NEUTROSOPHIC MULTISET STRUCTURES

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The Neutrosophic Multisets and the Neutrosophic Multiset Algebraic Structures were introduced by Florentin Smarandache [2, 4] in 2016: **http://fs.unm.edu/NeutrosophicMultisets.htm**

1. Definition of Neutrosophic Multiset

Let $\mathcal{U}$ be a universe of discourse, and $M \subset \mathcal{U}$.

A *Neutrosophic Multiset* is a neutrosophic set where one or more elements are repeated with the same neutrosophic components, or with different neutrosophic components.

2. Examples

$$A = \{a(0.6, 0.3, 0.1), b(0.8, 0.4, 0.2), c(0.5, 0.1, 0.3)\}$$

is a neutrosophic set (not a neutrosophic multiset).

But

$$B = \{a(0.6, 0.3, 0.1), a(0.6, 0.3, 0.1), b(0.8, 0.4, 0.2)\}$$

is a neutrosophic multiset, since the element *a* is repeated; we say that the element *a* has *neutrosophic multiplicity 2* with the same neutrosophic components.

While

$$C = \left\{ \begin{matrix} a(0.6, 0.3, 0.1), a(0.7, 0.1, 0.2), \\ a(0.5, 0.4, 0.3), c(0.5, 0.1, 0.3) \end{matrix} \right\}$$

is also a neutrosophic multiset, since the element *a* is repeated (it has *neutrosophic multiplicity 3*), but with different neutrosophic components, since, for example, during the time, the neutrosophic membership of an element may change.

If the element is repeated times keeping the same neutrosophic components (t_a, i_a, f_a), we say that *a* has *multiplicity* .

But if there is some change in the neutrosophic components of *a*, we say that *a* has the *neutrosophic multiplicity* .

Therefore, we define in general the *Neutrosophic Multiplicity Function*:

$$nm: \mathcal{U} \to \mathbb{N},$$

where $\mathbb{N} = \{1, 2, 3, \ldots, \infty\}$,

and for any $a \in A$ one has

$$nm(a) = \{(k_1, \langle t_1, i_1, f_1 \rangle), (k_2, \langle t_2, i_2, f_2 \rangle), \ldots, (k_j, \langle t_j, i_j, f_j \rangle), \ldots\}$$

which means that *a* is repeated k_1 times with the neutrosophic components $\langle t_1, i_1, f_1 \rangle$; *a* is repeated k_2 times with the neutrosophic components $\langle t_2, i_2, f_2 \rangle$, ..., *a* is repeated k_j times with the neutrosophic components $\langle t_j, i_j, f_j \rangle$, ..., and so on.

Of course, all $k_1, k_2, \dots, k_j, \dots \in \mathbb{N}$, and $\langle t_p, i_p, f_p \rangle \neq \langle t_r, i_r, f_r \rangle$, for $p \neq r$, with $p, r \in \mathbb{N}$.

Also, all neutrosophic components are with respect to the set . Then, a neutrosophic multiset *A* can be written as:

$$(A, nm(a))$$

or $\{(a, nm(a), \text{for } a \in A)\}$.

3. Examples of operations with neutrosophic multisets.

Let's have:

$$A = \{5_{\langle 0.6,0.3,0.2 \rangle}, 5_{\langle 0.6,0.3,0.2 \rangle}, 5_{\langle 0.4,0.1,0.3 \rangle}, 6_{\langle 0.2,0.7,0.0 \rangle}\}$$

$$B = \{5_{\langle 0.6,0.3,0.2 \rangle}, 5_{\langle 0.8,0.1,0.1 \rangle}, 6_{\langle 0.9,0.0,0.0 \rangle}\}$$

$$C = \{5_{\langle 0.6,0.3,0.2 \rangle}, 5_{\langle 0.6,0.3,0.2 \rangle}\}.$$

Then:

1.3.1. Intersection of Neutrosophic Multisets.

$$A \cap B = \{5_{\langle 0.6,0.3,0.2 \rangle}\}.$$

1.3.2. Union of Neutrosophic Multisets.

$$A \cup B = \left\{ \begin{matrix} 5_{\langle 0.6,0.3,0.2 \rangle}, 5_{\langle 0.6,0.3,0.2 \rangle}, 5_{\langle 0.4,0.1,0.3 \rangle}, 5_{\langle 0.8,0.1,0.1 \rangle}, \\ 6_{\langle 0.2,0.7,0.0 \rangle}, 6_{\langle 0.9,0.0,0.0 \rangle} \end{matrix} \right\}.$$

1.3.3. Inclusion of Neutrosophic Multisets.

$C \subset A$, but $C \not\subset B$

4. Cardinality of Neutrosophic Multisets.

$Card(A) = 4$, and $Card(B) =$ 3, where $Card(\cdot)$ means cardinal.

5. Cartesian Product of Neutrosophic Multisets.

$$B \times C = \left\{ \begin{matrix} \left(5_{(0.6,0.3,0.2)}, 5_{(0.6,0.3,0.2)}\right), \left(5_{(0.6,0.3,0.2)}, 5_{(0.6,0.3,0.2)}\right), \\ \left(5_{(0.8,0.1,0.1)}, 5_{(0.6,0.3,0.2)}\right), \left(5_{(0.8,0.1,0.1)}, 5_{(0.6,0.3,0.2)}\right), \\ \left(6_{(0.9,0.0,0.0)}, 5_{(0.6,0.3,0.2)}\right), \left(6_{(0.9,0.0,0.0)}, 5_{(0.6,0.3,0.2)}\right) \end{matrix} \right\}.$$

6. Difference of Neutrosophic Multisets.

$$A - B = \left\{5_{(0.6,0.3,0.2)}, 5_{(0.4,0.1,0.3)}, 6_{(0.2,0.7,0.0)}\right\}$$

$$A - C = \left\{5_{(0.4,0.1,0.3)}, 6_{(0.2,0.7,0.0)}\right\}$$

$$C - B = \left\{5_{(0.6,0.3,0.2)}\right\}$$

7. Sum of Neutrosophic Multisets.

$$A \uplus B = \left\{ \begin{matrix} 5_{(0.6,0.3,0.2)}, 5_{(0.6,0.3,0.2)}, 5_{(0.6,0.3,0.2)}, 5_{(0.4,0.1,0.3)}, 5_{(0.8,0.1,0.1)}, \\ 6_{(0.2,0.7,0.9)}, 6_{(0.9,0.0,0.0)} \end{matrix} \right\}$$

$$B \uplus B = \left\{ \begin{matrix} 5_{(0.6,0.3,0.2)}, 5_{(0.6,0.3,0.2)}, 5_{(0.8,0.1,0.1)}, 5_{(0.8,0.1,0.1)}, \\ 6_{(0.9,0.0,0.0)}, 6_{(0.9,0.0,0.0)} \end{matrix} \right\}.$$

Let's compute the neutrosophic multiplicity function, with respect to several of the previous neutrosophic multisets.

$$nm_A : A \to \mathbb{N}$$

$$nm_A(5) = \{(2, \langle 0.6, 0.3, 0.2 \rangle), (1, \langle 0.4, 0.1, 0.3 \rangle)\}$$

$$nm_A(6) = \{(1, \langle 0.2, 0.7, 0.0 \rangle)\}.$$

$$nm_B : B \rightarrow \mathbb{N}$$

$$nm_B(5) = \{(1, \langle 0.6, 0.3, 0.2 \rangle), (1, \langle 0.8, 0.1, 0.1 \rangle)\}$$

$$nm_B(6) = \{(1, \langle 0.2, 0.7, 0.0 \rangle)\}.$$

$$nm_C : C \rightarrow \mathbb{N}$$

$$nm_C(5) = \{(2, \langle 0.6, 0.3, 0.2 \rangle)\}$$

References

[1] Eric W. Weisstein, *Multiset*, MathWorld, CRC Encyclopedia of Mathematics, Boca Raton, FL, USA.

[2] F. Smarandache, ***Neutrosophic Theory and Applications***, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[3] F. Smarandache, *Neutrosphic Multiset Applied in Physical Processes*, Actualization of the Internet of Things, a FIAP Industrial Physics Conference, Monterey, California, Jan. 2017.

[4] F. Smarandache, Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications. Pons Editions, Bruxelles, 323 p., 2017;

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