Neutrosophic Nano ideal topological structure

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Abstract: This paper addressed the concept of Neutrosophic nano ideal topology which is induced by the two literature, they are nano topology and ideal topological spaces. We defined its local function, closed set and also defined and give new dimension to codense ideal by incorporating it to ideal topological structures. we investigate some properties of neutrosophic nano topology with ideal.

Keywords: neutrosophic nano ideal, neutrosophic nano local function, topological ideal, neutrosophic nano topological ideal.

1 Introduction and Preliminaries

In 1983, K. Atanassov [1] proposed the concept of IFS(intuitionistic fuzzy set) which is a generalization of FS(fuzzy set) [17], where each element has true and false membership degree. Smarandache [15] coined the concept of NS (neutrosophic set) which is new dimension to the sets. Neutrosophic set is classified into three independently related functions namely, membership, indeterminacy function and non-membership function. Lellis Thivagar [8], introduced the new notion of neutrosophic nano topology, which consist of upper, lower approximation and boundary region of a subset of a universal set using an equivalence class on it. There have been wide range of studies on neutrosophic sets, ideals and nano ideals [9, 10, 11, 12, 13, 14]. Kuratowski [7] and Vaidyanathaswamy [16] introduced the new concept in topological spaces, called ideal topological spaces and also local function in ideal topological space was defined by them. Afterwards the properties of ideal topological spaces studied by Hamlett and Jankovic[5,6].
In this paper, we introduce the new concept of neutrosophic nano ideal topological structures, which is a
generalized concept of neutrosophic nano and ideal topological structure. Also defined the codense ideal in
neutrosophic nano topological structure.

We recall some relevant basic definitions which are useful for the sequel and in particular, the work of M. L.
Thivagar [8], Parimala et al [9], F. Smarandache [15].

**Definition 1.1.** Let \( U \) be universe of discourse and \( R \) be an indiscernibility relation on \( U \). Then \( U \) is divided
into disjoint equivalence classes. The pair \((U, R)\) is said to be the approximation space. Let \( F \) be a NS in \( U \)
with the true \( \mu_F \), the indeterminacy \( \sigma_F \) and the false function \( \nu_F \). Then,

(i) The lower approximation of \( F \) with respect to equivalence class \( R \) is the set denoted by \( \overline{N}(F) \) and
defined as follows
\[
\overline{N}(F) = \{ \langle a, \mu_{R(F)}(a), \sigma_{R(F)}(a), \nu_{R(F)}(a) \rangle \mid y \in [a]_R, a \in U \}
\]

(ii) The higher approximation of \( F \) with respect to equivalence class \( R \) is the set denoted by \( \overline{N}(F) \) and
defefined as follows, \( \overline{N}(F) = \{ \langle a, \mu_{R(F)}(a), \sigma_{R(F)}(a), \nu_{R(F)}(a) \rangle \mid y \in [a]_R, a \in U \}
(iii) The boundary region of \( F \) with respect to equivalence class \( R \) is the set of all objects is denoted by \( B(F) \)
and defined by \( B(F) = \overline{N}(F) - \overline{N}(F) \).

where,
\[
\mu_{R(F)}(a) = \bigcup_{y_1 \in [a]_R} \mu_F(y_1), \quad \sigma_{R(F)}(a) = \bigcup_{y_1 \in [a]_R} \sigma_F(y_1),
\]
\[
\nu_{R(F)}(a) = \bigcap_{y_1 \in [a]_R} \nu_F(y_1), \quad \mu_{R(F)}(a) = \bigcap_{y_1 \in [a]_R} \mu_F(y_1),
\]
\[
\sigma_{R(F)}(a) = \bigcap_{y_1 \in [a]_R} \sigma_F(y_1), \quad \nu_{R(F)}(a) = \bigcap_{y_1 \in [a]_R} \nu_F(y_1).
\]

**Definition 1.2.** Let \( U \) be a nonempty set and the neutrosophic sets \( X \) and \( Y \) in the form \( X = \{ \langle a, \mu_X(a), \sigma_X(a), \nu_X(a) \rangle, a \in U \} \)
and \( Y = \{ \langle a, \mu_Y(a), \sigma_Y(a), \nu_Y(a) \rangle, a \in U \} \). Then the following statements hold:

(i) \( 0_N = \{ \langle a, 0, 0, 1 \rangle, a \in U \} \) and \( 1_N = \{ \langle a, 1, 1, 0 \rangle, a \in U \} \).

(ii) \( X \subseteq Y \) if and only if \( \mu_X(a) \leq \mu_Y(a), \sigma_X(a) \leq \sigma_Y(a), \nu_X(a) \geq \nu_Y(a) \) for all \( a \in U \).

(iii) \( X = Y \) if and only if \( X \subseteq Y \) and \( Y \subseteq X \).

(iv) \( X^C = \{ \langle a, \nu_X(a), 1 - \sigma_X(a), \mu_X(a) \rangle, a \in U \} \).

(v) \( X \cap Y \) if and only if \( \mu_X(a) \land \mu_X(a), \sigma_X(a) \land \sigma_Y(a), \nu_X(a) \lor \nu_Y(a) \) for all \( a \in U \).

(vi) \( X \cup Y \) if and only if \( \mu_Y(a) \lor \mu_Y(a), \sigma_X(a) \lor \sigma_Y(a), \nu_X(a) \land \nu_Y(a) \) for all \( a \in U \).

(vii) \( X - Y \) if and only if \( \mu_X(a) \land \nu_Y(a), \sigma_X(a) \land 1 - \sigma_Y(a), \nu_X(a) \lor \mu_Y(a) \) for all \( a \in U \).

**Definition 1.3.** Let \( X \) be a non-empty set and \( I \) is a neutrosophic ideal \((NI \) for short) on \( X \) if
In this section we introduce a new type of local function in neutrosophic nano topological space. Before that we shall consider the following concepts.

Neutrosophic nano ideal topological space (in short NNI) is denoted by \((U, \tau_N(F), I)\), where \((U, \tau_N(F), I)\) is a neutrosophic nano topological space (in short NNT) \((U, \tau_N(F))\) with an ideal \(I\) on \(U\).

**Definition 2.1.** Let \((U, \tau_N(F), I)\) be a NNI with an ideal \(I\) on \(U\) and \((.)^*_N\) be a set of operator from \(P(U)\) to \(P(U) \times P(U)\) \((P(U)\) is the set of all subsets of \(U)\). For a subset \(X \subseteq U\), the neutrosophic nano local function \(X^*_N(I, \tau_N(F))\) of \(X\) is the union of all neutrosophic nano points (NNP, for short) \(C(\alpha, \beta, \gamma)\) such that \(X^*_N(I, \tau_N(F)) = \bigvee\{C(\alpha, \beta, \gamma) \in U : X \cap G \notin J\text{ for all } G \in N(C(\alpha, \beta, \gamma))\}\). We will simply write \(X^*_N\) for \(X^*_N(I, \tau_N(F))\).

**Example 2.2.** Let \((U, \tau_N(F))\) be a neutrosophic nano topological space with an ideal \(I\) on \(U\) and for every \(X \subseteq U\).

(i) If \(I = \{0_\infty\}\), then \(X^*_N = Ncl(X)\),

(ii) If \(I = P(U)\), then \(X^*_N = 0_\infty\).

**Theorem 2.3.** Let \((U, \tau_N(F))\) be a NNT with ideals \(I, I'\) on \(U\) and \(X, B\) be subsets of \(U\). Then

(i) \(X \subseteq B \Rightarrow X^*_N \subseteq B^*_N\),

(ii) \(I \subseteq I' \Rightarrow X^*_N (I') \subseteq X^*_N (I)\),

(iii) \(X^*_N = Ncl(X^*_N) \subseteq Ncl(X)\) \((X^*_N\) is a neutrosophic nano closed subset of \(Ncl(X)\)),

(iv) \((X^*_N)^*_N \subseteq X^*_N\),

(v) \(X^*_N \cup B^*_N = (X \cup B)^*_N\),

(vi) \(X^*_N - B^*_N = (X - B)^*_N - B^*_N \subseteq (X - B)^*_N\),

(vii) \(V \in \tau_N(F) \Rightarrow V \cap X^*_N = V \cap (V \cap X)^*_N \subseteq (V \cap X)^*_N\) and

(viii) \(J \in I \Rightarrow (X \cup J)^*_N = X^*_N = (X - J)^*_N\).

**Proof.** (i) Let \(X \subseteq B\) and \(a \in X^*_N\). Assume that \(a \notin B^*_N\). We have \(G_N \cap B \in I\) for some \(G_N \in G_N(a)\). Since \(G_N \cap X \subseteq G_N \cap B\) and \(G_N \cap B \in I\), we obtain \(G_N \cap X \in I\) from the definition of ideal. Thus, we have \(a \notin X^*_N\). This is a contradiction. Clearly, \(X^*_N \subseteq B^*_N\).

(ii) Let \(I \subseteq I'\) and \(a \in X^*_N (I')\). Then we have \(G_N \cap X \notin J\) for every \(G_N \in G_N(a)\). By hypothesis, we obtain \(G_N \cap X \notin J\). So \(a \in X^*_N (I)\).

(iii) Let \(a \in X^*_N\). Then for every \(G_N \in G_N(a), G_N \cap X \notin J\). This implies that \(G_N \cap X \neq 0_\infty\). Hence
Proof. The proofs are clear from Theorem 2.3 and the definition of neutrosophic nano set is called neutrosophic nano closure and interior of \((\tau_N)^*(X)\), respectively in \(N\) and so

\[ \text{Proof.} \quad \text{The proofs of the other conditions are also obvious.} \]

\[ \text{Theorem 2.4.} \quad \text{If} (U, \tau_N(F), \beta) \text{is a NNT with an ideal} \quad \beta \text{on} \quad U \quad \text{and} \quad X \subseteq X_N^*, \quad \text{then} \quad X_N^* = \tau_N(F)(X_N^*) = \tau_N(F)(X). \]

\[ \text{Proof.} \quad \text{We have to show that for a given space} \quad (U, \tau_N(F), \beta) \quad \text{and an ideal} \quad \beta \quad \text{on} \quad U \quad \text{and} \quad X \subseteq X_N^*, \quad \text{then} \quad X_N^* = \tau_N(F)(X_N^*) = \tau_N(F)(X). \]

\[ \text{Remark 2.7.} \quad (i) \quad \text{We know from Example 2.2 that if} \quad I = \{0_\infty\} \quad \text{then} \quad X_N^* = \tau_N(F)(X). \quad \text{In this case,} \quad \tau_N(F)(X) = \tau_N(F)(X). \]

\[ \text{(ii) If} \quad (U, \tau_N(F), \beta) \quad \text{is a NNI with} \quad I = \{0_\infty\}, \quad \text{then} \quad \tau_N(F)(X) = \tau_N(F)(X). \]

\[ \text{Definition 2.8.} \quad \text{A basis} \quad \beta(I, \tau_N(F)) \quad \text{for} \quad \tau_N(F)^*(X) \quad \text{can be described as follows:} \]

\[ \beta(I, \tau_N(F)) = \{X - B : X \in \tau_N(F), B \in I\}. \]

\[ \text{Theorem 2.9.} \quad \text{Let} \quad (U, \tau_N(F), \beta) \quad \text{be a NNT and} \quad I \quad \text{be an ideal on} \quad U. \quad \text{Then} \quad \beta(I, \tau_N(F)) \quad \text{is a basis for} \quad \tau_N(F)^*(X). \]

\[ \text{Proof.} \quad \text{We have to show that for a given space} \quad (U, \tau_N(F), \beta) \quad \text{and an ideal} \quad I \quad \text{on} \quad U, \quad \beta(I, \tau_N(F)) \quad \text{is a basis for} \quad \tau_N(F)^*(X). \quad \text{If} \quad \beta(I, \tau_N(F)) \quad \text{is itself a neutrosophic nano topology, then we have} \quad \beta(I, \tau_N(F)) = \tau_N(F)^*(X) \quad \text{and all the open sets of} \quad \tau_N(F)^*(X) \quad \text{are of simple form} \quad X - B \quad \text{where} \quad X \in \tau_N(F) \quad \text{and} \quad B \in I. \]

\[ \text{Theorem 2.10.} \quad \text{Let} \quad (U, \tau_N(F), \beta) \quad \text{be a NNT with an ideal} \quad I \quad \text{on} \quad U \quad \text{and} \quad X \subseteq U. \quad \text{If} \quad X \subseteq X_N^*, \quad \text{then} \]

\[ \text{(i)} \quad \tau_N(F)(X) = \tau_N(F)(X), \]

\[ \text{(ii)} \quad \tau_N(F)(U - X) = \tau_N(F)(U - X). \]

\[ \text{Proof.} \quad (i) \quad \text{Follows immediately from Theorem 2.4.} \]

\[ \text{(ii) If} \quad X \subseteq X_N^*, \quad \text{then} \quad \tau_N(F)(X) = \tau_N(F)(X) \quad \text{by (i) and so} \quad U - \tau_N(F)(X) = U - \tau_N(F)(X). \quad \text{Therefore,} \]

\[ \tau_N(F)(U - X) = \tau_N(F)(U - X). \]
Theorem 2.11. Let \((U, \tau_N(F), I)\) be a NNT with an ideal \(I\) on \(U\) and \(X \subseteq X\). If \(X \subseteq X_N\), then \(X_N^* = Ncl(X_N^*) = ncl(X) = Ncl^*(X)\).

Definition 2.12. A subset \(A\) of a neutrosophic nano ideal topological space \((U, \tau_N(F), I)\) is \(N^*\)-dense in itself (resp. \(N^*\)-perfect) if \(X \subseteq X_N\) (resp. \(X = X_N\)).

Remark 2.13. A subset \(X\) of a neutrosophic nano ideal topological space \((U, \tau_N(F), I)\) is \(N^*\)-closed if and only if \(X_N^* \subseteq X\).

For the relationship related to several sets defined in this paper, we have the following implication:

\(N^*\)-dense in itself \(\iff N^*\)-perfect \(\Rightarrow N^*\)-closed

The converse implication are not satisfied as the following shows.

Example 2.14. Let \(U\) be the universe, \(X = \{P_1, P_2, P_3, P_4, P_5\} \subseteq U\), \(U/R = \{\{P_1, P_2\}, \{P_3\}, \{P_4, P_5\}\}\) and \(\tau_N(F) = \{1, 0, \overline{N}, \overline{N}, B\}\) and the ideal \(I = 0, 1\). For \(X = \{< P_1, (.5, .4, .7), < P_2, (.6, .4, .5) >, < P_3, (.4, .5, .4) >, < P_4, (.7, .3, .4) >, < P_5, (.8, .5, .2) >\}\), \(\overline{N}(X) = \{P_1, P_2, P_3, P_4, P_5\}\).

\(\overline{N}(X) = \{P_1, P_2, P_3, P_4, P_5\}\), \(B(X) = \{P_1, P_2, P_3, P_4, P_5\}\). If \(I = 0\), then \(X_N^* = Ncl(a)\). Thus \(X \subseteq X_N\). Hence \(X_N^*\) is \(N^*\)-dense but not \(N^*\)-perfect.

If \(I = 1\), then \(X_N^* = 0\). Thus \(X \supseteq X_N\). Hence \(X_N^*\) is \(N^*\)-closed but not \(N^*\)-perfect.

Lemma 2.15. Let \((U, \tau_N(F), I)\) be a NNI and \(X \subseteq U\). If \(X\) is \(N^*\)-dense in itself, then \(X_N^* = Ncl(X_N^*) = Ncl(X) = Ncl^*(X)\).

Proof. Let \(X\) be \(N^*\)-dense in itself. Then we have \(X \subseteq X_N^*\) and using Theorem 2.11 we get \(X_N^* = Ncl(X_N^*) = Ncl(X) = Ncl^*(X)\).

Lemma 2.16. If \((U, \tau_N(F), I)\) is a NNT with an ideal \(I\) and \(X \subseteq U\), then \(X_N(I, \tau_N(F)) = X_N^*(I, \tau_N(F))\) and hence \(\tau_N(F)^* = \tau_N(F)^{**}\).

3 \(\tau_N(F)^*\)-codense ideal

In this section we incorporated codense ideal [5] in ideal topological space and introduce similar concept in neutrosophic nano ideal topological spaces.

Definition 3.1. An ideal \(I\) in a space \((U, \tau_N(F), I)\) is called \(\tau_N(F)^*\)-codense ideal if \(\tau_N(F) \cap I = \{0\}\).

Following theorems are related to \(\tau_N(F)^*\)-codense ideal.

Theorem 3.2. Let \((U, \tau_N(F), I)\) be an NNI and \(I\) is \(\tau_N(F)^*\)-codense with \(\tau_N(F)\). Then \(U = U_N^*\).

Proof. It is obvious that \(U_N^* \subseteq U\). For converse, suppose \(a \in U\) but \(a \not\in U_N^*\). Then there exists \(G_x \in \tau_N(F)(a)\) such that \(G_x \cap U \subseteq I\). That is \(G_x \in I\), a contradiction to the fact that \(\tau_N(F) \cap I = \{0\}\). Hence \(U = U_N^*\).

Theorem 3.3. Let \((U, \tau_N(F), I)\) be a NNI. Then the following conditions are equivalent:

(i) \(U = U_N^*\).
(ii) $\tau_N(F) \cap I = \{0_\sim\}$.

(iii) If $J \in I$, then $\mathcal{N}int(J) = 0_\sim$.

(iv) For every $X \in \tau_N(F)$, $X \subseteq X^*_N$.

**Proof.** By Lemma 2.16, we may replace ‘$\tau_N(F)$’ by ‘$\tau_N(F)^*$’ in (ii), ‘$\mathcal{N}int(J) = 0_\sim$’ by ‘$\mathcal{N}int^*(J) = 0_\sim$’ in (iii) and ‘$X \in \tau_N(F)$’ by ‘$X \in \tau_N(F)^*$’ in (iv).

## 4 Conclusions

In this paper, we introduced the notion of neutrosophic nano ideal topological structures and investigated some relations over neutrosophic nano topology and neutrosophic nano ideal topological structures and studied some of its basic properties. In future, it motivates to apply this concepts in graph structures.

## References


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