Int. J. Math. And Appl., 9(2)(2021), 127–136
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Neutrosophic Micro Ideal Topological Structure

S. Ganesan[1,*], C. Alexander[1], A. Pandi[2] and F. Smarandache[3]

1 PG & Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai, Tamil Nadu, India. (Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India).

2 Department of Mathematics, Rathinam Technical Campus, Coimbatore, Tamil Nadu, India.

3 Mathematics & Science Department, University of New Maxico, 705 Gurley Ave, Gallup, NM 87301, USA.

Abstract: This article addressed the concept of neutrosophic micro ideal topology which is induced by the two litereture, they are ideal topological spaces and micro topology. We defined its local function, closed set and also defined and give new dimnesion to codense ideal by incorporating it to ideal topological structures. we investigate some properties of neutrosophic micro topology with ideal. Also we introduce a new definition of neutrosophic micro topological space like neutrosophic micro α-open, neutrosophic micro pre-open, neutrosophic micro semi-open, neutrosophic micro b-open, neutrosophic micro β-open, neutrosophic micro regular-open and neutrosophic micro π-open.

MSC: 03E72

Keywords: Neutrosophic set, ideal topological spaces, neutrosophic micro topology, neutrosophic micro ideal local function.

1. Introduction

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A \Rightarrow B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^* : \wp(X) \to \wp(X)$, called a local function [8] of A with respect to τ and $\mathcal{I}$ is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau \mid x \in U\}$. We will make use of the basic facts about the local functions [[7], Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I}, \tau)$, called the $\star$-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\mathcal{I}, \tau)$ [13]. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{I}, \tau)$ and τ^* for $\tau^*(\mathcal{I}, \tau)$. If $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal space. $\mathcal{N}$ is the ideal of all nowhere dense subsets in (X, τ). A subset A of an ideal space $(X, \tau, \mathcal{I})$ is $\star$-closed [7] (resp. $\star$-dense in itself [5]) if $A^* \subseteq A$ (resp. $A \subseteq A^*$). By a space, we always mean a topological space (X, τ) with no separation properties assumed. If $A \subseteq X$, $cl(A)$ and $int(A)$ will, respectively, denote the closure and interior of A in (X, τ) and $int^*(A)$ will denote the interior of A in (X, τ^*). L. A. Zadeh's [14] Fuzzy set laid the foundation of many theories such as intuitionistic fuzzy set and neutrosophic set, rough sets etc. Later, researchers developed K. T. Atanassov's [1] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computerscience and so on. F. Smarandache [11, 12] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. S. Chandrasekar [2] introduced and studied the new concept of micro topological spaces like micro interior, micro closure, micro continuous respectively.

* E-mail: sgsgsgsgsg77@mail.com

The notion of a neutrosophic micro topology was introduced and studied by S. Ganesan et al [3] which was defined neutrosophic micro closed, neutrosophic micro open, neutrosophic micro interior, neutrosophic micro closure and neutrosophic micro continuopus. The main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic micro ideal topology. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this article is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of neutrosophic micro ideal topology is investigated with some properties and also defined the codense ideal in neutrosophic micro topological structure. Also we introduce a new definition of neutrosophic micro topological space like neutrosophic micro α-open, neutrosophic micro pre-open, neutrosophic micro semi-open, neutrosophic micro b-open, neutrosophic micro β-open, neutrosophic micro regular-open and neutrosophic micro π-open.

2. Preliminary

Definition 2.1 ([11, 12]). *A neutrosophic set (in short ns) K on a set $X \neq \emptyset$ is defined by $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$, where $P_K : X \to [0,1], Q_K : X \to [0,1]$ and $R_K : X \to [0,1]$ denotes the membership of an object, indeterminacy and non-membership of an object, for each $a \in X$ to K, respectively and $0 \leq P_K(a) + Q_K(a) + R_K(a) \leq 3$ for each $a \in X$.*

Definition 2.2 ([10]). *Let $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$ be a ns. We must introduce the ns $0_\sim$ and $1_\sim$ in X as follows:*

$0_\sim$ *may be defined as:*

(1). $0_\sim = \{\prec x, 0, 0, 1 \succ : x \in X\}$

(2). $0_\sim = \{\prec x, 0, 1, 1 \succ : x \in X\}$

(3). $0_\sim = \{\prec x, 0, 1, 0 \succ : x \in X\}$

(4). $0_\sim = \{\prec x, 0, 0, 0 \succ : x \in X\}$

$1_\sim$ *may be defined as:*

(1). $1_\sim = \{\prec x, 1, 0, 0 \succ : x \in X\}$

(2). $1_\sim = \{\prec x, 1, 0, 1 \succ : x \in X\}$

(3). $1_\sim = \{\prec x, 1, 1, 0 \succ : x \in X\}$

(4). $1_\sim = \{\prec x, 1, 1, 1 \succ : x \in X\}$

Proposition 2.3 ([10]). *For any ns S, then the following conditions are holds:*

(1). $0_\sim \leq S$, $0_\sim \leq 0_\sim$.

(2). $S \leq 1_\sim$, $1_\sim \leq 1_\sim$.

Definition 2.4 ([10]). *Let $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$ be a ns.*

(1). A ns K is an empty set i.e., $K = 0_\sim$ if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e., $0_\sim = \{x, (0, 0, 1) : x \in X\}$.

(2). A ns K is a universal set i.e., $K = 1_\sim$ if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively. $1_\sim = \{x, (1,1,0) : x \in X\}$.

(3). $K_1 \cup K_2 = \{a, \max\{P_{K_1}(a), P_{K_2}\}(a)\}, \max\{Q_{K_1}(a), Q_{K_2}(a)\}, \min\{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$.

(4). $K_1 \cap K_2 = \{a, \min\{P_{K_1}(a), P_{K_2}\}(a)\}, \min\{Q_{K_1}(a), Q_{K_2}(a)\}, \max\{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$.

(5). $K_1^C = \{\prec a, R_K(a), 1 - Q_K(a), P_K(a) \succ : a \in X\}$.

Definition 2.5 ([10]). *A neutrosophic topology (nt) in Salama's sense on a nonempty set X is a family τ of ns in X satisfying three axioms:*

(1). Empty set ($0_\sim$) and universal set ($1_\sim$) are members of τ.

(2). $K_1 \cap K_2 \in \tau$ *where* $K_1, K_2 \in \tau$.

(3). $\cup K_\delta \in \tau$ *for every* $\{K_\delta : \delta \in \Delta\} \leq \tau$.

Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.6 ([9]). *Let U be a non-empty set and R be an equivalence relation on U. Let F be a neutrosophic set in U with the membership function μ_F, the indeterminancy function σ_F and the non-membership function ν_F. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of F in the approximation (U, R) denoted by $\underline{N}(F)$, $\overline{N}(F)$ and $BN(F)$ are respectively defined as follows:*

(1). $\underline{N}(F) = \{\prec x, \mu_{\underline{R}A}(x)\sigma_{\underline{R}A}(x)\nu_{\underline{R}A}(x) \succ : y \in [X]_R, x \in U\}$.

(2). $\overline{N}(F) = \{\prec x, \mu_{\overline{R}A}(x)\sigma_{\overline{R}A}(x)\nu_{\overline{R}A}(x) \succ : y \in [X]_R, x \in U\}$.

(3). $BN(F) = \overline{N}(F) - \underline{N}(F)$.

where $\mu_{\underline{R}A}(x) = \bigwedge_{y\in[X]_R} \mu A(y)$, $\sigma_{\underline{R}A}(x) = \bigwedge_{y\in[X]_R} \sigma A(y)$, $\nu_{\underline{R}A}(x) = \bigwedge_{y\in[X]_R} \nu A(y)$. $\mu_{\overline{R}A}(x) = \bigwedge_{y\in[X]_R} \mu A(y)$, $\sigma_{\overline{R}A}(x) = \bigwedge_{y\in[X]_R} \sigma A(y)$, $\nu_{\overline{R}A}(x) = \bigwedge_{y\in[X]_R} \nu A(y)$.

Definition 2.7 ([9]). *Let U be an universe, R be an equivalence relation on U and F be a neutrosophic set in U and if the collection $\tau_N(F) = \{0_N, 1_N, \underline{N}(F), \overline{N}(F), BN(F)\}$ forms a topology then it is said to be a neutrosophic nano topology. We call $(U, \tau_N(F))$ as the neutrosophic nano topological space. The elements of $\tau_N(F)$ are called neutrosophic nano open sets.*

Definition 2.8 ([9]). *Let U be a nonempty set and the neutrosophic sets A and B in the form* $A = \{\prec x : \mu A(x), \sigma A(x), \nu A(x) \succ, x \in U\}$, $B = \{\prec x : \mu B(x), \sigma B(x), \nu B(x) \succ, x \in U\}$. *Then the following statements hold*

(1). $0_N = \{x, (0,0,1) : x \in U\}$.

(2). $1_N = \{x, (1,1,0) : x \in U\}$.

(3). $A \vee B = \{a, \max\{\mu A(x), \mu B(x)\}, \max\{\sigma A(x), \sigma B(x)\}, \min\{\nu A(x), \nu B(x)\} : x \in U\}$.

(4). $A \wedge B = \{a, \min\{\mu A(x), \mu B(x)\}, \min\{\sigma A(x), \sigma B(x)\}, \max\{\nu A(x), \nu B(x)\} : x \in U\}$.

(5). $A^C = \{\prec a, \nu A(x), 1 - \sigma A(x), \mu A(x) \succ : x \in U\}$.

(6). $A \subseteq B$ *iff* $\mu A(x) \leq \mu B(x)$, $\sigma A(x) \leq \sigma B(x)$, $\nu A(x) \leq \nu B(x)$ *for all* $x \in U$.

(7). $A = B$ iff $A \leq B$, $B \leq A$.

Definition 2.9 ([9]). *If $(U, \tau_N(F))$ is a neutrosophic nano topological space with respect to neutrosophic subset of U and if A be any neutrosophic subset of U, then the neutrosophic nano interior of A is defined as the union of all neutrosophic nano open subsets of A and it is denoted by $N_F int(A)$. That is, $N_F int(A)$ is the largest neutrosophic nano open subset of A. The neutrosophic nano closure of A is defined as the intersection of all neutrosophic nano closed sets containing A and it is denoted by $N_F cl(A)$. That is, $N_F cl(A)$ is the smallest neutrosophic nano closed set containing A.*

Definition 2.10 ([3]). *Let $(U, \tau_N(F))$ be a neutrosophic nano topological space. Then, $\lambda_N(F) = \{S \vee (\acute{S} \wedge \lambda) : S, \acute{S} \in \tau_N(F)$ and $\lambda \notin \tau_N(F)\}$ is called the neutrosophic micro topology on U with respect to F. The triplet $(U, \tau_N(F), \lambda_N(F))$ is called neutrosophic micro topological space.*

Definition 2.11 ([3]). *The neutrosophic micro topology $\lambda_N(F)$ satisfies the following axioms*

(1). U, $\phi \in \lambda_N(F)$.

(2). The union of the elements of any sub-collection of $\lambda_N(F)$ is in $\lambda_N(F)$.

(3). The intersection of the elements of any finite sub collection of $\lambda_N(F)$ is in $\lambda_N(F)$.

Then $\lambda_N(F)$ is called the neutrosophic micro topology on U with respect to F. The triplet $(U, \tau_N(F), \lambda_N(F))$ is called neutrosophic micro topological spaces and the elements of $\lambda_N(F)$ are called neutrosophic micro open sets and the complement of a neutrosophic micro open set is called a neutrosophic micro closed set.

Definition 2.12 ([3]). *If $(U, \tau_N(F), \lambda_N(F))$ is a neutrosophic micro topological space with respect to neutrosophic subset of U and if A be any neutrosophic subset of U, then the neutrosophic nano interior of A is defined as the union of all neutrosophic micro open subsets of A and it is denoted by N_{mi}-int(A). That is, N_{mi}-int(A) is the largest neutrosophic micro open subset of A. The neutrosophic micro closure of A is defined as the intersection of all neutrosophic micro closed sets containing A and it is denoted by N_{mi}-cl(A). That is, N_{mi}-cl(A) is the smallest neutrosophic micro closed set containing A.*

Theorem 2.13 ([3]). *Let $(U, \tau_N(F), \lambda_N(F))$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U. Let A and B be neutrosophic subsets of U. Then the following statements hold :*

(1). $A \leq N_{mi} - cl(A)$.

(2). A is a neutrosophic micro closed set if and only if $N_{mi} - cl(A) = A$.

(3). $N_{mi} - cl(0_\sim) = 0_\sim$ and $N_{mi} - cl(1_\sim) = 1_\sim$.

(4). $A \leq B$ implies $N_{mi} - cl(A) \leq N_{mi} - cl(B)$.

(5). $N_{mi} - cl(A \vee B) \leq N_{mi} - cl(A) \vee N_{mi} - cl(B)$.

(6). $N_{mi} - cl(A \wedge B) \leq N_{mi} - cl(A) \wedge N_{mi} - cl(B)$.

(7). $N_{mi}(N_{mi} - cl(A)) = N_{mi} - cl(A)$.

(8). $N_{mi} - int(U - A) = X - N_{mi} - cl(A)$.

Theorem 2.14 ([3]). *Let $(U, \tau_N(F), \lambda_N(F))$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U. Let A and B be neutrosophic subsets of U. Then the following statements hold :*

(1). $A \leq N_{mi} - int(A)$.

(2). A is a neutrosophic micro open set if and only if $N_{mi} - int(A) = A$.

(3). $N_{mi} - int(0_\sim) = 0_\sim$ *and* $N_{mi} - int(1_\sim) = 1_\sim$.

(4). $A \leq B$ *implies* $N_{mi} - int(A) \leq N_{mi} - int(B)$.

(5). $N_{mi} - int(A \vee B) \leq N_{mi} - int(A) \vee N_{mi} - int(B)$.

(6). $N_{mi} - int(A \wedge B) \leq N_{mi} - int(A) \wedge N_{mi} - int(B)$.

(7). $N_{mi}(N_{mi} - int(A)) = N_{mi} - int(A)$.

(8). $N_{mi} - cl(U - A) = X - N_{mi} - int(A)$.

Theorem 2.15 ([3]). *$(U, \tau_N(F), \lambda_N(F))$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U. Let A be a neutrosophic subset of U. Then*

(1). $1_\sim - N_{mi} - int(A) = N_{mi} - cl(1_\sim - A)$.

(2). $1_\sim - N_{mi} - cl(A) = N_{mi} - int(1_\sim - A)$.

Remark 2.16 ([3]). *Taking complements on either side of (1) and (2), Theorem 2.15, we get* $(N_{mi} - int(A)) = 1_\sim - N_{mi} - cl(1_\sim - A))$ *and* $(N_{mi} - cl(A)) = 1_\sim - (N_{mi} - int(1_\sim - A))$.

3. Neutrosophic Micro Ideal Topological Spaces

In 1990, Jankovic and Hamlett [6, 7] have considered the local function in ideal topological space any they have obtained a new topology. In this section we shall introduce a similar type with the local function in neutrosophic micro topological spaces. Before starting the discussion we shall consider the following concepts. Let $(K, \mathcal{S}, \mathcal{G})$ be a neutrosophic micro topological space, where $\mathcal{S} = \tau_N(F)$ and $\mathcal{G} = \lambda_N(F)$.

A neutrosophic micro topological space $(K, \mathcal{S}, \mathcal{G})$ with an ideal $\mathcal{I}$ on K is called a neutrosophic micro ideal topological space and is denoted by $(K, \mathcal{S}, \mathcal{G}, \mathcal{I})$.

Let $(K, \mathcal{G})$ be a neutrosophic micro topological space and $V(_m)_\mathcal{N}(k_1, k_2, k_3) = \{V(_m)_\mathcal{N} \mid (k_1, k_2, k_3) \in V(_m)_\mathcal{N}, V(_m)_\mathcal{N} \in \mathcal{G}\}$ be the family of neutrosophic micro open sets which contain (k_1, k_2, k_3). If we take, $(k_1, k_2, k_3) = k$.

Definition 3.1. *Let $(K, \mathcal{S}, \mathcal{G})$ be a neutrosophic micro topological space with an ideal $\mathcal{I}$ on K and if $\wp(K)$ is the set of all subsets of K, a set operator $(.)^*_\mathcal{N} : \wp(K) \to \wp(K)$. For a subset $A \subset K$, $A^*_\mathcal{N}(\mathcal{I}, \mathcal{G}) = \{(k_1, k_2, k_3) \in K : V(_m)_\mathcal{N} \wedge A \notin \mathcal{I}$, for every $V(_m)_\mathcal{N} \in V(_m)_\mathcal{N}(k_1, k_2, k_3)\}$ is called the neutrosophic micro local function (brielfy, $\mathcal{N}_m$-local function) of A with repect to $\mathcal{I}$ and $\mathcal{G}$. We will simply write $A^*_\mathcal{N}$ for $A^*_\mathcal{N}$ $(\mathcal{I}, \mathcal{G})$.*

Example 3.2. *$(K, \mathcal{S}, \mathcal{G})$ be a neutrosophic micro topological space with an ideal $\mathcal{I}$ on K and for every $A \leq K$.*

(1). If $\mathcal{I} = \{0_\sim\}$ *then* $A^*_\mathcal{N} = N_{mi} - cl(A)$,

(2). If $\mathcal{I} = P(K)$, *then* $A^*_\mathcal{N} = 0_\sim$.

Theorem 3.3. *Let $(K, \mathcal{S}, \mathcal{G})$ be a neutrosophic micro topological space with ideal $\mathcal{I}$, $\mathcal{I}'$ on K and A, B be subsets of K. Then*

(1). $A \leq B \Rightarrow A^*_{\mathcal{N}} \leq B^*_{\mathcal{N}}$,

(2). $\mathcal{I} \leq \mathcal{I}' \Rightarrow A^*_{\mathcal{N}}(\mathcal{I}') \leq A^*_{\mathcal{N}}(\mathcal{I})$,

(3). $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) \leq N_{mi} - cl(A)(A^*_{\mathcal{N}}$ is a neutrosophic micro closed subset of $N_{mi} - cl(A))$,

(4). $(A^*_{\mathcal{N}})^*_{\mathcal{N}} \leq A^*_{\mathcal{N}}$,

(5). $A^*_{\mathcal{N}} \vee B^*_{\mathcal{N}} = (A \vee B)^*_{\mathcal{N}}$,

(6). $A^*_{\mathcal{N}} - B^*_{\mathcal{N}} = (A - B)^*_{\mathcal{N}} - B^*_{\mathcal{N}} \leq (A - B)^*_{\mathcal{N}}$,

(7). $U \in \mathcal{G} \Rightarrow U \wedge A^*_{\mathcal{N}} = U \wedge (U \wedge A)^*_{\mathcal{N}} \leq (U \wedge A)^*_{\mathcal{N}}$ and

(8). $F \in \mathcal{I} \Rightarrow (A \vee F)^*_{\mathcal{N}} = A^*_{\mathcal{N}} = (A - F)^*_{\mathcal{N}}$, and so $A^*_{\mathcal{N}} = 0_{\sim}$, if $A \in \mathcal{I}$.

Proof.

(1). Let $A \leq B$ and $k \in A^*_{\mathcal{N}}$. Assume that $k \notin B^*_{\mathcal{N}}$. We have $V(_m)_{\mathcal{N}} \wedge B \in \mathcal{I}$ for some $V(_m)_{\mathcal{N}} \in V(_m)_{\mathcal{N}}(k)$. Since $V(_m)_{\mathcal{N}} \wedge A \leq V(_m)_{\mathcal{N}} \wedge B$ and $V(_m)_{\mathcal{N}} \wedge B \in \mathcal{I}$, we obtain $V(_m)_{\mathcal{N}} \wedge A \in \mathcal{I}$ from the definition of ideal. Thus, we have $k \notin A^*_{\mathcal{N}}$. This is a contradiction. Clearly, $A^*_{\mathcal{N}} \leq B^*_{\mathcal{N}}$.

(2). Let $\mathcal{I} \leq \mathcal{I}'$ and $k \in A^*_{\mathcal{N}}(\mathcal{I}')$. Then we have $V(_m)_{\mathcal{N}} \wedge A \notin \mathcal{I}'$ for every $V(_m)_{\mathcal{N}} \in V(_m)_{\mathcal{N}}(k)$. By hypothesis, we obtain $V(_m)_{\mathcal{N}} \wedge A \notin \mathcal{I}$. So $k \in A^*_{\mathcal{N}}(\mathcal{I})$.

(3). Let $k \in A^*_{\mathcal{N}}$. Then for every $V(_m)_{\mathcal{N}} \in V(_m)_{\mathcal{N}}(k)$, $V(_m)_{\mathcal{N}} \wedge A \in \mathcal{I}$. This implies that $V(_m)_{\mathcal{N}} \wedge A \neq 0_{\sim}$. Hence $k \in N_{mi} - cl(A)$.

(4). From (3), $(A^*_{\mathcal{N}})^*_{\mathcal{N}} \leq N_{mi} - cl(A^*_{\mathcal{N}}) = A^*_{\mathcal{N}}$, since $A^*_{\mathcal{N}}$ is a neutrosophic micro closed set.

The proof of the other conditions are also obvious. □

Theorem 3.4. *Let $(K, \mathcal{S}, \mathcal{G})$ be an neutrosophic micro topological space with an ideal $\mathcal{I}$ and $A \leq A^*_{\mathcal{N}}$, then $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) = N_{mi} - cl(A)$.*

Proof. For every subset A of K, we have $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) = N_{mi} - cl(A)$, by Theorem 3.3 (3), $A \leq A^*_{\mathcal{N}}$ implies that $N_{mi} - cl(A) \leq N_{mi} - cl(A^*_{\mathcal{N}})$ and so $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) = N_{mi} - cl(A)$. □

Definition 3.5. *Let $(K, \mathcal{S}, \mathcal{G})$ be an neutrosophic micro topological space with an ideal $\mathcal{I}$ on K. The set operator $N_{mi} - cl^*$ is called a neutrosophic micro $\star$-closure and is defined as $N_{mi} - cl^*(A) = A \vee A^*_{\mathcal{N}}$ for $A \leq K$.*

Theorem 3.6. *The set operator $N_{mi} - cl^*$ satisfies the following conditions:*

(1). $A \leq N_{mi} - cl^*(A)$,

(2). $N_{mi} - cl^*(0_{\sim}) = 0_{\sim}$ and $N_{mi} - cl^*(1_{\sim}) = 1_{\sim}$,

(3). If $A \leq B$, then $N_{mi} - cl^*(A) \leq N_{mi} - cl^*(B)$,

(4). $N_{mi} - cl^*(A) \vee N_{mi} - cl^*(B) = N_{mi} - cl^*(A \vee B)$,

(5). $N_{mi} - cl^*(N_{mi} - cl^*(A)) = N_{mi} - cl^*(A)$.

Proof. The proof is clear from Theorem 3.3 and Definition 3.5. □

Now, $\mathcal{G}^*(\mathcal{I},\mathcal{G}) = \{U \subset K; N_{mi} - cl^*(K-U) = K-U\}$. $\mathcal{G}^*(\mathcal{I},\mathcal{G})$ is called neutrosophic micro $\star$-topology which is finer than $\mathcal{G}$ (we simply write $\mathcal{G}^*$ for $\mathcal{G}^*(\mathcal{I},\mathcal{G})$. The elements of $\mathcal{G}^*(\mathcal{I},\mathcal{G})$ are called neutrosophic micro $\star$-open (briefly, $\mathcal{N}m\star$-open) set and the complement of an $\mathcal{N}m\star$-open set is called is called neutrosophic micro $\star$-closed (briefly, $\mathcal{N}m\star$-closed) set. Here $N_{mi} - cl^*(A)$ and $N_{mi} - int^*(A)$ will denote the closure and interior of A in $(K,\mathcal{G}^*)$.

Remark 3.7.

(1). See Example 3.2 (1), if $\mathcal{I} = \{0_\sim\}$ then $A^*_{\mathcal{N}} = N_{mi} - cl(A)$. In this case, $N_{mi} - cl^*(A) = N_{mi} - cl(A)$.

(2). If $(K,\mathcal{N},\mathcal{G},\mathcal{I})$ is a neutrosophic micro ideal topological space with $\mathcal{I} = \{0_\sim\}$, then $\mathcal{G}^* = \mathcal{G}$.

Definition 3.8. *A basis $\beta(\mathcal{I},\mathcal{G})$ for $\mathcal{G}^*$ can be described as follows: $\beta(\mathcal{I},\mathcal{G}) = \{V - F : V \in \mathcal{G}, F \in \mathcal{I}\}$.*

Lemma 3.9. *Let $(K,\mathcal{S},\mathcal{G})$ be a neutrosophic micro topological space and $\mathcal{I}$ be an ideal on K. Then β $(\mathcal{I},\mathcal{G})$ is a basis for $\mathcal{G}^*$.*

We have to show that for a given space $(K,\mathcal{S},\mathcal{G})$ and an ideal $\mathcal{I}$ on K, $\beta(\mathcal{I},\mathcal{G})$ is a basis for $\mathcal{G}^*$. If $\beta(\mathcal{I},\mathcal{G})$ is itself a neutrosophic micro topology, then we have If $\beta(\mathcal{I},\mathcal{G}) = \mathcal{G}^*$ and all the neutrosophic micro open sets of $\mathcal{G}^*$ are of simple form $V - F$, where $V \in \mathcal{G}$ and $F \in \mathcal{I}$.

Lemma 3.10. *If $(K,\mathcal{S},\mathcal{G})$ is a neutrosophic micro topological space and $\mathcal{I}$ be an ideal on K and if $F \in \mathcal{I}$, then F is $\mathcal{N}m\star$-closed and if $A \subset K$ is $\mathcal{N}m\star$-closed then $A^*_{\mathcal{N}} \leq A$.*

Theorem 3.11. *$(K,\mathcal{S},\mathcal{G})$ be an neutrosophic micro topological space with an ideal $\mathcal{I}$ on K and for every $A \leq K$. If $A \leq A^*_{\mathcal{N}}$, then*

(1). $N_{mi} - cl(A) = N_{mi} - cl^*(A)$,

(2). $N_{mi} - int(K-A) = N_{mi} - int^*(K-A)$,

(3). $N_{mi} - cl(K-A) = N_{mi} - cl^*(K-A)$,

(4). $N_{mi} - int(A) = N_{mi} - int^*(A)$.

Proof.

(1). Follows immediately from Theorem 3.4.

(2). If $A \leq A^*_{\mathcal{N}}$, then $N_{mi} - cl(A) = N_{mi} - cl^*(A)$ by (1) and so $K - N_{mi} - cl(A) = K - N_{mi} - cl^*(A)$. Therefore, $N_{mi} - int(K-A) = N_{mi} - int^*(K-A)$.

(3). Follows by replacing A by $K - A$ in(1).

(4). If $A \leq A^*_{\mathcal{N}}$ then $N_{mi} - cl(K-A) = N_{mi} - cl^*(K-A)$ by (3) and so $K - N_{mi} - cl(K-A) = K - N_{mi} - cl^*(K-A)$. Therefore, $N_{mi} - int(A) = N_{mi} - int^*(A)$. □

Theorem 3.12. *$(K,\mathcal{S},\mathcal{G})$ be an neutrosophic micro topological space with an ideal $\mathcal{I}$ on K and for every $A \leq K$. If $A \leq A^*_{\mathcal{N}}$, then $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) = N_{mi} - cl(A) = N_{mi} - cl^*(A)$.*

Proof. Follows from Thoerem 3.4 and Theorem 3.11 (1). □

Definition 3.13. *A subset A of a neutrosophic micro ideal topological space $(K, \mathcal{N}, \mathcal{G}, \mathcal{I})$ is $\mathcal{N}m\star$ dense in itself (resp. $\mathcal{N}m\star$-perfect, $\mathcal{N}m\star$-closed) if $A \leq A^*_{\mathcal{N}}$ (resp. $A = A^*_{\mathcal{N}}$, $A^*_{\mathcal{N}} \leq A$).*

Remark 3.14. *We have the following diagram*

$$\mathcal{N}m\star \text{ dense in itself} \Leftarrow \mathcal{N}m\star\text{-perfect} \Rightarrow \mathcal{N}m\star\text{-closed}$$

The following examples show that the converse implications of the diagram are not satisfied.

Example 3.15. *Let $K = \{P\}$ be the universe of discourse. Let $K/R = \{P\}$ be an equivalence relation on K and $A = \prec (0.8, 0.5, 0.5) \succ$ be a neutrosophic set on K then $\underline{N}(A) = \{\prec (0.2, 0.5, 0.5) \succ\}$, $\overline{N}(A) = \{\prec (0.8, 0.5, 0.5) \succ\}$, $B(A) = \{\prec (0.5, 0.5, 0.5) \succ\}$. Then the collection $\tau_N(A) = \{0_\sim, \prec (0.2, 0.5, 0.5) \succ, \prec (0.8, 0.5, 0.5) \succ, \prec (0.5, 0.5, 0.5) \succ, 1_\sim\}$. Then $\lambda = \prec (0.6, 0.7, 0.8) \succ$. Neutrosophic micro open $= \lambda_N(A) = \{0_\sim, \prec (0.6, 1, 0.8) \succ, \prec (0.2, 1, 0.8) \succ, \prec (0.5, 1, 0.8) \succ, \prec (1, 0.7, 0) \succ, \prec (1, 0.5, 0) \succ, \prec (0.2, 0.7, 0.5) \succ, \prec (0.6, 0.5, 0.5) \succ, \prec (0.5, 0.5, 0.5) \succ, \prec (0.8, 0.7, 0.5) \succ, \prec (0.8, 0.5, 0.5) \succ, \prec (0.5, 0.7, 0.5) \succ, 1_\sim\}$ is a neutrosophic micro topology on K and neutrosophic micro closed $= (\lambda_N(A))^C = \{1^C_\sim, \prec (0.8, 0, 0.6) \succ, \prec (0.8, 0, 0.2) \succ, \prec (0.8, 0, 0.5) \succ, \prec (0, 0.3, 1) \succ, \prec (0, 0.5, 1) \succ, \prec (0.5, 0.3, 0.2) \succ, \prec (0.5, 0.5, 0.6) \succ, \prec (0.5, 0.5, 0.5) \succ, \prec (0.5, 0.3, 0.8) \succ, \prec (0.5, 0.5, 0.8) \succ, \prec (0.5, 0.3, 0.5) \succ, 0^C_\sim\}$ is also a neutrosophic micro topology on K. We know that $0_\sim = \{\prec x, 0, 0, 1 \succ : x \in K\}$, $1_\sim = \{\prec x, 1, 1, 0 \succ : x \in K\}$ and $0^C_\sim = \{\prec x, 1, 1, 0 \succ : x \in K\}, 1^C_\sim = \{\prec x, 0, 0, 1 \succ : x \in K\}$ and the ideal $\mathcal{I} = 0_\sim, 1_\sim$. If $\mathcal{I} = 0_\sim$ then $A^*_{\mathcal{N}} = N_{mi} - cl(A)$. Thus $A \leq A^*_{\mathcal{N}}$. Hence A is $\mathcal{N}m\star$ dense in itself but not $\mathcal{N}m\star$-perfect. If $\mathcal{I} = 1_\sim$ then $A^*_{\mathcal{N}} = 0_\sim$. Thus $A \geq A^*_{\mathcal{N}}$. Hence $A^*_{\mathcal{N}}$ is $\mathcal{N}m\star$-closed but not $\mathcal{N}m\star$-perfect.*

Theorem 3.16. *$(K, \mathcal{S}, \mathcal{G})$ be an neutrosophic micro topological space with an ideal $\mathcal{I}$ on K and for every $A \leq K$. If A is $\mathcal{N}m\star$ dense in itself, then $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) = N_{mi} - cl(A) = N_{mi} - cl^*(A)$.*

Proof. Let A be $\mathcal{N}m\star$ dense in itself. Then we have $A \leq A^*_{\mathcal{N}}$ and using Theorem 3.12, we get $A^*_{\mathcal{N}} = N_{mi} - cl(A^*_{\mathcal{N}}) = N_{mi} - cl(A) = N_{mi} - cl^*(A)$. ☐

Lemma 3.17. *$(K, \mathcal{S}, \mathcal{G})$ be an neutrosophic micro topological space with an ideal $\mathcal{I}$ on K and for every $A \leq K$ then $A^*_{\mathcal{N}}(\mathcal{I}, \mathcal{G}) = A^*_{\mathcal{N}}(\mathcal{I}, \mathcal{G}^*)$ and hence $\mathcal{G}^* = \mathcal{G}^{**}$.*

The study of ideal got new dimension when codence ideal [7] has been incorporated in ideal topological space. Now we introduce similar concept in neutrosophic micro ideal topological spaces.

Definition 3.18. *An ideal $\mathcal{I}$ in a space $(K, \mathcal{S}, \mathcal{G}, \mathcal{I})$ is called $\mathcal{G}$-codense ideal if $\mathcal{G} \wedge \mathcal{I} = \{0_\sim\}$.*

Theorem 3.19. *Let $(K, \mathcal{S}, \mathcal{G}, \mathcal{I})$ be an neutrosophic micro ideal topological space and $\mathcal{I}$ is $\mathcal{G}$- codense with $\mathcal{G}$. Then $K = K^*_{\mathcal{N}}$.*

Proof. It is obvious that $K^*_{\mathcal{N}} \leq K$. For converse, suppose $k \in K$ but $k \notin K^*_{\mathcal{N}}$. Then there exists $V(_k)_{\mathcal{N}} \in \mathcal{G}(k)$ such that $V(_k)_{\mathcal{N}} \wedge K \in \mathcal{I}$. That is $V(_k)_{\mathcal{N}} \in \mathcal{I}$, a contradiction to the fact that $\mathcal{G} \wedge \mathcal{I} = \{0_\sim\}$. Hence $K = K^*_{\mathcal{N}}$. ☐

Lemma 3.20. *If $(K, \mathcal{S}, \mathcal{G}, \mathcal{I})$ is any neutrosophic micro ideal topological space, then the following are equivalent*

(1). $K = K^*_{\mathcal{N}}$,

(2). $\mathcal{G} \wedge \mathcal{I} = \{0_\sim\}$,

(3). If $F \in \mathcal{I}$ then $N_{mi} - int(F) = 0_\sim$,

(4). for every $A \in \mathcal{G}$, $A \leq G^*_m$.

Proof. $A^*_{\mathcal{N}}(\mathcal{I},\mathcal{G}) = A^*_{\mathcal{N}}(\mathcal{I},\mathcal{G}^*)$ [by Lemma 3.17], we many replace $\mathcal{G}$ by $\mathcal{G}^*$ in (2), $N_{mi}-int(F) = 0_\sim by N_{mi}-int^*(F) = 0_\sim$ in (3) and $A \in \mathcal{G}$ by $A \in \mathcal{G}^*$ in (4). □

We introduce a new definition of neutrosophic micro topological space as follows.

Definition 3.21. *Let $(K,\mathcal{S},\mathcal{G})$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of K and $A \leq K$. Then,*

(1). A is called neutrosophic micro α-open (short, N_{mi}-α-open) if $A \leq N_{mi}$-$int(N_{mi}$-$cl(N_{mi}$-$int(A)))$.

(2). A is called neutrosophic micro pre-open (short, N_{mi}-pre-open) if $A \leq N_{mi}$-$int(N_{mi}$-$cl(A))$.

(3). A is called neutrosophic micro semi-open (short, N_{mi}-semi-open) if $A \leq N_{mi}$-$cl(N_{mi}$-$int(A))$.

(4). A is called neutrosophic micro b-open (short, N_{mi}-b-open) if $A \leq N_{mi}$-$int(N_{mi}$-$cl(A)) \vee N_{mi}$-$cl(N_{mi}$-$int(A))$.

(5). A is called neutrosophic micro β-open (short, N_{mi}-β-open) if $A \leq N_{mi}$-$cl(N_{mi}$-$int(N_{mi}$-$cl(A)))$.

(6). A is called neutrosophic micro regular-open (short, N_{mi}-regular-open) if $A = N_{mi}$-$int(N_{mi}$-$cl(A))$.

(7). A is called neutrosophic micro π-open if A is the finite union of neutrosophic micro regular-open.

The complement of above mentioned neutrosophic micro open sets are called their respective neutrosophic micro closed sets. The family of all neutrosophic micro semi-open (resp. neutrosophic micro regular-open) sets in $(K,\mathcal{S},\mathcal{G})$ is denoted by $N_{mi}SO(K,\mathcal{S},\mathcal{G})$ (resp. $N_{mi}RO(K,\mathcal{S},\mathcal{G})$.

Lemma 3.22. *If $(K,\mathcal{S},\mathcal{G})$ be a neutrosophic micro topological space with an ideal $\mathcal{I}$ on K, then the following are equivalent.*

(1). $N_{mi}RO(K,\mathcal{S},\mathcal{G}) \wedge \mathcal{I} = \{0_\sim\}$,

(2). $\mathcal{G} \wedge \mathcal{I} = \{0_\sim\}$.

Proof. Since $N_{mi}RO(K,\mathcal{S},\mathcal{G}) \subset \mathcal{G}$, it is enough to prove that (1) ⇒ (2). Suppose $A \in \mathcal{G} \wedge \mathcal{I}$. By Lemma 3.20 (3), $A \in \mathcal{I}$ implies that $N_{mi} - int(A) = 0_\sim$ and so, since $A \in \mathcal{G}$, $A = 0_\sim$. Therefore, $\mathcal{G} \wedge \mathcal{I} = \{0_\sim\}$. □

Lemma 3.23. *If $(K,\mathcal{S},\mathcal{G})$ be a neutrosophic micro topological space with an ideal $\mathcal{I}$ on K, then the following are equivalent.*

(1). $\mathcal{G} \wedge \mathcal{I} = \{0_\sim\}$,

(2). $N_{mi}SO(K,\mathcal{S},\mathcal{G}) \wedge \mathcal{I} = \{0_\sim\}$.

Proof. Since $\mathcal{G} \subset N_{mi}SO(K,\mathcal{S},\mathcal{G})$, it is enough to prove that (1) ⇒ (2). Suppose $A \in N_{mi}SO(K,\mathcal{S},\mathcal{G}) \wedge \mathcal{I}$. By Lemma 3.20 (3), $A \in \mathcal{I}$ implies that $N_{mi} - int(A) = 0_\sim$ and so, since $A \in N_{mi}SO(K,\mathcal{S},\mathcal{G})$, $A = 0_\sim$. Therefore, $N_{mi}SO(K,\mathcal{S},\mathcal{G}) \wedge \mathcal{I} = \{0_\sim\}$. □

Theorem 3.24. *If $(K,\mathcal{S},\mathcal{G},\mathcal{I})$ is any neutrosophic micro ideal topological space, then the following are equivalent*

(1). $K = K^*_{\mathcal{N}}$,

(2). for every $A \in \mathcal{G}$, $A \leq A^*_{\mathcal{N}}$,

(3). for every $A \in N_{mi}SO(K,\mathcal{S},\mathcal{G})$, $A \leq A^*_{\mathcal{N}}$,

(4). For every neutrosophic micro regular closed set F, $F = F^*_m$.

Proof. (1) and (2) are equivalent by Lemma 3.20.

(2) ⇒ (3) Suppose $A \in N_{mi}SO(K, \mathcal{S}, \mathcal{G})$. Then there exists an neutrosophic micro open set P such that $P \leq A \leq N_{mi}-cl(P)$. Since P is neutrosophic micro open, $P \leq P^*_m$ and so by Theorem 3.4, $A \leq N_{mi} - cl(P) \leq N_{mi} - cl(P^*_m) = P^*_m \leq A^*_{\mathcal{N}}$. Hence $A \leq A^*_{\mathcal{N}}$.

(3) ⇒ (4) If F is neutrosophic micro regular closed then F is neutrosophic micro-semi-open and neutrosophic micro closed. F is neutrosophic micro semi-open ⇒ $F \leq F^*_m$. F is neutrosophic micro closed implies that F is $\mathcal{N}m\star$-closed and so $F^*_m \leq F$, by Lemma 3.10. Hence $F = F^*_m$.

(4) ⇒ (1) It is clear. □

4. Conclusion

Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set. Since the world is full of indeterminacy, the neutrosophic micro ideal topology found its place into contemporary research world. This article can be further developed into several possible such as Geographical Information Systems (GIS) field including remote sensing, object reconstruction from airborne laser scanner, real time tracking, routing applications and modeling cognitive agents. In GIS there is a need to model spatial regions with indeterminate boundary and under indeterminacy. Hence this neutrosophic micro ideal topological spaces can also be extended to a neutrosophic spatial region.

References

[1] K. T. Atanassov, *Intuitionstic fuzzy sets*, Fuzzy sets and systems, 20(1986), 87-96.

[2] S. Chandrasekar, *On micro topological spaces*, Journal of New Theory, 26(2019), 23-31.

[3] S. Ganesan, S. Jafari and F. Smarandache, *Neutrosophic micro topological spaces*, (communicated).

[4] S. Ganesan and F. Smarandache, *Some new classes of neutrosophic minimal open sets*, Asia Mathematika, 5(1)(2021), 103-112.

[5] E. Hayashi, *Topologies defined by local properties*, Math. Ann., 156(1964), 205-215.

[6] T. R. Hamlett and D. Jankovic, *Ideals in topological space and the set operator ψ*, Bull. U. M. I., (7)(4-B)(1990), 863-874.

[7] D. Jankovic and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4)(1990), 295-310.

[8] K. Kuratowski, *Topology*, Vol. I, Academic Press, New york, (1966).

[9] M. Lellis Thivagar, Saeid Jafari V. Sutha Devi and V. Antonysamy, *A novel approach to nano topology via neutrosophic sets*, Neutrosophic Sets and Systems, 20(2018), 86-94.

[10] A. A. Salama and S. A. Alblowi, *Neutrosophic Set and Neutrosophic Topological Spaces*, IOSR J. Math., 3(2012), 31-35.

[11] F. Smarandache, *Neutrosophy and Neutrosophic Logic. First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics*, University of New Mexico, Gallup, NM, USA, (2002).

[12] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutro-sophic Probability*, American Research Press: Rehoboth, NM, USA, (1999).

[13] R. Vaidyanathaswamy, *Set Topology*, Chelsea Publishing Company, (1946).

[14] L. A. Zadeh, Fuzzy Sets, *Information and Control*, 18(1965), 338-353.