal of N^m . □

4 Direct product of Neutrosophic fuzzy ideals of near-rings

Definition 14. Let A and B be neutrosophic fuzzy subsets of near-rings N_1 and N_2 respectively. Then, the direct product of neutrosophic fuzzy subsets of near-rings is defined by

$$A \times B : N_1 \times N_2 \rightarrow [0, 1] \text{ such that } A \times B = \{ \langle (x, y), T_{A \times B}(x, y), I_{A \times B}(x, y), F_{A \times B}(x, y) \rangle : x \in N_1, y \in N_2 \}$$

$$\text{where, } T_{A \times B}(x, y) = \min(T_A(x), T_B(y)), I_{A \times B}(x, y) = \max(I_A(x), I_B(y)) \text{ and } F_{A \times B}(x, y) = \max(F_A(x), F_B(y))$$

Definition 15. Let A and B be neutrosophic fuzzy subsets of near-rings N_1 and N_2 respectively. Then $A \times B$ is a neutrosophic fuzzy ideal of $N_1 \times N_2$ if it satisfies the following conditions:

(i)

$$\begin{aligned} T_{A \times B}((x_1, x_2) - (y_1, y_2)) &\geq \min(T_{A \times B}(x_1, x_2), T_{A \times B}(y_1, y_2)), \\ I_{A \times B}((x_1, x_2) - (y_1, y_2)) &\leq \max(I_{A \times B}(x_1, x_2), I_{A \times B}(y_1, y_2)), \\ F_{A \times B}((x_1, x_2) - (y_1, y_2)) &\leq \max(F_{A \times B}(x_1, x_2), F_{A \times B}(y_1, y_2)) \end{aligned}$$

(ii)

$$\begin{aligned} T_{A \times B}((y_1, y_2) + (x_1, x_2) - (y_1, y_2)) &\geq \min(T_{A \times B}(x_1, x_2)), \\ I_{A \times B}((y_1, y_2) + (x_1, x_2) - (y_1, y_2)) &\leq \max(I_{A \times B}(x_1, x_2)), \\ F_{A \times B}((y_1, y_2) + (x_1, x_2) - (y_1, y_2)) &\leq \max(F_{A \times B}(x_1, x_2)) \end{aligned}$$

(iii)

$$\begin{aligned} T_{A \times B}((x_1, x_2)(y_1, y_2)) &\geq T_{A \times B}(y_1, y_2), \\ I_{A \times B}((x_1, x_2)(y_1, y_2)) &\leq I_{A \times B}(y_1, y_2), \\ F_{A \times B}((x_1, x_2)(y_1, y_2)) &\leq F_{A \times B}(y_1, y_2) \end{aligned}$$

(iv)

$$\begin{aligned} T_{A \times B}([(x_1, x_2) + (z_1, z_2)](y_1, y_2) - (x_1, x_2)(y_1, y_2)) &\geq T_{A \times B}(z_1, z_2), \\ I_{A \times B}([(x_1, x_2) + (z_1, z_2)](y_1, y_2) - (x_1, x_2)(y_1, y_2)) &\leq I_{A \times B}(z_1, z_2) \\ F_{A \times B}([(x_1, x_2) + (z_1, z_2)](y_1, y_2) - (x_1, x_2)(y_1, y_2)) &\leq T_{A \times B}(z_1, z_2) \end{aligned}$$

Theorem 16. Let A and B be neutrosophic fuzzy ideals of N_1, N_2 respectively. Then $A \times B$ is a neutrosophic fuzzy ideal of $N_1 \times N_2$.

Proof. Let A and B be neutrosophic fuzzy ideals of N_1, N_2 respectively. Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in N_1 \times N_2$

(i)

$$\begin{aligned} T_{A \times B}((x_1, x_2) - (y_1, y_2)) &= T_{A \times B}(x_1 - y_1, x_2 - y_2) \\ &= \min(T_A(x_1 - y_1), T_B(x_2 - y_2)) \\ &\geq \min\{\min[T_A(x_1), T_A(y_1)], \min[T_B(x_2), T_B(y_2)]\} \\ &= \min\{\min[T_A(x_1), T_B(x_2)], \min[T_A(y_1), T_B(y_2)]\} \\ &= \min(T_{A \times B}(x_1, x_2), T_{A \times B}(y_1, y_2)) \\ I_{A \times B}((x_1, x_2) - (y_1, y_2)) &= I_{A \times B}(x_1 - y_1, x_2 - y_2) \\ &= \max(I_A(x_1 - y_1), I_B(x_2 - y_2)) \\ &\leq \max\{\max[I_A(x_1), I_A(y_1)], \max[I_B(x_2), I_B(y_2)]\} \\ &= \max\{\max[I_A(x_1), I_B(x_2)], \max[I_A(y_1), I_B(y_2)]\} \\ &= \max(I_{A \times B}(x_1, x_2), I_{A \times B}(y_1, y_2)) \\ F_{A \times B}((x_1, x_2) - (y_1, y_2)) &= F_{A \times B}(x_1 - y_1, x_2 - y_2) \\ &= \max(F_A(x_1 - y_1), F_B(x_2 - y_2)) \\ &\leq \max\{\max[F_A(x_1), F_A(y_1)], \max[F_B(x_2), F_B(y_2)]\} \\ &= \max\{\max[F_A(x_1), F_B(x_2)], \max[F_A(y_1), F_B(y_2)]\} \\ &= \max(F_{A \times B}(x_1, x_2), F_{A \times B}(y_1, y_2)) \end{aligned}$$

(ii)

$$\begin{aligned} T_{A \times B}((y_1, y_2) + ((x_1, x_2)) - (y_1, y_2)) &= T_{A \times B}B((y_1 + x_1 - y_1), (y_2 + x_2 - y_2)) \\ &= \min[T_A(y_1 + x_1 - y_1), T_B(y_2 + x_2 - y_2)] \\ &\geq \min(T_A(x_1), T_B(x_2)) \\ &= T_{A \times B}(x_1, x_2)I_{A \times B}((y_1, y_2) + ((x_1, x_2)) - (y_1, y_2)) \\ &= I_{A \times B}((y_1 + x_1 - y_1), (y_2 + x_2 - y_2)) \\ &= \max[I_A(y_1 + x_1 - y_1), I_B(y_2 + x_2 - y_2)] \\ &\leq \max(I_A(x_1), I_B(x_2)) \\ &= I_{A \times B}(x_1, x_2)F_{A \times B}((y_1, y_2) + ((x_1, x_2)) - (y_1, y_2)) \\ &= F_{A \times B}((y_1 + x_1 - y_1), (y_2 + x_2 - y_2)) \\ &= \max[F_A(y_1 + x_1 - y_1), F_B(y_2 + x_2 - y_2)] \\ &\leq \max(F_A(x_1), F_B(x_2)) \\ &= F_{A \times B}(x_1, x_2) \end{aligned}$$

(iii)

$$\begin{aligned}
 T_{A \times B}(x_1, x_2)(y_1, y_2) &= T_{A \times B}(x_1y_1, x_2y_2) \\
 &= \min(T_A(x_1y_1), T_B(x_2y_2)) \\
 &\geq \min(T_A(y_1), T_B(y_2)) \\
 &= T_{A \times B}((y_1, y_2)) \\
 I_{A \times B}(x_1, x_2)(y_1, y_2) &= I_{A \times B}(x_1y_1, x_2y_2) \\
 &= \max(I_A(x_1y_1), I_B(x_2y_2)) \\
 &\leq \max(I_A(y_1), I_B(y_2)) \\
 &= I_{A \times B}(y_1, y_2) \\
 F_{A \times B}(x_1, x_2)(y_1, y_2) &= F_{A \times B}(x_1y_1, x_2y_2) \\
 &= \max(F_A(x_1y_1), F_B(x_2y_2)) \\
 &\leq \max(FI_A(y_1), F_B(y_2)) \\
 &= F_{A \times B}(y_1, y_2)
 \end{aligned}$$

(iv)

$$\begin{aligned}
 T_{A \times B}([(x_1, x_2) + (z_1, z_2) - (y_1, y_2) - (x_1, x_2)](y_1, y_2) &= T_{A \times B}([x_1 + z_1]y_1 - x_1y_1, \\
 & \quad [x_2 + z_2]y_2 - x_2y_2) \\
 &= \min(T_A[x_1 + z_1]y_1 - x_1y_1), \\
 & \quad T_B[x_2 + z_2]y_2 - x_2y_2) \\
 &\geq \min(T_A(z_1), T_B(z_2)) \\
 &= T_{A \times B}(z_1, z_2) \\
 I_{A \times B}([(x_1, x_2) + (z_1, z_2) - (y_1, y_2) - (x_1, x_2)](y_1, y_2) &= I_{A \times B}([x_1 + z_1]y_1 - x_1y_1, \\
 & \quad [x_2 + z_2]y_2 - x_2y_2) \\
 &= \max(I_A(x_1 + z_1]y_1 - x_1y_1), \\
 & \quad I_B[x_2 + z_2]y_2 - x_2y_2) \\
 &\leq \max(I_A(z_1), I_B(z_2)) \\
 &= I_{A \times B}(z_1, z_2) \\
 F_{A \times B}([(x_1, x_2) + (z_1, z_2) - (y_1, y_2) - (x_1, x_2)](y_1, y_2) &= F_{A \times B}([x_1 + z_1]y_1 - x_1y_1, \\
 & \quad [x_2 + z_2]y_2 - x_2y_2) \\
 &= \max(F_A(x_1 + z_1]y_1 - x_1y_1), \\
 & \quad F_B[x_2 + z_2]y_2 - x_2y_2) \\
 &\leq \max(F_A(z_1), F_B(z_2)) \\
 &= F_{A \times B}(z_1, z_2)
 \end{aligned}$$

Therefore $A \times B$ is a neutrosophic fuzzy ideal of $N_1 \times N_2$ □

5 Homomorphism of Neutrosophic fuzzy ideals of near-rings

Definition 17. Let R and S be two near-rings. Then the mapping $f : R \rightarrow S$ is called near-ring homomorphism if for all $x, y \in R$, the following holds

- (i) $f(x + y) = f(x) + f(y)$
- (ii) $f(xy) = f(x)f(y)$.

Definition 18. Let X and Y be two non-empty sets and $f : X \rightarrow Y$ be a function.

- (i) If B is a Neutrosophic fuzzy set in Y , then the preimage of B under f , denoted by $f^{-1}(B)$, is the Neutrosophic fuzzy set in X defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x)) \rangle : x \in X \}$$

where $f^{-1}(T_B(x)) = T_B(f(x))$ and so on.

- (ii) If A is a Neutrosophic fuzzy set in X , then the image of A under f , denoted by $f(A)$, is the Neutrosophic fuzzy set in Y defined by $f(A) = \{ \langle y, f(T_A(y)), f(I_A(y)), f(F_A(y)) \rangle : y \in Y \}$, where

$$f(T_A(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} T_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 0 & \text{otherwise} \end{cases}$$

$$f(I_A(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} I_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 0 & \text{otherwise} \end{cases}$$

$$f(F_A(y)) = \begin{cases} \inf_{x \in f^{-1}(y)} F_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 1 & \text{otherwise} \end{cases}$$

where $f(F_A(y)) = (1 - f(1 - F_A))(y)$

Theorem 19. Let N and N' be any two near-rings and f be a homomorphism of N on to N' . If A' is a Neutrosophic fuzzy ideal of N' , then $f^{-1}(A')$ is a Neutrosophic fuzzy ideal of N .

Proof. Let $x, y, z \in N$.

- (i)

$$\begin{aligned} f^{-1}(T_{A'})(x - y) &= T_{A'}(f(x - y)) \\ &= T_{A'}(f(x) - f(y)) \\ &\geq \min(T_{A'}(f(x)), T_{A'}(f(y))) \\ &= \min(f^{-1}(T_{A'})(x), f^{-1}(T_{A'})(y)) \\ f^{-1}(I_{A'})(x - y) &= I_{A'}(f(x - y)) \\ &= I_{A'}(f(x) - f(y)) \\ &\leq \max(I_{A'}(f(x)), I_{A'}(f(y))) \\ &= \max(f^{-1}(I_{A'})(x), f^{-1}(I_{A'})(y), f^{-1}(F_{A'})(x - y)) \\ &= F_{A'}(f(x - y)) \\ &= F_{A'}(f(x) - f(y)) \\ &\leq \max(F_{A'}(f(x)), F_{A'}(f(y))) \\ &= \max(f^{-1}(F_{A'})(x), f^{-1}(F_{A'})(y)) \end{aligned}$$

(ii)

$$\begin{aligned}
 f^{-1}(T_A)(y + x - y) &= T_A(f(y + x - y)) \\
 &= T_A(f(y) + f(x) - f(y)) \\
 &\geq \min(T_A(f(x))) \\
 &= \min(f^{-1}(T_A)(x)f^{-1}(I_A)(y + x - y)) \\
 &= I_A(f(y + x - y)) \\
 &= I_A(f(y) + f(x) - f(y)) \\
 &\leq \max(I_A(f(x))) \\
 &= \max(f^{-1}(I_A)(x), f^{-1}(F_A)(y + x - y)) \\
 &= F_A(f(y + x - y)) \\
 &= F_A(f(y) + f(x) - f(y)) \\
 &\leq \max(F_A(f(x))) \\
 &= \max(f^{-1}(F_A)(x))
 \end{aligned}$$

(iii)

$$\begin{aligned}
 f^{-1}(T_A)(xy) &= T_A(f(xy)) \\
 &= T_A(f(x)f(y)) \\
 &\geq \min(T_A(f(y))) \\
 &= \min(f^{-1}(T_A)(y)) \\
 f^{-1}(I_A)(xy) &= I_A(f(xy)) \\
 &= I_A(f(x)f(y)) \\
 &\leq \max(I_A(f(y))) \\
 &= \max(f^{-1}(I_A)(y)) \\
 f^{-1}(F_A)(xy) &= F_A(f(xy)) \\
 &= F_A(f(x)f(y)) \\
 &\leq \max(F_A(f(y))) \\
 &= \max(f^{-1}(F_A)(y))
 \end{aligned}$$

(iv)

$$\begin{aligned}
 f^{-1}(T_A)((x + z)y - xy) &= T_A(f[(x + z)y - xy]) \\
 &= T_A([f(x) + f(z)]f(y) - f(x)f(y)) \\
 &\geq \min(T_A(f(z))) \\
 &= \min(f^{-1}(T_A)(z)f^{-1}(I_A)((x + z)y - xy)) \\
 &= I_A(f[(x + z)y - xy]) \\
 &= I_A([f(x) + f(z)]f(y) - f(x)f(y)) \\
 &\leq \max(I_A(f(z))) \\
 &= \max f^{-1}(I_A)(z), f^{-1}(F_A)((x + z)y - xy)) \\
 &= F_A(f[(x + z)y - xy]) \\
 &= F_A([f(x) + f(z)]f(y) - f(x)f(y)) \\
 &\leq \max(F_A(f(z))) \\
 &= \max f^{-1}(F_A)(z).
 \end{aligned}$$

Therefore, $f^{-1}(A)$ is a Neutrosophic fuzzy ideal of N . □

Theorem 20. *Let N_1 and N_2 be any two near-rings and f be a homomorphism of N_1 on to N_2 . If A is a Neutrosophic fuzzy ideal of N_1 , then $f(A)$ is a Neutrosophic fuzzy ideal of N_2 .*

Proof. Let $y_1, y_2, y_3 \in N_2$ and $x_1, x_2, x_3 \in N_1$

(i)

$$\begin{aligned} f(T_A(y_1 - y_2)) &= \sup_{x_1, x_2 \in f^{-1}(N_2)} T_A(x_1 - x_2) \\ &\geq \sup_{x_1, x_2 \in f^{-1}(N_2)} \min\{T_A(x_1), T_A(x_2)\} \\ &= \min\left(\sup_{x_1 \in f^{-1}(N_2)} T_A(x_1), \sup_{x_2 \in f^{-1}(N_2)} T_A(x_2)\right) \\ &= \min\{f(T_A(y_1)), f(T_A(y_2))\} \end{aligned}$$

(ii)

$$\begin{aligned} f(T_A(y_1 + y_2 - y_1)) &= \sup_{x_1, x_2 \in f^{-1}(N_2)} T_A(x_1 + x_2 - x_1) \\ &\geq \sup_{x_1 \in f^{-1}(N_2)} T_A(x_1) \\ &= f(T_A(y_1)) \end{aligned}$$

(iii)

$$\begin{aligned} f(T_A(y_1 y_2)) &= \sup_{x_1, x_2 \in f^{-1}(N_2)} T_A(x_1 x_2) \\ &\geq \sup_{x_1 \in f^{-1}(N_2)} T_A(x_2) \\ &= f(T_A(y_2)) \end{aligned}$$

(iv)

$$\begin{aligned} f(T_A)[(y_1 + y_3)y_2 - y_1 y_2] &= \sup_{x_1, x_2, x_3 \in f^{-1}(N_2)} T_A[(x_1 + x_3)x_2 - x_1 x_2] \\ &\geq \sup_{x_3 \in f^{-1}(N_2)} T_A(x_3) = f(T_A(y_3)) \end{aligned}$$

Similarly we can prove that

$$\begin{aligned} f(I_A(y_1 - y_2)) &\geq \min\{f(I_A(y_1)), f(I_A(y_2))\}, \\ f(I_A(y_1 + y_2 - y_1)) &\geq f(I_A(y_1)), \\ f(I_A(y_1 y_2)) &\geq f(I_A(y_2)), f(I_A[(y_1 + y_3)y_2 - y_1 y_2]) \\ &\geq f(I_A(y_3)) \end{aligned}$$

. Also, we can prove that

$$\begin{aligned} f(F_A(y_1 - y_2)) &\leq \max\{f(F_A(y_1)), f(F_A(y_2))\}, \\ f(F_A(y_1 + y_2 - y_1)) &\leq f(F_A(y_1)), \\ f(F_A(y_1 y_2)) &\leq f(F_A(y_2)), \\ f(F_A[(y_1 + y_3)(y_2 - y_1 y_2)]) &\leq f(F_A(y_3)). \end{aligned}$$

Hence, $f(A)$ is a Neutrosophic fuzzy ideal of N_2 . □

6 Conclusion:

In this paper we have studied the notion of neutrosophic fuzzy sub near-ring, neutrosophic fuzzy ideals of near-rings and discussed some algebraic properties. We have proved that union of two neutrosophic fuzzy ideals of near-ring is a neutrosophic fuzzy ideal of that near-ring. Also we have proved that the positive integral powers of neutrosophic fuzzy ideal of near-ring is a neutrosophic fuzzy ideal. We have defined the direct product on neutrosophic fuzzy ideals of near-rings and proved that the direct product of any two neutrosophic fuzzy ideals of near-rings is a neutrosophic fuzzy ideal. Also, we can extend the result for finite number of neutrosophic fuzzy ideals of near-rings.

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