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RESEARCH ARTICLE

NEUTROSOPHIC RW CONTINUITY, NEUTROSOPHIC RW-OPEN MAPS AND CLOSED MAPS

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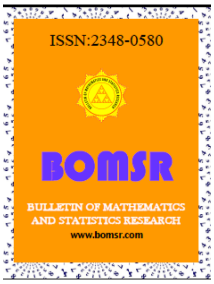


ABSTRACT

In this paper we introduce the concept of neutrosophic rw-continuity, neutrosophic rw-open maps and closed maps in neutrosophic topological spaces and some of their properties are discussed.

Keywords: Neutrosophic RW continuous map, neutrosophic RW- irresolute map, neutrosophic RW- open map and neutrosophic RW- closed map.

AMS Classification : 03E72

1. INTRODUCTION

The idea of "Neutrosophic set" was initiated by F. Smarandache [11] which is based on K.Atanassov's Intuitionistic fuzzy sets. A.A.Salama introduced neutrosophic topological spaces as a generalization of Intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2007, S.S. Benchalli and R.S. Wali**[4]**introduced RW-Closed sets in topological Spaces. The authors D. Savithiri and C. Janaki [12] introduced Neutrosophic RW-Closed sets in neutrosophic topological spaces.

In this article, we introduce Neutrosophic RW- continuous and irresolute maps, neutrosophic RW-open and closed maps and also we discuss some of its properties.

2. TERMINOLOGIES

We recall some important basic preliminaries, and in particular, the work of Smarandache [5] and Salama[10].

Definition 2.1:[5] Let X be a non-empty fixed set a Neutrosophic set (NS for short) A is an object having the form A = < x, $\mu_A(x),\sigma_A(x),\gamma_A(x)$> , x∈X where $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element x∈X to the set A.

Remark 2.2: [7] For the sake of simplicity A neutrosophic set A = {x, $\mu_A(x),\sigma_A(x),\gamma_A(x)$> ; x∈X} can be identified to be an ordered triple < μ_A,σ_A,γ_A> .

Definition 2.3:[10] The neutrosophic subsets 0_N and I_N in X are defined as follows:

0_N may be defined as:

(0_1) 0_N = { <x, 0, 0, 1> ; x ∈X}

(0_2) 0_N = { <x, 0, 1, 1> ; x ∈X}

(0_3) 0_N = { <x, 0, 1, 0> ; x ∈X}

(0_4) 0_N = { <x, 0, 0, 0> ; x ∈X}

I_N may be defined as :

(I_1) I_N = { <x, 1, 0, 0> ; x ∈X}

(I_2) I_N = { <x, 1, 0, 1> ; x ∈X}

(I_3) I_N = { <x, 1, 1, 0> ; x ∈X}

(I_4) I_N = { <x, 1, 1, 1> ; x ∈X}

Definition 2.4: [10] Let A = < μ_A,σ_A,γ_A> be a NS on X , then the complement of the set A [C(A) for short] may be defined as three kinds of complements :

(C_1) C(A) = { <x, 1- $\mu_A(x)$, 1-$\sigma_A(x)$, 1-$\gamma_A(x)$> ; x∈ X}

(C_2) C(A) = { <x, $\gamma_A(x)$, $\sigma_A(x)$, $\mu_A(x)$ > ; x∈ X}

(C_3) C(A) = { <x, $\gamma_A(x)$, 1-$\sigma_A(x)$, $\mu_A(x)$ > ; x∈ X}

Definition 2.5: [10] Let X be a nonempty set and neutrosophic sets A and B in the form A = {< x, $\mu_A(x),\sigma_A(x),\gamma_A(x)$> , x∈X} and B = {< x, $\mu_B(x),\sigma_B(x),\gamma_B(x)$> , x∈X}.

Then we may consider two possible definitions for subsets (A ⊆ B). **A ⊆ B may be defined as :**

(1) A ⊆ B ⇔ $\mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \le \sigma_B(x)$, and $\gamma_A(x) \ge \gamma_B(x)$ ∀ x ∈ X

(2) A ⊆ B ⇔ $\mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \ge \sigma_B(x)$, and $\gamma_A(x) \ge \gamma_B(x)$ ∀ x ∈ X

Proposition 2.6: [10] For any neutrosophic set A, the following conditions are holds :

(1) $0_N \subseteq A$, $0_N \subseteq I_N$

(2) $A \subseteq I_N$, $I_N \subseteq I_N$

Definition 2.7: [10] Let X be a nonempty set. Let A = {< x, $\mu_A(x),\sigma_A(x),\gamma_A(x)$> , x∈X} and B = {< x, $\mu_B(x),\sigma_B(x),\gamma_B(x)$> , x∈X} are NS sets. Then

(1) A ∩ B may be defined as :

(I_1) A ∩ B = < x, min($\mu_A(x),\mu_B(x)$), min($\sigma_A(x),\sigma_B(x)$) , max($\gamma_A(x)$, $\gamma_B(x)$) >

(I_2) A $\cap$ B = < x, min(μ_A(x),μ_B(x)), max(σ_A(x),σ_B(x)) , max(γ_A(x), γ_B(x)) >

(2) A $\cup$ B may be defined as :

(I_1) A $\cup$ B = < x, max(μ_A(x),μ_B(x)), max (σ_A(x),σ_B(x)) , min (γ_A(x), γ_B(x)) >

(I_2) A $\cup$ B = < x, max (μ_A(x),μ_B(x)), min(σ_A(x),σ_B(x)) , min (γ_A(x), γ_B(x)) >

We can easily generalize the operations of intersection and union in definition 2.7 to the arbitrary family of NSs as follows:

Definition 2.8: [11] Let {A_j : j$\in$J } be a arbitrary family of NS sets in X , then

(1) $\cap$ A_j may be defined as

(i) $\cap$ A_j = < x, $\wedge_{j\in J}$ μ_{A_j} (x) , $\wedge_{j\in J}$ σ_{A_j} (x) , $\vee_{j\in J}$ γ_{A_j} (x)

(ii) $\cap$ A_j = < x, $\wedge_{j\in J}$ μ_{A_j} (x) , $\vee_{j\in J}$ σ_{A_j} (x) , $\vee_{j\in J}$ γ_{A_j} (x)

(2) $\cup$ A_j may be defined as

(i) $\cup$ A_j = < x, $\vee_{j\in J}$ μ_{A_j} (x) , $\vee_{j\in J}$ σ_{A_j} (x) , $\wedge_{j\in J}$ γ_{A_j} (x)

(ii) $\cup$ A_j = < x, $\vee_{j\in J}$ μ_{A_j} (x) , $\wedge_{j\in J}$ σ_{A_j} (x) , $\wedge_{j\in J}$ γ_{A_j} (x)

Proposition 2.9: [10] For two NS sets A and B the following conditions are true:

(1) C (A $\cap$ B) = C (A) $\cup$ C (B)

(2) C (A $\cup$ B) = C (A) $\cap$ C (B)

Definition 2.10: [10] A neutrosophic topology [NT for short] is a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

(1) 0_N , I_N $\in$ τ,

(2) $G_1 \cap G_2 \in \tau$ for any G_1, $G_2 \in \tau$,

(3) $\cup$ $G_i \in \tau$ for every { G_i : i $\in$ J } $\subseteq \tau$

In this case, the pair (X,τ) is called a neutrosophic topological space [NTS for short]. The elements of τ are called neutrosophic open sets [NOS for short]. A neutrosophic set F is closed if and only if C(F) is neutrosophic open.

Definition 2.11: [10] The complement of A [C(A) for short] of NOS is called a neutrosophic closed set [NCS for short] in X.

Now, we define neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces:

Definition 2.12: [10] Let (X,τ) be NTS and A = < x, μ_A(x),σ_A(x),γ_A(x)> be a NS in X. Then the neutrosophic closure and neutrosophic interior of A are defined by

NCl (A) = $\cap$ { K : K is a NCS in X and A $\subseteq$ K }

NInt (A) = $\cup$ { G : G is a NOS in X and G $\subseteq$ A }

Proposition 2.13: [10] For any neutrosophic set A in (X,τ) we have

(a) NCl (C(A)) = C(NInt(A))

(b) NInt (C(A)) = C(NCl(A))

Proposition 2.14: [11] Let (X,τ) be a NTS and A,B be two neutrosophic sets in X. Then the following properties are holds :

a) NInt (A) $\subseteq$ A

b) A $\subseteq$ NCl (A).

c) A $\subseteq$ B $\Rightarrow$ NInt (A) $\subseteq$ NInt (B)

d) A $\subseteq$ B $\Rightarrow$ NCl(A) $\subseteq$ NCl(B)

e) NInt (NInt (A)) = NInt (A).

f) NCl (NCl (A)) = NCl (A).

g) NInt (A$\cap$B) = NInt (A) $\cap$ NInt (B)

h) NCl (A$\cup$B) = NCl (A) $\cup$ NCl (B)

i) NInt (0_N) = 0_N

j) NInt (I_N) = I_N

k) NCl (0_N) = 0_N

l) NCl (I_N) = I_N

m) A $\subseteq$ B $\Rightarrow$ C (B) $\subseteq$ C(A)

n) (n) NCl (A$\cap$B) $\subseteq$ NCl(A) $\cap$ NCl (B)

o) NInt (A$\cup$B) $\subseteq$ NInt (A) $\cup$ NInt (B)

Definition 2.15: [4] Let A be a neutrosophic set of a NTS X. Then A is said to be a

i) neutrosophic regular open set **(shortly N_r – open set)** if A = NInt(NCl(A)).

ii) neutrosophic regular closed set **(shortly N_r - closed set)** if A = NCl(NInt(A)).

iii) neutrosophic rg- Closed set [4] **(shortly N_{rg} – closed set)** of X if there exists a neutrosophic regular open set U such that NCl(A) $\subseteq$ U whenever A $\subseteq$ U.

iv) neutrosophic rwg- closed set [6] **(shortly N_{rwg} – closed set)** of X if there exists a neutrosophic regular open set U such that NCl(NInt(A)) $\subseteq$ U whenever A $\subseteq$ U.

v) neutrosophic w-closed set [6] **(shortly N_w – closed set)** of X if there exists a neutrosophic semi-open set U such that NCl(A) $\subseteq$ U whenever A $\subseteq$ U.

vi) neutrosophic g-closed set [6] **(shortly Ng – closed set)** of X if there exists a neutrosophic open set U such that NCl(A) $\subseteq$ U whenever A $\subseteq$ U.

vii) neutrosophic π-open set [6] **(shortly Nπ – open set)** of X if there exists a finite union of neutrosophic regular open sets.

viii) The complement of N_π-open set is called N_π-closed set.

ix) neutrosophic gpr- Closed set [4] **(shortly N_g – closed set)** of X if there exists a neutrosophic regular open set U such that NCl(A) $\subseteq$ U whenever A $\subseteq$ U.

Definition 2.16: [12] Let (X,τ) and (Y,σ) be any two Neutrosophic topological spaces

i) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic continuous** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic closed set in (X,τ).

ii) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic rg-continuous (shortly N_{rg} - continuous)** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic rg closed set in (X,τ).

iii) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic rwg-continuous (shortly N_{rwg} - continuous)** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic rwg closed set in (X,τ).

iv) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic w-continuous (shortly N_w - continuous)**if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic wclosed set in (X,τ).

v) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic gpr-continuous (shortly N_{gpr} - continuous)**if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic gpr closed set in (X,τ).

vi) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic π-continuous (shortly N_π- continuous)** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic πclosed set in (X,τ).

vii) A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic regular continuous (shortly N_r - continuous)** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic regular closed set in (X,τ).

Definition 2.17: [13] Let A be a neutrosophic set of a NTS X. Then A is said to be Neutrosophic RW-Closed set **(shortly N_{rw} – closed set)** of X if there exist a neutrosophic regular semi-open set U such that $NCl(A) \subseteq U$ whenever $A \subseteq U$.

The complement of N_{rw} – closed set is known as **N_{rw} – open** set.

Definition 2.18:[12] (i) If $B = (\mu_B,\sigma_B,\gamma_B)$ is a

NS in Y, then the preimage of B under f denoted by $f^{-1}(B) = \langle f^{-1}(\mu_B), f^{-1}(\sigma_B), f^{-1}(\gamma_B) \rangle$.

(ii) If $A= (\mu_A,\sigma_A,\gamma_A)$ is a NS in X, the image of A under f denoted by f(A), is a NS in Y defined by $f(A) = \langle f(\mu_A),f(\sigma_A),f(\gamma_A) \rangle$.

Corollary 2.18:[12] Let A, $\{A_i : i \in J\}$, be NSs in X and B, $\{B_j : j \in K\}$ NS in Y, and f: $X \to Y$ be a function. Then

a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,

b) $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

c) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f(f(A))$.

d) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B$.

e) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$.

f) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$; and if f is injective, then $f(\cap A_i) = \cap f(A_i)$

g) $f^{-1}(I_N) = I_N$, $f^{-1}(0_N) = 0_N$.

h) $f(I_N) = I_N$, $f(0_N) = 0_N$ if f is surjective.

3. NEUTROSOPHIC RW CONTINUOUS AND IRRESOLUTE MAPS

Definition 3.1: A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic rw-continuous (shortly N_{rw} - continuous)** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic rw closed set in (X,τ).

Example 3.2: Let X = {a,b}, Y = {x,y}. The NSs U = ⟨(0.5,0.5, 0.6), (0.7,0.5,0.6)⟩, V = ⟨(0.3,0.5,0.9), (0.5, 0.5,0.7)⟩. Then $\tau = \{0_N, U, I_N\}$, $\sigma = \{0_N, V, I_N\}$. Clearly (X,τ) and (Y,σ) are Neutrosophic topological spaces. Define a map f :$(X,\tau) \to (Y,\sigma)$ by f(a) = x, f(b) = y. Then f is neutrosophic rw-continuous map.

Theorem 3.3: Every neutrosophic continuous map is N_{rw} – continuous.

Proof: Straight forward.

Remark 3.4: The converse of the above theorem is not true as shown in the following example.

Example 3.5: In the example 3.2, f is a N_{rw}- continuous map but it is not neutrosophic continuous since λ= ⟨(0.9, 0.5,0.3), (0.7,0.5,0.5)⟩ is NC set in (Y,σ) but $f^{-1}(\lambda)$ is N_{rw}- closed but it is not a neutrosophic closed set in (X,τ).

Theorem 3.6: A mapping f: $(X,\tau) \to (Y,\sigma)$ is N_{rw}- continuous if and only if the inverse image of every neutrosophic open set of Y is neutrosophic rw- open in X.

Proof: Obvious because $f^{-1}(U^c) = [f^{-1}(U)]^c$ for every neutrosophic set U of Y.

Theorem 3.7: If a function f: $(X,\tau) \to (Y,\sigma)$ is N_{rw}-continuous, then $f(N_{rw}Cl(A)) \subseteq NClf(A)$ for every subset A of X.

Proof: Let A be a subset of (X,τ). Then NCl(f(A)) is N_{rw}- closed in (Y,σ) and $A \subseteq f^{-1}(NClf(A)))$, i.e., $f^{-1}(NClf(A)))$ is N_{rw}-closed subset of X containing A. Hence $N_{rw}Cl(A) \subseteq f^{-1}(NClf(A)))$, implies $f(N_{rw}Cl(A)) \subseteq NClf(A)$.

Theorem 3.8: Every N_{rw} – continuous map is (i) N_{rg} – continuous (ii) N_{rwg} – continuous (iii) N_{gpr} – continuous.

Proof: Obvious.

Remark 3.9: The converse of the above theorem need not be true as seen in the following example.

**Example 3.10: ***Let X = {a,b}, Y = {x,y}. Neutrosophic open sets are U = ⟨(0.1,0.5,0.9), (0.2,0.5,0.3)⟩,V = ⟨(0.8,0.3,0.1), (0.7,0.3,0.2)⟩ Let $\tau = \{0_N,U,I_N\}$ and $\sigma = \{0_N,V,I_N\}$ be neutrosophic topologies on X and Y respectively. Define f: $(X,\tau) \to (Y,\sigma)$ by f(a) = x, f(b) = y then f is both N_{rg} – and N_{rwg}- continuous map but it is not N_{rw}-continuous.

* Let X = {a,b}, Y = {x,y}. Neutrosophic open sets are U = ⟨(0.3,0.5,0.7), (0.1,0.5,0.9)⟩ , V = ⟨(0.8,0.4.0.1), (0.9,0.4,0.1)⟩. Let $\tau = \{0_N,U,I_N\}$ and $\sigma = \{0_N,V,I_N\}$ be neutrosophic topologies on X and Y respectively. Define f: $(X,\tau) \to (Y,\sigma)$ by f(a) = x, f(b) = y. Then f is N_{gpr}- continuous but it is not N_{rw}- continuous.

Theorem 3.11: Every (i) N_w – continuous (ii) N_π- continuous (iii) N_r – continuous function is N_{rw} – continuous.

Proof: Straight forward.

Remark 3.12: The converse of the above theorem is not true as shown in the following example.

Example 3.13: * Let X = {a}, Y = {x}. The neutrosophic open sets are $A_1 = \langle(0.6,0.6, 0.5)\rangle$,$A_2 = \langle(0.5,0.7,0.9)\rangle$, $A_3 = \langle(0.6,0.7,0.5)\rangle$, $A_4 = \langle(0.5,0.6,0.9)\rangle$, $B_1 = \langle(0.9,0.4,0.5)\rangle$. Then $\tau = \{0_N, A_1, A_2, A_3, A_4, I_N\}$, $\sigma = \{0_N, B_1, I_N\}$ are neutrosophic topologies for X and Y respectively. Define f: $(X,\tau) \to (Y,\sigma)$ by f (a) = x. Then f is N_{rw} – continuous but it is not N_w – continuous.

* Let X = {a,b}, Y = {x,y}. Neutrosophic open sets are U= ⟨(0.7,0.5,0.8), (0.5,0.5,0.4)⟩, V = ⟨(0.5,0.5,0.4), (0.7,0.5,0.8)⟩. Let $\tau = \{0_N, U, I_N\}$ and $\sigma = \{0_N, V, I_N\}$ be neutrosophic topologies on X and Y respectively. Define f: $(X,\tau) \to (Y,\sigma)$ by f(a) = y, f(b) = x then f is N_{rw} – continuous but it is not N_π - continuous and N_r – continuous since $\lambda = \langle(0.4,0.5,0.5), (0.8,0.5,0.7)\rangle$ is NC set in (Y,σ) but $f^{-1}(\lambda)$ is N_{rw}- closed but it is not a neutrosophic πclosed and regular closed set in (X,τ).

Remark 3.14: The composition of two neutrosophic rw-continuous map need not be a neutrosophic rw-continuous map which is shown below by an example.

Example 3.15: Let X = {a}, Y = {x}, Z = {c}. The neutrosophic open sets are $A_1 = \langle(0.6,0.6, 0.5)\rangle$,$A_2 = \langle(0.5,0.7,0.9)\rangle$, $A_3 = \langle(0.6,0.7,0.5)\rangle$, $A_4 = \langle(0.5,0.6,0.9)\rangle$, $B_1 = \langle(0.9,0.4,0.5)\rangle$, $C_1 = \langle(0.9,0.4,0.4)\rangle$. Let $\tau = \{0_N, A_1, A_2, A_3, A_4, I_N\}$, $\sigma = \{0_N, B_1, I_N\}$, $\eta = \{0_N, C_1, I_N\}$. Define maps f and g as f(a) = b, g(b) = c. Then f and g are N_{rw} – continuous but $g^\circ f : (X,\tau) \to (Z,\eta)$ defined by $g^\circ f(a) = c$ is not N_{rw} – continuous.

Theorem 3.16: If f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw continuous and g: $(Y,\sigma) \to (Z,\eta)$ is neutrosophic continuous then $g^\circ f$: $(X,\tau) \to (Z,\eta)$ is N_{rw} – continuous.

Proof: Let A be a NC set in (Z,η), then $g^{-1}(A)$ is NC set by hypothesis. Since f is N_{rw} - continuous, $g^\circ f^{-1}(A) = f^{-1}(g^{-1}(A))$ is N_{rw}- closed in X, Hence $g^\circ f$ is N_{rw} – continuous. **Theorem 3.17:** If f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw continuous and g: $(Y,\sigma) \to (Z,\eta)$ is neutrosophic g-continuous and (Y,σ) is neutrosophic $T_{1/2}$ then $g^\circ f$: $(X,\tau) \to (Z,\eta)$ is N_{rw} – continuous.

Proof: Let A be a neutrosophic closed set in (Z,η), then by hypothesis, $g^{-1}(A)$ is N_g – closed in Y. Since Y is neutrosophic $T_{1/2}$, $g^{-1}(A)$ is NC set in Y. Hence $g^\circ f^{-1}(A) = f^{-1}(g^{-1}(A))$ is N_{rw} – closed in X. Hence $g^\circ f$ is N_{rw}- continuous.

Definition 3.18: A map f: $(X,\tau) \to (Y,\sigma)$ is **neutrosophic rw-irresolute (shortly N_{rw} - irresolute)** if the inverse image of every neutrosophic rw closed set in (Y,σ) is neutrosophic rw closed set in (X,τ).

Theorem 3.19: If a map f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-irresolute, then it is neutrosophic rw continuous.

Proof: Obvious from the definitions.

Remark 3.20: The converse of the above theorem need not be true as seen in the following example.

Example 3.21: Let X = {a,b}, Y = {x,y}. Neutrosophic open sets are U = ⟨(0.4,0.4,0.4), (0.3,0.3,0.3)⟩,V_1= ⟨(0.4,0.6,0.5), (0.7,0.3,0.6)⟩ , V_2 = ⟨(0.3,0.6,0.8), (0.6,0.3,0.6)⟩ Let $\tau = \{0_N, U, I_N\}$ and $\sigma = \{0_N, V_1, V_2, I_N\}$ be neutrosophic topologies on X and Y respectively. Define f: $(X,\tau) \to (Y,\sigma)$ by f(a) = x, f(b) = y, then f is N_{rw} – continuous but it is not N_{rw} – irresolute.

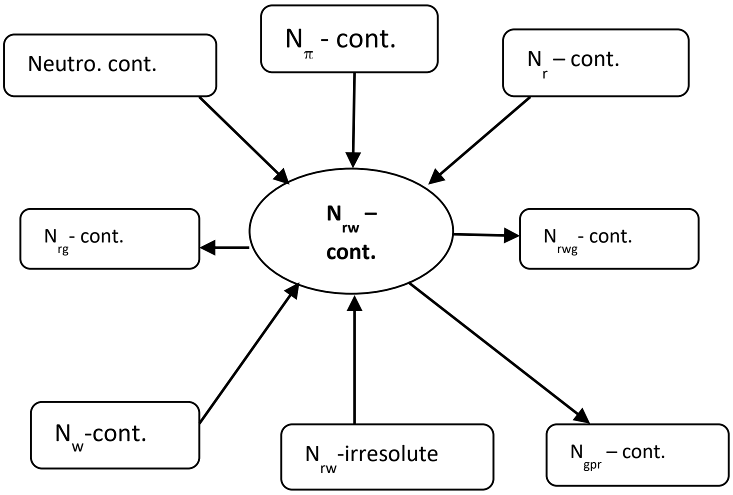
Theorem 3.22: If f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\eta)$ are both neutrosophic irresolute maps then $g^\circ f$: $(X,\tau) \to (Z,\eta)$ is N_{rw} – irresolute.

Proof: Obvious.

Theorem 3.23: If f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw irresolute and g: $(Y,\sigma) \to (Z,\eta)$ is neutrosophic continuous map then their composition $g^\circ f$: $(X,\tau) \to (Z,\eta)$ is N_{rw} – continuous.

Proof: Straight forward.

The above discussions are implicated in the following diagram.



4. NEUTROSOPHIC RW OPEN MAPS AND CLOSED MAPS

Definition 4.1: A mapping f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-open **(shortly N_{rw}-open)** map if the image of every neutrosophic open set of X is neutrosophic rw-open set in Y.

Theorem 4.2: A mapping f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-open if and only if for every neutrosophic open set U of X, $f(NInt(U)) \subseteq N_{rw}Int(f(U))$.

Proof: Necessity: Let f be a N_{rw} – open mapping and U is a NO set in X. Now $NInt(U) \subseteq U$ which implies that $f(NInt(U)) \subseteq f(U)$. Since f is N_{rw} – open map, f(NInt(U)) is neutrosophic rw open set in Ysuch that $f(NInt(U)) \subseteq f(U)$. Therefore $f(NInt(U)) \subseteq N_{rw}Int(f(U))$.

Sufficiency: Suppose that U is a NO set of X. Then $f(U) = f(NInt(U)) \subseteq N_{rw}Int(f(U))$. But $N_{rw}Int(f(U)) \subseteq f(U)$. Consequently $f(U) = N_{rw}Int(U)$ which implies that f(U) is a N_{rw} – open set of Y. Thus f is neutrosophic rw-open.

Theorem 4.3: A mapping f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-open if and only if for every neutrosophic set S of Y and for each NC set U of X containing $f^{-1}(S)$ there is a N_{rw}-closed set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is N_{rw}-open map. Let S be a NC set of Y and U be a NC set of X such that $f^{-1}(S) \subseteq U$. Then $V = f^{-1}(U^c)^c$ is a N_{rw} closed set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: Let F be a NO set of X. Then $f^{-1}(f(F))^c \subseteq F^c$ and F^c is a NC set in X. By hypothesis there is a N_{rw}- closed set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ i.e., $f(F) = V^c$ which is N_{rw}-open in Y and thus f is N_{rw}-open map.

Theorem 4.4: If a mapping f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-open then $NInt(f^{-1}(G)) \subseteq f^{-1}(N_{rw}Int(G))$ for every neutrosophic set G of Y.

Proof: Let G be neutrosophic set of Y. Then $NIntf^{-1}(G)$ is a NO set in X. Since f is N_{rw} – open $f(NIntf^{-1}(G)) \subseteq N_{rw}Int(f(f^{-1}(G)) \subseteq N_{rw}Int(G))$. Thus $NInt(f^{-1}(G)) \subseteq f^{-1}(N_{rw}Int(G))$.

Definition 4.5: A mapping f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-closed **(shortly N_{rw}-closed)** map if the image of every neutrosophic closed set of X is neutrosophic rw-closed set in Y.

Theorem 4.6: A mapping f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic rw-closed if and only if for every neutrosophic set S of Y and for each NO set U of X containing $f^{-1}(S)$ there is a N_{rw}-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is N_{rw}-closed map. Let S be a NC set of Y and U be a NO set of X such that $f^{-1}(S) \subseteq U$. Then $V = f^{-1}(U^c)^c$ is a N_{rw} closed set of Y such that $f^{-1}(V) \subseteq U$. Then $V = f^{-1}(U^c)$is a N_{rw}-open set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: Let F be a NC set of X. Then $f^{-1}(f(F))^c \subseteq F^c$ and F^c is a NO set in X. By hypothesis there is a N_{rw}- open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ i.e., $f(F) = V^c$ which is N_{rw}-closed in Y and thus f is N_{rw}-closed map.

Theorem 4.7: If f: $(X,\tau) \to (Y,\sigma)$ is neutrosophic closed map and g: $(Y,\sigma) \to (Z,\eta)$ is neutrosophic rw-closed map then their composition $g^\circ f$: $(X,\tau) \to (Z,\eta)$ is N_{rw} – closed map.

Proof: Let F be a NC set in X. Then by hypothesis, f(F) is NC set in Y. Since g is N_{rw}-closed map, $g^\circ f(F) = g(f(F))$ is N_{rw}-closed set in Z. Thus $g^\circ f$: $(X,\tau) \to (Z,\eta)$ is N_{rw} – closed map.

Conclusion

In this paper using neutrosophic rw closed sets, we have defined neutrosophic rw continuous maps and some of its characteristics have been discussed. Further we have introduced neutrosophic rw open and closed maps. This concept can be extended further in future.

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