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NEUTROSOPHIC PROJECT EVALUATION AND REVIEW TECHNIQUES

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Abstract: One of the most important and challenging jobs that any manager can take in the management of a large scale project that requires coordinating numerous activities throughout the organization. Initially, the activity times are static within the CPM technique and probabilistic within the PERT technique. Since neutrosophic set is the generalization of fuzzy set and intuitionistic fuzzy set, a new method of project evaluation and review technique for a project network in neutrosophic environment is proposed in this paper. Considering single valued neutrosophic number as the time of each activity in the project network, neutrosophic expected task time, neutrosophic variance, neutrosophic critical path and the neutrosophic total expected time for completing the project network are calculated here. The main concept of Neutrosophic Project Evaluation and Review Technique(NPERT) method is to solve the ambiguities in the activity times of a project network easily than other existing methods like classical PERT, Fuzzy PERT etc. The proposed method is explained by an illustrative example and the results are discussed here.

Keywords: Neutrosophic set, Single Valued Neutrosophic Numbers, Neutrosophic critical path, Neutrosophic expected task times, Neutrosophic variance.

1. Introduction

The success of any large-scale project is very much dependent upon the quality of the planning, scheduling, and controlling of various phases of the project. Unless some type of planning and coordinating tool is used, the number of phases does not need to be very large before management starts losing control. Project Evaluation and Review Technique (PERT) is the best project management tool used to schedule, organize and coordinate the tasks in such type of large-scale project[3]. It is originally designed to plan a manufacturing project by employing a network of interrelated activities, coordinating optimum cost and time. It also emphasizes the relationship between the time of each activity, the costs associated with each phase, and the resulting time and cost for the anticipated completion of the entire project (Harry, 2004). PERT is also an integrated project man-agement system to manage the complexities of major manufacturing projects and the time deadlines created by defence industry projects. Most of these management systems were developed following World War II, and each has its advantages. PERT was first developed in 1958 by the U.S. Navy Special Projects office on the Polaris missile system. Existing integrated planning on such a large scale was deemed inadequate, so the Navy pulled in the Lockheed Aircraft Corporation and the management consulting firm of Booz, Allen, and Hamilton. Traditional techniques such as line of balance, Gantt charts and other systems were eliminated and PERT evolved as a means to deal with various time periods and it takes to finish the critical activities of an overall project. All defence contractors adopted PERT to manage the massive one-time projects associated with the industry after 1960. Smaller businesses awarded defence related government contracts, found it necessary to use PERT[9]. A typical PERT network consists of activities and events. An event is the completion of one program component at a particular time. An activity is defined as the time and resources required to move from one event to another. Therefore, when events and activities are clearly defined, progress of a program is easily monitored, and the path of the project proceeds toward termination. PERT mandates that each preceding event be completed before succeeding events and thus the final project can be considered complete. The critical path is a combination of events and activities. Slack time is defined as the difference between the total expected activity time for the project and the actual time for the entire project. Slack time is the spare time experienced in the PERT (Ghaleb, 2001). PERT plays a major role whenever uncertainity occurs in activity times of a project network[3]. Several researchers are introduced and discussed the concept of PERT/CPM in various situations[1,2,5,6,7,10,14,16,17,18,19]. Neutrosophic sets have been introduced as a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets to represent uncertain, inconsistent and incomplete information about real world problems. Elements of neutrosophic set are characterized by a truth-membership, falsity-membership and indeterminacy membership functions[11,12]. Neutrosophic set theory is applied in multi attribute decision making[15]. The subtraction and division of neutrosophic numbers were discussed in [13]. In this paper, new algorithm for finding project evaluation and review technique(NPERT) by neutro-sophic numbers for a given network is introduced in a better way than other existing methods. Neutrosophic critical path and their variance of a project network are calculated here. The neutrosophic expected task times for completing the project and the probability of time for completing the project within a expected period of time are also derived.

2.Preliminaries

Some basic definitions in neutrosophic set and neutrosophic numbers which are very useful in the construction of NPERT presented here.

Definition 2.1. [8]. Let E be a universe. A neutrosophic set A in E is characterized by a truth-membership function T_A , a indeterminacy-membership function I_A and a falsity-membership function F_A . $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

$$\begin{split} A &= \{<x,\,(T_A(x),\,I_A(x),\,F_A(x))>:x\in E;\,T_A(x),\,I_A(x),\,F_A(x)\in]\ 0;\ 1^+[\ \} \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq \qquad T_A(x)+I_A(x)+F_A(x){\leq}3^+: \ 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq \qquad T_A(x)+I_A(x)+F_A(x){\leq}3^+: \ 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ and } F_A(x). \text{ So, } 0\leq 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x) \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction on the sum of } T_A(x)\,I_A(x), \text{ for } 1 \\ \text{There is no restriction$$

Definition 2.2. [8]. Let E be a universe. A single valued neutrosophic set A, which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function T_A , an indeterminacymembership function I_A and a falsity-membership function F_A . $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

A = { < x; (
$$T_A(x)$$
, $I_A(x)$, $F_A(x)$) >: x \in E, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0; 1]$ }

Definition 2.3. [4]. Let $\tilde{a} = \langle (a_1, b_1, c_1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a \rangle$, and $\tilde{b} = \langle (a_2, b_2, c_2); \tilde{w}_b, \tilde{u}_b, \tilde{y}_b \rangle$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

1. $\widetilde{a} + \widetilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); \widetilde{w}_a \wedge \widetilde{w}_b, \widetilde{u}_a \vee \widetilde{u}_b, \widetilde{y}_a \vee \widetilde{y}_b \rangle$ 2. $\widetilde{a} - \widetilde{b} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2); \widetilde{w}_a \wedge \widetilde{w}_b, \widetilde{u}_a \vee \widetilde{u}_b, \widetilde{y}_a \vee \widetilde{y}_b \rangle$

$$3. \quad \widetilde{a}\widetilde{b} = \begin{cases} <(a_{1}a_{2}, b_{1}b_{2}, c_{1}c_{2}); \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} > 0; c_{2} > 0) \\ <(a_{1}c_{2}, b_{1}b_{2}, c_{1}a_{2}); \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} < 0; c_{2} > 0) \\ <(c_{1}c_{2}, b_{1}b_{2}, a_{1}a_{2}); \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} < 0; c_{2} < 0) \end{cases}$$

$$4. \quad \frac{\widetilde{a}}{\widetilde{b}} = \begin{cases} <(a_{1}/c_{2}, b_{1}/b_{2}, c_{1}/a_{2}); \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} < 0; c_{2} < 0) \end{cases}$$

$$<(c_{1}/c_{2}, b_{1}/b_{2}, c_{1}/c_{2}); \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} < 0; c_{2} > 0) \end{cases}$$

$$<(c_{1}/c_{2}, b_{1}/b_{2}, a_{1}/c_{2});) \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} < 0; c_{2} > 0)$$

$$<(c_{1}/a_{2}, b_{1}/b_{2}, a_{1}/c_{2});) \ \widetilde{w}_{a} \land \widetilde{w}_{b}, \ \widetilde{u}_{a} \lor \widetilde{u}_{b}, \ \widetilde{y}_{a} \lor \widetilde{y}_{b} > (c_{1} < 0; c_{2} < 0) \end{cases}$$

$$5. \quad \gamma \widetilde{a} = \qquad \left\{ \begin{array}{c} <(\gamma a_{1}, \gamma b_{1}, \gamma c_{1}); \ \widetilde{w}_{a}, \ \widetilde{u}_{a}, \ \widetilde{y}_{a} > (\gamma > 0) \end{array} \right\}$$

$$\begin{cases} < (\gamma c1, \gamma b1, \gamma a1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a > (\gamma > 0) \end{cases}$$

6. $\tilde{a}^{-1} = \langle (1/c1, 1/b1, 1/a1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a \rangle (\tilde{a} \neq 0)$

Definition 2.4. [8]. Let $\tilde{A}_1 = \langle T_1, I_1, F_1 \rangle$ be a single valued neutrosophic number. Then, the score function $s(\tilde{A}_1)$, accuracy function $a(\tilde{A}_1)$, and certainty function $c(\tilde{A}_1)$ of an single valued neutrosophic numbers are defind 1. $s(\tilde{A}_1) = (T_1 + 1 - I_1 + 1 - F_1)/3$

2. $a(\tilde{A}_1) = T_1 - F_1$ 3. $c(\tilde{A}_1) = T_1$

Definition 2.5. [13]. Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be two single-valued neutrosophic numbers, where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0; 1]$, and $0 \le t_1, i_1, f_1 \le 3$ and $0 \le t_2, i_2, f_2 \le 3$. The division of neutrosophic numbers A and B is defined as follows:

A \emptyset B = (t₁, i₁, f₁) \emptyset (t₂, i₂, f₂) = $(\frac{t_1}{t_2}, \frac{i_1 - i_2}{1 - i_2}, \frac{f_1 - f_2}{1 - f_2})$, where t₁, i₁, f₁, t₂, i₂, f₂ \in [0; 1], with the restriction that $t_2 \neq 0, i_2 \neq 0$

1 and $f_2 \neq 1$

Similarly, the division of neutrosophic numbers only partially works, i.e. when $t_2 \neq 0$, $i_2 \neq 1$ and $f_2 \neq 1$. In the same way, the restriction that

 $\left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right) \in ([0, 1], [0, 1])$ is set when the traditional case occurs, when the neutrosophic number components t,

i, f are in the interval [0, 1].

3 NPERT Analysis

NPERT computations are the same as those of NCPM[8]. The main difference is that instead of activity duration we use neutrosphic expected time for the activity. Activity times are represented by a neutrosphic probability distribution. This neutrosphic probability distribution is based on three different time estimates are as follows:

1. Neutrosophic Optimistic Time [$t_o^N = (t_o^T, t_o^I, t_o^F)^N$]:

In this time, each and every activity of a network is going well without any disturbance like shortage of money, labour and raw materials etc., and the project will be completed within a period of expected time.

2. Neutrosophic Pessimistic Time [$t_p^N = (t_p^T, t_p^I, t_p^F)^N$]:

In this time, most of the activities in a project are disturbed when the work is going. So, the project will not be completed in an expected period of time. The time of completion of the project will go more than the expected time.

3. Neutrosophic Most likely Time [$t_m^N = (t_m^T, t_m^I, t_m^F)^N$]:

In this time, some of the activities are disturbed when the work is going. So the time of completion will extend slightly more than the expected time

Also the neutrosophic expected mean time $[t_e^N = (t_e^T, t_e^T, t_e^T, t_e^F)^N$ and the neutrosophic variance $\sigma^{2^N} = (\sigma^{T^2}, \sigma^{T^2}, \sigma^{F^2})^N$ of the project network are given as follows:

$$t_e^N = \frac{1}{6} [t_0^N + 4t_m^N + t_p^N] \text{ and } \sigma^{2^N} = \frac{(t_p^N - t_0^N)}{6}$$

The neutrosophic earliest and latest start time as well as neutrosophic earliest and latest finish time of each activity for finding neutrosophic critical path of a project are derived by using forward pass and backward pass calculations as follows:

(a). Forward Pass Calculations:

Here, we start from the initial node 1 with starting time zero in increasing order and end at final node n. At each node, neutrosophic earliest start and finish times are calculated by considering $E(\mu_i)^N$.

Step:1 Set $E(\mu_1)^N = <0, 0, 0>$, i=1 for the initial node.

Step:2 Set neutrosophic earliest start for each activity as $ES(\mu_{ij})^N = E(\mu_i)$ for all activities (i,j) that start at node i.

Step:3 Compute neutrosophic earliest finish for each activity as $EF(\mu_{ij})^N = E(\mu_{ij})^N + (t_{ij})^N = E(\mu_i)^N + (t_{ij})^N$, for all activities (i,j) that start at node i and move on to next node.

Step:4 If node j > i, compute neutrosophic earliest occurance for each node j using $E(\mu_j)^N = \max \{EF(\mu_{ij})^N\} = \max \{E(\mu_i)^N + (t_{ij})^N\}$, for all immediate predecessor activities.

Step:5 If j=n(final node), then the neutrosophic earliest finish time for the project is given by $E(\mu_n)^N = \max \{E(\mu_{n-1})^N + (t_{ij})^N\}:$

(b). Backward Pass Calculations:

Here, we start from last(final) node n of the network in decreasing order and end at initial node 1. At every node, neutrosophic latest finish and start times for each activity are calculated by considering $L(\mu_j)^N$.

Step:1 $L(\mu_n)^N = E(\mu_n)^N$, j=n.

Step:2 Set neutrosophic latest finish time for each activity as LF $(\mu_{ij})^N = L(\mu_j)$ that ends at node j.

Step:3 Compute neutrosophic latest occurence time for all activities ends at j as $LS(\mu_{ij})^N = LF(\mu_{ij})^N (t_{ij})^N = L(\mu_{ij})^N (t_{ij})^N$, for all activities (i,j) that start at node i and move on to next node.

Step:4 If node i < j, compute neutrosophic latest occurence time for each node i, using $L(\mu_i)^N = \min\{LS(\mu_{ij})^N\} = \min\{L(\mu_i)^N - (t_{ij})^N\}$ by proceeding backward process from node j to node 1. **Step:5** If j=1(initial node), then we have $L(\mu_1)^N = \min\{LS(\mu_{ij})^N\} = \min\{L(\mu_2)^N - (t_{ij})^N\}$:

From the above calculations, an activity (i,j) will be critical if it satisfies the following conditions: (i). $E(\mu_i)^N = L(\mu_i)^N$ and $E(\mu_j)^N = L(\mu_j)^N$ (ii). $E(\mu_j)^N - E(\mu_i)^N = L(\mu_j)^N - L(\mu_i)^N = (t_{ij})^N$. An activity which does not satisfies the above conditions is called non critical activity.

3.1 Neutrosophic Float or Slack of an activity and event:

The neutrosophic time of an activity which makes delay in its completion time without affecting the total project completion time is called neutrosophic float. Neutrosophic event float and neutrosophic activity float are two types of neutrosophic floats.

1. Neutrosophic Event float:

The neutrosophic float of an event is the difference between its neutrosophic latest and earliest time. (i.e). Neutrosophic event float = $L(\mu_i)^N - E(\mu_i j)^N$

2. Neutrosophic activity float:

Neutrosophic total float and neutrosophic free float are two types of neutrosophic activity floats. They are calculated as follows:

a. Neutrosophic total float(NTF)

Neutrosophic total float is the positive difference between the neutrosophic earliest finish(start) time and neutrosophic latest finish(start) time of an activity. Neutrosophic total float (NTF) = LF $(\mu_{ij})^N$ - EF $(\mu_{ij})^N$ (or) LS $(\mu_{ij})^N$ - ES $(\mu_{ij})^N$

b. Neutrosophic free float(NFF)

The delay in neutrosophic time of an activity which does not cause delay in its immediate successor activities is called neutrosophic free float of an activity. Neutrosophic free float(NFF) = ($E(\mu_j)^N - E(\mu_i)^N$) - $(t_{ij})^N$

3.2 Neutrosophic Project Evaluation an Review Technique Algorithm:

In this section, NPERT algorithm for a project network is established. This algorithm is used to find the neutrosophic critical path, neutrosophic expected time to complete each activity in a project and to calculate the probability of the neutrosophic expected time to compete the total project within a given period of time when it is not able to find best solution using existing methods in uncertain environment.

Step:1 Calculate neutrosophic earliest and latest work time for every activity using forward pass and backward pass calculations.

Step:2 Using neutrosophic earliest and latest work time of every activity, determine neutrosophic critical path for the given network.

Step:3 Calculate neutrosophic expected time of every activity for the given network.

Step:4 Calculate neutrosophic expected variance.

Step:5 Calculate neutrosophic total float of every activity.

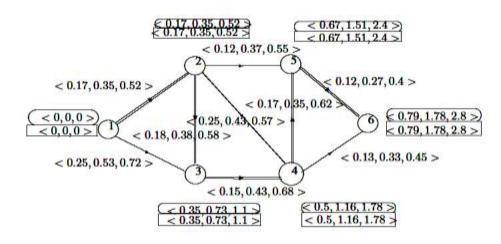
Step:6 Find neutrosophic standard normal $Z^N = \frac{D^N - E(\mu_c^N)}{\sigma^N}$ using the neutrosophic critical path.

Step:7 Finally estimate the probability of completing the project within a due date.

Illustrative Example:

The neutrosophic estimate times for all activities in a project network are given in the following table.

Task	Neutro. optimistic time (t_0^N)	Neutro. Most likely time(t _m ^N)	Neutro. Pessimistic time(tp ^N)
A(1,2)	< 0.1, 0.7, 0.8 >	< 0.2, 0.3, 0.5 >	< 0.1, 0.2, 0.3 >
A(1,3)	< 0.2, 0.8, 0.9>	< 0.3, 0.5, 0.7 >	< 0.1, 0.4, 0.6 >
A(2,3)	< 0.1, 0.4, 0.7 >	< 0.2, 0.4, 0.6 >	< 0.2, 0.3, 0.4 >
A(2,4)	< 0.2, 0.6, 0.7 >	< 0.3, 0.4, 0.5 >	< 0.1, 0.4, 0.7 >
A(2,5)	< 0.1, 0.3, 0.6 >	< 0.1, 0.4, 0.5 >	< 0.2, 0.5, 0.7 >
A(3,4)	< 0.2, 0.5, 0.9 >	< 0.1, 0.4, 0.6 >	< 0.3, 0.5, 0.8 >
A(4,5)	< 0.1, 0.2, 0.4 >	< 0.2, 0.4, 0.7 >	< 0.1, 0.3, 0.5 >
A(4,6)	< 0.2, 0.4, 0.6 >	< 0.1, 0.3, 0.4 >	< 0.3, 0.4, 0.5 >
A(5,6)	< 0.1, 0.5, 0.7 >	< 0.1, 0.2, 0.3 >	< 0.2, 0.3, 0.5 >



Determine the following:

i. Neutrosophic earliest and neutrosophic latest expected times for each node

- ii. Neutrosophic critical path
- iii. Neutrosophic expected task times and their variance
- iv. The probability that the project will be completed within $[D^{N} =] < 0.8, 2, 3 >$

Solution:

i(a).Neutrosophic earliest expected task times:

$E(\mu_1)^N$	$= \langle 0, 0, 0 \rangle$
$E(\mu_2)^N$	= < 0, 0, 0 > + < 0.17, 0.35, 0.52 >
	$= \langle 0.17, 0.35, 0.52 \rangle$
$E(\mu_3)^N$	$= \max\{<0, 0, 0>+<0.25, 0.53, 0.72>, <0.17, 0.35, 0.52>+<0.18, 0.38, 0.58>\}$
	= < 0.35, 0.73, 1.1 >
Similarly,	
$E(\mu_4)^N$	= < 0.5, 1.16, 1.78 >
$E(\mu_5)^N$	= < 0.67, 1.51, 2.4 >
$E(\mu_6)^N$	= < 0.79, 1.78, 2.8 >

i(b). Neutrosophic latest expected task times:

$L(\mu_6)^N$	= < 0.79, 1.78, 2.8 >
$L(\mu_5)^N$	= < 0.79, 1.78, 2.8 > - < 0:12, 0:27, 0:4 >
$L(\mu_4)^N$	$= \min\{<0.79, 1.78, 2.8 > -<0.13, 0.33, 0.45 >, <0.69, 1.51, 2.4 > -<0.17, 0.35, 0.62 >\}$
	= < 0.5, 1.16, 1.78 >
milarly	

Similarly,

$L(\mu_3)^N$	= < 0.35, 0.73, 1.1 >
$L(\mu_2)^N$ $L(\mu_1)^N$	= < 0.17, 0.35, 0.52 >
$L(\mu_1)^N$	= < 0, 0, 0 >

ii. From i(a) and i(b), we conclude that the neutrosophilic critical path is $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$.

iii. From above neutrosophic critical path, the total neutrosophic expected task time for completing the project is < 0.79, 1.78, 2.8 >

Also, the neutrosophic variance for the critical activities are:

Task	$t_{e} N = \frac{1}{6} [t_{o}^{N} + 4t_{m}^{N} + t_{p}^{N}]$	$\boldsymbol{\sigma^{2^N}} = (\begin{array}{c} t_p^N t_o^N \\ \hline 6 \end{array})^2$
(1,2)	< 0.17, 0.35, 0.52 >	< 0, 0.0064, 0.0064 >
(2,3)	< 0.18, 0.38, 0.58 >	< 0.0004, 0.0004, 0.0025 >
(3,4)	< 0.15, 0.43, 0.68 >	< 0.0004, 0, 0.0004 >
(4,5)	< 0.17, 0.35, 0.62 >	< 0, 0.0004, 0.0004 >
(5,6)	< 0.12, 0.27, 0:4 >	< 0.0004, 0.0009, 0.0009 >
Total	< 0.79, 1.78, 2.8 >	< 0.0012, 0.0081, 0.0106 >

Table.1

Hence, from table(1), the neutrosophic expected mean time [E(μ_e^N)] = < 0.79, 1.78, 2.8 > and the neutrosophic expected valance [σ^{2^N}] =<0.0012, 0.0081, 0.0106>

So, $\sigma^N = < 0.0346, 0.09, 0.103 > .$

Now,

$$Z^{N} = \frac{D^{N} - \mathcal{E}(\mu e^{N})}{e^{N}}$$

= $\frac{\langle 0.8, 2, 3 \rangle - \langle 0.79, 1.78, 2.8 \rangle}{\langle 0.0346, 0.09, 0.103 \rangle}$
= $\frac{\langle 0.01, 0.22, 0.2 \rangle}{\langle 0.0346, 0.09, 0.103 \rangle}$
= $\langle 0.29, 0.143, 0.108 \rangle$
= 0.6796

by score function in definition 2.4.

iv. From the table of area under normal curve, $P(Z^N \le 0.6796) = 0.5 + 0.2517 = 0.7517$. Therefore, the requried probability that the project will be completed within the time < 0.8, 2, 3 > is 0.7517.

4 Applications

This method is very useful than other existing methods like PERT, fuzzy PERT and intuitionistic fuzzy PERT etc., whenver uncertainty occurs in various activities like planning, scheduling, developing, designing, testing, maintaining and advertising for the fields of administration, construction, manufacturing and marketing etc.,

5 Advantages

- 1. It gives the better accuracy than other methods for each and every activity of a project.
- 2. Due to its accuracy, it is easy to find the best optimum schedule for every project.
- 3. Also the level of performance of each and every activity will be increased by interrelating them.

4. Controlling each activity in a project become very simple.

Conclusion

In this paper, NPERT calculation with single valued neutrosophic numbers for finding the total neutrosophic expected task time for completing a project network is introduced. The proposed method helps the users to take right decisions in scheduling, organizing and completing the project within a minimum duration. Also, it helps to find the probability of neutrosophic estimate time of a project. Comparing with other existing meth-ods, this method gives better results and also the NPERT algorithm is explained by an example using a set of neutrosophic numbers as length of arcs in a network. The applications and advantages of proposed method are also given. In future, we have planned to use this NPERT method in various network models.

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