Neutrosophic Regular Weakly Generalized Homoeomorphism

R. Suresh*1, S. Palaniammal*2

1Department of Science and Humanities, Sri Krishna College of Engineering and Technology, Coimbatore, Tamil Nadu, India.
2 Principal, Sri Krishna Adithya College of Arts and Science, Coimbatore, Tamil Nadu, India.

Abstract— Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic mapping is called Neutrosophic Regular Weakly Generalized Homoeomorphism in Neutrosophic topological spaces and also discussed about properties and characterization of Neutrosophic Regular Weakly Generalized Homoeomorphism.

Keywords— (NS(R)WG open set, (NS(R)WG closed set, (NS(R)WG homeomorphism, Neutrosophic topological spaces

I. INTRODUCTION

A.A. Salama introduced Neutrosophic topological spaces by using Smarandache’s Neutrosophic sets. I.Arokiarani.[2] et al. introduced Neutrosophic α-closed sets. P. Ishwarya, [8] et al. introduced and studied about Neutrosophic semi-open sets in Neutrosophic topological spaces. Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic mapping is called Neutrosophic Regular Weakly Generalized Homoeomorphism in Neutrosophic topological spaces and also discussed about properties and characterization of Neutrosophic Regular Weakly Generalized Homoeomorphism.

II. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Definition 2.1 [7]

Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form

\[ A = \{(x, \eta_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \]

Where \( \eta_A(x), \sigma_A(x) \) and \( \gamma_A(x) \) which represent Neutrosophic topological spaces the degree of membership function, the degree of indeterminacy and degree of non-membership function respectively of each element \( x \in X \) to the set \( A \).

Remark 2.2 [7]

A Neutrosophic set \( A = \{(x, \eta_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \) can be identified to an ordered triple \( <\eta_A, \sigma_A, \gamma_A> \) in \( ]0,1[ \) on \( X \).

Remark 2.3 [7]

We shall use the symbol \( A = <\eta_A, \sigma_A, \gamma_A> \) for the Neutrosophic set \( A = \{(x, \eta_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \).

Example 2.4 [7]

Every Neutrosophic set \( A \) is a non-empty set in \( X \) is obviously on Neutrosophic set having the form \( A = \{(x, \eta_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \). Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topological mapping, we construct Neutrosophic set \( 0_N \) and \( 1_N \) in \( X \) as follows:

\[ 0_N = \{(x, 0, 0, 0) : x \in X\} \]
\[ 1_N = \{(x, 1, 1, 1) : x \in X\} \]

Definition 2.5 [7]

Let \( A = <\eta_A, \sigma_A, \gamma_A> \) be a Neutrosophic set on \( X \), then the complement of the set \( A \) \( A^c \) defined as

\[ A^c = \{(x, \gamma_A(x), \sigma_A(x), \eta_A(x)) : x \in X\} \]

Definition 2.6 [7]

Let \( X \) be a non-empty set, and Neutrosophic sets \( A \) and \( B \) in the form

\[ A = \{(x, \eta_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \]
\[ B = \{(x, \eta_B(x), \sigma_B(x), \gamma_B(x)) : x \in X\} \]

Then we consider definition for subsets \( A \subseteq \mathbb{B} \).

\[ A \subseteq \mathbb{B} \iff \eta_A(x) \leq \eta_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x) \text{ for all } x \in X \]
Definition 2.8 [7]
Let X be a non-empty set, and $A = \{x, \eta_B(x), \sigma_A(x), \gamma_A(x) : x \in X\}$, $B = \{x, \eta_B(x), \sigma_B(x), \gamma_B(x) : x \in X\}$ be two Neutrosophic sets. Then

(i) $A \cap B$ defined as $A \cap B = \{x, \eta_B(x)/\eta_B(x), \sigma_A(x)/\sigma_B(x), \gamma_A(x)/\gamma_B(x) : x \in X\}$

(ii) $A \cup B$ defined as $A \cup B = \{x, \eta_B(x)/\eta_B(x), \sigma_A(x)/\sigma_B(x), \gamma_A(x)/\gamma_B(x) : x \in X\}$

Proposition 2.9 [7]
For all A and B are two Neutrosophic sets then the following condition are true:

(i) $(A \cap B)^{\bar{c}} = A^{\bar{c}} \cup B^{\bar{c}}$

(ii) $(A \cup B)^{\bar{c}} = A^{\bar{c}} \cap B^{\bar{c}}$

Definition 2.10 [11]
A Neutrosophic topology is a non-empty set X is a family $\tau_X$ of Neutrosophic subsets in X satisfying the following axioms:

(i) $0_N \in \tau_X$, $1_N \in \tau_X$

(ii) $G_1 \cap G_2 \in \tau_X$ for any $G_1, G_2 \in \tau_X$

(iii) $\bigcup G \in \tau_X$ for any family $\{G_i \in \tau_X \}$.

The pair $(X, \tau_X)$ is called a Neutrosophic topological space.

The element Neutrosophic topological spaces of $\tau_X$ are called Neutrosophic open sets.

A Neutrosophic set $A$ is closed if and only if $A^c$ is Neutrosophic open.

Example 2.11 [11]
Let $X = \{x\}$ and $A_1 = \{x, 0.6, 0.6, 0.5 : x \in X\}$
$A_2 = \{x, 0.5, 0.7, 0.9 : x \in X\}$
$A_3 = \{x, 0.6, 0.7, 0.5 : x \in X\}$
$A_4 = \{x, 0.5, 0.6, 0.9 : x \in X\}$

Then the family $\tau_X = \{0_N, 1_N, A_1, A_2, A_3, A_4\}$ is called a Neutrosophic topological space on X.

Definition 2.12 [11]
Let $(X, \tau_X)$ be Neutrosophic topological spaces and $A = \{x, \eta_A(x), \sigma_A(x), \gamma_A(x) : x \in X\}$ be a Neutrosophic set in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by

$\text{Neu-c}(A) = \cap \{K : K \text{ is a Neutrosophic closed set in } X \text{ and } A \subseteq K\}$

$\text{Neu-int}(A) = \cup \{G : G \text{ is a Neutrosophic open set in } X \text{ and } G \subseteq A\}$

Definition 2.13
Let $(X, \tau_X)$ be a Neutrosophic topological space. Then A is called

(i) Neutrosophic regular Closed set [2] (Neu-RCS in short) if $A = \text{Neu-c}(\text{Neu-int}(A))$

(ii) Neutrosophic $\alpha$-Closed set[2] (Neu-$\alpha$CS in short) if $A = \text{Neu-c}(\text{Neu-int}(\text{Neu-c}(A))) \subseteq A$

(iii) Neutrosophic semi Closed set [8] (Neu-SCS in short) if $A = \text{Neu-c}(\text{Neu-cl}(\text{Neu-int}(A))) \subseteq A$

(iv) Neutrosophic pre Closed set [12] (Neu-PCS in short) if $A = \text{Neu-c}(\text{Neu-int}(\text{Neu-int}(A))) \subseteq A$

Definition 2.14
Let $(X, \tau_X)$ be a Neutrosophic topological space. Then A is called

(i). Neutrosophic regular open set [2](Neu-ROS in short) if $A = \text{Neu-int}(\text{Neu-c}(A))$

(ii). Neutrosophic $\alpha$-open set [2](Neu-$\alpha$OS in short) if $A = \text{Neu-int}(\text{Neu-cl}(\text{Neu-int}(A)))$

(iii). Neutrosophic semi open set [8](Neu-SOS in short) if $A = \text{Neu-cl}(\text{Neu-int}(\text{Neu-int}(A)))$

(iv). Neutrosophic pre open set [13] (Neu-POS in short) if $A = \text{Neu-int}(\text{Neu-cl}(\text{Neu-int}(A)))$

Definition 2.15
Let $(X, \tau_X)$ be a Neutrosophic topological space. Then A is called

(i).Neutrosophic generalized closed set[4](Neu-GCS in short) if $\text{Neu-cl}(\text{Neu-int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is a Neu-OS in X ,

(ii).Neutrosophic generalized semi closed set[12] (Neu-GSCS in short) if $\text{Neu-cl}(\text{Neu-int}(\text{Neu-cl}(\text{Neu-int}(A)))) \subseteq U$ whenever $A \subseteq U$ and $U$ is a Neu-OS in X ,

(iii).Neutrosophic generalized semi closed set [9](Neu-$\alpha$GCS in short) if $\text{Neu-cl}(\text{Neu-int}(\text{Neu-int}(\text{Neu-int}(A)))) \subseteq U$ whenever $A \subseteq U$ and $U$ is a Neu-OS in X ,

(iv).Neutrosophic generalized alpha closed set [5] (Neu-$\alpha$CS in short) if $\text{Neu-cl}(\text{Neu-int}(\text{Neu-int}(\text{Neu-int}(A)))) \subseteq U$ whenever $A \subseteq U$ and $U$ is a Neu-$\alpha$OS in X .

The complements of the above mentioned Neutrosophic closed sets are called their respective Neutrosophic open sets.

Definition 2.16. [7] An IFS A is said to be an Neutrosophic regular weakly generalized closed set (NS(RWG)CS in short) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is an NSROS in X .

Definition 2.17. [9] A mapping $f : (X, NS_X) \rightarrow (Y, NS_Y)$ is called an Neutrosophic regular weakly generalized continuous (NSRWWGC in short) if $f^{-1}(B)$ is an NSRWWGC in $(X, NS_X)$ for every NSCS B of $(Y, NS_Y)$

Definition 2.18. [8] A mapping $f : (X, NS_X) \rightarrow (Y, NS_Y)$ is called an Neutrosophic regular weakly generalized irresolute (NSRWWIR in short) if $f^{-1}(B)$ is an NSRWWGC in $f^{-1}(B)$ for every NSRWWGC B of $(Y, NS_Y)$.

3. Neutrosophic regular weakly generalized homeomorphism

Definition 3.1
A bijective mapping $f : (X, NS_X) \rightarrow (Y, NS_Y)$ is called an Neutrosophic regular weakly generalized homeomorphism (NS(R)WG homeomorphism in short) if $f$ and $f^{-1}$ are NS(R)WG continuous mappings.
Example 3.2.
Let \( X = \{a, b, c\}, Y = \{u, v, w\} \) and
\[
G_1 = \langle \begin{pmatrix} \frac{2}{10} & \frac{5}{10} & \frac{5}{10} \\ \frac{3}{10} & \frac{5}{10} & \frac{6}{10} \\ \frac{3}{10} & \frac{5}{10} & \frac{7}{10} \end{pmatrix} \rangle >,
\]
\[
G_2 = \langle \begin{pmatrix} \frac{9}{10} & \frac{5}{10} & \frac{1}{10} \\ \frac{8}{10} & \frac{5}{10} & \frac{1}{10} \\ \frac{4}{10} & \frac{5}{10} & \frac{1}{10} \end{pmatrix} \rangle >.
\]
Then \( N_S_1 = \{0_{NS}, G_1, 1_{NS}\} \) and \( N_S_\sigma = \{0_{NS}, G_2, 1_{NS}\} \) are NSTS on X and Y respectively. Consider a bijective mapping \( f: (X, N_S) \rightarrow (Y, N_S) \) defined as \( f(a) = u, f(b) = v \) and \( f(c) = w \). Then \( f \) and \( f^{-1} \) are NS(R)WG continuous mappings. Hence \( f \) is a NS(R)WG homeomorphism.

Theorem 3.3.
Every NS homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:
Let \( f: (X, N_S) \rightarrow (Y, N_S) \) be an NS homeomorphism. Then \( f \) and \( f^{-1} \) are NS continuous mappings. This implies \( f \) and \( f^{-1} \) are NS(R)WG continuous mappings. Hence \( f \) is a NS(R)WG homeomorphism.

Example 3.4.
Let \( X = \{a, b, c\}, Y = \{u, v, w\} \) and
\[
G_1 = \langle \begin{pmatrix} \frac{2}{10} & \frac{5}{10} & \frac{7}{10} \\ \frac{3}{10} & \frac{5}{10} & \frac{7}{10} \\ \frac{4}{10} & \frac{5}{10} & \frac{6}{10} \end{pmatrix} \rangle >,
\]
\[
G_2 = \langle \begin{pmatrix} \frac{9}{10} & \frac{5}{10} & \frac{1}{10} \\ \frac{6}{10} & \frac{5}{10} & \frac{3}{10} \\ \frac{2}{10} & \frac{5}{10} & \frac{3}{10} \end{pmatrix} \rangle >.
\]
Then \( N_S_1 = \{0_{NS}, G_1, 1_{NS}\} \) and \( N_S_\sigma = \{0_{NS}, G_2, 1_{NS}\} \) are NSTS on X and Y respectively. Consider a bijective mapping \( f: (X, N_S) \rightarrow (Y, N_S) \) defined as \( f(a) = u, f(b) = v \) and \( f(c) = w \). Then \( f \) is a NS(R)WG homeomorphism but not an NS homeomorphism, since \( f \) and \( f^{-1} \) are not NS continuous mappings.

Theorem 3.5.
Let \( f: (X, N_S) \rightarrow (Y, N_S) \) be an NS(R)WG homeomorphism from an NSTS \( (X, N_S) \) into an NSTS \( (Y, N_S) \). Then \( f \) is a NS homeomorphism if \( (X, N_S) \) and \( (Y, N_S) \) are \( NRWTS \) spaces.

Proof:
Let \( B \) be an NSCS in Y. By hypothesis, \( f^{-1} (B) \) is a NS(R)WG CS in X. Since \( (X, N_S) \) is a NS \( RWTS \) space, \( f^{-1} (B) \) is a NSCS in X. Hence \( f \) is a NS continuous mapping. Also by hypothesis, \( f^{-1}: (Y, N_S) \rightarrow (X, N_S) \) is a NS(R)WG continuous mapping. Let \( A \) be an NSCS in X. Then \( (f^{-1})^{-1}(A) = f(A) \) is a NS(R)WG CS in Y, by hypothesis. Since \( (Y, N_S) \) is a NS \( RWTS \) space, \( f(A) \) is a NSCS in Y. Hence \( f^{-1} \) is a NS continuous mapping. Thus \( f \) is a NS homeomorphism.

Theorem 3.6.
Every NS \( \alpha \) homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:
Let \( f: (X, N_S) \rightarrow (Y, N_S) \) be an NS \( \alpha \) homeomorphism. Then \( f \) and \( f^{-1} \) are NS \( \alpha \) continuous mappings. This implies \( f \) and \( f^{-1} \) are NS(R)WG continuous mappings. Hence \( f \) is a NS(R)WG homeomorphism.

Example 3.7.
\( X = \{a, b, c\}, Y = \{u, v, w\} \) and
\[
G_1 = \langle \begin{pmatrix} \frac{4}{10} & \frac{5}{10} & \frac{5}{10} \\ \frac{4}{10} & \frac{5}{10} & \frac{6}{10} \\ \frac{5}{10} & \frac{5}{10} & \frac{5}{10} \end{pmatrix} \rangle >,
\]
\[
G_2 = \langle \begin{pmatrix} \frac{7}{10} & \frac{5}{10} & \frac{3}{10} \\ \frac{8}{10} & \frac{5}{10} & \frac{2}{10} \\ \frac{6}{10} & \frac{5}{10} & \frac{2}{10} \end{pmatrix} \rangle >.
\]
Then \( N_S = \{0_{NS}, G_1, 1_{NS}\} \) and \( N_S_\sigma = \{0_{NS}, G_2, 1_{NS}\} \) are NSTS on X and Y respectively. Consider a bijective mapping \( f: (X, N_S) \rightarrow (Y, N_S) \) defined as \( f(a) = u, f(b) = v \) and \( f(c) = w \). Then \( f \) is a NS(R)WG homeomorphism but not an NS \( \alpha \) homeomorphism, since \( f \) and \( f^{-1} \) are not NS \( \alpha \) continuous mappings.

Theorem 3.8.
Every NS(G) homeomorphism is a NS(R)WG homeomorphism but not conversely.

Proof:
Let \( f: (X, N_S) \rightarrow (Y, N_S) \) be an NS(G) homeomorphism. Then \( f \) and \( f^{-1} \) are NS(G) continuous mappings. This implies \( f \) and \( f^{-1} \) are NS(R)WG continuous mappings. Hence \( f \) is a NS(R)WG homeomorphism.

Example 3.9.
\( X = \{a, b, c\}, Y = \{u, v, w\} \) and
\[
G_1 = \langle \begin{pmatrix} \frac{2}{10} & \frac{5}{10} & \frac{8}{10} \\ \frac{2}{10} & \frac{5}{10} & \frac{7}{10} \\ \frac{1}{10} & \frac{5}{10} & \frac{8}{10} \end{pmatrix} \rangle >,
\]
\[
G_2 = \langle \begin{pmatrix} \frac{9}{10} & \frac{5}{10} & \frac{1}{10} \\ \frac{8}{10} & \frac{5}{10} & \frac{1}{10} \\ \frac{9}{10} & \frac{5}{10} & \frac{1}{10} \end{pmatrix} \rangle >.
\]
Then \( N_S = \{0_{NS}, G_1, 1_{NS}\} \) and \( N_S_\sigma = \{0_{NS}, G_2, 1_{NS}\} \) are NSTS on X and Y respectively. Consider a bijective mapping \( f: (X, N_S) \rightarrow (Y, N_S) \) defined as \( f(a) = u, f(b) = v \) and \( f(c) = w \). Then \( f \) is a NS(R)WG homeomorphism but not an NS(G) homeomorphism, since \( f \) and \( f^{-1} \) are not NS(G) continuous mappings.
Theorem 3.10.
Every NS(RwG) homeomorphism is an NS(RwG) homeomorphism but not conversely.

Proof:
Let \( f : (X, NS_r) \rightarrow (Y, NS_s) \) be an NS\( R \)G homeomorphism. Then \( f \) and \( f^{-1} \) are NS\( R \)G continuous mappings. This implies \( f \) and \( f^{-1} \) are NS\( R \)WG continuous mappings.

Example 3.11.
\( X = \{a, b, c\}, Y = \{u, v, w\} \) and
\[
\begin{align*}
G_1 &= <a \left( \begin{array}{cc} 5 & 3 \\ 10 & 10 \\ 10 \\ 10 \\
6 & 4 \\ 10 \\ 10 \\
5 & 3 \\ 10 \\ 10 \
\end{array} \right) >, \\
G_2 &= <a \left( \begin{array}{cc} 5 & 3 \\ 10 & 10 \\ 10 \\ 10 \\
6 & 4 \\ 10 \\ 10 \\
5 & 3 \\ 10 \\ 10 
\end{array} \right) >.
\end{align*}
\]
Then \( NS_r = \{0_{NS_r}, G_1, I_{NS_r}\} \) and \( NS_s = \{0_{NS_s}, G_2, I_{NS_s}\} \) are NSTS on \( X \) and \( Y \) respectively. Consider a bijective mapping \( f : (X, NS_r) \rightarrow (Y, NS_s) \) as defined as \( f(a) = u, f(b) = v \) and \( f(c) = w \).

Theorem 3.12.
Let \( f : (X, NS_r) \rightarrow (Y, NS_s) \) be a NS\( R \)G homeomorphism but not an NS\( R \)G homeomorphism, since \( f \) and \( f^{-1} \) are not NS\( R \)G continuous mappings.

Corollary 3.13.
Let \( f : (X, NS_r) \rightarrow (Y, NS_s) \) be a NS\( R \)G continuous mapping, then the following statements are equivalent.

(a) \( f \) is a NS\( R \)G OM, 
(b) \( f \) is a NS\( R \)WGM, 
(c) \( f^{-1} : (Y, NS_s) \rightarrow (X, NS_r) \) is a NS\( R \)G continuous mapping.

Proof:
(a) \( \Rightarrow \) (b): Let \( A \) be an NSCS in \( X \), then \( A^c \) is a NS OS in \( X \). By hypothesis, \( f(A^c) = (f(A))^c \) is a NS\( R \)G OS in \( Y \). Therefore \( f(A) \) is a NS\( R \)WG CS in \( Y \). Hence \( f \) is a NS\( R \)G CM.

The composition of two NS\( R \)WG homeomorphism need not be an NS\( R \)WG homeomorphism in general.

Theorem 3.14.
Let \( X = \{a, b\}, Y = \{c, d\} \) and \( Z = \{u, v\} \) and
\[
\begin{align*}
G_1 &= <a \left( \begin{array}{cc} 2 & 2 \\ 10 & 10 \\
6 & 4 \\ 10 \\
5 & 3 \\ 10 \\
\end{array} \right) >, \\
G_2 &= <a \left( \begin{array}{cc} 2 & 2 \\ 10 & 10 \\
6 & 4 \\ 10 \\
5 & 3 \\ 10 
\end{array} \right) >. \\
G_3 &= <a \left( \begin{array}{cc} 2 & 2 \\ 10 & 10 \\
6 & 4 \\ 10 \\
5 & 3 \\ 10 
\end{array} \right) >.
\end{align*}
\]
Then \( NS_r = \{0_{NS_r}, G_1, I_{NS_r}\}, NS_s = \{0_{NS_s}, G_2, I_{NS_s}\} \) and \( NS_d = \{0_{NS_d}, G_3, I_{NS_d}\} \) are NSTS on \( X, Y \) and \( Z \) respectively. Consider a bijective mapping \( f : (X, NS_r) \rightarrow (Y, NS_s) \) as defined as \( f(a) = c, f(b) = d \) and \( g : (Y, NS_s) \rightarrow (Z, NS_d) \) by \( g(c) = u, g(d) = v \). Then \( f \) and \( f^{-1} \) are NS\( R \)WG continuous mappings. Also \( g \) and \( g^{-1} \) are NS\( R \)WG continuous mappings. Hence \( f \) and \( g \) are NS\( R \)WG homeomorphism. But the composition \( g \circ f \) is not an NS\( R \)WG homeomorphism, since \( g \circ f \) is not an NS\( R \)WG continuous mapping.

Theorem 3.15.
Let \( f : (X, NS_r) \rightarrow (Y, NS_s) \) and \( g : (Y, NS_s) \rightarrow (Z, NS_d) \) be two NS\( R \)WG homeomorphisms and \( (Y, NS_s) \) an NS \( rwT \) space.

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4. Neutrosophic regular weakly generalized $i^*$ homeomorphism

Definition 4.1.

A bijective mapping $f: (X, NS_a) \rightarrow (Y, NS_{a'})$ is called an Neutrosophic regular weakly generalized $i^*$ homeomorphism (NS WRGi* homeomorphism in short) if $f$ and $f^{-1}$ are $NS(R)WG$ irresolute mappings.

Theorem 4.2.

Every $NS(R)WG$ $i^*$ homeomorphism is a $NS(R)WG$ homeomorphism but not conversely.

Proof:

Let $f: (X, NS_a) \rightarrow (Y, NS_{a'})$ be an $NS(R)WG$ $i^*$ homeomorphism. Let $B$ be an NSCS in $Y$. This implies $B$ is a $NS(R)WG$ CS in $Y$.

By hypothesis, $f^{-1}(B)$ is a $NS(R)WG$ CS in $X$. Hence $f$ is a $NS(R)WG$ continuous mapping. Similarly we can prove $f^{-1}$ is a $NS(R)WG$ continuous mapping. Hence $f$ and $f^{-1}$ are $NS(R)WG$ continuous mapping. Thus $f$ is a $NS(R)WG$ homeomorphism.

Example 4.3.

Let $A = \{a, b, c\}$, $Y = \{u, v, w\}$ and $G_1 = \langle \frac{3}{5}, \frac{5}{10}, \frac{5}{10}, \frac{5}{10}, \frac{5}{10}, \frac{5}{10} \rangle$.

$G_2 = \langle \frac{2}{10}, \frac{5}{10}, \frac{5}{10}, \frac{5}{10}, \frac{5}{10}, \frac{5}{10} \rangle$.

Then $NS_{a'} = \{0_{NS}, G_1, 1_{NS}\}$. Consider a bijective mapping $f: (X, NS_a) \rightarrow (Y, NS_{a'})$ defined as $f(a) = u$, $f(b) = v$ and $f(c) = w$.

Then $f$ is a $NS(WG)$ homeomorphism.
(iii). Since the identity is NS(R)WG i* homeomorphism, it is a identity element of NS(R)WG i* (X, NS₂).
(iv). As the element of NS(R)WG i* (X, NS₂) are bijection f⁻¹ exist in NS(R)WG i* (X, NS₂). Hence NS(R)WG * (X, NS₂) forms a group under composition of mappings.

**Theorem 4.9.**
If f: X → Y is NS(R)WG i* then it is induces an isomorphism f* from the group NS(R)WG i* (X, NS₂) onto NS(R)WG i*(Y, NS₂) given by f*(h) = f.h.f⁻¹ for every h ∈ NS(R)WG i* (X, NS₂).

**Proof:** By usual arguments the proof follows.

**Theorem 4.10.** Let X and Y be NSTS and let f be a bijective mapping from X onto Y. Then f is NS(rwg) open and NS(rwg) continuous if and only if f is NS(rwg) homeomorphism.

**Proof:** Let f be NS(rwg) open and NS(rwg) continuous. Let A be an open set in X. Then f(A) is NS(rwg) open in Y. i.e (f⁻¹)⁻¹(A) = f(A) is NS(rwg) open in Y. Hence f⁻¹ is NS(rwg) continuous. Conversely, assume that f be a NS(rwg) homeomorphisms and f⁻¹ = f. Since f is bijective, g is also bijective. If A is an open set g⁻¹(A) is a NS(rwg) open set for g is NS(rwg) continuous. That is f(A) is NS(rwg) open. Hence f is NS(rwg) open.

**Theorem 4.11.** Let X and Y be NSTS and let f be a bijective mapping from X onto Y. Then f is NS(rwg) homeomorphism if and only if f is NS(rwg) closed and NS(rwg) continuous.

**Proof:**
Assume that f is NS(rwg) homeomorphisms, let A be a closed set in X. Then X-A is open and since f = g⁻¹ is NS(rwg) continuous, g⁻¹(X-A) is NS(rwg) open. That is g⁻¹ g⁻¹(X-A) = Y - g⁻¹(F) is NS(rwg) open. Thus g⁻¹ (F) is NS(rwg) closed, that is f(F) is NS(rwg) closed. Hence f is NS(rwg) closed map. Conversely assume that f is NS(rwg) closed and NS(rwg) continuous. Let B be an open set. Then X-B is closed. Since f is closed f(X-B) = NS(rwg) closed. That is g⁻¹ (X-B) = Y - g⁻¹(F) is NS(rwg) closed. That is g⁻¹(X-B) = Y - g⁻¹ is NS(rwg) closed, implies g⁻¹ (G) is NS(rwg) open. Thus inverse image under g of every open set is NS(rwg) open. That is g = f⁻¹ is NS(rwg) continuous. Thus f is NS(rwg) homeomorphisms.

**CONCLUSIONS**
Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In this paper, we introduced the concept of NS(R)WG homeomorphisms in Neutrosophic Topological Spaces.. This shall be extended in the future Research with some applications

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