Neutrosophic M/M/1, M/M/c, M/M/1/b Queueing Systems

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صفوف الانتظار النيتروسوفكية M/M/1, M/M/c, M/M/1/b

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الملخص

قمنا في هذا البحث بتقديم ودراسة صفوف الانتظار M/M/1 و M/M/c و M/M/1/b حسب المفهوم النيتروسوفكي. قمنا بإيجاد الاحتمال النيتروسوفكي لوجود k زبوناً في النظام عند وصول زبون جديد، ثم أوجدنا مقاييس الأداء النيتروسوفكية لصفوف الانتظار المذكورة حيث قمنا باستخدام الأعداد النيتروسوفكية الإحصائية لوصف عدم الدقة في معدلات الوصول ومعدلات التخديم.

هذه الدراسة تفتح الأفق للتعامل مع الحالات التي تكون فيها معالم أنظمة صفوف الانتظار غير محددة بدقة.

كلمات مفتاحية: عملية بواسون، التوزيع الأسي، نظرية صفوف الانتظار، المنطق النيتروسوفكي، الأعداد النيتروسوفكية الإحصائية.

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Neutrosophic M/M/1, M/M/c, M/M/1/b Queueing Systems

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Abstract

In this paper we introduce and study M/M/1 queue, M/M/c queue and M/M/1/b queue in neutrosophic philosophy. We have derived the neutrosophic probability that arriving customers will find k customers in the system and then we have derived the neutrosophic performance measures in the mentioned neutrosophic queues. We have used statistical neutrosophic numbers to describe imprecise arriving rates and imprecise serving (departure) rates. This study opens the way to deal with cases in which parameters of queues are not specified accurately.

Keywords: Poisson Process, Exponential Distribution, Queueing Theory, Neutrosophic Logic, Neutrosophic Statistical Numbers

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1. Introduction

Queueing theory was developed by Erlang in 1909 to model waiting lines and develop efficient systems that reduce customers waiting times which makes it possible to serve more customers and increase profits to the organizations. The problem of classical queueing theory is the assumption of well and clear knowledge of the parameters of queueing system (e.g. arrival rate, departure rate) which is often impossible [1,2].

Many researchers studied fuzzy queueing theory which deals with uncertainty in parameters of queueing systems like in [3,4], but the problem is that possibilities and alpha-cut ranges in queueing theory is meaningless in many real-life problems (like 0 alpha cut of performance measure) [3,4].

Neutrosophic logic introduced by F. Smarandache in 1995 is a generalization of fuzzy logic and intuitionistic fuzzy logic [5,6,7,8]. This logic makes dealing with indeterminant data easier, clearer and more realistic. So modelling queues using neutrosophic parameters makes decisions more efficient [9,10,11,12,13,14].

In Neutrosophic probability, an event is said to be (t) true, (i) indeterminate, and (f) false, where t, i, f are real values from the ranges T, I, F in]-0, 1+[, with no restriction on the sum t+i+f, but in many applications these components may be not presented clearly and neutrosophic numbers of the form N=D+I could be used, where D is the determinant part of the number, and I is the indeterminacy [11,12,14,15].

Lots of researches extended statistical models and probability distributions letting them take imprecise parameters depending on concept of neutrosophy, like binomial distribution, normal distribution, exponential distribution, Poisson distribution, Weibull distribution, central tendency measures, measures of dispersion, confidence intervals, hypothesis testing, time series models, etc. [9,11,12,15,16]

Extension of classical queueing theory to neutrosophic queueing theory means that parameters of queues can take imprecise values presented in ranges, which allows dealing with vagueness, e.g. we can let λ the arrival rate be in the form $\lambda_N = \lambda + I$ and μ the departure rate be in the form $\mu_N = \mu + I$ where I notes indeterminant part of the given numbers.

In this paper we show the power of neutrosophic crisp sets theory [9,10] to deal with imprecise parameters of some queueing models that is: M/M/1, M/M/c, M/M/1/b, and we derive neutrosophic performance measures of the mentioned queues, also solved examples showing the power of this extension are presented.

2. Preliminaries

We can use the following form to describe queueing systems: **(A/B/c/b):(SP/CS/b)** where A is the interarrival times distribution, B is servicing or processing times distribution, c is number of parallel servers, SP is the service policy, CS is the calling size and b is the system capacity. Also, we can use the short notion that is **(A/B/c/b)**

Amon this paper, NP stands for neutrosophic probability, NM stands for neutrosophic exponential distribution, NL_S, NL_q, NW_S, NW_q stand for neutrosophic expected number of customers in system, neutrosophic expected number of customers in queue, neutrosophic mean waiting time in system and neutrosophic mean waiting time in queue respectively; which are neutrosophic performance measures.

We will use neutrosophic statistical numbers presented in [11] and we will depend on interval arithmetic to define summation, subtraction, multiplication and division of neutrosophic numbers.

2.1 Interval Arithmetic [17]:

Let $[a_1, b_1], [a_2, b_2]$ be two Intervals where $a_1, a_2, b_1, b_2 \in \mathbb{R}$ and for practical cases set $a_1 > 0, a_2 > 0, b_1 > 0, b_2 > 0$ then:

- $[a_1, b_1] + [a_2, b_2] = [a_1 + a_2, b_1 + b_2]$
- $[a_1, b_1] - [a_2, b_2] = [a_1 - b_2, b_1 - a_2]$
- $[a_1, b_1] * [a_2, b_2] = [a_1 a_2, b_1 b_2]$
- $[a_1, b_1]/[a_2, b_2] = [a_1, b_1] * \frac{1}{[a_2, b_2]} = \left[\frac{a_1}{b_2}, \frac{b_1}{a_2}\right]$

2.2 Crisp M/M/1 Queue [1,2]:

In M/M/1 queue, the interarrival times have exponential distribution with parameter λ and the servicing times also have exponential distribution with parameter μ. Customers are being served by one server according to FCFS policy which means first comes first served. The calling source and the system size are infinite.

The probability that arriving customer will find k customers in the queue will be:

$$P(k) = (1-\rho)\rho^k ; k = 0,1,... \tag{1}$$

The performance measures will be then:

$$L_S = \rho/(1-\rho) \tag{2}$$

$$L_q = \rho^2/(1-\rho) \tag{3}$$

$$W_S = 1/(\mu - \lambda) \tag{4}$$

$$W_q = \rho/(\mu - \lambda) \tag{5}$$

2.3 Crisp M/M/c Queue [1,2]:

M/M/c queue is the same as M/M/1 queue except that in M/M/c customers are being served by c parallel homogeneous servers according to FCFS policy.

The probability that arriving customer will find k customers in the queue will be:

$$P(k) = \begin{cases} \frac{(c\rho)^k}{k!} P(0); k < c \\ \frac{\rho^k c^c}{c!} P(0); k \geq c \end{cases} \tag{6}$$

Where:

$$P(0) = \left(\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \cdot \frac{1}{1-\rho} \right)^{-1} \tag{7}$$

The performance measures will be then:

$$L_S = c\rho + \frac{(c\rho)^c \rho}{c!(1-\rho)^2} P(0) \tag{8}$$

$$L_q = \frac{(c\rho)^c \rho}{c!(1-\rho)^2} NP(0) \tag{9}$$

$$W_q = \frac{1}{\lambda} L_q \tag{10}$$

$$W_S = \frac{1}{\lambda} L_S \tag{11}$$

2.4 Crisp M/M/1/b Queue [1,2]:

Interarrival times and serving times follow exponential distribution, customers are being served by one server according to FCFS policy, the calling source is infinite and system size is finite and determined by b including the one being served.

The probability that arriving customer will find k customers in the queue will be:

$$P(k) = \frac{\rho^k (1-\rho)}{(1-\rho^{b+1})}; k = 0..b \tag{12}$$

The performance measures will be then:

$$L_q = \frac{\rho^2\left[1-b\rho^{b-1}+(b-1)\rho^b\right]}{(1-\rho)\left(1-\rho^{b+1}\right)} \quad (13)$$

$$L_S = L_q + Eff\ \rho\ ;\ Eff\ \rho = \frac{Eff\ \lambda}{\mu}\ ,\ Eff\ \lambda = \lambda\left(1-P(b)\right) \quad (14)$$

Where $Eff\ \rho$ is effective rho and $Eff\ \lambda$ is effective λ that is according to the finite system size so when the queue is full no new customers can join the queue.

$$W_q = \frac{1}{Eff\ \lambda} L_q \quad (15)$$

$$W_S = \frac{1}{Eff\ \lambda} L_S \quad (16)$$

3. Neutrosophic Queues

3.1 Definition

Depending on [18,19], A queueing system is said to be a neutrosophic queue if its parameters are neutrosophic numbers, i.e. average rate of customers entering the queueing system λ and average rate of customers being served μ are neutrosophic numbers.

We'll denote neutrosophic λ by $\lambda_N = \left[\lambda^L, \lambda^U\right]$, and neutrosophic μ by $\mu_N = \left[\mu^L, \mu^U\right]$, so if we have c servers, we'll call $\rho_N = \lambda_N / c\mu_N = \frac{\left[\lambda^L, \lambda^U\right]}{c\left[\mu^L, \mu^U\right]} = \left[\frac{\lambda^L}{c\mu^U}, \frac{\lambda^U}{c\mu^L}\right]$ the neutrosophic traffic intensity that is depending on interval arithmetic.

3.2 (NM/NM/1):(FCFS/∞/∞) Queue

3.2.a Deriving the Neutrosophic Queueing Formulas

In NM/NM/1 queue, the interarrival times have neutrosophic exponential distribution with parameter λ_N and the servicing times also have neutrosophic exponential distribution with parameter μ_N. Customers are being served by one server according to FCFS policy which means first comes first served. The calling source and the system size are infinite.

The neutrosophic probability that arriving customer will find k customers in the queue will be after replacing crisp parameters by neutrosophic parameters in (1):

$$NP(k) = (1-\rho_N)\rho_N{}^k ; k = 0,1,...$$

$$NP(k)=\left(1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]^k \ ;k=0,1,\ldots$$

$$NP(k)=\left[1-\frac{\lambda^U}{\mu^L},1-\frac{\lambda^L}{\mu^U}\right]\left[\left(\frac{\lambda^L}{\mu^U}\right)^k,\left(\frac{\lambda^U}{\mu^L}\right)^k\right];k=0,1,\ldots$$

$$NP(k)=\left[\left(1-\frac{\lambda^U}{\mu^L}\right)\left(\frac{\lambda^L}{\mu^U}\right)^k,\left(1-\frac{\lambda^L}{\mu^U}\right)\left(\frac{\lambda^U}{\mu^L}\right)^k\right];k=0,1,\ldots \tag{17}$$

The neutrosophic performance measures will be then by substituting neutrosophic parameters in (2),(3),(4),(5):

- Neutrosophic expected number of customers in system:

$$NL_S=\rho_N\big/(1-\rho_N)$$

$$NL_S=\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\Big/\left(1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)$$

$$NL_S=\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\Big/\left[1-\frac{\lambda^U}{\mu^L},1-\frac{\lambda^L}{\mu^U}\right]$$

$$NL_S=\left[\frac{\frac{\lambda^L}{\mu^U}}{1-\frac{\lambda^L}{\mu^U}},\frac{\frac{\lambda^U}{\mu^L}}{1-\frac{\lambda^L}{\mu^U}}\right] \tag{18}$$

- Neutrosophic expected number of customers in queue:

$$NL_q=\rho_N{}^2\big/(1-\rho_N)$$

$$NL_q=\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]^2\Big/\left(1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)$$

$$NL_q=\left[\left(\frac{\lambda^L}{\mu^U}\right)^2,\left(\frac{\lambda^U}{\mu^L}\right)^2\right]\Big/\left[1-\frac{\lambda^U}{\mu^L},1-\frac{\lambda^L}{\mu^U}\right]$$

$$NL_q=\left[\frac{\left(\frac{\lambda^L}{\mu^U}\right)^2}{1-\frac{\lambda^L}{\mu^U}},\frac{\left(\frac{\lambda^U}{\mu^L}\right)^2}{1-\frac{\lambda^U}{\mu^L}}\right] \tag{19}$$

- Neutrosophic expected waiting time in system:

$$NW_S=1\big/(\mu_N-\lambda_N)$$

$$NW_S=1\big/\left(\left[\mu^L,\mu^U\right]-\left[\lambda^L,\lambda^U\right]\right)$$

$$NW_S=1\big/\left[\mu^L-\lambda^U,\mu^U-\lambda^L\right]$$

$$NW_S = \left[\frac{1}{\mu^U - \lambda^L}, \frac{1}{\mu^L - \lambda^U} \right] \quad (20)$$

- Neutrosophic expected waiting time in queue:

$$NW_q = \rho_N / (\mu_N - \lambda_N)$$

$$NW_q = \left[\frac{\frac{\lambda^L}{\mu^U}}{\mu^U - \lambda^L}, \frac{\frac{\lambda^U}{\mu^L}}{\mu^L - \lambda^U} \right] \quad (21)$$

Using equations (17-21) we can calculate neutrosophic probability of finding k customer in NM/NM/1 queue and neutrosophic performance measures.

Even though M/M/1 queue is the simplest queueing system, but we can clearly see how hard is finding it's formulas in neutrosophic environment, so for other complex queues we may not be able to present the explicit formulas of neutrosophic probabilities and neutrosophic performance measures.

3.2.b Example

In a one-person hair salon, customers arrive according to Poisson process with a mean arrival rate between 2 and 4 customers per hour. Customers are served according to FCFS policy. Customer processing time is exponentially distributed with an average between 10 and 12 min.

We're going to find the probability that an arriving customer will find the salon empty, the probability that an arriving customer will find at least one customer, and then we're going to calculate the salon performance measures.

Solution:

$\lambda_N = [2, 4]$ customers per hour. $\mu_N = [1/12, 1/10]$ customers per minute, i.e. $\mu_N = [5, 6]$ customers per hour.

$\rho_N = \lambda_N / \mu_N = [0.33, 0.80]$

- The probability that arriving customer will find the salon empty:

$$NP(0) = (1 - \rho_N)\rho_N^{\ 0} = (1 - [0.33, 0.80]) = [0.20, 0.67]$$

Thus, the probability that an arriving customer will find the salon empty ranges between 0.2 and 0.67

- The probability that arriving customer will find at least one customer:

$1 - NP(0) = 1 - [0.20, 0.67] = [0.33, 0.80]$ which is the same traffic intensity and so we can say that the probability that arriving customer will find at least one customer ranges between 0.33 and 0.80

- The performance measures:

$$NL_S = \rho_N / (1-\rho_N) = [0.33, 0.80]/[0.20, 0.67] = [0.49, 4]$$

Which means that expected number of customers in system ranges between 0.49 and 4

$$NL_q = \rho_N{}^2 / (1-\rho_N) = [0.33, 0.80]^2 / [0.20, 0.67] = [0.1089, 0.64]/[0.20, 0.67] = [0.1625, 3.2]$$

Which means that expected number of customers in queue ranges between 0.1625 and 3.2

$$NW_S = 1/(\mu_N - \lambda_N) = 1/[1, 4] = [0.25, 1]$$

Which means that mean waiting time in system ranges between 15 mins and 1 hour

$$NW_q = \rho_N / (\mu_N - \lambda_N) = [0.33, 0.80] \cdot [0.25, 1] = [0.0825, 0.80]$$

Which means that mean waiting time in queue ranges between 4.95 mins and 48 mins

3.3 (NM/NM/c):(FCFS/∞/∞) Queue

3.3.a Deriving the Neutrosophic Formulas

NM/NM/c queue is the same as NM/NM/1 queue except that in NM/NM/c customers are being served by c parallel homogeneous servers according to FCFS policy.

The neutrosophic probability that arriving customer will find k customers in the queue will be:

$$NP(k) = \begin{cases} \dfrac{(c\rho_N)^k}{k!} NP(0); k < c \\ \dfrac{\rho_N{}^k c^c}{c!} NP(0); k \geq c \end{cases}$$

$$NP(k) = \begin{cases} \dfrac{\left(c\left[\dfrac{\lambda^L}{\mu^U}, \dfrac{\lambda^U}{\mu^L}\right]\right)^k}{k!} NP(0); k < c \\ \dfrac{\left[\dfrac{\lambda^L}{\mu^U}, \dfrac{\lambda^U}{\mu^L}\right]^k c^c}{c!} NP(0); k \geq c \end{cases} \quad (22)$$

Where:

$$NP(0) = \left(\sum_{n=0}^{c-1} \frac{(c\rho_N)^n}{n!} + \frac{(c\rho_N)^c}{c!} \cdot \frac{1}{1-\rho_N} \right)^{-1}$$

$$NP(0)=\left(\sum_{n=0}^{c-1}\frac{\left(c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^n}{n!}+\frac{\left(c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^c}{c!}\cdot\frac{1}{1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]}\right)^{-1} \tag{23}$$

The neutrosophic performance measures will be then:

$$NL_S=c\rho_N+\frac{(c\rho_N)^c\rho_N}{c!(1-\rho_N)^2}NP(0)$$

$$NL_S=c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]+\frac{\left(c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]}{c!\left(1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^2}NP(0) \tag{24}$$

$$NL_q=\frac{(c\rho_N)^c\rho_N}{c!(1-\rho_N)^2}NP(0)$$

$$NL_q=\frac{\left(c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^c\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]}{c!\left(1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^2}NP(0) \tag{25}$$

$$NW_q=\frac{1}{\lambda_N}NL_q$$

$$NW_q=\left[\frac{1}{\lambda^U},\frac{1}{\lambda^L}\right]NL_q \tag{26}$$

$$NW_S=\frac{1}{\lambda_N}NL_S$$

$$NW_S=\left[\frac{1}{\lambda^U},\frac{1}{\lambda^L}\right]NL_S \tag{27}$$

Deriving exact analytical solutions is very hard as we mentioned before, but we can use rules of interval arithmetic to calculate probabilities and performance measures as we will show in the following example:

3.3.b Example

A car wash station consists of two washing machines, each machine takes between 10 and 12 mins to wash a car. Cars are arriving to the station 6 to 9 cars per hour according to Poisson distribution.

We're going to calculate the probability that an arriving car will find the station empty and then find the performance measures of the car wash station.

Solution:

$\lambda_N = [6,9]$ cars per hour. $\mu_N = [5,6]$ cars per hour.

$$\rho_N = \lambda_N / 2\mu_N = [6,9] / (2\cdot[5,6]) = [6,9]/[10,12] = [0.5, 0.90]$$

- The probability that an arriving car will find the station empty

$$NP(0) = \left(\sum_{n=0}^{1} \frac{(2\rho_N)^n}{n!} + \frac{(2\rho_N)^2}{2!} \cdot \frac{1}{1-\rho_N} \right)^{-1} = \left(1 + 2\rho_N + 2\rho_N{}^2 \cdot \frac{1}{1-\rho_N} \right)^{-1}$$

$$= \left(1 + 2[0.5, 0.90] + 2[0.5, 0.90]^2 \cdot \frac{1}{1-[0.5, 0.90]} \right)^{-1} = \left(1 + [1, 1.8] + [0.5, 1.62] \cdot \frac{1}{[0.1, 0.5]} \right)^{-1}$$

$$= ([2, 2.8] + [1, 1.62])^{-1} = [3, 4.42]^{-1} = [0.2262, 0.3333]$$

Thus:

Thus, the probability that an arriving car will find the station empty ranges between 0.2262 and 0.3333

- The performance measure:

$$NL_S = 2[0.5, 0.90] + \frac{(2[0.5, 0.90])^2 [0.5, 0.90]}{2!(1-[0.5, 0.90])^2} [0.2262, 0.3333]$$

$$= [1, 1.8] + \frac{[0.05655, 0.4859514]}{[0.01, 0.25]} = [1, 1.8] + [0.2262, 48.59514] = [1.2262, 50.39514]$$

Which means that expected number of cars in the station ranges between 1.2262 and 50.9514

$$NL_q = \frac{(2[0.5, 0.90])^2 [0.5, 0.90]}{2!(1-[0.5, 0.90])^2} [0.2262, 0.3333] = [0.2262, 48.59514]$$

Which means that expected number of cars in queue ranges between 0.2262 and 48.59514

$$NW_S = \frac{1}{[6,9]} [1.2262, 50.39514] = [0.1362, 8.3992]$$

Which means that mean waiting time in the station ranges between 0.13 hour and 8.3992 hour

$$NW_q = \frac{1}{\lambda_N} NL_q = \frac{1}{[6,9]} [0.2262, 48.59514] = [0.0251, 8.0992]$$

Which means that mean waiting time in queue ranges between 0.0251 hour and 8.0992 hour

3.4 (NM/NM/1):(FCFS/∞/b) Queue

3.3.a Deriving the Neutrosophic Formulas

Interarrival times and serving times follow neutrosophic exponential distribution, customers are being served by one server according to FCFS policy, the calling source is infinite and system size is finite and determined by b including the one being served.

The neutrosophic probability that arriving customer will find k customers in the queue will be:

$$NP(k) = \frac{\rho_N{}^k (1-\rho_N)}{\left(1-\rho_N{}^{b+1}\right)}; k = 0..b$$

$$NP(k) = \frac{\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^k \left(1-\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]\right)}{\left(1-\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^{b+1}\right)}; k = 0..b \qquad (28)$$

The neutrosophic performance measures will be then:

$$NL_q = \frac{\rho_N{}^2 \left[1-b\rho_N{}^{b-1}+(b-1)\rho_N{}^b\right]}{(1-\rho_N)\left(1-\rho_N{}^{b+1}\right)}$$

$$NL_q = \frac{\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^2 \left[1-b\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^{b-1}+(b-1)\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^b\right]}{\left[1-\frac{\lambda^U}{\mu^L}, 1-\frac{\lambda^L}{\mu^U}\right]\left(1-\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^{b+1}\right)} \qquad (29)$$

$$NL_S = NL_q + Eff\ \rho_N \ ; Eff\ \rho_N = \frac{Eff\ \lambda_N}{\mu_N}, Eff\ \lambda_N = \lambda_N\left(1-NP(b)\right)$$

$$NL_S = NL_q + Eff\ \rho_N \ ; Eff\ \rho_N = \frac{Eff\ \lambda_N}{\left[\mu^L, \mu^U\right]}, Eff\ \lambda_N = \left[\lambda^L, \lambda^U\right]\left(1-NP(b)\right)$$

$$NL_S = NL_q + Eff\ \rho_N \ ; Eff\ \rho_N = \frac{Eff\ \lambda_N}{\left[\mu^L, \mu^U\right]}, Eff\ \lambda_N = \left[\lambda^L, \lambda^U\right]\left(1-NP(b)\right) \qquad (30)$$

Where $Eff\ \rho_N$ is effective rho and $Eff\ \lambda_N$ is effective λ_N that is according to the finite system size so when the queue is full no new customers can join the queue.

$$NW_q = \frac{1}{Eff\ \lambda_N} NL_q$$

$$NW_S = \frac{1}{Eff\ \lambda_N} NL_S$$

Here also we cannot represent the explicit solutions of the mentioned neutrosophic queueing system, so we will use the interval arithmetic rule to find neutrosophic probabilities and neutrosophic performance measures as in the following example:

3.3.b Example

A two-line telephone service with one operator receiving information from a caller, when operator is on talking to a caller on a line, the other line is open for one customer to wait to talk to the operator. When all the two lines are busy new calls are lost.

The telephone service has a rate of 4 to 5 calls per hour following Poisson distribution. Each call takes a duration ranges between 5 to 6 mins.

We're going to calculate the probability that a new call is going to be lost, then we are going to calculate the performance measures.

Solution:

From the problem we can see that $b=2$ and we have then (NM/NM/1):(FCFS/∞/2) queue.

$\lambda_N = [4,5]$ calls per hour. $\mu_N = [60/6, 60/5] = [10,12]$ calls per hour.

$$\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[4,5]}{[10,12]} = [0.33, 0.5]$$

- The probability that a new call is going to be lost is the probability that the system is full, i.e. probability that there are two calls, that is:

$$NP(2) = \frac{\rho_N{}^2(1-\rho_N)}{(1-\rho_N{}^{2+1})} = \frac{[0.33,0.5]^2(1-[0.33,0.5])}{1-[0.33,0.5]^3} = \frac{[0.1089,0.25][0.5,0.67]}{1-[0.035937,0.125]} = \frac{[0.05445,0.1675]}{[0.875,0.964063]}$$

$$= [0.05648, 0.19143]$$

Which means that the probability that new call is going to be lost ranges between 0.05648 and 0.19143

- To calculate he neutrosophic performance measures we must first calculate the effective lambda and the effective rho:

$$Eff\ \lambda_N = \lambda_N(1-NP(b)) = \lambda_N(1-NP(2)) = [4,5](1-[0.05648,0.19143]) = [3.23428, 4.7176]$$

$$Eff\ \rho_N = \frac{[3.23428, 4.7176]}{[10,12]} = [0.269523, 0.47176]$$

So, the performance measures will be:

$$NL_q = \frac{\rho_N{}^2\left[1-b\rho_N{}^{b-1}+(b-1)\rho_N{}^b\right]}{(1-\rho_N)(1-\rho_N{}^{b+1})} = \frac{[0.33,0.5]^2\left[1-2[0.33,0.5]+[0.33,0.5]^2\right]}{(1-[0.33,0.5])\left(1-[0.33,0.5]^3\right)}$$

$$= \frac{[0.01556181, 0.1475]}{[0.4375, 0.61699392]} = [0.02522, 0.33714]$$

Means that average number of waiting calls will be between 0.02522 and 0.33714

$$NL_S = NL_q + Eff\ \rho_N = [0.02522, 0.33714] + [0.269523, 0.47176] = [0.294743, 0.8089]$$

Means that average number of calls in the system will be between 0.294743 and 0.8089

$$NW_q = \frac{1}{Eff\ \lambda_N} NL_q = \frac{[0.02522, 0.33714]}{[3.23428, 4.7176]} = [0.005346, 0.1042396]$$

Means that expected holding time to the new call will be between 0.005346 and 0.1042396 hour or between 19 secs and 6.25 mins

$$NW_S = \frac{1}{Eff\ \lambda_N} NL_S = \frac{[0.294743, 0.8089]}{[3.23428, 4.7176]} = [0.062477, 0.250102]$$

Means that expected time of a call duration including holding time will be between 0.062477 and 0.250102 hour or between 3.75 and 15 mins.

4. Difference between Neutrosophic Queueing Systems and Crisp Queueing Systems:

In crisp queueing systems we estimate parameters of queueing systems by its rates or means e.g. λ, μ, but in reality, we must take impreciseness in consideration, i.e. we must replace these parameters by intervals. This problem is solved using neutrosophic sets theory by replacing these parameters using neutrosophic numbers $\lambda_N = [\lambda^L, \lambda^U], \mu_N = [\mu^L, \mu^U]$. In the following theorem we are going to prove that crisp queue is a special case of neutrosophic queue.

Theorem: Neutrosophic performance measures are generalized form of crisp performance measures:

It's easy to see that crisp number is a special case of neutrosophic number when upper and lower limits of the intervals are equal. We will prove theorem 1 for M/M/1 queue and other queues can be proven in same way:

Proof:

By substituting $\lambda^L = \lambda^U = \lambda$, $\mu^L = \mu^U = \mu$ in (17-21) we find:

$$NP(k)=\left[\left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^k,\left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^k\right]$$

$$=\left[(1-\rho)(\rho)^k,(1-\rho)(\rho)^k\right]=(1-\rho)(\rho)^k=P(k)$$

$$NL_S=\left[\frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}},\frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}}\right]=\left[\frac{\rho}{1-\rho},\frac{\rho}{1-\rho}\right]=\frac{\rho}{1-\rho}=L_S$$

$$NL_q=\left[\frac{\left(\frac{\lambda}{\mu}\right)^2}{1-\frac{\lambda}{\mu}},\frac{\left(\frac{\lambda}{\mu}\right)^2}{1-\frac{\lambda}{\mu}}\right]=\left[\frac{\rho^2}{1-\rho},\frac{\rho^2}{1-\rho}\right]=\frac{\rho^2}{1-\rho}=L_q$$

$$NW_S=\left[\frac{1}{\mu-\lambda},\frac{1}{\mu-\lambda}\right]=\frac{1}{\mu-\lambda}=W_S$$

$$NW_q=\left[\frac{\frac{\lambda}{\mu}}{\mu-\lambda},\frac{\frac{\lambda}{\mu}}{\mu-\lambda}\right]=\left[\frac{\rho}{\mu-\lambda},\frac{\rho}{\mu-\lambda}\right]=\frac{\rho}{\mu-\lambda}=W_q$$

So crisp queueing theory is special case of neutrosophic queueing theory.

5. Conclusions

We conclude that neutrosophic queueing theory is more general and can solve problems cannot be solved using classical logic specially when we deal with imprecise data and unknown exactly parameters. We have presented three types of queues in neutrosophic environment, (NM/NM/1):(FCFS/∞/∞) queue, (NM/NM/c):(FCFS/∞/∞) queue and (NM/NM/1):(FCFS/∞/b) queue.

For (NM/NM/1):(FCFS/∞/∞) queue we have presented the explicit solutions including neutrosophic probabilities and neutrosophic performance measures, but for other complex queues solutions was not presented explicitly. We have presented solved examples after each queueing system. We look forward to study other types of queueing systems in neutrosophic logic like phase type queues, batch arrivals queues, etc...

6. References

1. F Shortle J.; M Thompson J.; Gross D.; M Harris C., Fundamentals of Queueing Theory, 5th ed.; Wiley: United States of America, 2018; pp. 1–27, 77–96.
2. Nick T. Thomopoulos; Fundamentals of Queuing Systems, 1st ed.; Springer: United States of America, 2012; pp. 19–26, 41–48.
3. Mohamed Bisher Zeina, Khudr Al-Kridi and Mohammed Taher Anan, New Approach to FM/FM/1 Queue's Performance Measures, Journal of King Abdulaziz University: "Science", Vol.30, No. 1., 2017
4. Chen, S.P., Eur. J., Parametric Nonlinear Programming for Analyzing Fuzzy Queues with Infinite Capacity, Oper. Res., Vol. 157, 2009
5. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.
6. Smarandache, F, Neutrosophic set a generalization of the intuitionistic fuzzy sets. Inter. J. Pure Appl. Math., 24, 287 – 297, 2005.
7. Smarandache. F, Introduction to Neutrosophic Measure, Integral, Probability. Sitech Education publisher, 2015.
8. Smarandache, F, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA,2002.
9. Rafif Alhabib, Moustafa Mzher Ranna, Haitham Farah, A.A. Salama, Some Neutrosophic Probability Distributions. Neutrosophic Sets and Systems, Vol. 22, 2018.
10. Salama.A.A, Smarandache. F, Neutrosophic Crisp Set Theory. Education Publishing, Columbus, 2015.
11. Patro.S.K, Smarandache. F. the Neutrosophic Statistical Distribution, More Problems, More Solutions. Neutrosophic Sets and Systems, Vol. 12, 2016.
12. Smarandache. F, Neutrosophical statistics, Sitech & Education publishing, 2014.
13. Smarandache, F. & Pramanik, S. (Eds). (2018). New trends in neutrosophic theory and applications, Vol.2. Brussels: Pons Editions.
14. Smarandache, F. & Pramanik, S. (Eds). (2016). New trends in neutrosophic theory and applications. Brussels: Pons Editions.

15. Alhabib, R., A. A. Salama,Using Moving Averages To Pave The Neutrosophic Time Series, International Journal of Neutrosophic Science, Vol. 3, Issue 1, pp. 14-20 , 2020
16. Kawther Fawzi Hamza Alhasan, F. Smarandache, “Neutrosophic Weibull distribution and Neutrosophic Family Weibull Distribution”, Neutrosophic Sets and Systems, vol. 28, pp. 191-199, 2019
17. Bohlender. G; Kulisch. U, Definition of the Arithmetic Operations and Comparison Relations for an Interval Arithmetic Standard, Institut fur Angewandte und Numerische Mathematik, Universitat Karlsurhe, Germany, 2010.
18. Mohamed Bisher Zeina, Neutrosophic Event-Based Queueing Model, International Journal of Neutrosophic Science, Vol. 6, No. 1, PP. 48-55, 2020
19. Mohamed Bisher Zeina, Erlang Service Queueing Model with Neutrosophic Parameters, International Journal of Neutrosophic Science, Vol. 6, No. 2, PP. 106-112, 2020.