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Neutrosophic $\aleph -$ideals in semigroups

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Abstract: The aim of this paper is to introduce the notion of neutrosophic $\aleph -$ideals in semigroups and investigate their properties. Conditions for neutrosophic $\aleph -$structure to be a neutrosophic $\aleph -$ideal are provided. We also discuss the concept of characteristic neutrosophic $\aleph -$structure of semigroups and its related properties.

Keywords: Semigroup; neutrosophic $\aleph -$ structure; neutrosophic $\aleph -$ ideals, neutrosophic $\aleph -$product.

1. Introduction

Throughout this paper, S denotes a semigroup and for any subsets A and B of S, the multiplication of A and B is defined as $AB = \{ab | a \in A \text{ and } b \in B\}$. A nonempty subset A of S is called a subsemigroup of S if $A^2 \subseteq A$. A subsemigroup A of S is called a left (resp., right) ideal of S if $AX \subseteq A$ (resp., $XA \subseteq A$). A subset A of S is called two-sided ideal or ideal of S if it is both a left and right ideal of S.

L.A. Zadeh introduced the concept of fuzzy subsets of a well-defined set in his paper [17] for modeling the vague concepts in the real world. K. T. Atanassov [1] introduced the notion of an Intuitionistic fuzzy set as a generalization of a fuzzy set. In fact from his point of view for each element of the universe there are two degrees, one a degree of membership to a vague subset and the other is a degree of non-membership to that given subset. Many researchers have been working on the theory of this subject and developed it in interesting different branches.

As a more general platform which extends the notions of the classic set and fuzzy set, intuitionistic fuzzy set and interval valued (intuitionistic) fuzzy set, Smarandache introduced the notion of neutrosophic sets (see [15, 16]), which is useful mathematical tool for dealing with incomplete, inconsistent and indeterminate information. This concept has been extensively studied and investigated by several authors in different fields (see [2 – 8] and [10 – 14]).

For further particulars on neutrosophic set theory, we refer the readers to the site http://fs.gallup.unm.edu/FlorentinSmarandache.htm

In [9], M. Khan et al. introduced the notion of neutrosophic $\aleph -$subsemigroup in semigroup and investigated several properties. It motivates us to define the notion of neutrosophic $\aleph -$ideal in semigroup. In this paper, the notion of neutrosophic $\aleph -$ideals in semigroups is introduced and several properties are investigated. Conditions for neutrosophic $\aleph -$structure to be neutrosophic $\aleph -$ideal are provided. We also discuss the concept of characteristic neutrosophic $\aleph -$structure of semigroups and its related properties.

2. Neutrosophic ℵ - structures

This section explains some basic definitions of neutrosophic $\aleph-$ structures of a semigroup S that have been used in the sequel and introduce the notion of neutrosophic $\aleph-$ ideals in semigroups.

The collection of function from a set S to $[-1,0]$ is denoted by $\mathfrak{J}(S,[-1,0])$. An element of $\mathfrak{J}(S,[-1,0])$ is called a negative-valued function from S to $[-1,0]$ (briefly, $\aleph-$ function on S). By a $\aleph-$structure, we mean an ordered pair (S,g) of S and a $\aleph-$function g on S.

For any family $\{x_i \mid i \in \Lambda\}$ of real numbers, we define:

$$\bigvee\{x_i \mid i \in \Lambda\} := \begin{cases} max\{x_i \mid i \in \Lambda\} & if\ \Lambda\ is\ finite \\ sup\{x_i \mid i \in \Lambda\} & if\ \Lambda\ is\ infinite \end{cases}$$

and

$$\bigwedge\{x_i \mid i \in \Lambda\} := \begin{cases} min\{x_i \mid i \in \Lambda\} & if\ \Lambda\ is\ finite \\ inf\{x_i \mid i \in \Lambda\} & if\ \Lambda\ is\ infinite \end{cases}$$

For any real numbers x and y, we also use $x \vee y$ and $x \wedge y$ instead of $\vee\{x,y\}$ **and** $\wedge\{x,y\}$ respectively.

Definition 2.1. [9] A neutrosophic $\aleph-$ structure over S defined to be the structure:

$$S_N := \frac{S}{(T_N,\ I_N,\ F_N)} = \left\{\frac{x}{T_N(x),\ I_N(x),\ F_N(x)} \mid x \in S\right\},$$

where T_N, I_N and F_N are $\aleph-$ functions on S which are called the negative truth membership function, the negative indeterminacy membership function and the negative falsity membership function, respectively, on S. It is clear that for any neutrosophic $\aleph-$ structure S_N over S, we have $-3 \le T_N(y) + I_N(y) + F_N(y) \le 0$ for all $y \in S$.

Definition 2.2. [9] Let $S_N := \frac{S}{(T_N,\ I_N,\ F_N)}$ and $S_M := \frac{S}{(T_M,\ I_M,\ F_M)}$ be neutrosophic $\aleph-$structures over S. Then

(i) S_N is called a neutrosophic $\aleph-$ substructure of S_M over S, denote by $S_N \subseteq S_M$, if $T_N(s) \ge T_M(s)$, $I_N(s) \le I_M(s)$, $F_N(s) \ge F_M(s)$ for all $s \in S$.

If $S_N \subseteq S_M$ and $S_M \subseteq S_N$, then we say that $S_N = S_M$.

(ii) The neutrosophic $\aleph-$ product of S_N and S_M is defined to be a neutrosophic $\aleph-$structure over S

$$S_N \odot S_M := \frac{S}{(T_{N\circ M},\ I_{N\circ M},\ F_{N\circ M})} = \left\{\frac{s}{T_{N\circ M}(s),\ I_{N\circ M}(s),\ F_{N\circ M}(s)} \mid s \in S\right\},$$

where

$$T_{N\circ M}(s) = \begin{cases} \bigwedge_{s=uv}\{T_N(u) \vee T_M(v)\} & if\ \exists\ u,v \in S\ such\ that\ s = uv \\ 0 & otherwise, \end{cases}$$

$$I_{N\circ M}(s) = \begin{cases} \bigvee_{s=uv}\{I_N(u) \wedge I_M(v)\} & if\ \exists\ u,v \in S\ such\ that\ s = uv \\ 0 & otherwise, \end{cases}$$

$$F_{N\circ M}(s) = \begin{cases} \bigwedge_{s=uv}\{F_N(u) \vee F_M(v)\} & if\ \exists\ u,v \in S\ such\ that\ s = uv \\ 0 & otherwise. \end{cases}$$

For $s \in S$, the element $\frac{s}{(T_{N\circ M},\ I_{N\circ M},\ F_{N\circ M})}$ is simply denoted by $(S_N \odot S_M)(s) = (T_{N\circ M}(s),\ I_{N\circ M}(s),\ F_{N\circ M}(s))$ for the sake of convenience.

(iii) The union of S_N and S_M is defined to be a neutrosophic $\aleph-$structure over S

$$S_{N\cup M} = (S; T_{N\cup M},\ I_{N\cup M},\ F_{N\cup M}),$$

where

$$T_{N\cup M}(a) = T_N(a) \wedge T_M(a),$$

$$I_{N\cup M}(a) = I_N(a) \vee I_M(a),$$
$$F_{N\cup M}(a) = F_N(a) \wedge F_M(a) \text{ for all } a \in S.$$

(iv) The intersection of S_N and S_M is defined to be a neutrosophic $\aleph$ −structure over S

$$S_{N\cap M} = (S; T_{N\cap M},\ I_{N\cap M},\ F_{N\cap M}),$$

where

$$T_{N\cap M}(a) = T_N(a) \vee T_M(a),$$
$$I_{N\cap M}(a) = I_N(a) \wedge I_M(a),$$
$$F_{N\cap M}(a) = F_N(a) \vee F_M(a) \text{ for all } a \in S.$$

Definition 2.3. [9] A neutrosophic $\aleph-$ structure S_N over *S* is called a neutrosophic $\aleph$ −subsemigroup of *S* if it satisfies:

$$(\forall\, a, b \in S)\begin{pmatrix} T_N(ab) \le T_N(a) \vee T_N(b) \\ I_N(ab) \ge I_N(a) \vee I_N(b) \\ F_N(ab) \le F_N(a) \vee F_N(b) \end{pmatrix}.$$

Definition 2.4. A neutrosophic $\aleph$ −structure S_N over *S* is called a neutrosophic $\aleph$ −left (resp., right) ideal of *S* if it satisfies:

$$(\forall\, a, b \in S)\begin{pmatrix} T_N(ab) \le T_N(a)\ (resp., T_N(ab) \le T_N(b)) \\ I_N(ab) \ge I_N(a)\ (resp.,\ I_N(ab) \ge I_N(b)) \\ F_N(ab) \le F_N(a)\ (resp.,\ F_N(ab) \le F_N(b)) \end{pmatrix}.$$

If S_N is both a neutrosophic $\aleph-$ left and neutrosophic $\aleph$ −right ideal of *S,* then it called a neutrosophic $\aleph$ −ideal of *S.*

It is clear that every neutrosophic $\aleph$ −left and neutrosophic $\aleph$ −right ideal of *S* is a neutrosophic $\aleph-$ subsemigroup of *S,* but neutrosophic $\aleph-$ subsemigroup of *S* is need not to be either a neutrosophic $\aleph$ −left or a neutrosophic $\aleph$ −right ideal of *S* as can be seen by the following example.

Example 2.5. Let *S*= $\{0, 1, 2, 3, 4, 5\}$ be a semigroup with the following multiplication table:

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	1	1	1	1
2	0	1	2	3	1	1
3	0	1	1	1	2	3
4	0	1	4	5	1	1
5	0	1	1	1	4	5

Then $S_N = \left\{\frac{0}{(-0.9,-0.1,-0.8)}, \frac{1}{(-0.5,-0.2,-0.6)}, \frac{2}{(-0.1,-0.8,-0.1)}, \frac{3}{(-0.3,-0.6,-0.4)}, \frac{4}{(-0.1,-0.8,-0.1)}, \frac{5}{(-0.4,-0.3,-0.5)}\right\}$ is a neutrosophic $\aleph-$ subsemigroup of S, but not a neutrosophic $\aleph-$ left ideal of S as $T_N(3.5) \nleq T_N(5), I_N(3.5) \ngeq I_N(5)$ and $F_N(3.5) \nleq F_N(5)$.

□

Example 2.6. Let $S = \{a, b, c,\ d\}$ be a semigroup with the following multiplication table:

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

Then $S_N = \left\{\frac{a}{(-0.9,\ -0.1,\ -0.8)}, \frac{b}{(-0.5,\ -0.2,\ -0.6)}, \frac{c}{(-0.3,\ -0.3,\ -0.4)}, \frac{d}{(-0.4,\ -0.2,\ -0.5)}\right\}$ is a neutrosophic $\aleph$ −ideal of *S*. □

Definition 2.7. For a subset A of S, consider the neutrosophic $\aleph$ −structure

$$\chi_A(S_N) = \frac{S}{(\chi_A(T)_N, \chi_A(I)_N, \chi_A(F)_N)}$$

where

$$\chi_A(T)_N : S \to [-1,0],\ s \to \begin{cases} -1 \text{ if } s \in A \\ 0 \text{ otherwise} \end{cases}$$

$$\chi_A(I)_N : S \to [-1,0],\ s \to \begin{cases} 0 \text{ if } s \in A \\ -1 \text{ otherwise} \end{cases}$$

$$\chi_A(F)_N : S \to [-1,0],\ s \to \begin{cases} -1 \text{ if } s \in A \\ 0 \text{ otherwise} \end{cases}$$

which is called the characteristic neutrosophic $\aleph$ −structure of S.

Definition 2.8. [9] Let S_N be a neutrosophic $\aleph -$ structure over S and let $\alpha, \beta, \gamma \in [-1, 0]$ be such that $-3 \leq \alpha + \beta + \gamma \leq 0$. Consider the following sets:

$$T_N^{\alpha} = \{s \in S: T_N(s) \leq \alpha\},$$
$$I_N^{\beta} = \{s \in S: I_N(s) \geq \beta\},$$
$$F_N^{\gamma} = \{s \in S: F_N(s) \leq \gamma\}.$$

The set $S_N(\alpha,\beta,\gamma) := \{s \in S \mid T_N(s) \leq \alpha,\ I_N(s) \geq \beta,\ F_N(s) \leq \gamma\}$ is called a (α, β, γ)-level set of S_N. Note that $S_N(\alpha,\beta,\gamma) = T_N^{\alpha} \cap I_N^{\beta} \cap F_N^{\gamma}$.

3. Neutrosophic $\aleph -$ ideals

Theorem 3.1 Let S_N be a neutrosophic $\aleph -$ structure over S and let $\alpha, \beta, \gamma \in [-1, 0]$ be such that $-3 \leq \alpha + \beta + \gamma \leq 0$. If S_N is a neutrosophic $\aleph -$ left (resp., right) ideal of S, then $(\alpha,\beta,\gamma) -$ level set of S_N is a neutrosophic left (resp., right) ideal of S whenever it is non-empty.

Proof: Assume that $S_N(\alpha,\beta,\gamma) \neq \emptyset$ for $\alpha,\beta,\gamma \in [-1,0]$ with $-3 \leq \alpha + \beta + \gamma \leq 0$. Let S_N be a neutrosophic $\aleph -$ left ideal of S and let $x, y \in S_N(\alpha,\beta,\gamma)$. Then $T_N(xy) \leq T_N(x) \leq \alpha; I_N(xy) \geq I_N(x) \geq \beta$ and $F_N(xy) \leq F_N(x) \leq \gamma$ which imply $xy \in S_N(\alpha,\beta,\gamma)$. Therefore $S_N(\alpha,\beta,\gamma)$ is a neutrosophic $\aleph -$ left ideal of S. □

Theorem 3.2. Let S_N be a neutrosophic $\aleph -$ structure over S and let $\alpha,\beta,\gamma \in [-1,0]$ be such that $-3 \leq \alpha + \beta + \gamma \leq 0$. If T_N^{α}; I_N^{β} and F_N^{γ} are left (resp., right) ideals of S, then S_N is a neutrosophic $\aleph$ −left (resp., right) ideal of S whenever it is non-empty.

Proof: If there are $a,\ b \in S$ such that $T_N(ab) > T_N(a)$. Then $T_N(ab) > t_{\alpha} \geq T_N(a)$ for some $t_{\alpha} \in [-1, 0)$. Thus $a \in T_N^{t_{\alpha}}(a)$, but $ab \notin T_N^{t_{\alpha}}(a)$, a contradiction. So $T_N(ab) \leq T_N(a)$. Similar way we can get $T_N(ab) \leq T_N(b)$.

If there are $a, b \in S$ such that $I_N(ab) < I_N(a)$. Then $I_N(ab) < t_{\beta} \leq I_N(a)$ for some $t_{\beta} \in (-1,\ 0]$. Thus $a \in I_N^{t_{\beta}}(a)$, but $ab \notin I_N^{t_{\beta}}(a)$, a contradiction. So $I_N(ab) \geq I_N(a)$. Similar way we can get $I_N(ab) \geq I_N(b)$.

If there are $a, b \in S$ such that $F_N(ab) > F_N(a)$. Then $F_N(ab) > t_{\gamma} \geq F_N(a)$ for some $t_{\gamma} \in [-1, 0)$. Thus $a \in F_N^{t_{\gamma}}(a)$, but $ab \notin F_N^{t_{\gamma}}(a)$, a contradiction. So $F_N(ab) \leq F_N(a)$. Similar way we can get $F_N(ab) \leq F_N(b)$.

Hence S_N is a neutrosophic $\aleph$ −left ideal of S. □

Theorem 3.3. Let S be a semigroup. Then the intersection of two neutrosophic $\aleph$ −left (resp., right) ideals of S is also a neutrosophic $\aleph$ −left (resp., right) ideal of S.

Proof: Let $S_N := \frac{S}{(T_N,\ I_N,\ F_N)}$ and $S_M := \frac{S}{(T_M,\ I_M,\ F_M)}$ be neutrosophic $\aleph$ −left ideals of S. Then for any $x, y \in S$, we have

$$T_{N\cap M}(xy) = T_N(xy) \vee T_M(xy) \le T_N(y) \vee T_M(y) = T_{N\cap M}(y),$$
$$I_{N\cap M}(xy) = I_N(xy) \wedge I_M(xy) \ge I_N(y) \wedge I_M(y) = I_{N\cap M}(y),$$
$$F_{N\cap M}(xy) = F_N(xy) \vee F_M(xy) \le F_N(y) \vee F_M(y) = F_{N\cap M}(y).$$

Therefore $X_{N\cap M}$ is a neutrosophic $\aleph$ −left ideal of *S*.

□

Corollary 3.4. Let *S* be a semigroup. Then $\{X_{N_i} | i \in \mathbb{N}\}$ is a family of neutrosophic $\aleph$ −left (resp., right) ideals of *S*, then so is $X_{\cap N_i}$.

Theorem 3.5. For any non-empty subset *A* of *S*, the following conditions are equivalent:

(i) A is a neutrosophic $\aleph$ −left (resp., right) ideal of *S*,

(ii) The characteristic neutrosophic $\aleph$ −structure $\chi_A(X_N)$ over *S* is a neutrosophic $\aleph$ −left (resp., right) ideal of *S*.

Proof: Assume that *A* is a neutrosophic $\aleph$ −left ideal of *S*. For any $x, y \in A$.

If $y \notin A$, then $\chi_A(T)_N(xy) \le 0 = \chi_A(T)_N(y)$; $\chi_A(I)_N(xy) \ge -1 = \chi_A(I)_N(y)$ and $\chi_A(F)_N(xy) \le 0 = \chi_A(F)_N(y)$. Otherwise $y \in A$. Then $xy \in A$, so $\chi_A(T)_N(xy) = -1 = \chi_A(T)_N(y)$; $\chi_A(I)_N(xy) = 0 = \chi_A(I)_N(y)$ and $\chi_A(F)_N(xy) = -1 = \chi_A(F)_N(y)$. Therefore $\chi_A(S_N)$ is a neutrosophic $\aleph$ −left ideal of *S*.

Conversely, assume that $\chi_A(S_N)$ is a neutrosophic $\aleph$ −left ideal of *S*. Let $a \in A$ and $x \in S$. Then $\chi_A(T)_N(xa) \le \chi_A(T)_N(a) = -1$, $\chi_A(I)_N(xa) \ge \chi_A(I)_N(a) = 0$ and $\chi_A(F)_N(xa) \le \chi_A(F)_N(a) = -1$. Thus $\chi_A(T)_N(xa) = -1$, $\chi_A(I)_N(xa) = 0$ and $\chi_A(F)_N(xa) = -1$ and hence $xa \in A$. Therefore *A* is a neutrosophic $\aleph$ −left ideal of *S*. □

Theorem 3.6. Let $\chi_A(S_N)$ and $\chi_B(S_N)$ be characteristic neutrosophic $\aleph$ −structure over *S* for subsets A and B of *S*. Then

(i) $\chi_A(S_N) \cap \chi_B(S_N) = \chi_{A\cap B}(S_N)$.

(ii) $\chi_A(S_N) \odot \chi_B(S_N) = \chi_{AB}(S_N)$.

Proof: (i) Let s $\in S$.

If s $\in A \cap B$, then

$$(\chi_A(T)_N \cap \chi_B(T)_N)(s) = \chi_A(T)_N(s) \vee \chi_B(T)_N(s) = -1 = \chi_{A\cap B}(T)_N(s),$$
$$(\chi_A(I)_N \cap \chi_B(I)_N)(s) = \chi_A(I)_N(s) \wedge \chi_B(I)_N(s) = 0 = \chi_{A\cap B}(I)_N(s),$$
$$(\chi_A(F)_N \cap \chi_B(F)_N)(s) = \chi_A(F)_N(s) \vee \chi_B(F)_N(s) = -1 = \chi_{A\cap B}(F)_N(s).$$

Hence $\chi_A(S_N) \cap \chi_B(S_N) = \chi_{A\cap B}(S_N)$.

If s $\notin A \cap B$, then s $\notin A$ or s $\notin B$. Thus

$$(\chi_A(T)_N \cap \chi_B(T)_N)(s) = \chi_A(T)_N(s) \vee \chi_B(T)_N(s) = 0 = \chi_{A\cap B}(T)_N((s)),$$
$$(\chi_A(I)_N \cap \chi_B(I)_N)(s) = \chi_A(I)_N(s) \wedge \chi_B(I)_N(s) = -1 = \chi_{A\cap B}(I)_N(s),$$
$$(\chi_A(F)_N \cap \chi_B(F)_N)(s) = \chi_A(F)_N(s) \vee \chi_B(F)_N(s) = 0 = \chi_{A\cap B}(F)_N(s).$$

Hence $\chi_A(S_N) \cap \chi_B(S_N) = \chi_{A\cap B}(S_N)$.

(ii) Let $x \in S$. If $x \in AB$, then $x = ab$ for some $a \in A$ and $b \in B$.

Now

$$\begin{aligned}
(\chi_A(T)_N \circ \chi_B(T)_N)(x) &= \wedge_{x=st}\{\chi_A(T)_N(s) \vee \chi_B(T)_N(t)\} \\
&\le \chi_A(T)_N(a) \vee (\chi_B(T)_N(b) \\
&= -1 = \chi_{AB}(T)_N(x), \\
(\chi_A(I)_N \circ \chi_B(I)_N)(x) &= \bigvee_{x=st}\{\chi_A(I)_N(s) \vee \chi_B(I)_N(t)\} \\
&\ge \chi_A(I)_N(a) \vee \chi_B(I)_N(b) \\
&= 0 = \chi_{AB}(I)_N(x), \\
(\chi_A(F)_N \circ \chi_B(F)_N)(x) &= \bigwedge_{x=st}\{\chi_A(F)_N(s) \vee \chi_B(F)_N(t)\}
\end{aligned}$$

$$\leq \chi_A(F)_N(a) \vee (\chi_B(F)_N (b)$$
$$= -1 = \chi_{AB}(F)_N(x).$$

Therefore $\chi_A(S_N) \odot \chi_B(S_N) = \chi_{AB}(S_N)$. □

Note 3.7. Let $S_N := \frac{S}{(T_N,\ I_N,\ F_N)}$ and $S_M := \frac{S}{(T_M,\ I_M,\ F_M)}$ be neutrosophic $\aleph$ −structures over *S*. Then for any subsets *A* and *B* of *S*, we have

(i) $\chi_{A\cap B}(S_N \cap S_M) = (S: \chi_{A\cap B}(T)_{N\cap M}, \chi_{A\cap B}(I)_{N\cap M}, \chi_{A\cap B}(F)_{N\cap M})$,

where

$$\chi_{A\cap B}(T)_{N\cap M}(s) = \chi_{A\cap B}(T)_N(s) \vee \chi_{A\cap B}(T)_M(s),$$
$$\chi_{A\cap B}(I)_{N\cap M}(s) = \chi_{A\cap B}(I)_N(s) \wedge \chi_{A\cap B}(I)_M(s),$$
$$\chi_{A\cap B}(F)_{N\cap M}(s) = \chi_{A\cap B}(F)_N(s) \vee \chi_{A\cap B}(F)_M(s) \ for\ s\ \varepsilon\ S.$$

(ii) $\chi_{A\cup B}(S_N \cap S_M) = (S: \chi_{A\cup B}(T)_{N\cup M}, \chi_{A\cup B}(I)_{N\cup M}, \chi_{A\cup B}(F)_{N\cup M})$,

where

$$\chi_{A\cup B}(T)_{N\cup M}(s) = \chi_{A\cup B}(T)_N(s) \wedge \chi_{A\cup B}(T)_M(s),$$
$$\chi_{A\cup B}(I)_{N\cup M}(s) = \chi_{A\cup B}(I)_N(s) \vee \chi_{A\cup B}(I)_M(s),$$
$$\chi_{A\cup B}(F)_{N\cup M}(s) = \chi_{A\cup B}(F)_N(s) \wedge \chi_{A\cup B}(F)_M(s) \ for\ s\ \varepsilon\ S.$$

Theorem 3.8. Let S_M be a neutrosophic $\aleph-$ structure over *S*. Then S_M is a neutrosophic $\aleph-$ left ideal of *S* if and only if $S_N \odot S_M \subseteq S_M$ for any neutrosophic $\aleph-$ structure S_N over *S*.

Proof: Assume that S_M is a neutrosophic $\aleph-$ left ideal of *S* and let $s, t, u \in S$. If s$= tu$, then

$T_M(s) = T_M(tu) \leq T_M(u) \leq T_M(t) \vee T_M(u)$ which implies $T_M(s) \leq T_{N\circ M}(s)$. Otherwise s$\neq tu$. Then $T_M(s) \leq 0 = T_{N\circ M}(s)$.

$I_M(s) = I_M(tu) \geq I_M(u) \geq I_M(t) \wedge I_M(t)$ which implies $I_M(s) \geq I_{N\circ M}(s)$. Otherwise s $\neq tu$. Then $I_M(s) \geq -1 = I_{N\circ M}(s)$.

$F_M(s) = F_M(tu) \leq F_M(u) \leq F_M(t) \vee F_M(u)$ which implies $F_M(s) \leq F_{N\circ M}(s)$. Otherwise s$\neq tu$. Then $F_M(s) \leq 0 = F_{N\circ M}(s)$.

Conversely, assume that S_M is a neutrosophic $\aleph-$ structure over *S* such that $S_N \odot S_M \subseteq S_M$ for any neutrosophic $\aleph-$ structure S_N over *S*. Let $x, y \in S$. If $a = xy$, then

$$T_M(xy) = T_M(a) \leq (\chi_X(T)_N \circ T_M)(a) = \bigwedge_{a=st}\{\chi_X(T)_N(s) \vee T_M(t)\} \leq \chi_X(T)_N(x) \vee T_M(y) = T_M(y),$$

$$I_M(xy) = I_M(a) \geq (\chi_X(I)_N \circ I_M)(a) = \bigvee_{a=st}\{\chi_X(I)_N(s) \wedge I_M(t)\} \geq \chi_X(I)_N(x) \vee I_M(y) = I_M(y),$$

$$F_M(xy) = F_M(a) \leq (\chi_X(F)_N \circ F_M)(a) = \bigwedge_{a=st}\{\chi_X(F)_N(s) \vee F_M(t)\} \leq \chi_X(F)_N(x) \vee F_M(y) = F_M(y).$$

Therefore S_M is a neutrosophic $\aleph-$ left ideal of *S*. □

Similarly, we have the following.

Theorem 3.9. Let S_M be a neutrosophic $\aleph-$ structure over *S*. Then S_M is a neutrosophic $\aleph-$ left ideal of *S* if and only if $S_M \odot S_N \subseteq S_M$ for any neutrosophic $\aleph-$ structure S_N over *S*.

Theorem 3.10. Let S_M and S_N be neutrosophic $\aleph-$ structures over *S*. If S_M is a neutrosophic $\aleph-$ left ideal of *S*, then so is the $S_M \odot S_N$.

Proof: Assume that S_M is a neutrosophic $\aleph-$ left ideal of *S* and let $x, y \in S$. If there exist $a, b \in S$ such that $y = ab$, then $xy = x(ab) = (xa)b$.

Now,

$$(T_N \circ T_M)(y) = \bigwedge_{y=ab}\{T_N(a) \vee T_M(b)\}$$

$$\leq \bigwedge_{xy=(xa)b} \{T_N(xa) \vee T_M(b)$$

$$= \bigwedge_{xy=cb} \{T_N(c) \vee T_M(b)\} = (T_N \circ T_M)(xy),$$

$$(I_N \circ I_M)(y) = \bigvee_{y=ab} \{I_M(b) \wedge I_M(b)\}$$

$$\geq \bigvee_{xy=(xa)b} \{I_M(xa) \wedge I_M(b)\}$$

$$= \bigvee_{xy=cb} \{I_M(c) \wedge I_M(b)\} = (I_N \circ I_M)(xy),$$

$$(F_N \circ F_M)(y) = \bigwedge_{y=ab} \{F_N(a) \vee F_M(b)\}$$

$$\leq \bigwedge_{xy=(xa)b} \{F_N(xa) \vee F_M(b)$$

$$= \bigwedge_{xy=cb} \{F_N(c) \vee F_M(b)\} = (F_N \circ F_M)(xy).$$

Therefore $S_M \odot S_N$ is a neutrosophic $\aleph -$ left ideal of *S*. □

Similarly, we have the following.

Theorem 3.11. Let S_M and S_N be neutrosophic $\aleph -$ structures over *S*. If S_M is a neutrosophic $\aleph -$ right ideal of *S*, then so is the $S_M \odot S_N$.

Conclusions

In this paper, we have introduced the notion of neutrosophic $\aleph -$ideals in semigroups and investigated their properties, and discussed characterizations of neutrosophic $\aleph -$ideals by using the notion of neutrosophic $\aleph -$ product, also provided conditions for neutrosophic $\aleph -$structure to be a neutrosophic $\aleph -$ ideal in semigroup. We have also discussed the concept of characteristic neutrosophic $\aleph -$structure of semigroups and its related properties. Using this notions and results in this paper, we will define the concept of neutrosophic $\aleph -$bi-ideals in semigroups and study their properties in future.

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