Neutrosophic Regular Weakly Generalized Homoeomorphism

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***Abstract*— Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic mapping is called Neutrosophic Regular Weakly Generalized Homoeomorphism in Neutrosophic topological spaces and also discussed about properties and characterization Neutrosophic Regular Weakly Generalized Homoeomorphism .**

***Keywords*—** (NS(R)WG open set, (NS(R)WG closed set ,(NS(R)WG homeomorphism, Neutrosophic topological spaces

I. Introduction

A.A.Salama introduced Neutrosophic topological spaces by using Smarandache's Neutrosophic sets. I.Arokiarani.[2] et al, introduced Neutrosophic α-closed sets.P. Ishwarya, [8]et.al, introduced and studied about on Neutrosophic semi-open sets in Neutrosophic topological spaces. Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic mapping is called Neutrosophic Regular Weakly Generalized Homoeomorphism in Neutrosophic topological spaces and also discussed about properties and characterization Neutrosophic Regular Weakly Generalized Homoeomorphism .

II. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Definition 2.1 [7]

Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form

$A = \{<x, \eta_A(x), \sigma_A(x) ,\gamma_A(x) >:x\in X\}$

Where $\eta_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent Neutrosophic topological spaces the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.2 [7]

A Neutrosophic set $A=\{<x, \eta_A(x), \sigma_A(x), \gamma_A(x) >: x\in X\}$ can be identified to an ordered triple $<\eta_A, \sigma_A, \gamma_A>$ in]-0,1+[on X.

Remark 2.3[7]

We shall use the symbol

$A =<x, \eta_A, \sigma_A, \gamma_A>$ for the Neutrosophic set $A = \{<x, \eta_A(x),\sigma_A(x),\gamma_A(x) >:x\in X\}$.

Example 2.4 [7]

Every Neutrosophic set A is a non-empty set in X is obviously on Neutrosophic set having the form $A=\{ <x, \eta_A(x), 1-((\eta_A(x) + \gamma_A(x)), \gamma_A(x) >:x\in X\}$**.** Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N=\{<x, 0, 0, 1>: x \in X\}$

(0_2) $0_N=\{<x, 0, 1, 1>: x \in X\}$

(0_3) $0_N =\{<x, 0, 1, 0 >:x\in X\}$

(0_4) $0_N=\{<x, 0, 0, 0>: x \in X\}$

1_N may be defined as :

(1_1) $1_N =\{<x, 1, 0, 0>: x\in X\}$

(1_2) $1_N =\{<x, 1, 0, 1 >: x\in X\}$

(1_3) $1_N =\{<x, 1, 1, 0 >: x\in X\}$

(1_4) $1_N =\{<x, 1, 1, 1 >: x\in X\}$

Definition 2.5 [7]

Let $A=<\eta_A, \sigma_A,\gamma_A>$ be a Neutrosophic set on X, then the complement of the set A

A^C defined as

$A^C=\{<x , \gamma_A(x) ,1- \sigma_A(x), \eta_A(x) >: x \in X\}$

Definition 2.6 [7]

Let X be a non-empty set, and Neutrosophic sets A and B in the form

$A =\{<x, \eta_A(x), \sigma A(x), \gamma A(x)>:x\in X\}$ and

$B =\{<x, \eta_B(x), \sigma_B(x), \gamma_B(x)>: x\in X\}$.

Then we consider definition for subsets ($A\subseteq B$).

$A\subseteq B$ defined as: $A\subseteq B \Leftrightarrow \eta_A(x) \leq \eta_B(x), \sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x\in X$

Proposition 2.7 [7]
For any Neutrosophic set A, then the following condition are holds:
(i) $0_N \subseteq A$, $0_N \subseteq 0_N$
(ii) $A \subseteq 1_N$, $1_N \subseteq 1_N$
Definition 2.8 [7]
Let X be a non-empty set, and A=<x, $\eta_B(x)$,$\sigma_A(x)$, $\gamma_A(x)$> , B =<x, $\eta_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$> be two Neutrosophic sets. Then
(i) A∩B defined as :A∩B =<x, $\eta_A(x)\wedge\eta_B(x)$, $\sigma_A(x)\wedge\sigma_B(x)$,$\gamma_A(x)\vee\gamma_B(x)$>
(ii) A∪B defined as :A∪B =<x, $\eta_A(x)\vee\eta_B(x)$, $\sigma_A(x)\vee\sigma_B(x)$, $\gamma_A(x)\wedge\gamma_B(x)$>
Proposition 2.9 [7]
For all A and B are two Neutrosophic sets then the following condition are true:
(i) $(A\cap B)^C=A^C\cup B^C$
(ii) $(A\cup B)^C=A^C\cap B^C$.
Definition 2.10 [11]
A Neutrosophic topology is a non-empty set X is a family τ_N of Neutrosophic subsets in X satisfying the following axioms:
(i) $0_N, 1_N \in \tau_N$,
(ii) $G_1\cap G_2\in\tau_N$ for any $G_1, G_2\in\tau_N$,
(iii) $\cup G_i\in\tau_N$ for any family $\{G_i \mid i\in J\}\subseteq\tau_N$.
the pair (X, τ_N) is called a Neutrosophic topological space.
The element Neutrosophic topological spaces of τ_N are called Neutrosophic open sets.
A Neutrosophic set A is closed if and only if A^C is Neutrosophic open.
Example 2.11[11]
Let X={x} and
A_1= {<x, 0.6, 0.6, 0.5>:x∈X}
A_2= {<x, 0.5, 0.7, 0.9>:x∈X}
A_3= {<x, 0.6, 0.7, 0.5>:x∈X}
A_4= {<x, 0.5, 0.6, 0.9>:x∈X}
Then the family $\tau_N=\{0_N, 1_N, A_1, A_2, A_3, A_4\}$is called a Neutrosophic topological space on X.
Definition 2.12[11]
Let (X, τ_N) be Neutrosophic topological spaces and A={<x,$\eta_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$>:x∈X} be a Neutrosophic set in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by
Neu-cl(A)=∩{K:K is a Neutrosophic closed set in X and A⊆K}
Neu-int(A)=∪{G:G is a Neutrosophic open set in X and G⊆A}.
Definition 2.13
Let (X, τ_N) be a Neutrosophic topological space. Then A is called
(i) Neutrosophic regular Closed set [2] (Neu-RCS in short) if A=Neu-Cl(Neu-Int(A)),
(ii) Neutrosophic α-Closed set[2] (Neu-αCS in short) if Neu-Cl(Neu-Int(Neu-Cl(A)))⊆A,
(iii) Neutrosophic semi Closed set [8] (Neu-SCS in short) if Neu-Int(Neu-Cl(A))⊆A ,
(iv) Neutrosophic pre Closed set [12] (Neu-PCS in short) if Neu-Cl(Neu-Int(A))⊆A,
Definition 2.14
Let (X, τ_N) be a Neutrosophic topological space. Then A is called
(i). Neutrosophic regular open set [2](Neu-ROS in short) if A=Neu-Int(Neu-Cl(A)),
(ii). Neutrosophic α-open set [2](Neu-αOS in short) if A⊆Neu-Int(Neu-Cl(Neu-Int(A))),
(iii). Neutrosophic semi open set [8](Neu-SOS in short) if A⊆Neu-Cl(Neu-Int(A)),
(iv).Neutrosophic pre open set [13] (Neu-POS in short) if A⊆Neu-Int(Neu-Cl(A)),
Definition 2.15
Let (X, τ_N) be a Neutrosophic topological space. Then A is called
(i).Neutrosophic generalized closed set[4](Neu-GCS in short) if Neu-cl(A)⊆U whenever A⊆U and U is a Neu-OS in X ,
(ii).Neutrosophic generalized semi closed set[12] (Neu-GSCS in short) if Neu-scl(A)⊆U Whenever A⊆U and U is a Neu-OS in X,
(iii).Neutrosophic α generalized closed set [9](Neu-αGCS in short) if Neu-αcl(A)⊆U whenever A⊆U and U is a Neu-OS in X ,
(iv).Neutrosophic generalized alpha closed set [5] (Neu-GαCS in short) if Neu-αcl(A)⊆U whenever A⊆U and U is a Neu-αOS in X .
The complements of the above mentioned Neutrosophic closed sets are called their respective Neutrosophic open sets.
Definition 2.16. [7] An IFS A is said to be an *Neutrosophic regular weakly generalized closed set* (NS(RWG)CS in short) if cl(int(A)) ⊆ U whenever A ⊆ U and U is an NSROS in X.
Definition 2.17. [9] A mapping f: $(X, NS_\tau) \rightarrow (Y, NS_\sigma)$ is called an *Neutrosophic regular weakly generalized continuous* (NSRWG continuous in short) if $f^{-1}(B)$ is an NSRWGCS in (X, NS_τ) for every NSCS B of (Y, NS_σ)
Definition 2.18. [8] A mapping f: $(X, NS_\tau) \rightarrow (Y, NS_\sigma)$ is called an *Neutrosophic regular weakly generalized irresolute* (NSRWG irresolute in short) if $f^{-1}(B)$ is an NSRWGCS in f: (X, NS_τ) for every NSRWGCS B of (Y, NS_σ).
3. Neutrosophicregular weakly generalized homeomorphism
Definition 3.1.
A bijective mapping f: $(X, NS_\tau) \rightarrow (Y, NS_\sigma)$ is called an Neutrosophic regular weakly generalized homeomorphism (NS(R)WG homeomorphism in short) if f and f^{-1} are NS(R)WG continuous mappings.

Example 3.2.
Let X = {a, b, c}, Y = {u, v, w} and
G_1 = <x, $\left(\frac{2}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right)$ >,
G_2 = <y, $\left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right)$ >.
Then NS_τ ={0_{NS},G_1, 1_{NS}} and NS_σ = {0_{NS},G_2, 1_{NS}} are NSTS on X and Y respectively. Consider a bijective mapping
f: $(X, NS_\tau) \to (Y, NS_\sigma)$ defined as f(a)= u f(b) = v and f(c) = w. Then f and f^{-1} are NS(R)WG continuous mappings. Hence f is a NS(R)WG homeomorphism.

Theorem 3.3.
Every NS homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:
Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ be an NS homeomorphism. Then f and f^{-1} are NS continuous mappings. This implies f and f^{-1} are NS(R)WG continuous mappings. Hence f is a NS(R)WG homeomorphism

Example 3.4.
Let X = {a, b, c}, Y = {u, v, w} and
G_1 = <x, $\left(\frac{2}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right)$ >,
G_2 = <y, $\left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{7}{10},\frac{5}{10},\frac{3}{10}\right)$ >.
Then NS_τ ={0_{NS},G_1, 1_{NS}} and NS_σ = {0_{NS},G_2, 1_{NS}} are NSTS on X and Y respectively. Consider a bijective mapping
f: $(X, NS_\tau) \to (Y, NS_\sigma)$ defined as f(a)= u f(b) = v and f(c) = w. Then f is a NS(R)WG homeomorphism but not an NS homeomorphism, since f and f^{-1} are not NS continuous mappings.

Theorem 3.5.
Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ be an NS(R)WG homeomorphism from an NSTS (X, NS_τ) into an NSTS (Y, NS_σ). Then f is a NS homeomorphism if (X, NS_τ) and (Y, NS_σ) are NS rw$T_{\frac{1}{2}}$ spaces.

Proof:
Let B be an NSCS in Y. By hypothesis, f^{-1} (B) is a NS(R)WG CS in X. Since (X, NS_τ) is a NS rw$T_{\frac{1}{2}}$ space, f^{-1} (B) is a NSCS in X. Hence f is a NS continuous mapping. Also by hypothesis, f^{-1}: $(Y, NS_\sigma) \to (X, NS_\tau)$ is a NS(R)WG continuous mapping. Let A be an NSCS in X. Then $(f^{-1})^{-1}$(A) = f (A) is a NS(R)WG CS in Y,by hypothesis. Since (Y, NS_σ) is a NS rw$T_{\frac{1}{2}}$ space, f (A) is a NSCS in Y. Hence f^{-1} is a NS continuous mapping. Thus f is a NS homeomorphism.

Theorem 3.6.
Every NS α homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:
Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ be an NS α homeomorphism. Then f and f^{-1} are NS α continuous mappings. This implies f and f^{-1} are NS(R)WG continuous mappings.Hence f is a NS(R)WG homeomorphism.

Example 3.7.
X = {a, b, c}, Y = {u, v, w} and
G_1 = <x, $\left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right)$ >,
G_2 = <y, $\left(\frac{7}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right)$ >.
Then
NS_τ={0_{NS},G_1, 1_{NS}} and NS_σ={0_{NS},G_2, 1_{NS}} are NSTS on X and Y respectively.
Consider a bijective mapping f : $(X, NS_\tau) \to (Y, NS_\sigma)$ defined as f(a)= u f(b) =v and f(c) = w. Then f is a NS(R)WG homeomorphism but not an NS α homeomorphism, since f and f^{-1} are not NS α continuous mappings.

Theorem 3. 8.
Every NS(G) homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:
Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ be an NS G homeomorphism. Then f and f^{-1} are NS G continuous mappings. This implies f and f^{-1} are NS(R)WG continuous mappings.Hence f is a NS(R)WG homeomorphism.

Example 3.9.
X = {a, b, c}, Y = {u, v, w} and
G_1 = <x, $\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{8}{10}\right)$ >,
G_2 = <y, $\left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right)$ >.
Then NS_τ= {0_{NS},G_1 , 1_{NS}} and NS_σ = {0_{NS},G_2, 1_{NS}} are NSTS on X and Y respectively. Consider a bijective mapping
f: $(X, NS_\tau) \to (Y, NS_\sigma)$ defined as f(a)= u f(b) = v and f(c) = w. Then f is a NS(R)WG homeomorphism but not an NS G homeomorphism, since f and f^{-1} are not NS G continuous mappings.

Theorem 3.10.
Every NS(αG) homeomorphism is a NS(R)WG homeomorphism but not conversely.
Proof:
Let f: (X, NS_τ) →(Y, NS_σ) be an NSαG homeomorphism. Then f and f^{-1} are NSαG continuous mappings. This implies f and f^{-1} are NS(R)WG continuous mappings.
Example 3.11.
X = {a, b, c}, Y = {u, v, w} and
G_1 = <x,$\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{7}{10},\frac{5}{10},\frac{3}{10}\right)$ >,
G_2 = <y, $\left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{4}{10}\right)$ >.
Then NS_τ = {0_{NS},G_1, 1_{NS} } and NS_σ = {0_{NS},G_2 , 1_{NS}} are NSTS on X and Y respectively. Consider a bijective mapping f: (X, NS_τ) →(Y, NS_σ)defined as f(a)= u f(b) = v and f(c) = w.
Then f is a NS(R)WG homeomorphism but not an NSαG homeomorphism, since f and f^{-1} are not NSαG continuous mappings.
Theorem 3 .12.
Let f: (X, NS_τ) →(Y, NS_σ) be a bijective mapping from an NSTS (X, NS_τ) into an NSTS (Y, NS_σ), then the following statements are equivalent.
(a) f is a NS(R)WG OM,
(b) f is a NS(R)WG CM,
(c) f^{-1}: (Y, NS_σ)→ (X, NS_τ)is a NS(R)WG continuous mapping.
Proof:
(a) ⇒ (b): Let A be an NSCS in X, then A^C is a NS OS in X. By hypothesis, f(A^C) = $(f(A))^C$ is a NS(R)WG OS in Y. Therefore f(A) is a NS(R)WG CS in Y. Hence f is a NS(R)WG CM.
(b) ⇒ (c): Let B be an NSCS in X. Since f is a NS(WG)CM, f (A) = $(f^{-1})^{-1}$ (A) is a NS(R)WG CS in Y. Hence f^{-1} is a NS(R)WG continuous mapping.
(c) ⇒ (a): Let A be an NS OS in X. By hypothesis, $(f^{-1})^{-1}$(A) = f(A) is a NS(R)WG OS in Y. Hence f is a NS(R)WG OM.
Corollary 3.13.
Let f: (X, NS_τ) →(Y, NS_σ) be a bijective mapping from an NSTS (X, NS_τ) into an NSTS (Y, NS_σ). If f is a NS(R)WG continuous mapping, then the following statements are equivalent.
(a). f is a NS(R)WG CM,
(b). f is a NS(R)WG OM,
(c). f is a NS(R)WG homeomorphism.
Theorem 3.14.
The composition of two NS(R)WG homeomorphism need not be an NS(R)WG homeomorphism in general.
Proof:
Let X = {a, b} , Y = {c, d} and Z = {u, v} and
G_1 = <x,$\left(\frac{8}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)$ >,
G_2 = <y, $\left(\frac{5}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{4}{10}\right)$ >.
G3 = <z, $\left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right)$ >.
Then NS_τ = {0_{NS},G_1, 1_{NS}}, NS_σ = {0_{NS},G_2, 1_{NS}} and NS_δ = {0_{NS}, G_3, 1_{NS}} are NSTS on X, Y and Z respectively. Consider a bijective mapping f: (X, NS_τ) →(Y, NS_σ)defined as f(a) = c,f(b) = d and g : (Y, NS_σ)→ (Z, NS_δ) by g(c) = u, g(d) = v. Then f and f^{-1} are NS(R)WG continuous mappings. Also g and g^{-1} are NS(R)WG continuous mappings. Hence f and g are NS(R)WG homeomorphism. But the composition gof: X → Z is not an NS(R)WG homeomorphism, since gof is not an NS(R)WG continuous mapping.
Theorem 3 . 15.
Let f: (X, NS_τ) →(Y, NS_σ) and g: (Y, NS_σ)→ (Z, NS_δ) be two NS(R)WG homeomorphisms and (Y, NS_σ)an NS rw$T_{\frac{1}{2}}$space. Then g o f is a NS(R)WG homeomorphism.
Proof:
Let A be an NSCS in Z. Since g: (Y, NS_σ)→ (Z, NS_δ) is a NS(R)WG continuous mapping, g^{-1} (A) is a NS(R)WG CS in Y. Then g^{-1} (A) is a NSCS in Y as (Y, NS_σ)is a NSrw$T_{\frac{1}{2}}$space. Also since f: (X, NS_τ) →(Y, NS_σ) is a NS(R)WG continuous mapping, f^{-1} (g^{-1} (A)) = $(gof)^{-1}$ (A) is a NS(R)WG CS in X. Hence gof is a NS(R)WG continuous mapping. Let A be an NSCS in X. Since f^{-1}: (Y, NS_σ)→ (X, NS_τ) is a NS(R)WG continuous mapping, $(f^{-1})^{-1}$ (A) = f(A) is a NS(R)WG CS in Y. Then f(A) is a NSCS in Y as (Y, NS_σ)is a NS rw$T_{\frac{1}{2}}$space. Also since g^{-1}: (Z, NS_δ) →(Y, NS_σ)is a NS(R)WG continuous mapping,
$(g^{-1})^{-1}$(f (A))=g(f (A))=(gof)(A) is a NS(R)WG CS in Z. Therefore $((gof)^{-1})^{-1}$(A)=(gof)(A) is a NS(R)WG CS in Z. Hence $(gof)^{-1}$ is an NS(R)WG continuous mapping. Thus gof is a NS(R)WG homeomorphism.

4. Neutrosophic regular weakly generalized i* homeomorphism

Definition 4.1.

A bijective mapping f: $(X, NS_\tau) \to (Y, NS_\sigma)$ is called an Neutrosophic regular weakly generalized i* homeomorphism (NS WRGi* homeomorphism in short) if f and $f^{-1}$are NS(R)WG irresolute mappings.

Theorem 4 . 2.

Every NS(R)WG i* homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:

Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ be an NS(R)WG i* homeomorphism. Let B be an NSCS in Y. This implies B is a NS(R)WG CS in Y. By hypothesis, f^{-1}(B) is a NS(R)WG CS in X. Hence f is a NS(R)WG continuous mapping. Similarly we can prove f^{-1} is a NS(R)WG continuous mapping. Hence f and f^{-1} are NS(R)WG continuous mapping. Thus f is a NS(R)WG homeomorphism.

Example 4.3.

X = {a, b, c}, Y = {u, v, w} and

$G_1 = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$,

$G_2 = \langle y, \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right) \rangle$.

Then $NS_\tau = \{0_{NS}, G_1, 1_{NS}\}$ and $NS_\sigma = \{0_{NS}, G_2, 1_{NS}\}$ are NSTS on X and Y respectively. Consider a bijective mapping

f: $(X, NS_\tau) \to (Y, NS_\sigma)$ defined as f(a)= u f(b) = v and f(c) = w.

Then f is a NS(WG) homeomorphism.

Let $A = \langle y, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$.

be an NSS in Y. Clearly A is a NS(R)WG CS in Y. But f^{-1} (A) is not an NS(R)WG CS in X.This implies f is not an NS(WG) irresolute mapping. Hence f is not an NS(R)WG i* homeomorphism.

Theorem 4.4.

Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ be a bijective mapping from an NSTS (X, NS_τ) into an NSTS (Y, NS_σ), then the following statements are equivalent.

(a). f is a NS(R)WG i* homeomorphism,

(b). f is a NS(R)WG irresolute and NS(R)WG i*OM,

(c). f is a NS(R)WG irresolute and NS(R)WG i*CM.

Proof:

(a) ⇒ (b): Let f be an NS(R)WG i* homeomorphism. Then f and f^{-1} are NS(R)WG irresolute mappings. To prove that f is a NS(R)WG i*OM, let A be an NS(R)WG OS in X. Since

f^{-1}: $(Y, NS_\sigma) \to (X, NS_\tau)$is a NS(R)WG irresolute mapping, $(f^{-1})^{-1}$ (A) = f (A) is a NS(R)WG OS in Y. Hence f is a NS(R)WG i*OM.

(b) ⇒ (a): Let f be an NS(R)WG irresolute and NS(R)WG i*OM.

To prove that f^{-1}: $(Y, NS_\sigma) \to (X, NS_\tau)$ is a NSRFWG irresolute mapping, let A be an NS(R)WG OS in X. Since f is aS(R)WG i*OM, f(A) is a NS(R)WG OS in Y. Now $(f^{-1})^{-1}$ (A) = f(A) is a NS(R)WG OS in Y. Therefore f^{-1}: $(Y, NS_\sigma) \to (X, NS_\tau)$ is a NS(R)WG irresolute mapping.

Hence f is a NS(R)WG i* homeomorphism.

(b) ⇒ (c): Let f be an NS(R)WG irresolute and NS(R)WG i*OM. To prove that f is a NS(R)WG *C, let B be an NS(R)WG CS in X. Then B^C is a NS(R)WG OS in X. Since f is a NS(R)WG *OM, $f(B^C) = (f(B))^C$ is a NS(R)WG OS in Y. Therefore f(B) is a NS(R)WG CS in Y. Hence f is a NS(R)WG i*CM.

(c) ⇒ (b):

Let f be an NS(R)WG irresolute and NS(R)WG i*CM. To prove that f is a NS(R)WG i*OM, let A be an NS(R)WG OS in X. Then A^C is a NS(R)WG CS in X. Since f is a NS(R)WG i*CM, $f(A^C) = (f(A))^C$ is a NS(R)WG CS in Y. Therefore f(A) is a NS(R)WG OS in Y. Hence f is a NS(R)WG i*OM.

Theorem 4.6.

The composition of two NS(R)WG i* homeomorphism is a NS(R)WG i*homeomorphism in general.

Proof:

Let f: $(X, NS_\tau) \to (Y, NS_\sigma)$ and g: $(Y, NS_\sigma) \to (Z, NS_\delta)$ be any two NS(R)WG i*homeomorphisms. Let A be an NS(R)WG CS in Z. Since g: $(Y, NS_\sigma) \to (Z, NS_\delta)$ is a NS(R)WG irresolute mapping, g^{-1}(A) is a NS(WG)CS in Y. Also since f: $(X, NS_\tau) \to (Y, NS_\sigma)$ is a NS(R)WG irresolute mapping, f^{-1} (g^{-1} (A)) = $(gof)^{-1}$(A) is a NS(R)WG CS in X. Hence gof is a NS(R)WG irresolute mapping. Again, let A be an NS(WG)CS in X.Since f^{-1}: $(Y, NS_\sigma) \to (X, NS_\tau)$ is a NS(WG) irresolute mapping, $(f^{-1})^{-1}$(A) = f(A) is a NS(R)WG CS in Y. Also since g^{-1}: $(Z, NS_\delta) \to (Y, NS_\sigma)$ is a NS(R)WG irresolute mapping,$(g^{-1})^{-1}$(f (A)) = g(f(A)) = (gof)(A) is NS(R)WG CS in Z. Therefore $((gof)^{-1})^{-1}$ (A) = (gof)(A) is a NS(R)WG CS in Z.Hence $(gof)^{-1}$ is a NS(R)WG irresolute mapping. Thus gof is a NS(R)WG i*homeomorphism.

Remark 4.7.

The family of all NS(R)WG i* homeomorphism from (X, NS_τ) onto itself is denoted by NS(R)WG i* (X, NS_τ).

Theorem 4.8:

The set NS(R)WG i* (X, NS_τ) forms a group under composition of mappings.

Proof:

(i). Operation is closed.

(ii). The composition of two NS(R)WG i* homeomorphism is a NS(R)WG i*homeomorphism in general. Hence associative axiom is satisfied.

(iii). Since the identity is NS(R)WG i* homeomorphism, it is a identity element of NS(R)WG i* (X, NS_τ).
(iv). As the element of NS(R)WG i* (X, NS_τ) are bijection f^{-1} exist in NS(R)WG i* (X, NS_τ).Hence NS(R)WG * (X, NS_τ) forms a group under composition of mappings.

Theorem 4.9.
If f: X → Y is NS(R)WG i* then it is induces an isomorphism f* from the group NS(R)WG i* (X, NS_τ) onto NS(R)WG i*(Y, NS_σ)given by f*(h)=f.h.f^{-1} for every h∈NS(R)WG i* (X, NS_τ).

Proof: By usual arguments the proof follows.

Theorem 4.10. Let X and Y be NSTS and let f be a bijective mapping from X onto Y. Then f is NS(rwg) open and NS(rwg) continuous if and only if f is NS(rwg) homeomorphism.

Proof: Let f be NS(rwg) open and NS(rwg) continuous. Let A be an open set in X.Then f(A) is NS(rwg) open in Y. i.e $(f^{-1})^{-1}(A)$ = f(A) is NS(rwg) open in Y. Hence f^{-1} is NS(rwg) continuous.Conversely, assume that f be a NS(rwg) homeomorphisms and f^{-1} = f. Since f is bijective, g is also bijective. If A is a open set g^{-1} (A) is a NS(rwg) open set for g is NS(rwg) continuous. That is f(A) is NS(rwg) open. Hence f is NS(rwg) open.

Theorem 4.11.
Let X and Y be NSTS and let f be a bijective mapping from X onto Y. Then f is NS(rwg) homeomorphism if and only if f is NS(rwg) closed and NS(rwg) continuous.

Proof:
Assume that f is NS(rwg) homeomorphisms, let A be a closed set in X. then X-A is open and since f = g^{-1} is NS(rwg) continuous, g^{-1}(X-A) is NS(rwg) open.That is gg^{-1} (X-A) = Y- g^{-1} (F) is NS(rwg) open. Thus g^{-1} (F) is NS(rwg) closed, that is f(F) is NS(rwg) closed. Hence f is NS(rwg) closed map.
Conversely assume that f is NS(rwg) closed and NS(rwg) continuous. Let B be an open set. Then X-B is closed. Since f is closed f(X-B) is NS(rwg) closed.That is g^{-1}(X-B) = Y - g^{-1} is NS(rwg) closed. That is g^{-1}(X-B) = Y - g^{-1} is NS(rwg) closed, implies g^{-1}(G) is NS(rwg) open. Thus inverse image under g of every open set is NS(rwg) open. That is g = f-1 is NS(rwg) continuous. Thus f is NS(rwg) homeomorphisms.

CONCLUSIONS

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, : In this paper, we introduced the concept of NS(R)WG homeomorphisms in Neutrosophic Topological Spaces.. This shall be extended in the future Research with some applications

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