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Neutrosophic Q-Fuzzy Subgroups

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Abstract: In this paper, the notation of concept of neutrosopy in Q-fuzzy set is introduced. Further some properties and results on neutrosophic Q-fuzzy subgroups are discussed.

Keywords: Neutrosophic Q-fuzzy set, neutrosophic Q-fuzzy subgroup (NQLFG), neutrosophic Q-fuzzy normal subgroup.

1. Introduction

Zadeh [11] introduced the notion of fuzzy sets and fuzzy set operations. Afterwards many researches were conducted on the notion of fuzzy sets. The study of algebraic structure of fuzzy sets was started by Rosenfield [7]. Smarandache [8, 9] introduced the notion of Neutrosophy as a new branch of philosophy. Neutrosophy is a base of Neutrosophic logic which is an extension of fuzzy logic in which indeterminacy is included. In Neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F. The theory of neutrosophic set have achieved great success in various fields like medical diagnosis, image processing decision making problem and so on. Arockiarani, and Martina Jency [2] consider the neutrosophic set with value from the subset of [0,1] and extended the research in fuzzy neutrosophic set. They [3] initiated the concept of subgroupoid in fuzzy neutrosophic set. Solairaju and Nagarajan [10] introduced and defined a new algebraic structure of Q-fuzzy groups. In this paper, the concept of neutrosophic Q-fuzzy set and derived the results on neutrosophic Q-fuzzy subgroups.

2. Preliminaries

Definition 2.1. *Let X be a non-empty set. A fuzzy set A is a map $A : X \to [0, 1]$.*

Definition 2.2. *Let X and Q be any two non-empty sets. A mapping $\mu : X \times Q \to [0, 1]$ is called a Q-fuzzy set in X.*

Definition 2.3. *A Neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, an indeterminacy function $I_A(x)$ and a falsity membership function $F_A(x)$ is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, where $T_A, I_A, F_A : X \to [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.*

Definition 2.4. *Let X, Y be two non-empty sets and $f : X \to Y$ be a function.*

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(1). If $B = \{\langle y, T_B(y), I_B(y), F_B(y)\rangle : y \in Y\}$ is a neutrosophic fuzzy set in Y then the preimage of B under f, denoted by $f^{-1}(B)$, is the neutrosophic fuzzy set in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x))\rangle : x \in X\}$ where $f^{-1}(T_B(x)) = T_B(f(x))$.

(2). If $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$ is a Neutrosophic fuzzy set in X, then the image f(A) of A under f is the neutrosophic fuzzy set in Y defined by $f(A) = \{\langle y, f(T_A(y)), f(I_A(y)), f(F_A(y))\rangle : y \in Y\}$,

where

$$f(T_A(y)) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} T_A(x); & \text{if } f^{-1}(y) \neq 0_N \\ 0; & \text{otherwise} \end{cases}$$

$$f(I_A(y)) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} I_A(x); & \text{if } f^{-1}(y) \neq 0_N \\ 0; & \text{otherwise} \end{cases}$$

$$f_{\sim}(F_A(y)) = \begin{cases} \inf\limits_{x\in f^{-1}(y)} F_A(x); & \text{if } f^{-1}(y) \neq 0_N \\ 1; & \text{otherwise} \end{cases},$$

where $f_{\sim}(F_A(y)) = (1 - f(1 - F_A))\, y$.

Definition 2.5. *A neutrosophic Q-fuzzy set is an object having the form* $A = \{\langle (x,q), T_A(x,q), I_A(x,q), F_A(x,q)\rangle : x \in X, q \in Q\}$, *where* $T_A : X \times Q \to [0,1]$, $I_A : X \times Q \to [0,1]$, $F_A : X \times Q \to [0,1]$ *denote the degree of truth membership function, degree of indeterminacy membership function and the degree of false membership function for each element* (x,q) *to the set* A *respectively, and* $0 \le T_A(x,q) + I_A(x,q) + F_A(x,q) \le 3$, *for all* $x \in X$, *and* $q \in Q$.

Definition 2.6. *Let* X *be a nonempty set, and* $A = \langle (x,q), T_A(x,q), I_A(x,q), F_A(x,q)\rangle$, $B = \langle (x,q), T_B(x,q), I_B(x,q), F_B(x,q)\rangle$ *are two neutrosophic Q-fuzzy sets on X. Then,*

(1). $A \subseteq B$, *if* $T_A(x,q) \le T_B(x,q)$, $I_A(x,q) \le I_B(x,q)$, $F_A(x,q) \ge F_B(x,q)$, *for all* $x \in X$, *and* $q \in Q$.

(2). $A \cup B = \langle (x,q), \max(T_A(x,q), T_B(x,q)), \max(I_A(x,q), I_B(x,q)), \min((F_A(x,q), F_B(x,q))\rangle$.

(3). $A \cap B = \langle (x,q), \min(T_A(x,q), T_B(x,q)), \min(I_A(x,q), I_B(x,q)), \max((F_A(x,q), F_B(x,q))\rangle$.

Definition 2.7. *The complement* A^c *of a neutrosophic Q-fuzzy subset* A *is defined by* $A^c = \{\langle (x,q), T_{A^c}(x,q), I_{A^c}(x,q), F_{A^c}(x,q)\rangle : x \in X, q \in Q\}$, *where* $T_{A^c}(x,q) = F_A(x,q)$, $I_{A^c}(x,q) = 1 - I_A(x,q)$, $F_{A^c}(x,q) = T_A(x,q)$, *for all* $x \in X$, *and* $q \in Q$.

3. Neutrosophic Q-Fuzzy Subgroups

Definition 3.1. *Let* $(G, .)$ *be a group and let* A *be a Neutrosophic Q-fuzzy subset in* G. *Then* A *is called a Neutrosophic Q-fuzzy sub group of* G *if it satisfies the conditions*

(1). $T_A(xy,q) \ge (T_A(x,q) \wedge (T_A(y,q)$, $I_A(xy,q) \ge (I_A(x,q) \wedge (I_A(y,q)$, $F_A(xy,q) \le (F_A(x,q) \vee (F_A(y,q))$.

(2). $T_A\left(x^{-1},q\right) = T_A(x,q)$, $I_A\left(x^{-1},q\right) = I_A(x,q)$, $F_A\left(x^{-1},q\right) = F_A(x,q)$, *for all* $x, y \in G$, $q \in Q$.

Theorem 3.2. *A neutrosophic Q-fuzzy subset* A *of* G *is a neutrosophic Q-fuzzy subgroup of* G *if and only if* $T_A\left(xy^{-1},q\right) \ge (T_A(x,q) \wedge (T_A(y,q)$, $I_A\left(xy^{-1},q\right) \ge (I_A(x,q) \wedge (I_A(y,q)$, $F_A\left(xy^{-1},q\right) \le (F_A(x,q) \vee (F_A(y,q))$.

Proof. Let A be a neutrosophic Q-fuzzy subgroup of G. $\Leftrightarrow T_A(xy,q) \geq T_A(x,q) \wedge T_{\boldsymbol{A}}(y,q)$, $I_A(xy,q) \geq I_A(x,q) \wedge I_A(y,q)$, $F_A(xy,q) \leq F_A(x,q) \vee F_A(y,q))$ and $T_A(x^{-1},q) = T_A(x,q)$, $I_A(x^{-1},q) = I_A(x,q)$, $F_A(x^{-1},q) = F_A(x,q)$ for all $x,y \in G$, and $q \in Q \Leftrightarrow T_A(xy^{-1},q) \geq T_A(x,q) \wedge T_A(y,q)$, $I_A(xy^{-1},q) \geq I_A(x,q) \wedge I_{\boldsymbol{A}}(y,q)$, $F_A(xy^{-1},q) \leq F_A(x,q) \vee F_{\boldsymbol{A}}(y,q)$. □

Definition 3.3. *A neutrosophic Q-fuzzy subgroup A of G is said to be Neutrosophic Q-fuzzy normal subgroup of G if* $T_A(xy,q) = T_A(yx,q)$, $I_A(xy,q) = I_A(yx,q)$, $F_A(xy,q) = F_A(yx,q)$ *or* $T_A(xyx^{-1},q) = T_A(y,q)$, $I_A(xyx^{-1},q) = I_A(y,q)$, $F_A(xyx^{-1},q) = F_A(y,q)$ *for all* $x,y \in G$, $q \in Q$.

Definition 3.4. *Let A be a neutrosophic Q-fuzzy subset of X. Let* $\alpha,\beta,\gamma \in [0,1]$ *with* $\alpha+\beta+\gamma \leq 3$. *Then* $[\alpha,\beta,\gamma]$*-Q-level subset of A is defined by* $[A]_{(\alpha,\beta,\gamma)} = \{x \in X, q \in Q : T_A(x,q) \geq \alpha, I_A(x,q) \geq \beta, F_A(x,q) \leq \gamma\}$.

Theorem 3.5. *If A is a NQFSG of G and* $\alpha,\beta,\gamma \in [0,1]$, *then* $[\alpha,\beta,\gamma]$*-Q-level subset* $[A]_{(\alpha,\beta,\gamma)}$ *of A is a subgroup of G where* $T_A(e,q) \geq \alpha$, $I_A(e,q) \geq \beta$, $F_A(e,q) \leq \gamma$, *where e is the identity element of G, and* $q \in Q$.

Proof. Since, $T_A(e,q) \geq \alpha$, $I_A(e,q) \geq \beta$, $F_A(e,q) \leq \gamma$, $e \in [A]_{(\alpha,\beta,\gamma)}$. Therefore $[A]_{(\alpha,\beta,\gamma)} \neq \{\ \}$. Let $x,y \in [A]_{(\alpha,\beta,\gamma)}$ and $q \in Q$. Then

$$T_A(x,q) \geq \alpha, I_A(x,q) \geq \beta, F_A(x,q) \leq \gamma, T_A(y,q) \geq \alpha, I_A(y,q) \geq \beta, F_A(y,q) \leq \gamma.$$
$$\Leftrightarrow T_A(x,q) \wedge T_A(y,q) \geq \alpha, I_A(x,q) \wedge I_A(y,q) \geq \beta, F_A(x,q) \vee F_A(y,q) \leq \gamma$$
$$\Leftrightarrow T_A(xy^{-1},q) \geq \alpha, I_A(xy^{-1},q) \geq \beta, F_A(xy^{-1},q) \leq \gamma$$
$$\Leftrightarrow xy^{-1} \in [A]_{(\alpha,\beta,\gamma)}$$
$$\Leftrightarrow [A]_{(\alpha,\beta,\gamma)} \quad \text{is a subgroup of G.}$$

□

Theorem 3.6. *If A is a neutrosophic Q-fuzzy subset of a group G, then A is a NQFSG of G if and only if* $[A]_{(\alpha,\beta,\gamma)}$ *is a subgroup of G for* $\alpha,\beta,\gamma \in [0,1]$.

Proof. Let R$x,y \in [A]_{(\alpha,\beta,\gamma)}$ and $q \in Q$. Let A is a neutrosophic Q-fuzzy subgroup of G.

$$\Leftrightarrow T_A(xy^{-1},q) \geq T_A(x,q) \wedge T_A(y,q), I_A(xy^{-1},q) \geq I_A(x,q) \wedge I_A(y,q), F_A(xy^{-1},q) \leq F_A(x,q) \vee F_A(y,q))$$
$$\Leftrightarrow T_A(xy^{-1},q) \geq \alpha, I_A(x,q) \geq \beta, F_A(xy^{-1},q) \leq \gamma$$
$$\Leftrightarrow xy^{-1} \in [A]_{(\alpha,\beta,\gamma)}.$$
$$\Leftrightarrow [A]_{(\alpha,\beta,\gamma)} \quad \text{is a subgroup of G.}$$

□

Theorem 3.7. *A is a neutrosophic Q-fuzzy subset of a group G. Then A is a neutrosophic Q-fuzzy normal subgroup of G if and only if* $[A]_{(\alpha,\beta,\gamma)}$ *is a normal subgroup of G.*

Proof. Let A be a neutrosophic Q-fuzzy normal subgroup of G. Then, $T_A(xyx^{-1},q) = T_A(y,q) \geq \alpha$, $I_A(xyx^{-1},q) = I_A(y,q) \geq \beta$, $F_A(xyx^{-1},q) = F_A(y,q) \leq \gamma$, for all $x,y \in G$, $q \in Q$. Hence $[A]_{(\alpha,\beta,\gamma)}$ is a normal subgroup of G. □

Theorem 3.8. *If* $A_1, A_2, \ldots, A_n$ *be neutrosophic Q-fuzzy subgroups of G. Then* $A = \bigcup_{i=1}^{n} A_i$ *is a Neutrosophic Q-fuzzy subgroup of G.*

Proof. Let $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G. Let $A = \bigcup_{i=1}^{n} A_i$, $x, y \in G$, $q \in Q$. Then,

$$\begin{aligned}
A\left(xy^{-1}, q\right) &= \bigcup_{i=1}^{n} A_i\left(xy^{-1}, q\right) \\
&= \langle (x,q), \left(T_{\bigcup_{i=1}^{n} A_I}\left(xy^{-1}, q\right)\right), \left(I_{\bigcup_{i=1}^{n} A_I}\left(xy^{-1}, q\right)\right), (F_{\bigcup_{i=1}^{n} A_I}\left(xy^{-1}, q\right))\rangle \\
T_{\bigcup_{i=1}^{n} A_I}\left(xy^{-1}, q\right) &= \vee T_{A_I}\left(xy^{-1}, q\right) \geq \vee(T_{A_I}(x,q) \wedge T_{A_I}(y,q)) \\
&= (\vee T_{A_I}(x,q)) \wedge (\vee T_{A_I}(y,q)) \\
&= T_{\bigcup_{i=1}^{n} A_I}(x,q) \wedge T_{\bigcup_{i=1}^{n} A_I}(y,q) \\
I_{\bigcup_{i=1}^{n} A_I}\left(xy^{-1}, q\right) &= \vee I_{A_I}\left(xy^{-1}, q\right) \geq \vee(I_{A_I}(x,q) \wedge I_{A_I}(y,q)) \\
&= (\vee I_{A_I}(x,q)) \wedge (\vee I_{A_I}(y,q)) \\
&= I_{\bigcup_{i=1}^{n} A_I}(x,q) \wedge I_{\bigcup_{i=1}^{n} A_I}(y,q) \\
F_{\bigcup_{i=1}^{n} A_I}\left(xy^{-1}, q\right) &= \wedge F_{A_I}\left(xy^{-1}, q\right) \leq \wedge(F_{A_I}(x,q) \vee F_{A_I}(y,q)) \\
&= (\wedge(F_{A_I}(x,q)) \vee (\wedge(F_{A_I}(y,q)) \\
&= F_{\bigcup_{i=1}^{n} A_I}(x,q) \vee F_{\bigcup_{i=1}^{n} A_I}(y,q)
\end{aligned}$$

Hence, $A = \bigcup_{i=1}^{n} A_i$ is a Neutrosophic Q-fuzzy subgroup of G. □

Theorem 3.9. *If $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G. Then $A = \bigcap_{i=1}^{n} A_i$ is a neutrosophic Q-fuzzy subgroup of G.*

Proof. Let $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G. Let $A = \bigcap_{i=1}^{n} A_i$, $x, y \in G$, $q \in Q$. Then,

$$\begin{aligned}
A\left(xy^{-1}, q\right) &= \bigcap_{i=1}^{n} A_i\left(xy^{-1}, q\right) \\
&= \langle (x,q), \left(T_{\bigcap_{i=1}^{n} A_I}\left(xy^{-1}, q\right)\right), \left(I_{\bigcap_{i=1}^{n} A_I}\left(xy^{-1}, q\right)\right), (F_{\bigcap_{i=1}^{n} A_I}\left(xy^{-1}, q\right)\rangle \\
T_{\bigcap_{i=1}^{n} A_I}\left(xy^{-1}, q\right) &= \wedge T_{A_I}\left(xy^{-1}, q\right) \geq \wedge(T_{A_I}(x,q) \wedge T_{A_I}(y,q)) \\
&= (\wedge T_{A_I}(x,q)) \wedge (\wedge T_{A_I}(y,q)) \\
&= T_{\bigcap_{i=1}^{n} A_I}(x,q) \wedge T_{\bigcap_{i=1}^{n} A_I}(y,q) \\
I_{\bigcap_{i=1}^{n} A_I}\left(xy^{-1}, q\right) &= \wedge I_{A_I}\left(xy^{-1}, q\right) \geq \wedge(I_{A_I}(x,q) \wedge I_{A_I}(y,q)) \\
&= (\wedge I_{A_I}(x,q)) \wedge (\wedge I_{A_I}(y,q)) \\
&= I_{\bigcap_{i=1}^{n} A_I}(x,q) \wedge I_{\bigcap_{i=1}^{n} A_I}(y,q) \\
F_{\bigcap_{i=1}^{n} A_I}\left(xy^{-1}, q\right) &= \vee F_{A_I}\left(xy^{-1}, q\right) \leq \vee(F_{A_I}(x,q) \vee F_{A_I}(y,q)) \\
&= (\vee(F_{A_I}(x,q)) \vee (\vee(F_{A_I}(y,q)) \\
&= F_{\bigcap_{i=1}^{n} A_I}(x,q) \vee F_{\bigcap_{i=1}^{n} A_I}(y,q)
\end{aligned}$$

Hence, $A = \bigcap_{i=1}^{n} A_i$ is a neutrosophic Q-fuzzy subgroup of G. □

4. Homomorphism of Neutrosophic Q-Fuzzy Subgroups of G

Definition 4.1. *Let G and G' be any two groups. The function $f : G \times Q \to G' \times Q$ is said to be group Q-homomorphism if*

(1). $f : G \to G'$ is a group homomorphism

(2). $f(xy, q) = f(x, q)\, f(y, q)$, for all $x, y \in G$, $q \in Q$.

Theorem 4.2. *Let G and G' be groups and f be a homomorphism of G onto G'. If A is a NQFSG of G', then $f^{-1}(A)$ is a NQFSG of G.*

Proof. Let A be a Neutrosophic fuzzy subgroup of G'. By definition, $f^{-1}(A) = (f^{-1}(T_A), f^{-1}(I_A), f^{-1}(F_A))$. Now for $x, y \in G$, $q \in Q$, we have

$$\begin{aligned}
f^{-1}(T_A)\left(xy^{-1}, q\right) &= T_A(f\left(xy^{-1}, q\right)) \\
&= T_A\left(f(x)\, f\left(y^{-1}\right), q\right) \quad \text{(since f is a homomorphism)} \\
&\geq T_A\left(f(x), q\right) \wedge T_A\left(f\left(y^{-1}\right), q\right) \\
&= f^{-1}(T_A)(x, q) \wedge f^{-1}(T_A)\left(y^{-1}, q\right). \\
f^{-1}(I_A)\left(xy^{-1}, q\right) &= I_A(f\left(xy^{-1}, q\right)) \\
&= I_A\left(f(x)\, f\left(y^{-1}\right), q\right) \quad \text{(since, f is a homomorphism)} \\
&\geq I_A\left(f(x), q\right) \wedge I_A\left(f\left(y^{-1}\right), q\right) \\
&= f^{-1}(I_A)(x, q) \wedge f^{-1}(I_A)\left(y^{-1}, q\right) \\
&= f^{-1}(I_A)(x, q) \wedge f^{-1}(I_A)(y, q). \\
f^{-1}(F_A)\left(xy^{-1}, q\right) &= F_A(f\left(xy^{-1}, q\right)) \\
&= F_A\left(f(x)\, f\left(y^{-1}\right), q\right) \quad \text{(since, f is a homomorphism)} \\
&\leq F_A\left(f(x), q\right) \vee F_A\left(f\left(y^{-1}\right), q\right) \\
&= f^{-1}(F_A)(x, q) \wedge f^{-1}(F_A)\left(y^{-1}, q\right) \\
&= f^{-1}(F_A)(x, q) \wedge f^{-1}(F_A)(y, q).
\end{aligned}$$

Hence, $f^{-1}(A)$ is a NQFSG of G. □

Theorem 4.3. *Let X and Y be any two groups and f be a homomorphism of X onto Y. If A is a Neutrosophic Q-fuzzy subgroup of X, then $f(A)$ is a Neutrosophic Q-fuzzy subgroup of Y.*

Proof. Let A be a Neutrosophic Q-fuzzy subgroup of X. By definition, $f(A) = (f(T_A), f(I_A), f(F_A))$. Now for $x_1, x_2 \in X$, $y_1, y_2 \in Y$, $q \in Q$

$$\begin{aligned}
f(T_A)(y_1 y_2, q) &= \sup_{x_1 x_2 \in f^{-1}(Y)} T_A(x_1 x_2, q) \geq \sup_{x_1, x_2 \in f^{-1}(Y)} (T_A(x_1, q) \wedge T_A(x_2, q)) \quad \text{(since A is a NQFSG)} \\
&= \sup_{x_1 \in f^{-1}(Y)} T_A(x_1, q) \wedge \sup_{x_2 \in f^{-1}(Y)} T_A(x_2, q) \\
&= f(T_A)(y_1, q) \wedge f(T_A)(y_2, q) \\
f(T_A)\left(y^{-1}, q\right) &= \sup_{x^{-1} \in f^{-1}(Y)} T_A\left(x^{-1}, q\right) = \sup_{x \in f^{-1}(Y)} T_A(x, q) = f(T_A)(y, q) \\
f(I_A)(y_1 y_2, q) &= \sup_{x_1 x_2 \in f^{-1}(Y)} I_A(x_1 x_2, q) \geq \sup_{x_1, x_2 \in f^{-1}(Y)} (I(x_1, q) \wedge I_A(x_2, q)) \quad \text{(since A is a NQFSG)} \\
&= \sup_{x_1 \in f^{-1}(Y)} I_A(x_1, q) \wedge \sup_{x_2 \in f^{-1}(Y)} I_A(x_2, q)
\end{aligned}$$

$$
\begin{aligned}
&= f\left(I_A\right)\left(y_1, q\right) \wedge f\left(I_A\right)\left(y_2, q\right) \\
f\left(I_A\right)\left(y^{-1}, q\right) &= \sup_{x^{-1} \in f^{-1}(Y)} I_A\left(x^{-1}, q\right) = \sup_{x \in f^{-1}(Y)} I_A\left(x, q\right) = f\left(I_A\right)\left(y, q\right) \\
f\left(F_A\right)\left(y_1 y_2, q\right) &= \inf_{x_1 x_2 \in f^{-1}(Y)} F_A\left(x_1 x_2, q\right) \leq \inf_{x_1, x_2 \in f^{-1}(Y)} \left(F_A\left(x_1, q\right) \vee F_A(x_2, q)\right) \ \text{(since A is a NQFSG)} \\
&= \inf_{x_1 \in f^{-1}(Y)} F_A(x_1, q) \vee \inf_{x_2 \in f^{-1}(Y)} F_A(x_2, q) \\
&= f\left(F_A\right)\left(y_1, q\right) \wedge f\left(F_A\right)\left(y_2, q\right) \\
f\left(F_A\right)\left(y^{-1}, q\right) &= \inf_{x^{-1} \in f^{-1}(Y)} F_A\left(x^{-1}, q\right) = \inf_{x \in f^{-1}(Y)} F_A\left(x, q\right) = f\left(F_A\right)\left(y, q\right).
\end{aligned}
$$

Therefore $f\left(A\right)$ is a Neutrosophic Q-fuzzy subgroup of Y. □

Lemma 4.4. *For all $a, b \in I$ and i is any positive integer, if $a \leq b$, then*

(1). $(a)^i \leq (b)^i$.

(2). $(a \wedge b)^i = (a)^i \wedge (b)^i$.

(3). $(a \vee b)^i = (a)^i \vee (b)^i$.

Theorem 4.5. *Let A be a NQFSG of G. Then $A^i = \{\langle (x, q), (T_A(x, q))^i, (I_A(x, q))^i, (F_A(x, q))^i \rangle : x \in G, q \in Q\}$ is a NQFSG of G^i, where i is a positive integer.*

Proof. If $(G, .)$ is a group, then $(G^i, .)$ is also a group. Let A be a NQFSG of G. Now for $x, y \in G$, $q \in Q$, we have

$$
\begin{aligned}
T_{A^i}\left(xy^{-1}, q\right) &= \left(T_A\left(xy^{-1}, q\right)\right)^i \\
&\geq \left(T_A\left(x, q\right) \wedge T_A\left(y, q\right)\right)^i \\
&= \left(T_A\left(x, q\right)\right)^i \wedge \left(T_A\left(y, q\right)\right)^i \\
&= T_{A^i}\left(x, q\right) \wedge T_{A^i}\left(y, q\right) \\
I_{A^i}\left(xy^{-1}, q\right) &= \left(I_A\left(xy^{-1}, q\right)\right)^i \\
&\geq \left(I_A\left(x, q\right) \wedge I_A\left(y, q\right)\right)^i \\
&= \left(I_A\left(x, q\right)\right)^i \wedge \left(I_A\left(y, q\right)\right)^i \\
&= I_{A^i}\left(x, q\right) \wedge I_{A^i}\left(y, q\right) \\
F_{A^i}\left(xy^{-1}, q\right) &= \left(F_A\left(xy^{-1}, q\right)\right)^i \\
&\leq \left(F_A\left(x, q\right) \vee F_A\left(y, q\right)\right)^i \\
&= \left(F_A\left(x, q\right)\right)^i \vee \left(F_A\left(y, q\right)\right)^i \\
&= F_{A^i}\left(x, q\right) \vee F_{A^i}\left(y, q\right)
\end{aligned}
$$

Therefore A^i is a NQFSG of G^i. □

5. Direct Product of Neutrosophic Q-fuzzy Subgroups

Definition 5.1. *Let A, B be neutrosophic Q-fuzzy subsets of X and Y respectively. Then the Cartesian product of A and B denoted by $A \times B$ is defined by*

$$A \times B = \{\langle ((x, y), q), T_{A \times B}((x, y), q), I_{A \times B}((x, y), q), F_{A \times B}((x, y), q) \rangle : x \in X, y \in Y, q \in Q\}$$

where $T_{A\times B}((x,y),q) = \min(T_A(x,q), T_B(y,q))$, $I_{A\times B}((x,y),q) = \min(I_A(x,q), I_B(y,q))$, $F_{A\times B}((x,y),q) = \max(F_A(x,q), F_B(y,q)$.

Theorem 5.2. *If A and B are neutrosophic Q-fuzzy subgroups of the group X and Y respectively, then $A\times B$ is a neutrosophic Q-fuzzy subgroup of $X\times Y$.*

Proof. Let A and B are neutrosophic Q-fuzzy subgroups of the group X and Y respectively. Now for $(x_1,y_1),(x_2,y_2)\in A\times B$, $q\in Q$

$$\begin{aligned}
T_{A\times B}((x_1,y_1)(x_2,y_2),q) &= T_{A\times B}((x_1x_2,y_1y_2),q)\\
&= T_A((x_1x_2,q))\wedge T_B((y_1y_2,q))\\
&\geq [T_A(x_1,q)\wedge T_A(x_2,q)]\wedge[T_B(y_1,q)\wedge T_B(y_2,q)] \text{ (since A and B are NQFSG)}\\
&= T_A(x_1,q)\wedge T_B(y_1,q)\wedge T_A(x_2,q)\wedge T_B(y_2,q)\\
&= T_{A\times B}((x_1,y_1),q)\wedge T_{A\times B}((x_2,y_2),q).\\
T_{A\times B}\left((x,y)^{-1},q)\right) &= T_{A\times B}\left((x^{-1},y^{-1}),q\right)\\
&= T_A\left(x^{-1},q\right)\wedge T_B(y^{-1},q)\\
&= T_A(x,q)\wedge T_B(y,q)\\
&= T_{A\times B}((x,y),q).\\
I_{A\times B}((x_1,y_1)(x_2,y_2),q) &= I_{A\times B}((x_1x_2,y_1y_2),q)\\
&= I_A((x_1x_2,q))\wedge I_B((y_1y_2,q))\\
&\geq [I_A(x_1,q)\wedge I_A(x_2,q)]\wedge[I_B(y_1,q)\wedge I_B(y_2,q)] \text{ (since A and B are NQFSG)}\\
&= I_A(x_1,q)\wedge I_B(y_1,q)\wedge I_A(x_2,q)\wedge I_B(y_2,q)\\
&= I_{A\times B}((x_1,y_1),q)\wedge I_{A\times B}((x_2,y_2),q).\\
I_{A\times B}\left((x,y)^{-1},q)\right) &= I_{A\times B}\left((x^{-1},y^{-1}),q\right)\\
&= I_A\left(x^{-1},q\right)\wedge I_B(y^{-1},q)\\
&= I_A(x,q)\wedge I_B(y,q)\\
&= I_{A\times B}((x,y),q).\\
F_{A\times B}((x_1,y_1)(x_2,y_2),q) &= F_{A\times B}((x_1x_2,y_1y_2),q)\\
&= F_A((x_1x_2,q))\vee F_B((y_1y_2,q))\\
&\leq [F_A(x_1,q)\vee F_A(x_2,q)]\vee[F_B(y_1,q)\vee F_B(y_2,q)] \text{ (since A and B are NQFSG)}\\
&= F_A(x_1,q)\vee F_B(y_1,q)\vee F_A(x_2,q)\vee F_B(y_2,q)\\
&= F_{A\times B}((x_1,y_1),q)\vee F_{A\times B}((x_2,y_2),q).\\
F_{A\times B}\left((x,y)^{-1},q)\right) &= F_{A\times B}\left((x^{-1},y^{-1}),q\right)\\
&= F_A\left(x^{-1},q\right)\vee F_B(y^{-1},q)\\
&= F_A(x,q)\vee F_B(y,q)\\
&= F_{A\times B}((x,y),q).
\end{aligned}$$

Hence $A\times B$ is a Neutrosophic Q-fuzzy subgroup of $X\times Y$. □

6. Conclusion

In this paper, the notion of neutrosophic Q-Fuzzy subgroup is introduced, and discussed some of its basic algebraic properties .Also the results on homomorphic image, pre image of neutrosophic Q- fuzzy subgroup are derived. Proved findings are that the direct product of any two neutrosophic Q-subgroups is a neutrosophic Q-fuzzy subgroup, and it is extended for finite number of neutrosophic groups.

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