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Neutrosophic reducible weighted Maclaurin symmetric mean for undergraduate teaching audit and evaluation

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ABSTRACT The undergraduate teaching audit and evaluation (UTAE) is critically important for university to promote the establishment of a quality assurance system and improve the quality of teaching. In considering the case of UTAE, the essential question that arises strong ambiguity and interaction. The Maclaurin symmetric mean (MSM), as a significant information integration tool, can seize the interrelation among multiple input values more effectively. A series of weighted MSMs have been developed to dispose of diverse neutrosophic information aggregation issues by reason that the attribute variables are frequently disparate. Nevertheless, these weighted form of MSM operators do not possess the idempotency. Moreover, the weight MSM cannot degrade into the MSM when their weights information are equivalent. In other words, it signifies without the reducibility. To resolve two issues, we develop the single-valued neutrosophic reducible weighted MSM (SVNRWMSM) operator and the single-valued neutrosophic reducible weighted dual MSM (SVNRWDMSM) operator. Meanwhile, certain interesting properties and some special cases of the SVNRWMSM and SVNRWDMSM operators are explored in detail. Afterwards, we develop two multiple attribute decision making (MADM) methods based on SVNRWMSM and SVNRWDMSM. The validity of algorithms are illustrated by a undergraduate teaching evaluation issue, along with the sensitivity analysis of diverse parameter values on the ranking. Finally, a comparison of the developed with the existing single-valued neutrosophic decision making algorithms has been executed for displaying their efficiency.

INDEX TERMS Single-valued neutrosophic set; Aggregation operator; Idempotency; reducible weighted MSM.

I. INTRODUCTION

THE "Five-in-One" undergraduate teaching evaluation system established in the new period is based on the self-evaluation of colleges and universities, with normal monitoring of teaching basic state data, evaluation of colleges and universities, professional certification and evaluation, international evaluation as the main content, and government, schools, specialized agencies and social multi-evaluation as the combination of teaching evaluation system. The undergraduate teaching audit and evaluation (UTAE) is a mode of evaluation in the "Five-in-One" evaluation system. The guiding ideology of audit evaluation can be summarized as "one insistence, two highlights and three intensification". For best performance, it should adhere to the two-cross policy of "promoting construction by evaluation, promoting reform by evaluation, promoting management by evaluation, combining evaluation with construction, focusing on construction". It stresses connotation construction, highlights characteristic development. And also strengthen the rational orientation of running a school, the position of talent training center and the construction of quality assurance system, and constantly improve the quality of talent training. Nevertheless, the influential factors of "UTAE" centers on not only the subject principle and developable principle but also on their diversity principle and empirical principle. In addition, other trait, such as objective principle, is also continually considered. A brief description of these principles is shown in FIGURE 1.

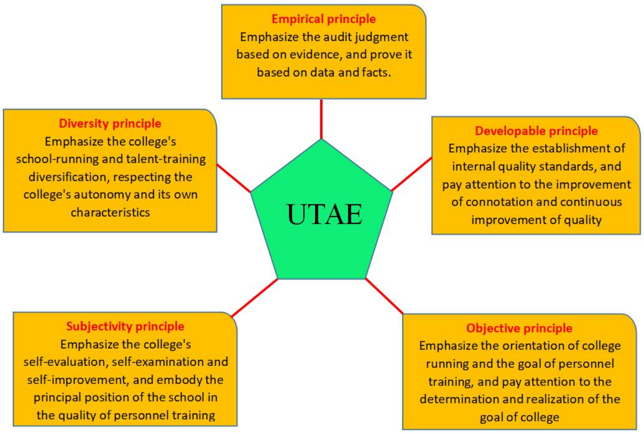

FIGURE 1: The five principles of UTAE.

Presently, UTAE has already been completely applied to evaluate the universities or colleges by ministry of education in China. China's President Xi Jinping once said: "Education determines our present and future. Human society demands for continuously cultivating the talents needed by the society through education. It is necessary to teach to know, update old knowledge, discover new knowledge, and explore the unknown, so that people can better understand the world and transform the world, and better create a better future for mankind." The universities want to seek their own quick development in the China's education ranking. Whereas, if they want have a higher ranking, it is not enough to depend on themselves alone. Hence, they had better select some universities to collaborate with. Therefore, the idea that regards the process to select the some ideal universities to collaborate with to the dominating multiple attribute decision making (MADM) problem comes to my mind. However, the increasingly complex decision-making environments and willy-nilly decision makers (DMs) have made it difficult in expressing decision information with simple numbers in the process of solving the above MADM problems.

Neutrosophic set (NS), examined by Smarandache [1], has treated as a more preferable means for describing absonant information in a philosophical perspective compared with the intuitionistic fuzzy set (IFS) [2]. From scientific point, the neutrosophic set and set-theoretic operators should be regulation, or it will be hard for employing practical environments. As a result, Wang et al. [3] put forward the notion of single valued neutrosophic set (SVNS) as well as some epoch-making properties of SVNSs. Up to now, SVNS has drawn much attention and achieved some infusive achievements [4].

One of the best-loved tools to solve MADM issues is aggregation operators, which integrate all the given individual parameters into a monolithic parameter. Consequently, it is always a hot topic in academic research. The presented research related to aggregation operators can be fallen into general categories: (1) Assume that the parameters of aggregation operators are independent of each other. The most frequently-used operators in this category are the weighted averaging (WA) operator and the weighted geometric (WG) operator [5]. In the last few decades, diverse generalized form of the WA operator and the WG operator have been presented, significantly striding the richness of aggregation operators, such as ordered WA (OWA) operator [6], ordered WG (OWG) operator [7], continuous OWA (COWA) operator[8], continuous OWG (COWG) operator [9], induced COWA (ICOWA) operator [10], induced COWG (ICOWG) operator [11], induced generalized COWA (IGCOWA) operator [12], induced generalized OWA operator [13], power OWA (POWA) operator [14], power OWG (POWG) operator [15]. (2) Assume that the parameters of aggregation operators are interactional and correlative. It is more closely matches the real decision making environment, on account of having correlation of decision making process. For example, if a man would like to buy a car, then there is a definite relationship between its fuel efficiency and price. Among them, the most cross-sectional operators are Bonferroni mean (BM) operator [16], geometric BM (GBM) operator [17], geometric Heronian mean (HM) operator [18], Dombi operator [19], Muirhead mean (MM) operator [20], Hamy mean (HM) operator [21]. Up to now, there are plentiful extensions of aggregation operators by applying them in diverse uncertain environment [22–26].

Nevertheless, it can only seize the pertinence between a fixed number of parameters by the above-discussed operators. For example, the HM operator only seizes the pertinence between two parameters. In order to boost the elasticity of information aggregation, Maclaurin [27] presented the Maclaurin symmetric mean (MSM), which can seize the pertinence between any number of parameters. Qin and Liu [28] firstly combined the MSM with the intuitionistic fuzzy environment in uncertain domain and initiated the weighted IF MSM (WIFMSM) for aggregating the decision evaluation information. Until now, there are abundant extensions of MSM operators by applying them in diverse uncertain environment [29–39].

By researching the current WMSM operators in diverse uncertain environment, we discover some counter-intuitive issues: (1) When weight information in whole attributes are equal, the related environment of WMSM operators [29–39] cannot degenerate into the homologous MSM operators, which is an essential trait of the classical weighted operators. (2) These form of weighted MSM operators [29–39] do not possess the properties of idempotency. In other words, it is unconscionable that the weighted average value of some identical integrated parameters depends on the weight values. Inspired by reducible weighted MSM operator and reducible weighted dual MSM (RWDMSM) operator [40], we combine the above operators with SVNS to integrate single-valued neutrosophic information and handle single-valued neutrosophic MADM issues by considering the merits of both.

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These considerations have guided us to consider the main targets as follows:

1) To develop two novel kinds of aggregation operators (single-valued neutrosophic RWMSM (SVNRWMSM) and single-valued neutrosophic RWDMSM (SVNRWDMSM)) for fusing the preferences of experts or decision makers;
2) To propose some algorithms for dealing the MADM issues by proposed aggregate operators for solving the antilogarithm by zero issue [41] and the division by zero issue [42];
3) To explore the sensitivity analysis of diverse parameter values on the final ordering;
4) To build a synthesize appraise system for UTAE;
5) To explicitly explore some examples for showing the superiority of proposed algorithms.

The rest of the paper is organized as follows: In Section II, we briefly review basic concepts of SVNSs, the MSM, DMSM, RWMSM and RWDMSM operator. In Section III, we propose the single-valued neutrosophic Maclaurin symmetric means operators, including SVNRWMSM operator and SVNRWDMSM operator. In Section IV, we propose two MADM methods based on the conceived SVNRWMSM operator and SVNRWDMSM operator with SVNNs. Moreover, we employ an example to illustrate the effectiveness of the presented MADM methods by discussing the effect of the different parameter values on the ordering of the objects. In Section V, a comparison with some existing algorithms is examined. Also, the characteristic comparisons of diverse single-valued neutrosophic aggregation operators and diverse uncertain environment are carried out. Finally, Section VI gives the concluding remarks.

II. PRELIMINARIES

A. SINGLE-VALUED NEUTROSOPHIC SET (SVNS)

Neutrosophic set is a part of neutrosophy, which explores the origin, and field of neutralities, as well as its interactions with different conceptual view [1]. It is a compellent common framework, which expands the IFS [2] from philosophical viewpoint. Smarandache [1] presented the definition of NS as follows:

Definition 1: [1] Let X be universe of discourse, with a crowd of elements in X denoted by x. A NS B in X is summarized by a truth membership function $T_B(x)$, an indeterminacy membership function $I_B(x)$, and a falsity membership function $F_B(x)$. The functions $T_B(x), I_B(x)$, and $F_B(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_B(x) : X \rightarrow]0^-, 1^+[$, $I_B(x) : X \rightarrow]0^-, 1^+[$, and $F_B(x) : X \rightarrow]0^-, 1^+[$.

There is restriction on the sum of $T_B(x), I_B(x)$, and $F_B(x)$, so $0^- \le \sup T_B(x) + \sup I_B(x) + \sup F_B(x) \le 3^+$.

As discussed above, it is hard to employ the NS in solving some practical problems. Hence, Wang et al. [3] presented SVNS, which is a subclass of the NS and mentioned the definition in the following.

Definition 2: [3] Let X be universe of discourse, with a crowd of elements in X presented by x. A SVNS N in X is summarized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then a SVNS N can be denoted as follows:

$$N = \{< x, T_N(x), I_N(x), F_N(x) >|\ x \in X\}, \qquad (1)$$

where $T_N(x), I_N(x), F_N(x) \in [0, 1]$ for $\forall x \in X$. Meanwhile, the sum of $T_N(x), I_N(x)$, and $F_N(x)$ fulfills the condition $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$. For a SVNS N in X, the triplet $(T_N(x), I_N(x), F_N(x))$ is called single-valued neutrosophic number (SVNN). For convenience, we can simply use $x = (T_x, I_x, F_x)$ to represent a SVNN as an element in the SVNS N.

Definition 3: [3, 41] Let $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ be two SVNNs, then operations can be defined as follows:

(1) $x^c = (F_x, 1 - I_x, T_x)$;
(2) $x \bigcup y = (\max\{T_x, T_y\}, \min\{I_x, I_y\}, \min\{F_x, F_y\})$;
(3) $x \bigcap y = (\min\{T_x, T_y\}, \max\{I_x, I_y\}, \max\{F_x, F_y\})$;
(4) $x \oplus y = (T_x + T_y - T_x * T_y, I_x * I_y, F_x * F_y)$;
(5) $x \otimes y = (T_x * T_y, I_x + I_y - I_x * I_y, F_x + F_y - F_x * F_y)$;
(6) $\lambda x = (1 - (1 - T_x)^\lambda, (I_x)^\lambda, (F_x)^\lambda), \lambda > 0$;
(7) $x^\lambda = ((T_x)^\lambda, 1 - (1 - I_x)^\lambda, 1 - (1 - F_x)^\lambda), \lambda > 0$.

Definition 4: [43] Let $x = (T_x, I_x, F_x)$ be a SVNN, then the score function $s(x)$, accuracy function $a(x)$ and certainty function $c(x)$ is defined as follows:

(1) $s(x) = \frac{2}{3} + \frac{T_x}{3} - \frac{I_x}{3} - \frac{F_x}{3}$;
(2) $a(x) = T_x - F_x$;
(3) $c(x) = T_x$.

For any two SVNNs x, y,

(1) if $s(x) > s(y)$, then $x \succ y$;
(2) if $s(x) = s(y)$ and $a(x) > a(y)$, then $x \succ y$;
(3) if $s(x) = s(y)$, $a(x) = a(y)$ and $c(x) > c(y)$, then $x \succ y$;
(4) if $s(x) = s(y)$, $a(x) = a(y)$ and $c(x) = c(y)$, then $x \sim y$.

B. REDUCIBLE WEIGHTED MACLAURIN SYMMETRIC MEANS

The Maclaurin symmetric mean (MSM), originally initiated by Maclaurin [27], can seize the interrelation among multiple input values more effectively. Now, over the last decade years, the MSM is employed in aggregating uncertain information in decision making process.

Definition 5: [27] Let $x_i (i = 1, 2, \cdots, n)$ be a series of nonnegative real numbers, and $k = 1, 2, \cdots, n$, then the

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Maclaurin symmetric mean (MSM) operator is defined as follows:

$$\mathrm{MSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} x_{i_j}}{C_n^k} \right)^{1/k}, \tag{2}$$

where $(i_1, i_2, \cdots, i_k)$ traverses all the $k-$permutations of $(1, 2, \cdots, n)$, and the C_n^k is the binomial coefficient satisfying following formula: $C_n^k = \frac{n!}{k!(n-k)!}$.

Definition 6: [29] Let $x_i (i = 1, 2, \cdots, n)$ be a series of nonnegative real numbers, and $k = 1, 2, \cdots, n$, then the dual Maclaurin symmetric mean (DMSM) operator is defined as follows:

$$\mathrm{DMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} x_{i_j}}{C_n^k} \right)^{1/k}, \tag{3}$$

where $(i_1, i_2, \cdots, i_k)$ traverses all the $k-$permutations of $(1, 2, \cdots, n)$, and the C_n^k is the binomial coefficient satisfying following formula: $C_n^k = \frac{n!}{k!(n-k)!}$.

In order to solve the issues of idempotency and reducibility, Shi and Xiao [40] developed the reducible weighted MSM (RWMSM) and the reducible weighted dual MSM (RWDMSM) in the following.

Definition 7: [40] Let $x_i (i = 1, 2, \cdots, n)$ be a series of nonnegative real numbers, $k = 1, 2, \cdots, n$, and $W = (w_1, w_2, \cdots, w_n)^T$ with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then the reducible weighted MSM (RWMSM) operator is defined as follows:

$$\mathrm{RWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} \right)^{1/k}. \tag{4}$$

Definition 8: [40] Let $x_i (i = 1, 2, \cdots, n)$ be a series of nonnegative real numbers, $k = 1, 2, \cdots, n$, and $W = (w_1, w_2, \cdots, w_n)^T$ with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then the reducible weighted DMSM (RWDMSM) operator is defined as follows:

$$\mathrm{RWDMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \frac{\prod\limits_{1 \le i_1 < \cdots < i_k \le n} \left(\sum_{j=1}^{k} x_{i_j} \right)^{\frac{\sum_{j=1}^{k} w_{i_j}}{\sum_{1 \le i_1 < \cdots < i_k \le n} \sum_{j=1}^{k} w_{i_j}}}}{k}. \tag{5}$$

III. SINGLE-VALUED NEUTROSOPHIC REDUCIBLE WEIGHTED MACLAURIN SYMMETRIC MEANS

In this section, based on the operational rules of SVNNs with the RWMSM and RWDMSM operators, we develop single-valued neutrosophic RWMSM (SVNRWMSM) and single-valued neutrosophic RWDMSM (SVNRWDMSM). Meanwhile, we will prove some interesting properties and discuss some special cases of proposed aggregation operators.

A. SINGLE-VALUED NEUTROSOPHIC REDUCIBLE WEIGHTED MSM OPERATOR

Definition 9: Let $x_i = (T_{x_i}, I_{x_i}, F_{x_i}) (i = 1, 2, \cdots, n)$ be a series of SVNNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. The SVNRWMSM: $\Omega^m \to \Omega$, a SVNRWMSM operator is given as follows:

$$\mathrm{SVNRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} \right)^{1/k} \tag{6}$$

where Ω is the set of all SVNNs, then SVNRWMSM is called the single-valued neutrosophic reducible weighted MSM operator.

According to the operational laws of the SVNNs described in Definition 3, from Eq. (6), we can achieve the integrated result presented in Theorem 1.

Theorem 1: Let $x_i = (T_{x_i}, I_{x_i}, F_{x_i}) (i = 1, 2, \cdots, n)$ be a series of SVNNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then the result of SVNRWMSM operator is still a SVNN.

Theorem 2: (Monotonicity) Let $x_i (i = 1, 2, \cdots, n)$ and $x'_i (i = 1, 2, \cdots, n)$ be two series of SVNNs, if $x'_i = (T_{x'_i}, I_{x'_i}, F_{x'_i})$, $x_i = (T_{x_i}, I_{x_i}, F_{x_i})$, $T_{x_i} \ge T_{x'_i}, I_{x_i} \le I_{x'_i}, F_{x_i} \le F_{x'_i}$ for all $i = 1, 2, \cdots, n$, then

$$SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \ge SVNRWMSM^{(k)}(x'_1, x'_2, \cdots, x'_n). \tag{8}$$

Theorem 3: (Commutativity) Let $(x'_1, x'_2, \cdots, x'_n)$ be any permutation of $(x_1, x_2, \cdots, x_n)$, then

$$SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) = SVNRWMSM^{(k)}(x'_1, x'_2, \cdots, x'_n). \tag{9}$$

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$$\mathrm{SVNRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \Bigg(\left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}} \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k},$$
$$1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} (1 - I_{x_{i_j}}) \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k}, \tag{7}$$
$$1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} (1 - F_{x_{i_j}}) \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \Bigg)$$

Proof:

$$\begin{aligned} &\mathrm{SVNRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) \\ &= \left(\frac{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} (\prod\limits_{j=1}^{k} w_{i_j})(\prod\limits_{j=1}^{k} x_{i_j})}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod\limits_{j=1}^{k} w_{i_j}} \right)^{1/k} \\ &= \left(\frac{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} (\prod\limits_{j=1}^{k} w'_{i_j})(\prod\limits_{j=1}^{k} x'_{i_j})}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod\limits_{j=1}^{k} w'_{i_j}} \right)^{1/k} \\ &= \mathrm{SVNRWMSM}^{(k)}(x'_1, x'_2, \cdots, x'_n). \end{aligned}$$

Theorem 4: (Idempotency) Let $x_i (i = 1, 2, \cdots, n)$ be a series of SVNNs, if $x_i = x = (T, I, F)$ for $\forall i$, then

$$SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) = x. \tag{10}$$

Proof:

$$\begin{aligned} &\mathrm{SVNRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) \\ &= \left(\frac{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} (\prod\limits_{j=1}^{k} w_{i_j})(\prod\limits_{j=1}^{k} x_{i_j})}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod\limits_{j=1}^{k} w_{i_j}} \right)^{1/k} \\ &= \left(\frac{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} (\prod\limits_{j=1}^{k} w_{i_j})(\prod\limits_{j=1}^{k} x)}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod\limits_{j=1}^{k} w_{i_j}} \right)^{1/k} \\ &= \left(\frac{x^k \sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod\limits_{j=1}^{k} w_{i_j}}{\sum\limits_{1 \le i_1 < \cdots < i_k \le n} \prod\limits_{j=1}^{k} w_{i_j}} \right)^{1/k} = x. \end{aligned}$$

Theorem 5: (Boundedness) Let $x_i (i = 1, 2, \cdots, n)$ be a series of SVNNs, and $x^+ = \left(\max\limits_{i=1}^{n} T_i, \min\limits_{i=1}^{n} I_i, \min\limits_{i=1}^{n} F_i \right)$, $x^- = \left(\min\limits_{i=1}^{n} T_i, \max\limits_{i=1}^{n} I_i, \max\limits_{i=1}^{n} F_i \right)$, then

$$x^- \le SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \le x^+. \tag{11}$$

Proof: According to the above monotonicity and idempotency, we can achieve
$SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \le SVNRWMSM^{(k)}(x^+, x^+, \cdots, x^+)$
and
$SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \ge SVNRWMSM^{(k)}(x^-, x^-, \cdots, x^-)$.
Consequently, we can obtain
$x^- \le SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \le x^+$.

The Proof of Theorem 1:

According to the Definition 3, we can have $\prod_{j=1}^{k} x_{i_j} = \left(\prod_{j=1}^{k} T_{x_{i_j}}, 1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}), 1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}) \right)$ and

$$\left(\prod_{j=1}^{k} w_{i_j}\right)\left(\prod_{j=1}^{k} x_{i_j}\right) = \left(1 - \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}}, \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}}, \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right).$$

Further,

$$\sum_{1 \le i_1 < \cdots < i_k \le n} \left(\prod_{j=1}^{k} w_{i_j}\right)\left(\prod_{j=1}^{k} x_{i_j}\right) = \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}}, \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}}, \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right).$$

Consequently, $\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} = \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}}, \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}}, \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right).$

Finally, we can have

$$\mathrm{SVNRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} \right)^{1/k} = \left(\left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k}, 1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k}, 1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \right)$$

, then we can know that Eq. (7) is correct.

It is easily known that $0 \le \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \le 1,$

$$0 \le 1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \le 1,$$

$$0 \le 1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \le 1.$$

Hence, we can conclude that the integrated result from Eq. (7) is a SVNN. The theorem is proved.

The Proof of Theorem 2:
Suppose that $SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) = (T, I, F)$, $SVNRWMSM^{(k)}\ (x'_1, x'_2, \cdots, x'_n) = (T', I', F')$.
Since $T_{x_i} \geq T'_{x_i}$, then we can easily obtain $1 - \prod_{j=1}^{k} T_{x_{i_j}} \leq 1 - \prod_{j=1}^{k} T_{x'_{i_j}}$.
Further, we can achieve

$$\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}T_{x_{i_j}}\right)^{\prod_{j=1}^{k}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k}w_{i_j}}}\right)^{1/k} \geq$$

$$\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}T_{x'_{i_j}}\right)^{\prod_{j=1}^{k}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k}w_{i_j}}}\right)^{1/k}.$$

Consequently, $T \geq T'$.
Similar to above, we can also prove that $I \leq I', F \leq F'$.
Finally, we have $(T, I, F) \geq (T', I', F')$.
In other words, $SVNRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \geq SVNRWMSM^{(k)}(x'_1, x'_2, \cdots, x'_n)$.

Next, we discuss some peculiar cases of the SVNRWMSM by changing the value of the parameter k.

Case 1: If $k = 1$, the SVNRWMSM reduces to a single-valued neutrosophic weighted averaging (SVNWA) operator (Peng et al. [43] and Liu [44]).

$$\mathrm{SVNRWMSM}^{(1)}(x_1, x_2, \cdots, x_n) =$$

$$\left(\left(1-\left(\prod_{1\le i_1\le n}\left(1-T_{x_{i_1}}\right)^{w_{i_1}}\right)^{\frac{1}{\sum_{1\le i_1\le n}w_{i_1}}}\right)^{1/1},\right.$$

$$1-\left(1-\left(\prod_{1\le i_1\le n}I_{x_{i_1}}^{w_{i_1}}\right)^{\frac{1}{\sum_{1\le i_1\le n}w_{i_1}}}\right)^{1/1},$$

$$\left.1-\left(1-\left(\prod_{1\le i_1\le n}F_{x_{i_1}}^{w_{i_1}}\right)^{\frac{1}{\sum_{1\le i_1\le n}w_{i_1}}}\right)^{1/1}\right)$$

$$= \left(1-\prod_{1\le i_1\le n}\left(1-T_{x_{i_1}}\right)^{w_{i_1}}, \prod_{1\le i_1\le n}I_{x_{i_1}}^{w_{i_1}}, \prod_{1\le i_1\le n}F_{x_{i_1}}^{w_{i_1}}\right)$$

$$= \mathrm{SVNWA}(x_1, x_2, \cdots, x_n)$$

Case 2: If $k = n$, the SVNRWMSM reduces to a single-valued neutrosophic geometric (SVNG) operator (Peng et al. [43] and Liu [44]).

$$\mathrm{SVNRWMSM}^{(n)}(x_1, x_2, \cdots, x_n) =$$

$$= \left(\left(1-\left(\left(1-\prod_{j=1}^{n}T_{x_{i_j}}\right)^{\prod_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\prod_{j=1}^{n}w_{i_j}}}\right)^{1/n},\right.$$

$$1-\left(1-\left(\left(1-\prod_{j=1}^{n}(1-I_{x_{i_j}})\right)^{\prod_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\prod_{j=1}^{n}w_{i_j}}}\right)^{1/n},$$

$$\left.1-\left(1-\left(\left(1-\prod_{j=1}^{n}(1-F_{x_{i_j}})\right)^{\prod_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\prod_{j=1}^{n}w_{i_j}}}\right)^{1/n}\right)$$

$$= \left(1-\prod_{1\le i_1\le n}\left(1-T_{x_{i_1}}\right)^{\frac{1}{n}}, \prod_{1\le i_1\le n}I_{x_{i_1}}^{\frac{1}{n}}, \prod_{1\le i_1\le n}F_{x_{i_1}}^{\frac{1}{n}}\right)$$

$$= \mathrm{SVNG}(x_1, x_2, \cdots, x_n)$$

B. SINGLE-VALUED NEUTROSOPHIC REDUCIBLE WEIGHTED DUAL MSM OPERATOR

Definition 10: Let $x_i = (T_{x_i}, I_{x_i}, F_{x_i})(i = 1, 2, \cdots, n)$ be a series of SVNNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. The SVNRWDMSM: $\Omega^m \to \Omega$, a SVNRWDMSM operator is given as follows:

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$$\mathrm{SVNRWDMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \frac{\prod\limits_{1 \le i_1 < \cdots < i_k \le n} \left(\sum\limits_{j=1}^{k} x_{i_j} \right)^{\frac{\sum_{j=1}^{k} w_{i_j}}{\sum_{1 \le i_1 < \cdots < i_k \le n} \sum_{j=1}^{k} w_{i_j}}}}{k} \tag{12}$$

where Ω is the set of all SVNNs, then SVNRWDMSM is called the single-valued neutrosophic reducible weighted dual MSM operator.

According to the operational laws of the SVNNs described in Definition 3, from Eq. (12), we can achieve the integrated result presented in Theorem 6.

Theorem 6: Let $x_i = (T_{x_i}, I_{x_i}, F_{x_i})(i = 1, 2, \cdots, n)$ be a series of SVNNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then the result of SVNRWDMSM operator is still a SVNN.

Remark 1: The SVNRWDMSM operator also has the properties of commutativity, idempotency, monotonicity and boundedness.

Next, we discuss some special cases of the SVNR-WDMSM by changing the value of the parameter k.

Case 1: If $k = 1$, the SVNRWDMSM reduces to a single-valued neutrosophic weighted geometric (SVNWG) operator (Peng et al. [43] and Liu [44]).

$$\begin{aligned}
&\mathrm{SVNRWDMSM}^{(1)}(x_1, x_2, \cdots, x_n) \\
&= \left(1 - \left(1 - \left(\prod_{1 \le i_1 \le n} \left(1 - \prod_{j=1}^{1} (1 - T_{x_{i_j}}) \right)^{\sum_{j=1}^{1} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 \le n} \sum_{j=1}^{1} w_{i_j}}} \right)^{1/1}, \right. \\
&\left(1 - \left(\prod_{1 \le i_1 \le n} \left(1 - \prod_{j=1}^{1} I_{x_{i_j}} \right)^{\sum_{j=1}^{1} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 \le n} \sum_{j=1}^{1} w_{i_j}}} \right)^{1/1}, \\
&\left. \left(1 - \left(\prod_{1 \le i_1 < \le n} \left(1 - \prod_{j=1}^{1} F_{x_{i_j}} \right)^{\sum_{j=1}^{1} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 \le n} \sum_{j=1}^{1} w_{i_j}}} \right)^{1/1} \right) \\
&= \left(\prod_{1 \le i_1 \le n} T_{x_{i_j}}^{w_{i_j}}, \prod_{1 \le i_1 \le n} 1 - (1 - I_{x_{i_j}})^{w_{i_j}}, \prod_{1 \le i_1 \le n} 1 - (1 - F_{x_{i_j}})^{w_{i_j}} \right) \\
&= \mathrm{SVNWG}(x_1, x_2, \cdots, x_n)
\end{aligned}$$

Case 2: If $k = n$, the SVNRWDMSM reduces to a single-valued neutrosophic averaging (SVNA) operator (Peng et al. [43] and Liu [44]).

$$\begin{aligned}
&\mathrm{SVNRWMSM}^{(n)}(x_1, x_2, \cdots, x_n) = \\
&\left(\left(1 - \left(1 - \left(\left(1 - \prod_{j=1}^{n} (1 - T_{x_{i_j}}) \right)^{\sum_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{n} w_{i_j}}} \right) \right)^{1/n}, \right. \\
&\left(1 - \left(\left(1 - \prod_{j=1}^{n} I_{x_{i_j}} \right)^{\sum_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{n} w_{i_j}}} \right)^{1/n}, \\
&\left. \left(1 - \left(\left(1 - \prod_{j=1}^{n} F_{x_{i_j}} \right)^{\sum_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{n} w_{i_j}}} \right)^{1/n} \right) \\
&= \left(1 - \prod_{1 \le i_1 \le n} (1 - T_{x_{i_1}})^{\frac{1}{n}}, \prod_{1 \le i_1 \le n} I_{x_{i_1}}^{\frac{1}{n}}, \prod_{1 \le i_1 \le n} F_{x_{i_1}}^{\frac{1}{n}} \right) \\
&= \mathrm{SVNA}(x_1, x_2, \cdots, x_n)
\end{aligned}$$

Remark 2: Notably, it is important that SVNRWMSM or SVNRWDMSM operator cannot seize the interrelation among many given arguments when $k = 1$ or $k = n$. In other words, both of them reduce to the independent operators such as SVNA, SVNG, SVNWA and SVNWG (Peng et al. [43] and Liu [44]).

IV. THE MADM METHODS BASED ON SVNRWMSM AND SVNRWDMSM OPERATORS

A. DESCRIPTION OF THE MADM PROBLEMS

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \cdots, C_n\}$ be a collection of n attributes, and $W = \{w_1, w_2, \cdots, w_n\}$ be a weight vector assigned to the attributes by the experts with the standard constraints $w_j \in [0, 1], \sum_{j=1}^{n} w_j = 1$. We assume that the global evaluation of the alternatives with respect to attributes is represented by a single-valued neutrosophic matrix $P = (p_{ij})_{m \times n} = (T_{ij}, I_{ij}, F_{ij})_{m \times n}$. By this we mean that the values associated with the alternatives for the modelization of MADM problems can be shown as in Table 1.

B. THE MADM METHOD BASED ON SVNRWMSM OR SVNRWDMSM OPERATOR

In order to make decisions in our setting, the framework for using the proposed method is shown in FIGURE 2.

Meanwhile, the following algorithm is self-explanatory:

C. A CASE OF UTAE

The report of the 18th National Congress of the Communist Party of China clearly stated that "trying to do a good job of the people's satisfaction with education", "focusing on

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$$\mathrm{SVNRWDMSM}^{(k)}(x_1,x_2,\cdots,x_n)=$$

$$\left(1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}(1-T_{x_{i_j}})\right)^{\sum_{j=1}^{k}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k}w_{i_j}}}\right)^{1/k},\right.$$
$$\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}I_{x_{i_j}}\right)^{\sum_{j=1}^{k}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k}w_{i_j}}}\right)^{1/k}, \tag{13}$$
$$\left.\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}F_{x_{i_j}}\right)^{\sum_{j=1}^{k}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k}w_{i_j}}}\right)^{1/k}\right)$$

TABLE 1: The single-valued neutrosophic MADM matrix.

	C_1	C_2	$\cdots$	C_n
A_1	(T_{11},I_{11},F_{11})	(T_{12},I_{12},F_{12})	$\cdots$	(T_{1n},I_{1n},F_{1n})
A_2	(T_{21},I_{21},F_{21})	(T_{22},I_{22},F_{22})	$\cdots$	(T_{2n},I_{2n},F_{2n})
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	(T_{m1},I_{m1},F_{m1})	(T_{m2},I_{m2},F_{m2})	$\cdots$	(T_{mn},I_{mn},F_{mn})

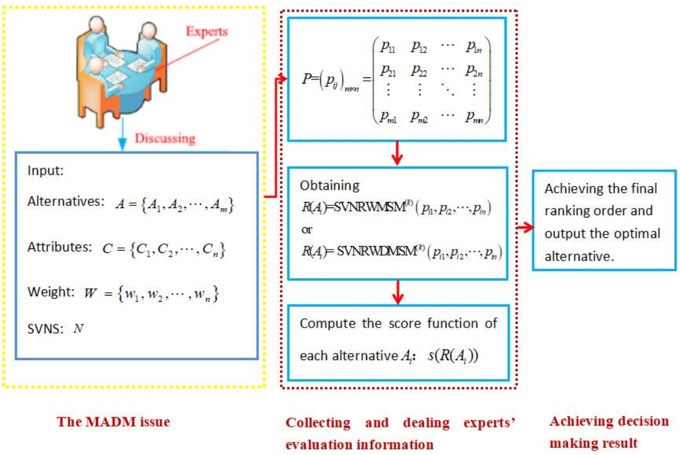

FIGURE 2: The framework for using the proposed method.

improving the quality of education" and "promoting the intensive development of higher education". It is a systematic project to promote colleges and universities to enter the connotative development road with improving quality as the core. The undergraduate teaching audit and evaluation (UTAE) is a mode of evaluation in the "Five-in-One" evaluation system. Audit evaluation is different from conformity assessment and level assessment. The conformity assessment is an assessment of the certification model and is passed when the criteria are met. The level assessment belongs to the evaluation of the selection model, mainly to see what level the participating schools are at, and the focus is on selecting "excellent". The evaluation mainly depends on whether the assessed object has achieved its own setting goals. The country does not have a unified evaluation standard. It uses its own ruler to measure itself, and the audit conclusion is not graded, forming a realistic audit report. The audit assessment aims to guide the school to establish a self-regulatory mechanism, strengthen self-improvement, and improve the school-running level and quality of education.

The audit assessment covers all aspects of the talent training process in colleges and universities. It mainly depends on the realization of the training objectives and training effects of the school personnel, and finally forms a realistic report regardless of the grade. The focus of the study is on the following five aspects, which are referred to as "five degrees".

(1) **The achievement degree** of school talent training effect and training goal

The goal of talent training is the starting point of a series of educational activities. In the absence of training objectives or unclear training objectives, education teaching activities are lack of pertinence, aimless and difficult to achieve results. Starting from the training goal, a training process was completed after a number of education teaching

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Algorithm 1 :SVNRWMSM or SVNRWDMSM operators

1: Sort the given alternatives and attributes, and achieve the single-valued neutrosophic matrix $P = (p_{ij})_{m\times n}$ which is shown in the format of Table 1.

2: Transform the matrix $P = (p_{ij})_{m\times n}$ into a normalized single-valued neutrosophic matrix $P' = (p'_{ij})_{n\times m}$ by Eq. (14).

$$p'_{ij} = \begin{cases} (T_{ij}, I_{ij}, F_{ij}), & C_j \text{ is benefit attribute} \\ (F_{ij}, 1-I_{ij}, T_{ij}), & C_j \text{ is cost attribute} \end{cases} \tag{14}$$

3: Employ the SVNRWMSM operator

$$R(A_i) = (T_i, I_i, F_i) = SVNRWMSM^{(k)}(p'_{i1}, p'_{i2}, \cdots, p'_{in}) \tag{15}$$

or

Employ the SVNRWDMSM operator

$$R(A_i) = (T_i, I_i, F_i) = SVNRWDMSM^{(k)}(p'_{i1}, p'_{i2}, \cdots, p'_{in}) \tag{16}$$

to determine the aggregated preference value.

4: Determine the score function $s(R(A_i))$ of the whole values $R(A_i)(i = 1, 2, \cdots, m)$.

5: Select the optimal alternative(s) by maximization of their scores, i.e., any alternative whose score $R(A_i)$ is at least as great as any other alternative's can be chosen.

activities to achieve the expected talent training goal. The result and the cultivation target overlap greatly, indicating the result and the goal attainment degree is high. If the overlapping surface is small or cannot be overlapped, the training objective is not practical, or if education teaching activities are not in place, the achievement degree is very low. The achievement of the training effect is not only reflected in the graduation rate and employment rate of the students, but also mainly reflected in the process of talent training, including classroom examination, graduation thesis (design), experimental internship, second classroom, social practice and other activities. It is necessary to attach great importance to these teaching links in order to ensure the quality of training.

(2) **The adaptability degree** of school orientation, talent training objectives and national and regional economic and social development needs

Colleges and universities are important bases for cultivating and transporting talents for the society. The talent training goals determined by higher education institutions must be reflected in economic construction services to meet the needs of social development for talents. National and regional economic and social development has diverse needs for talents, such as research talents, applied talents, skilled talents, and compound talents. Institutions of higher learning must rationally position themselves from the actual situation and formulate corresponding talent training goals. Develop training programs based on the training objectives and form a curriculum system that is compatible with them.

(3) **The guarantee degree** of teachers and teaching resource condition

Teaching resources refer to all the materialized, explicit or implicit teaching elements that can help students achieve learning goals and serve students' learning. Teachers and teaching resources usually include teaching infrastructure, teaching funds, teaching materials, teaching instruments and equipment, etc. Teachers and teaching resources are the basic conditions for running a university. The state has basic requirements for this, such as the ratio of students to teachers, the value of teaching and research equipment, the book per student, the administrative room for teaching per student, and the structure of teacher education. These indicators are the basic conditions for colleges and universities to guarantee the quality of talent cultivation. In addition, the talent cultivation goal set by the school according to its own situation should also provide people, financial and material conditions to support it, so as to promote the realization of the training goal. Teacher's morality, teaching level and scientific research ability are the embodiment of teacher's quality. High quality talent team and teaching resources are the guarantee to improve the quality of talent training.

(4) **The effective degree** of the teaching and quality assurance system

Internal quality assurance system is the key to guarantee the smooth development of teaching. The teaching quality assurance system is composed of quality standards, quality evaluation, quality control, information collection and feedback improvement. The school should establish its own professional standards, curriculum standards and quality standards of all major teaching links in accordance with the national higher education quality standards and relevant industry standards. Schools should teach according to these quality standards. The school should establish a self-assessment system to carry out department evaluation, professional evaluation and course evaluation on teaching conditions, teaching process and teaching effect. The school should establish the basic teaching status database, give full play to the normal monitoring effect of the teaching status data on the teaching work, and combine with the self-evaluation work to form and release the teaching quality report annually. The school shall collect relevant teaching information through self-assessment, supervision and inspection, and regular monitoring of teaching status data, and timely feed back to all links of teaching work, adjust and improve the work, and constantly improve the quality of talent cultivation.

(5) **The satisfied degree** of student and social employer

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The satisfaction of students and social employers is the fundamental measure of talent training quality. The school should establish a tracking investigation mechanism for social employers and graduates, understand the needs of social employers and graduates' reflection regularly, and adjust and improve the professional setting, training objectives, training specifications, training programs and teaching methods according to the feedback information. Students are the subject of learning and one of the evaluators of education teaching quality. The quality of education teaching activities and the service work of the school is reflected in whether the quality of students can be improved and whether the learning needs can be met. Student evaluation and satisfaction are the internal standards of school work quality. The school should insist on student-oriented, strengthen the guidance and service to students, and establish a scientific and effective evaluation system, regularly understand students' opinions and suggestions on teaching, management and service, and constantly improve teaching work to improve students' satisfaction.

The "five degrees" constitutes the assessment of students as the main line and student development as the center, covering the whole process of inputting and outputting talents from the entrance to the graduation of students. By examining whether the school's teaching design, resource allocation and teacher security can meet the needs of students' learning and growth in the process of training, and whether the students who train output can meet the needs of economic and social development, they can evaluate the quality of school personnel training.

Assessment experts make a comprehensive judgment on the school's talent cultivation and education quality through auditing these aspects.

Example 1: Suppose that there are five universities $A = \{A_1, A_2, A_3, A_4, A_5\}$ to be considered for the assessment in U-TAE. The expert elects the highly representative attribute set $C = \{C_1(\text{Achievement degree}), C_2(\text{Adaptability degree}), C_3(\text{Guarantee degree}), C_4(\text{Effective degree}), C_5(\text{Satisfied degree})\}$. According to the general evolving principle and the features of the UTAE, we can ascertain that whole attributes are benefit attributes. Assume that the expert has given the weight information as $w = (w_1, w_2, w_3, w_4, w_5) = (0.2, 0.3, 0.15, 0.15, 0.2)$. The evaluations for UTAE of universities arising from questionnaire investigation by the veteran expert group and generating the final single-valued neutrosophic matrix with its tabular form given in Table 2.

Next, we employ the algorithm proposed above in selecting optimal university to collaborate under single-valued neutrosophic text.

Step 1: Sort the alternatives and attributes, and obtain the single-valued neutrosophic matrix $P = (p_{ij})_{5\times 5}$ which is shown in the format of Table 2.

Step 2: No conversion is required, on account of all attributes are beneficial attributes.

Step 3: Employ the $SVNRWMSM^{(1)}$ operator to aggregate the preference value as follows:
$R(A_1) = (0.843083, 0.110957, 0.123114)$,
$R(A_2) = (0.769505, 0.174110, 0.185028)$,
$R(A_3) = (0.687798, 0.174110, 0.216330)$,
$R(A_4) = (0.581126, 0.174110, 0.208961)$,
$R(A_5) = (0.479877, 0.174110, 0.208961)$.
or
Employ the $SVNRWDMSM^{(1)}$ operator to aggregate the preference value as follows:
$R(A_1) = (0.833668, 0.115761, 0.131246)$,
$R(A_2) = (0.763471, 0.180931, 0.197174)$,
$R(A_3) = (0.676652, 0.180931, 0.233228)$,
$R(A_4) = (0.570848, 0.180931, 0.228699)$,
$R(A_5) = (0.469984, 0.180931, 0.228699)$.

Step 4: Compute the score function $s(R(A_i))$ of the whole values $R(A_i)(i = 1, 2, 3, 4, 5)$ as follows:
SVNRWMSM:
$s(R(A_1)) = 0.869671$, $s(R(A_2)) = 0.803456$,
$s(R(A_3)) = 0.765786$, $s(R(A_4)) = 0.732685$,
$s(R(A_5)) = 0.698935$.
SVNRWDMSM:
$s(R(A_1)) = 0.862220$, $s(R(A_2)) = 0.795122$,
$s(R(A_3)) = 0.754164$, $s(R(A_4)) = 0.720406$,
$s(R(A_5)) = 0.686785$.

Step 5: Based on above score function $s(R(A_i))(i = 1, 2, 3, 4, 5)$, we can obtain the ordering of the alternatives $\{A_1, A_2, A_3, A_4, A_5\}$ as follows: $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

D. SENSITIVITY ANALYSIS OF THE PARAMETER K ON THE ORDERING IN DEVELOPED ALGORITHMS

For analyzing the sensitivity of the parameters k on the score values, an experiment (Example 1) was executed by adopting diverse values of $k(k = 1, 2, 3, 4, 5)$.

Based on the SVNRWMSM and SVNRWDMSM operators, the final score values of five universities are summarized in Fig. 3 and Fig. 4. From two figures, some significant points have been come to a conclusion in the following.

(1) For SVNRWMSM algorithm, the score values of all five universities are firstly monotonically increases when $k \in [1, 4]$, and later monotonically decreases when $k \in [4, 5]$. Moreover, it is not very clear to see the final ordering on account of achieving the similar values which vary from 0.943918 to 0.999968 with difference value of 0.05 when $k = 4$. The final results all keep as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

(2) For SVNRWDMSM algorithm, the score values of all five universities are firstly monotonically decreases when $k \in [1, 4]$, and later monotonically increases when $k \in [4, 5]$.

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TABLE 2: The single-valued neutrosophic matrix in Example 1.

	C_1	C_2	C_3	C_4	C_5
A_1	$(0.9, 0.1, 0.1)$	$(0.8, 0.1, 0.2)$	$(0.8, 0.2, 0.1)$	$(0.9, 0.1, 0.1)$	$(0.8, 0.1, 0.1)$
A_2	$(0.8, 0.1, 0.2)$	$(0.8, 0.2, 0.2)$	$(0.7, 0.2, 0.3)$	$(0.8, 0.2, 0.2)$	$(0.7, 0.2, 0.1)$
A_3	$(0.7, 0.1, 0.2)$	$(0.7, 0.2, 0.3)$	$(0.6, 0.2, 0.1)$	$(0.8, 0.2, 0.3)$	$(0.6, 0.2, 0.2)$
A_4	$(0.5, 0.1, 0.2)$	$(0.6, 0.2, 0.3)$	$(0.6, 0.2, 0.2)$	$(0.7, 0.2, 0.3)$	$(0.5, 0.2, 0.1)$
A_5	$(0.4, 0.1, 0.2)$	$(0.5, 0.2, 0.3)$	$(0.5, 0.2, 0.2)$	$(0.6, 0.2, 0.3)$	$(0.4, 0.2, 0.1)$

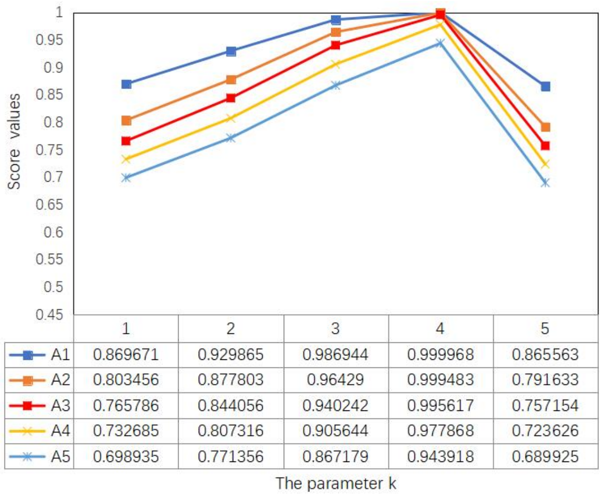

FIGURE 3: The total changing trend of parameter k in SVNRWMSM algorithm.

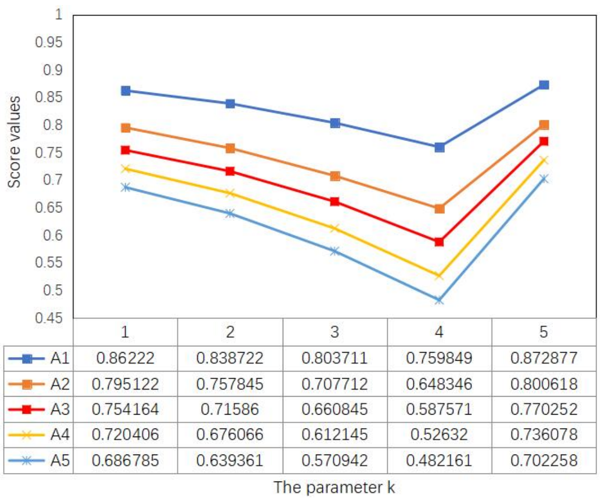

FIGURE 4: The total changing trend of parameter k in SVNRWDMSM algorithm.

Moreover, it is very clear to see the final ordering compared with the SVNRWMSM algorithm. The final results also all keep as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

(3) The reason for the inflection point is that $k = 1$ is the form of averaging operator at the beginning and $k = 5$ is the form of geometric operator at the end for SVNRWMSM algorithm. Similarly, for SVNRWDMSM algorithm, it experiences the transformation from geometric operator to averaging operator.

(4) The values at both ends ($k = 1$ and $k = 5$) have an interesting phenomenon. For SVNRWMSM algorithm, the score values of all five universities are the minimum compared with $k = 1, 2, 3, 4$ when $k = 5$ (geometric form). For SVNRWDMSM algorithm, the score values of all five universities are the maximal compared with $k = 1, 2, 3, 4$ when $k = 5$ (averaging form).

V. COMPARATIVE ANALYSIS AND DISCUSSION

In the following, some existing decision making methods [41–51] with their limitations and characteristics are discussed in detail. An example is given to show the advantages of our proposed algorithms.

Example 2: Continue to Example 1. Suppose that the assessments for universities arising from another group of experts are given which is shown in Table 3.

Remark 3: According to Table 4, we can find that the red background color indicates the illogical result on account of the division by zero problem which is probed in Peng and Dai [42]. That is to say, if the assessed value of one alternative in corresponding attribute is the largest in whole alternatives for proprietary attributes, then we can't give a decision. For Cross-entropy [41], it is easily lead to the antilogarithm by zero issue. In addition, we will see that our optimal alternative and the ranking are same as the existing aggregation methods [43–51].

In aggregation functions, only just the aggregation methods [46, 47, 50] take the interrelationship of the attributes into consideration. For better differentiate the characteristics of existing SVN aggregation operators, we make a summary of them presented in Table 5. From Table 5, we can find the presented aggregation operators are based on RWMSM operators with a parameter k. Hence, the initiated aggregation operators (SVNRWMSM and SVNRWDMSM) are more all-purpose than some existing aggregation operators. In the meantime, they can take the interrelation of the multiple attributes into consideration for handling MADM issues.

For a better comparison with some MSM operators in diverse uncertain environment [28–39], we make a summary of them presented in Table 6.

From Table 6, some existing weighted forms of MSM operators do not possess the idempotency. Moreover, the weight MSM cannot degrade into the MSM when their weights in-

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TABLE 3: The single-valued neutrosophic matrix in Example 2.

	C_1	C_2	C_3	C_4	C_5
A_1	$(0.9, 0.1, 0)$	$(0.8, 0.1, 0.2)$	$(0.8, 0.2, 0.1)$	$(0.9, 0.1, 0.1)$	$(0.7, 0.1, 0.1)$
A_2	$(0.8, 0.1, 0.2)$	$(0.8, 0.2, 0.3)$	$(0.7, 0.2, 0.3)$	$(0.8, 0.2, 0.2)$	$(0.7, 0.2, 0.1)$
A_3	$(0.7, 0.1, 0.2)$	$(0.7, 0.2, 0.3)$	$(0.6, 0.2, 0.1)$	$(0.8, 0.2, 0.3)$	$(0.6, 0.2, 0.2)$
A_4	$(0.5, 0.1, 0.2)$	$(0.6, 0.2, 0.3)$	$(0.6, 0.2, 0.2)$	$(0.7, 0.2, 0.3)$	$(0.5, 0.2, 0.1)$
A_5	$(0.4, 0.1, 0.2)$	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.2)$	$(0.6, 0.2, 0.3)$	$(0.4, 0.2, 0.1)$

TABLE 4: A comparison study with some existing methods in Example 2.

Algorithms	Ranking	Optimal alternative
Algorithm 1: SVNRWMSM	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Algorithm 1: SVNRWDMSM	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Cross-entropy [41]	LOG	*
TOPSIS [42]	N/A	*
ASVNNWA [43, 44]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
ASVNNWG [43, 44]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
ESVNNWA [43, 44]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
ESVNNWG [43, 44]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
HSVNNWA [44]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
HSVNNWG [44]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNFWA [45]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNFWG [45]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNNWBM^{p,q} [46]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
NNIGWHM^{p,q} [47]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
NNIGWGHM^{p,q} [47]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNDWAA [48]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNDWGA [48]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNDPWA [49]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNDPWG [49]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNWBPM^{p,q} [50]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
SVNWGBPM^{p,q} [50]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
GNNHWA [51]	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1

$k = 2$ in SVNRWMSM and SVNRWDMSM;
$\gamma = 2$ in HSVNNWA, HSVNNWG and GNNHWA, $\rho = 2$ in SVNDWAA and SVNDWGA;
$p = 2, q = 2$ in SVNNWBM, NNIGWHM, NNIGWGHM, SVNWBPM and SVNWGBPM;
LOG denotes it cannot calculate because the antilogarithm by zero problem (Gray background), and $*$ presents no result;
N/A denotes it cannot calculate because the division by zero problem (Red background), and $*$ presents no result.

TABLE 5: Characteristic comparisons of diverse single-valued neutrosophic aggregation operators.

Aggregation operators	Whether consider interrelationships between aggregating two attributes	Whether make the information aggregation process more flexible by a parameter	Whether consider interrelationships between aggregating multiple attributes
ASVNNWA [43, 44]	No	No	No
ASVNNWG [43, 44]	No	No	No
ESVNNWA [43, 44]	No	No	No
ESVNNWG [43, 44]	No	No	No
HSVNNWA [44]	No	No	No
HSVNNWG [44]	No	No	No
GNNHWA [51]	No	No	No
SVNFWA [45]	No	No	No
SVNFWG [45]	No	No	No
SVNDWAA [48]	No	Yes	No
SVNDWGA [48]	No	Yes	No
SVNDPWA [49]	No	Yes	No
SVNDPWG [49]	No	Yes	No
SVNNWBM^{p,q} [46]	Yes	No	No
NNIGWHM^{p,q} [47]	Yes	No	No
NNIGWGHM^{p,q} [47]	Yes	No	No
SVNWBPM^{p,q} [50]	Yes	No	No
SVNWGBPM^{p,q} [50]	Yes	No	No
SVNRWMSM	Yes	Yes	Yes
SVNRWDMSM	Yes	Yes	Yes

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TABLE 6: Characteristic comparisons of diverse uncertain environment of MSM.

Sets	Aggregation operators	Whether possess the property of idempotency	Whether possess the property of reducibility
SVNS	SVNRWMSM	Yes	Yes
SVNS	SVNRWDMSM	Yes	Yes
IFS	WIFMSM [28]	No	No
ULS	ULWDMSM [29]	No	No
ILS	WILMSM [30]	No	No
IULS	WIULMSM [30]	No	No
LIFS	WLIFMSM [31]	No	No
LIFS	WLIFDMSM [31]	No	No
PFS	PFIWMSM [32]	No	No
HFLS	HFLWMSM [33]	No	No
PFS	PFWMSM [34]	No	No
HFS	WHFMSM [35]	No	No
SVTNS	SVTNWMSM [36]	No	No
INLS	SVTNWMSM [37]	No	No
SVNI2TLS	SVNITLWMSM [38]	No	No
2TLS	2TLWMSM [39]	No	No

SVNS: Single-valued neutrosophic set, IFS: Intuitionistic fuzzy set, ULS: Uncertain linguistic set, ILS: Intuitionistic linguistic set, IULS: Intuitionistic uncertain linguistic set, LIFS: Linguistic intuitionistic fuzzy set, PFS: Pythagorean Fuzzy set, HFLS: Hesitant fuzzy linguistic set, HFS: Hesitant fuzzy set, SVTNS: Single-valued trapezoidal neutrosophic set, INLS: Interval neutrosophic linguistic set, SVNI2TLS: Single-valued neutrosophic interval 2-tuple linguistic set, 2TLS: 2-tuple linguistic set.

formation are equivalent. In other words, it signifies without the reducibility.

VI. CONCLUSION

The main contributions can be illustrated and reviewed as follows:

(1) Two fire-new SVN aggregation operators are developed such as SVNRWMSM operator and SVNRWDMSM operator.

(2) Some interesting properties such as monotonicity, commutativity, idempotency, boundedness and reducibility are discussed in detail under single-valued neutrosophic environment. Some existing MSM operators in diverse uncertain environment [28–39] are out of idempotency and reducibility.

(3) Two algorithms for solving single-valued neutrosophic decision making issue by SVNRWMSM and SVNRWDMSM are presented. The sensitivity analysis of diverse parameter value k on the ranking is explored in detailed (Figs. 3, 4). Compared with the existing single-valued neutrosophic decision making algorithms (Table 4), are (i) it has no division by zero issue [42]; (ii) it has no antilogarithm by zero problem [41].

(4) Utilize the modern statistics methods to build a synthesize appraise system of UTAE in five degrees (Achievement degree, Adaptability degree, Guarantee degree, Effective degree, Satisfied degree), and employ it in estimating the ideal cooperative universities.

In the future, we will take the SVNRWMSM and SVNRWDMSM operator in other ways such as gene selection [52]. Furthermore, we will also take RWMSM and RWDMSM into diverse fuzzy environment [53–61].

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