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# Neutrosophic Fuzzy Matrices and Some Algebraic Operations

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**Abstract:** In this article, we study neutrosophic fuzzy set and define the subtraction and multiplication of two rectangular and square neutrosophic fuzzy matrices. Some properties of subtraction, addition and multiplication of these matrices and commutative property, distributive property have been examined.
**Keywords:** Neutrosophic fuzzy matrix, Neutrosophic set. Commutativity, Distributive, Subtraction of neutrosophic matrices.

## 1. Introduction

Neutrosophic set was introduced by Florentin Smarandache [1] in 1998, where each element had three associated defining functions, namely the membership function ($T$), the non-membership ($F$) function and the indeterminacy function ($I$) defined on the universe of discourse $X$, the three functions are completely independent. Relative to the natural problems sometimes one may not be able to decide. After the development of the Neutrosophic set theory, one can easily take decision and indeterminacy function of the set is the nondeterministic part of the situation. The applications of the theory has been found in various field for dealing with indeterminate and inconsistent information in real world one may refer to [2,3,4]. Neutrosophic set is a part of neutrosophy which studied the origin, nature and scope of neutralities, as well as their interactions with ideational spectra. The neutrosophic set generalizes the concept of classical fuzzy set [10, 11], interval valued fuzzy set, intuitionistic fuzzy set and so on. In the recent years, the concept of neutrosophic set has been applied successfully by Broumi et al. [12, 13, 14] and Abdel-Basset et al. [15, 16, 17, 18]

The single-valued neutrosophic number which is a generalization of fuzzy numbers and intuitionistic fuzzy numbers. A single-valued neutrosophic number is simply an ordinary number whose precise value is somewhat uncertain from a philosophical point of view. There are two special forms of single-valued neutrosophic numbers such as single-valued trapezoidal neutrosophic numbers and single-valued triangular neutrosophic numbers.

The neutrosophic interval matrices have been defined by Vasantha Kandasamy and Florentin Smarandache in their book "Fuzzy interval matrices, Neutrosophic interval matrices, and applications". A neutrosophic fuzzy matrix $[a_{ij}]_{nxm}$, whose entries are of the form $a + Ib$ (neutrosophic number), where $a$, $b$ are the elements of the interval [0,1] and $I$ is an indeterminate such that $I^n = I$, $n$ being a positive integer.

So the difference between the neutrosophic number of the form $a + Ib$ and the single-valued neutrosophic numbers is that the generalization of fuzzy number and the single-valued neutrosophic components <*T*, *I*, *F*> is the generalization of fuzzy numbers and intuitionistic fuzzy numbers. Since fuzzy number lies between 0 to 1 so the component neutrosophic fuzzy number $a$ and $b$ lies in [0,1]. In the case of single-valued neutrosophic matrix components will be the true value, indeterminacy and fails value with three components in each element of a matrix [3, 4, 8].

We know the important role of matrices in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. Kandasamy and Smarandache [7] introduced fuzzy relational maps and neutrosophic relational maps. Thomason [8], introduced the fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. Dhar, Broumi and Smarandache [2] define Square Neutrosophic Fuzzy Matrices whose entries are of the form $a+Ib$, where a and b are fuzzy number from [0, 1] gives the definition of Neutrosophic Fuzzy Matrices multiplication.

In this paper our ambition is to define the subtraction of fuzzy neutrosophic matrices, rectangular fuzzy neutrosophic matrices and study some algebraic properties. We shall focus on all types of neutrosophic fuzzy matrices. The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set, neutrosophic matrices, Fuzzy integral neutrosophic matices and fuzzy matrix. Section 3 presents a new type of fuzzy neutrosophic matrices and investigated some properties such as subtraction, commutative property and distributive property.

## 2. Materials and Methods (proposed work with more details)

In this section we recall some concepts of neutrosophic set, neutrosophic matrices and fuzzy neutrosophic matrices proposed by Kandasamy and Smarandache in their monograph [3], and also the concept of fuzzy matrix (One may refer to [2])

**Definition 2.1 (**Smarandache [1]). Let $U$ be an universe of discourse then the neutrosophic set A is an object having the form $A = \{< x{:}T_A(x), I_A(x), F_A(x)>, x \in U\}$, where the functions $T, I, F : U \rightarrow ]^-0, 1^+[$ define respectively the degree of membership (or Truthness), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in U$ to the set $A$ with the condition.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

**Definition 2.2** (Dhar et al. [3]). Let $M_{mxn}= \{(a_{ij}) : a_{ij} \in K(I)\}$, where $K(I)$, is a neutrosophic field. We call $M_{mxn}$ to be the neutrosophic matrix.

**Example 2.1**: Let $R(I) = \langle R \cup I \rangle$be the neutrosophic field

$$M_{4x3} = \begin{pmatrix} 5 & 0 & 2.1I \\ 3.5I & 3 & 5 \\ 7 & 4I & 0 \\ 8 & -5I & I \end{pmatrix}$$

$M_{4x3}$ denotes the neutrosophic matrix, with entries from real and the indeterminacy.

**Definition 2.3** (Kandasamy and Smarandache [5])

Let $N = [0, 1] \cup I$ where $I$ is the indeterminacy. The $m \times n$ matrices $M_{mxn} = \{(a_{ij}) : a_{ij} \in [0, 1] \cup I\}$ is called the fuzzy integral neutrosophic matrices. Clearly the class of $m \times n$ matrices is contained in the class of fuzzy integral neutrosophic matrices.

The row vector $1 \times n$ and column vector $m \times 1$ are the fuzzy neutrosophic row matrices and fuzzy neutrosophic column matrices respectively.

**Example 2.2:** Let $M_{4x3} = \begin{pmatrix} 0.5 & 0 & 0.1I \\ I & 0.3 & 0.5 \\ 0.7 & 0.4I & 0 \\ 0.8 & 0.5I & I \end{pmatrix}$ be a 4 ×3 integral fuzzy neutrosophic matrix

**Definition 2.5** (Kandasamy and Smarandache [5]).

Let $N_s = [0, 1] \cup \{bI : b \in [0, 1]\}$; we call the set $N_s$ to be the fuzzy neutrosophic set. Let $N_s$ be the fuzzy neutrosophic set. $M_{mxn} = \{(a_{ij}): a_{ij} \in N_s\ i = 1 \text{ to } m \text{ and } j = 1 \text{ to } n\}$ we call the matrices with entries from $N_s$ to be the fuzzy neutrosophic matrices.

**Example 2.3**: Let $N_s = [0,1] \cup \{bI: b \in [0,1]\}$ be the fuzzy neutrosophic set and

$$P = \begin{pmatrix} 0.5 & 0 & 0.1I \\ I & 0.3 & 0.5 \\ 0 & I & 0.01 \end{pmatrix}$$

be a 3 ×3 fuzzy neutrosophic matrix.

**Definition 2.6** (Thomas [9]). A fuzzy matrix is a matrix which has its elements from the interval [0, 1], called the unit fuzzy interval. $A_{mxn}$ fuzzy matrix for which $m = n$ (i.e. the number of rows is equal to the number of columns) and whose elements belong to the unit interval [0, 1] is called a fuzzy square matrix of order $n$. A fuzzy square matrix of order two is expressed in the following way

$$A = \begin{pmatrix} x & y \\ t & z \end{pmatrix},$$

where the entries $x, y, t, z$ all belongs to the interval [0,1].

**Definition 2.7** (Kandasamy and Smarandache [5]). Let $A$ be a neutrosophic fuzzy matrix, whose entries is of the form $a + Ib$ (neutrosophic number), where $a$, $b$ are the elements of [0,1] and $I$ is an indeterminate such that $I^n = I$, $n$ being a positive integer.

$$A = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}$$

**Definition 2.8 Multiplication Operation of two Neutrosophic Fuzzy Matrices**

Consider two neutrosophic fuzzy matrices, whose entries are of the form *a* + *Ib* (neutrosophic number), where *a*, *b* are the elements of [0,1] and *I* is an indeterminate such that $I^n = I$, *n* being a positive integer, given by

$$A = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}, \quad B = \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$$

The Multiplication Operation of two Neutrosophic Fuzzy Matrices is given by

$$AB = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}\begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$$

$$D_{11} = [\max\{ \min(x_1, m_1), \min(x_2, m_3)\} + I \max\{ \min\{(y_1, n_1), \min(y_2, n_3)\}]$$

$$D_{21} = [\max \{\min(x_1, m_2), \min (x_2, m_4)\} + I \max \{\min(y_1, n_2), \min(y_2, n_4)\}]$$

$$D_{21} = [\max \{\min\{(x_3, m_1), \min (x_4, m_3)\} + I \max \{\min\{(y_3, n_1), \min (y_4, n_3)\}]$$

$$D_{22} = [\max \{\min\{(x_3, m_2), \min (x_4, m_4)\} + I \max\{ \min\{(y_3, n_2), \min (y_4, n_4)\}]$$

Hence, $AB = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$.

## 3. Results (examples / case studies related to the proposed work)

In this section we define the subtraction and distributive property of neutrosophic fuzzy matrices along with some properties associated with such matrices.

### 3.1 Subtraction Operation of two Neutrosophic Fuzzy Matrices

Consider two neutrosophic fuzzy matrices given by

$$A = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \\ x_5 + Iy_5 & x_6 + Iy_6 \end{pmatrix}$$

and $$B = \begin{pmatrix} t_1 + Iz_1 & t_2 + Iz_2 \\ t_3 + Iz_3 & t_4 + Iz_4 \\ t_5 + Iz_5 & t_6 + Iz_6 \end{pmatrix}.$$

Addition and multiplication between two neutrosophic fuzzy matrices have been defined in Smarandache [2]. We would like to define the subtraction of these two matrices as follows.

*A*- *B* = *C*,

where $c_{ij}$ are as follows

$c_{11} = \min\{x_1, t_1\} + I \min\{y_1, z_1\}$

$c_{12} = \min\{x_2, t_2\} + I \min\{y_2, z_2\}$

$c_{21} = \min\{x_3, t_3\} + I \min\{y_3, z_3\}$

$c_{21} = \min\{x_4, t_4\} + I \min\{y_4, z_4\}$

$c_{31} = \min\{x_5, t_5\} + I \min\{y_5, z_5\}$

$c_{32} = \min\{x_6, t_6\} + I \min\{y_6, z_6\}$

Since $\min\{a, b\} = \min\{b, a\}$ so based on this we have the following properties.

**Proposition 3.1.** The following properties hold in the case of neutrosophic fuzzy matrix for subtraction

(*i*) $A\text{-}B = B\text{-}A$

(*ii*) $(A - B) - C = A - (B - C) = (B- C) – A = (C – B) – A$.

**Proof.** Consider three neutrosophic fuzzy matrices $A$, $B$ and $C$ as follows.

$$A = \begin{pmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I \\ a_{31} + b_{31}I & a_{32} + b_{32}I \end{pmatrix},\ B = \begin{pmatrix} c_{11} + d_{11}I & c_{12} + d_{12}I \\ c_{21} + d_{21}I & c_{22} + d_{22}I \\ c_{31} + d_{31}I & c_{32} + d_{32}I \end{pmatrix}$$

and $$C = \begin{pmatrix} l_{11} + m_{11}I & l_{12} + m_{12}I \\ l_{21} + m_{21}I & l_{22} + m_{22}I \\ l_{31} + m_{31}I & l_{32} + m_{32}I \end{pmatrix}$$

$$A - B = \begin{pmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I \\ a_{31} + b_{31}I & a_{32} + b_{32}I \end{pmatrix} - \begin{pmatrix} c_{11} + d_{11}I & c_{12} + d_{12}I \\ c_{21} + d_{21}I & c_{22} + d_{22}I \\ c_{31} + d_{31}I & c_{32} + d_{32}I \end{pmatrix} = D \text{ (say)},$$

where,

$$D_{11} = \min\{a_{11}, c_{11}\} + I\min\{b_{11}, d_{11}\} = x_{11} + Iy_{11}$$

$$D_{12} = \min\{a_{12}, c_{12}\} + I\min\{b_{12}, d_{12}\} = x_{12} + Iy_{12}$$

$$D_{21} = \min\{a_{21}, c_{21}\} + I\min\{b_{21}, d_{21}\} = x_{21} + Iy_{21}$$

$$D_{22} = \min\{a_{22}, c_{22}\} + I\min\{b_{22}, d_{22}\} = x_{22} + Iy_{22}$$

$$D_{31} = \min\{a_{31}, c_{31}\} + I\min\{b_{31}, d_{31}\} = x_{31} + Iy_{31}$$

$$D_{32} = \min\{a_{32}, c_{32}\} + I\min\{b_{32}, d_{32}\} = x_{32} + Iy_{32}$$

$$D = \begin{pmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \\ x_{31} + Iy_{31} & x_{32} + Iy_{32} \end{pmatrix} \text{ and } B - A = \begin{pmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \\ x_{31} + Iy_{31} & x_{32} + Iy_{32} \end{pmatrix} = D,$$

[$\therefore$ $\min(a, c) = \min(c, a)$]

Hence, $A – B = B – A$.

Now we have,

$D – C = (A – B) – C$

$$= \begin{pmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \\ x_{31} + Iy_{31} & x_{32} + Iy_{32} \end{pmatrix} - \begin{pmatrix} l_{11} + m_{11}I & l_{12} + m_{12}I \\ l_{21} + m_{21}I & l_{22} + m_{22}I \\ l_{31} + m_{31}I & l_{32} + m_{32}I \end{pmatrix}$$

$= F$ (say),

where,

$F_{11}$ = min{$x_{11}$, $l_{11}$}+Imin{$y_{11}$, $m_{11}$} = min{$a_{11}$, $c_{11}$, $l_{11}$}+Imin{$b_{11}$, $d_{11}$, $m_{11}$} = $n_{11} + Ik_{11}$

$F_{12}$ = min{$x_{12}$, $l_{12}$}+Imin{$y_{12}$, $m_{12}$} = min{$a_{12}$, $c_{12}$, $l_{12}$}+Imin{$b_{11}$, $d_{12}$, $m_{12}$} = $n_{12} + Ik_{12}$

$F_{21}$ = min{$x_{21}$, $l_{21}$}+Imin{$y_{21}$, $m_{21}$} = min{$a_{21}$, $c_{21}$, $l_{21}$}+Imin{$b_{21}$, $d_{21}$, $m_{21}$}= $n_{21} + Ik_{21}$

$F_{22}$ = min{$x_{22}$, $l_{22}$}+Imin{$y_{22}$, $m_{22}$}= min{$a_{22}$, $c_{22}$, $l_{22}$}+Imin{$b_{22}$, $d_{22}$, $m_{22}$}= $n_{22} + Ik_{22}$

$F_{31}$ = min{$x_{31}$, $l_{31}$}+Imin{$y_{31}$, $m_{31}$} = min{$a_{31}$, $c_{31}$, $l_{31}$}+Imin{$b_{31}$, $d_{31}$, $m_{31}$}= $n_{31} + Ik_{31}$

$F_{32}$ = min{$x_{32}$, $l_{32}$}+Imin{$y_{32}$, $m_{32}$} = min{$a_{31}$, $c_{31}$, $l_{31}$}+Imin{$b_{31}$, $d_{31}$, $m_{31}$}= $n_{32} + Ik_{32}$

$$(A-B)-C = F = \begin{pmatrix} n_{11}+Ik_{11} & n_{12}+Ik_{12} \\ n_{21}+Ik_{21} & n_{22}+Ik_{22} \\ n_{31}+Ik_{31} & n_{32}+Ik_{32} \end{pmatrix}.$$

Next we have,

$$B-C = \begin{pmatrix} c_{11}+d_{11}I & c_{12}+d_{12}I \\ c_{21}+d_{21}I & c_{22}+d_{22}I \\ c_{31}+d_{31}I & c_{32}+d_{32}I \end{pmatrix} - \begin{pmatrix} l_{11}+m_{11}I & l_{12}+m_{12}I \\ l_{21}+m_{21}I & l_{22}+m_{22}I \\ l_{31}+m_{31}I & l_{32}+m_{32}I \end{pmatrix} = E \text{ (say)},$$

where

$E_{11}$ = min{$c_{11}$, $l_{11}$}+Imin{$d_{11}$, $m_{11}$} = $p_{11} + Iq_{11}$

$E_{12}$ = min{$c_{12}$, $l_{12}$}+Imin{$d_{12}$, $m_{12}$} = $p_{12} + Iq_{12}$

$E_{21}$ = min{$c_{21}$, $l_{21}$}+Imin{$d_{21}$, $m_{21}$} = $p_{21} + Iq_{21}$

$E_{22}$ = min{$c_{22}$, $l_{22}$}+Imin{$d_{22}$, $m_{22}$}= $p_{22} + Iq_{22}$

$E_{31}$ = min{$c_{31}$, $l_{31}$}+Imin{$d_{31}$, $m_{31}$} = $p_{31} + Iq_{31}$

$E_{32}$ = min{$c_{32}$, $l_{32}$}+Imin{$d_{32}$, $m_{32}$} = $p_{32} + Iq_{32}$.

We have

$$B-C = E = \begin{pmatrix} p_{11}+Iq_{11} & p_{12}+Iq_{12} \\ p_{21}+Iq_{21} & p_{22}+Iq_{22} \\ p_{31}+Iq_{31} & p_{32}+Iq_{32} \end{pmatrix}$$

$$A-(B-C) = \begin{pmatrix} a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{21}+b_{21}I & a_{22}+b_{22}I \\ a_{31}+b_{31}I & a_{32}+b_{32}I \end{pmatrix} - \begin{pmatrix} p_{11}+Iq_{11} & p_{12}+Iq_{12} \\ p_{21}+Iq_{21} & p_{22}+Iq_{22} \\ p_{31}+Iq_{31} & p_{32}+Iq_{32} \end{pmatrix},$$

where

min{$a_{11}$, $p_{11}$}+Imin{$b_{11}$, $q_{11}$} = min{$a_{11}$, $c_{11}$, $l_{11}$}+Imin{$b_{11}$, $d_{11}$, $m_{11}$}

min{$a_{12}$, $p_{12}$}+$I$min{$b_{12}$, $q_{12}$} = min{$a_{12}$, $c_{12}$, $l_{12}$}+$I$min{$b_{11}$, $d_{12}$, $m_{12}$}

min{$a_{21}$, $p_{21}$}+$I$min{$b_{21}$, $q_{21}$} = min{$a_{21}$, $c_{21}$, $l_{21}$}+$I$min{$b_{21}$, $d_{21}$, $m_{21}$}

min{$a_{22}$, $p_{22}$}+$I$min{$b_{22}$, $q_{22}$}= min{$a_{22}$, $c_{22}$, $l_{22}$}+$I$min{$b_{22}$, $d_{22}$, $m_{22}$}

min{$a_{31}$, $p_{31}$}+$I$min{$b_{31}$, $q_{31}$} = min{$a_{31}$, $c_{31}$, $l_{31}$}+$I$min{$b_{31}$, $d_{31}$, $m_{31}$}

min{$a_{32}$, $p_{32}$}+$I$min{$b_{32}$, $q_{32}$} = min{$a_{31}$, $c_{31}$, $l_{31}$}+$I$min{$b_{31}$, $d_{31}$, $m_{31}$}

$$F = \begin{pmatrix} n_{11} + Ik_{11} & n_{12} + Ik_{12} \\ n_{21} + Ik_{21} & n_{22} + Ik_{22} \\ n_{31} + Ik_{31} & n_{32} + Ik_{32} \end{pmatrix}$$

Therefore, $A - (B - C) = F = (A - B) - C$.

### 3.2 Identity element for subtraction

In the group theory under the operation "*" the identity element $I_N$ of a set is an element such that $I_N * A = A * I_N = A$.

Specially the identity element of neutrosophic set is $I_N = \{[a_{ij} + b_{ij}I]_{m\times n}: \quad a_{ij} = 1 = b_{ij}$ for all $i, j\}$.

**Result 3.1.** For a neutrosophic fuzzy matrix, $I_N$ is the identity matrix for subtraction.

Let $A= \begin{pmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I \\ a_{31} + b_{31}I & a_{32} + b_{32}I \end{pmatrix}$, and $I_N = \begin{pmatrix} 1+I & 1+I \\ 1+I & 1+I \\ 1+I & 1+I \end{pmatrix}$ be the neutrosophic identity matrix of order 3x2.

Then we have the following

$$A - I_N = \begin{pmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I \\ a_{31} + b_{31}I & a_{32} + b_{32}I \end{pmatrix} - \begin{pmatrix} 1+I & 1+I \\ 1+I & 1+I \\ 1+I & 1+I \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I \\ a_{31} + b_{31}I & a_{32} + b_{32}I \end{pmatrix} = I_N - A = A,$$

where

min{$a_{11}$, 1}+Imin{$b_{11}$,1} = $a_{11} + b_{11}I$

min{$a_{12}$, 1}+Imin{$b_{12}$,1}= $a_{12} + b_{12}I$

min{$a_{21}$, 1}+Imin{$b_{21}$,1} = $a_{21} + b_{21}I$

min{$a_{22}$, 1}+Imin{$b_{22}$,1} = $a_{22} + b_{22}I$

min{$a_{31}$, 1}+Imin{$b_{31}$,1} = $a_{31} + b_{31}I$

$\min\{a_{32}, 1\}+I\min\{b_{32},1\} = a_{32} + b_{32}I$

**3.3 Identity element for addition**

In neutrosophic matrix addition we can define a identity element $I_N$ such that $I_N = \{[a_{ij} + b_{ij}I]_{mxn}: \quad a_{ij} = 0 = b_{ij}$ for all $i, j\}$

Let $A=\begin{pmatrix} a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{21}+b_{21}I & a_{22}+b_{22}I \\ a_{31}+b_{31}I & a_{32}+b_{32}I \end{pmatrix}$, and $I_N = \begin{pmatrix} 0+0I & 0+0I \\ 0+0I & 0+0I \\ 0+0I & 0+0I \end{pmatrix}$ be the neutrosophic identity matrix of order 3x2.

Then we have the following

$$A \;- I_N = \begin{pmatrix} a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{21}+b_{21}I & a_{22}+b_{22}I \\ a_{31}+b_{31}I & a_{32}+b_{32}I \end{pmatrix} - \begin{pmatrix} 1+I & 1+I \\ 1+I & 1+I \\ 1+I & 1+I \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{21}+b_{21}I & a_{22}+b_{22}I \\ a_{31}+b_{31}I & a_{32}+b_{32}I \end{pmatrix}$$

$$= I_N - A = A,$$

where

$\max\{a_{11}, 0\}+I\max\{b_{11},0\} = a_{11} + b_{11}I$
$\max\{a_{12}, 0\}+I\max\{b_{12},0\}= a_{12} + b_{12}I$
$\max\{a_{21}, 0\}+I\max\{b_{21},0\} = a_{21} + b_{21}I$
$\max\{a_{22}, 0\}+I\max\{b_{22},0\} = a_{22} + b_{22}I$
$\max\{a_{31}, 0\}+I\max\{b_{31},0\} = a_{31} + b_{31}I$
$\max\{a_{32}, 0\}+I\max\{b_{32},0\} = a_{32} + b_{32}I$.

**Result 3.2.** The neutrosophic set forms a groupoid, semigroup, monaid and is commutative under the neutrosophic matrix operation of subtraction. The distributive law also holds for subtraction, i.e. $A(B - C) = AB - AC$.

**Result 3.3.** The neutrosophic set forms a groupoid, semigroup, monaid and commutative under the operation of addition. The distributive law also holds for addition, i.e.
$A(B + C) = AB + AC$.

Thus we have, $A(B \pm C) = AB \pm AC$.

## 4. Applications

The formation of neutrosophic group structure, neutrosophic matrix set and algebraic structure on this set, the results are applicable

## 5. Conclusions

In this paper we have established some neutrosophic algebraic property, and subtraction operation addition and multiplication of these matrices and commutative property, distributive property had been examine. This result can be applied further application of neutrosophic fuzzy matric theory. For the development of neutrosophic group and its algebraic property the results of this paper would be helpful.

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