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Neutrosophic Soft $\alpha-$Open Set in Neutrosophic Soft Topological Spaces

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ABSTRACT

In this paper, the notion of generalized neutrosophic soft open set (GNSOS) in neutrosophic soft open set (GN-SOS) in neutrosophic soft topological structures relative to neutrosophic soft points is introduced.The concept of generalized neutrosophic soft separation axioms in neutrosophic soft topological spaces with respect to

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1 Abstract continued

soft points. Several related properties, structural characteristics have been investigated. Then the convergence of sequence in neutrosophic soft topological space is defined and its uniqueness in generalized neutrosophic soft Hausdorff space (GNSHS) relative to soft points is examined. Neutrosophic monotonous soft function and its characteristics are switched over to different results. Lastly, generalized neutrosophic soft product spaces with respect to crisp points have been addressed.

2 Introduction

Cagman et al.[5] defined the concept of soft topology on a soft set, and presented its related properties. The authors also discussed the foundations of the theory of soft topological spaces. Shabir and Naz [18] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are introduced and their basic properties are investigated. It is shown that a soft topological space gives a parametrized family of topological spaces. Furthermore, with the help of an example it is established that the converse does not hold. The soft subspaces of a soft topological space are defined and inherent concepts as well as the characterization of soft open and soft closed sets in soft subspaces are investigated. Finally, soft T_i spaces and notions of soft normal and soft regular spaces are discussed in detail. A sufficient condition for a soft topological space to be a soft T_1 spaces is also presented.

Bayramov and Gunduz [1] investigated some basic notions of soft topological spaces by using new soft point concept. Later the authors addressed T_i soft space and the relationships between them are discussed in detail. Finally, the authors define soft compactness and explore some of its important properties. Khattak et al [7] introduced the concept of most generalized soft open $(\alpha,\ \beta)$ sets in soft single point topology.The authors generated soft separation axiom in soft single point topology with the application of generalized soft open $(\alpha,\ \beta)$ sets with respect to crisp point and soft points.The authors further discussed strong soft separation axioms in soft single points topology with respect to crisp points and soft points.At the end some hereditary characteristics with respect to crisp points and soft points of different soft structures are addressed. Zadeh [20] introduced the concept of fuzzy set. The author described that a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint. Atanassov [19] introduced the concept of intuitionistic fuzzy set (IFS) which is generalization of the concept fuzzy set. Various properties are proved, which are connected to the operations and relations over sets, and with modal and topological operators, defined over the set of IFS's. Bayramov and Gunduz [2] introduced some important properties of intuitionistic fuzzy soft topological spaces and define the intuitionistic fuzzy soft closure and interior of an intuitionistic fuzzy soft set. Furthermore, intuitionistic fuzzy soft continuous mapping are given and structural characteristics are discussed and studied. Deli and Broumi [6] defined for the first define a relation on neutrosophic soft sets which allows to compose two neutrosophic soft sets. It is devised to derive useful information through the composition of two neutrosophic soft sets. The authors then, examined symmetric, transitive and reflexive neutrosophic soft relations and many related concepts such as equivalent neutrosophic soft set relation, partition of neutrosophic soft sets, equivalence classes, quotient neutrosophic soft sets, neutrosophic soft composition are given and their propositions are discussed. Finally a decision making method on neutrosophic soft sets is presented. Bera and Mahapatra [3] Introduced the concept of Cartesian product and the relations on neutrosophic soft sets in a new approach. Some properties of this concept have been discussed and verified with suitable real life examples. The neutrosophic soft composition has been defined and verified with the help of example. Then, some basic properties to it have been established. After that the concept of neutrosophic soft function along with some of its basic properties have been introduced and verified by suitable examples. Injective, surjective, bijective, constant and identity neutrosophic soft functions have been defined. Finally, properties of inverse neutrosophic soft function have been discussed with proper example. Smarandache [17] for the first time initiated the concept of neutrosophic set which is generalization of the intuitionistic fuzzy set (IFS), para-consistent set, and intuitionistic set to the neutrosophic set (NS). many examples are presented. Peculiarities between NS and IFS are underlined.Maji [16] studied the concept of neutrosophic set of Smarandache. The author introduced this concept in soft sets and defined neutrosophic soft set. Some definitions and operations have been introduced on neutrosophic soft set. Some properties of this concept have been established. Bera and Mahapatra [4] introduced how to construct a topology on a neutrosophic soft set (NSS). the notion of neutrosophic soft interior, neutrosophic soft closure, neutrosophic soft neighborhood, and neutrosophic soft boundary, regular NSS are introduced and some of their basic properties are studied in this paper. Then the base for neutrosophic soft topology and subspace topology on NSS have been defined with suitable examples. Some related properties have been developed, too. Moreover, the concept of separation axioms on neutrosophic soft topological space have been introduced along with investigation of several structural characteristics.

Khattak et al. [9, 10] continued work on soft bi-topological structures. The authors discussed weak separation axioms and other separation axioms in soft bi-topological space with respect to crisp points and soft points of the spaces respectively.

Mehmood et al. [13] discussed soft α-connectedness, soft α-dis-connectedness and soft α-compact spaces in bi-polar soft topological spaces with respect to ordinary points. For better understanding the authors provided suitable examples.

Khattak et al. [11] introduced for the first time application soft semi open sets in binary topology. An important outcome of this work is a formal framework for the study of informations associated with ordered pairs of soft sets. Moreover the authors discussed five main results concerning binary soft topological spaces in the same article. Khattak et al.[8] for the first time bounced up the idea of NS b-open set, NS b-closed sets and their properties. Also the idea of neutrosophic soft b-neighborhood and neutrosophic soft b-separation axioms in NSTS are reflected here. Later on the important results are discussed related to these newly defined concepts with respect to soft points. The concept of neutrosophic soft b-separation axioms of NSTS is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft b-T_i -space (i = 0,1,2,3,4) and some associations between them are discussed.

Khattak et al. [12, 14, 15] continued has work on another structures known as neutrosophic soft topological structures. The authors introduced generalized and most generalized neutrosophic soft open sets in neutrosophic soft topological structures. They discussed different results with respect to soft point of the space.

The first aim of this paper is to reintroduce the concept of GNS-open set and develop a neutrosophic soft topology on GNS-open set with respect to soft points. Later the notions of neutrosophic soft point and their characteristics are addressed in connection with GNS-neighborhood. The concept of generalized separation axioms of neutrosophic soft topological spaces and related results are studied with respect to soft points. Study is slowly extended to neutrosophic soft Countability, NS sequences and their convergence in Hausdorff spaces. Furthermore, neutrosophic soft monotonous function and neutrosophic soft product spaces are addressed in connection with neutrosophic soft generalized open sets with respect to soft points. We hope that these results will be useful for future study on neutrosophic soft topology to carry out a general framework for most real-world applications.

3 Preliminaries

In this section we now state certain useful definitions, theorems, and several existing results for neutrosophic soft sets that we require in the next sections.

Definition 3.1. [8] NSS on father set $\langle X\rangle$ is characterized as $A^{neutrosophic} = [\lceil x, T_A^{ntophic}\langle X\rangle\ I_A^{ntophic}\langle X\rangle]: x \propto \langle X\rangle]$. $T : (x) \rightarrow]0^-, 1^+\ [I : \langle X\rangle \rightarrow]0^-, 1^+\ [F : \langle X\rangle \rightarrow]0^-, 1^+[$ So that's it $o^- \preccurlyeq \{T + I + F \preccurlyeq 3^+\}$

Definition 3.2. [8] let $\langle X\rangle$ be a fatherset, $\daleth^{paramater}$ be a set of all conditions, and $\pounds(\langle X\rangle)$ denote the efficiency set of $\langle X\rangle$. A pair $(f, \daleth^{paramater})$ is referred to as a soft set over $\langle X\rangle$ where f is a map given by $F : \daleth^{paramater}) \rightarrow \pounds(\langle X\rangle)$. For $n \propto \daleth^{paramater}$, $f(n)$ may be viewed as the set of n -softset elements $(f, \daleth^{paramater})$ or as a set of $n - estimated$the soft set components, i.e.$(f, \daleth^{paramater}) = \{n, f(n) : n \propto \daleth^{paramater}, F : \daleth^{paramater}) \rightarrow \pounds(\langle X\rangle)\}$

Definition 3.3. [8] let $\langle X\rangle$ be a fatherset, $\daleth^{paramater}$ be a set of all conditions, and $\pounds(\langle X\rangle)$ denote the efficiency set of $\langle X\rangle$. A pair $(f, \daleth^{paramater})$ is referred to as a soft set over $\langle X\rangle$ where f is a map given by $F : \daleth^{paramater}) \rightarrow \pounds(\langle X\rangle)$. Then a NS set $(\tilde{f}, \daleth^{paramater})$ over $\langle X\rangle$ is a set defined by a set of valued functions signifying a mapping $\tilde{f} : \daleth^{paramater} \rightarrow \pounds(\langle X\rangle)$ is referred to as the approximate NSset function $(\tilde{f} : \daleth^{paramater})$ In other words, the NSset is a group of conditions of certain elements of the set $\pounds(\langle X\rangle)$ so it can be written as a set of ordered pairs:$(\tilde{f} : \daleth^{paramater}) = \{((n, [x, I_{\tilde{f}}(x)^{(x)}, I_{\tilde{f}}(x)^{(x)}, F_{\tilde{f}}(x)^{(x)}, n :\propto \daleth^{paramater}\}\ T_{\tilde{f}}(x)^{(x)}, I_{\tilde{f}}(x)^{(x)}, F_{\tilde{f}}(x)^{(x)}\ [0, 1]$ are membership of truth, membership of indeterminacy and membership of falsehood $\tilde{f}(n)$ Since the supremum of eachT, I, f is 1 , the inequality that $0 \preceq \{T_f(x)^{(x)} + \preceq \{I_f(x)^{(x)} + \preceq \{F_f(x)^{(x)} \preceq +3^+\}$ is obvious

Definition 3.4. [8] Let $(\tilde{f} : \daleth^{paramater})$ be a NSS over the father set $\langle X\rangle$. The complement of $(\tilde{f} : \daleth^{paramater})$ is signified $(\tilde{f} : \daleth^{paramater})^c$ and is defined as follows:
$(\tilde{f} : \daleth^{paramater})^c = \{((n, [x, I_{\tilde{f}}(x)^{(x)}, I_{\tilde{f}}(x)^{(x)}, F_{\tilde{f}}(x)^{(x)}, n :\propto \daleth^{paramater}\}\ T_{\tilde{f}}(x)^{(x)}, I_{\tilde{f}}(x)^{(x)}, F_{\tilde{f}}(x)^{(x)}$ It's clear that $((\tilde{f} : \daleth^{paramater})^c)^c = (\tilde{f} : \daleth^{paramater})$.

Definition 3.5. [8] Let $(\tilde{f}, n)$ and $(\tilde{\rho}, n)$ two NSS over the father set (X) $(\tilde{f}, n)$ is supposed to be $NSSS$ of $(\tilde{\rho}, n)$ if $T_{\tilde{f}}(x)^{(x)} \preceq T_{\tilde{\rho}}(x)^{(x)}, I_{\tilde{f}}(x)^{(x)} \preceq T_{\tilde{\rho}}(x)^{(x)}, I_{\tilde{f}}(x)^{(x)} \succeq F_{\tilde{\rho}}(x)^{(x)}, n \propto \daleth^{paramater} and x \propto (x)$ It is signifies as $(\tilde{f}, n) \sqsubseteq (\tilde{\rho}, n)$.$(\tilde{f}, n)$ is is said to be NS equal to $(\tilde{\rho}, n)$ if $(\tilde{f}, n)$ is $NSSS$ of $(\tilde{\rho}, n)$ and $(\tilde{\rho}, n)$ is $NSSS$ of if $(\tilde{f}, n)$. It is symbolized as $(\tilde{f}, n) = (\tilde{\rho}, n)$

4 Neutrosophic soft points and their properties

Definition 4.1. [8] Let $(\tilde{f}_1, n)$ and $(\tilde{f}_2, n)$ be two $NSSS$ over father set $\langle X \rangle$ s.t. $(\tilde{f}_1, n)$ $\neq (\tilde{f}_2, n)$ and then their union is signifies as $(\tilde{f}_1, n)\tilde{\bigcup}(\tilde{f}_2, n)$=$(\tilde{f}_3, n)$ and is defined as $(\tilde{f}_3, n) = \{((n, [x, T_{\tilde{f}_3}(x)^{(x)}, I_{\tilde{f}_3}(x)^{(x)}, F_{\tilde{f}_3}(x)^{(x)}, n :\propto \exists^{paramater}\}$
Where,
$T_{\tilde{f}_3}(x)^{(x)} = max[T_{\tilde{f}_1}(x)^{(x)}, T_{\tilde{f}_2}(x)^{(x)} I_{\tilde{f}_3}(x)^{(x)} = max[I_{\tilde{f}_1}(x)^{(x)}, I_{\tilde{f}_2}(x)^{(x)}, F_{\tilde{f}_3}(x)^{(x)} = min[I_{\tilde{f}_1}(x)^{(x)}, I_{\tilde{f}_2}(x)^{(x)}$

Definition 4.2. [8] Let $(\tilde{f}_1, n)$ and $(\tilde{f}_2, n)$ be two $NSSS$ over father set $\langle X \rangle$ s.t. $(\tilde{f}_1, n)$ $\neq (\tilde{f}_2, n)$ and. Then their union is signifies as $(\tilde{f}_1, n)\tilde{\bigcap}(\tilde{f}_2, n)$=$(\tilde{f}_3, n)$ and is defined as $(\tilde{f}_3, n) = \{((n, [x, T_{\tilde{f}_3}(x)^{(x)}, I_{\tilde{f}_3}(x)^{(x)}, F_{\tilde{f}_3}(x)^{(x)}, n :\propto \exists^{paramater}\}$
Where,
$T_{\tilde{f}_3}(x)^{(x)} = max[T_{\tilde{f}_1}(x)^{(x)}, T_{\tilde{f}_2}(x)^{(x)} I_{\tilde{f}_3}(x)^{(x)} = max[I_{\tilde{f}_1}(x)^{(x)}, I_{\tilde{f}_2}(x)^{(x)}, F_{\tilde{f}_3}(x)^{(x)} = min[I_{\tilde{f}_1}(x)^{(x)}, I_{\tilde{f}_2}(x)^{(x)}$

Definition 4.3. [8] NS $(\tilde{f}_1, n)$ be a NSS over the father set $\langle X \rangle$ is said to be a vacuous set if $T_f(x)^{(x)}$ =0,$T_f(x)^{(x)}$ =0,$T_f(x)^{(x)}$=1,$\curlyvee e \propto nand \curlyvee x \propto (x)$ It is signifies as $0_{((x),n)}$.

Definition 4.4. [8] NS $(\tilde{f}_1, n)$ be a NSS over the father set X It is said to be an absolute neutrosophical softness if $T_f(x)^{(x)}$ =0,$T_f(x)^{(x)}$ =0,$T_f(x)^{(x)}$=1,$\curlyvee e \propto nand \curlyvee x \propto (x) Itissignifiesas 1_{((x),n)}$ clearly, $0_{((x),}n)^c = 1_{((x).n)} and 1_{((x).}n)^c = 0_{((x),n)}$

Definition 4.5. [8] Let NSS $(\langle \tilde{x} \rangle, \exists^{parameter})$ be the family of all NS sets over the father set $(\tilde{X}\ )$ and $\tau \subset$ NSS$(\langle \tilde{x} \rangle, \exists^{parameter})$. Then τ is said to be a NST on $(\tilde{X})$ if:
(1) $0_{(\langle X \rangle,n)}.1_{((x).n)} \propto \tau$
(2) The union of any number of NSS in τ belongs to τ
(3) The intersection of a finite number of NSS in τ belongs to τ Then NSS $(\langle \tilde{x} \rangle, \tau\exists^{parameter})$ is said to be a $NSTS$ over $(\tilde{X})$. Each member of τ is said to be a NSopen set.

Definition 4.6. [8] Let $(\langle \tilde{x} \rangle, \tau\exists^{parameter})$ be a $NSTS$ over $(\tilde{X}\ )$ and $(\tilde{f} : \exists^{paramater})$ be a neutrosophic soft set over $(\tilde{X}\ )$ Then $(\tilde{f} : \exists^{paramater})$ is supposed to be a NS closed set iff its complement is a $NSopen$ set..

Definition 4.7. [8] Let $(\langle \tilde{x} \rangle, \tau\exists^{parameter})$ be a $NSTS$ over $(\tilde{X}\ )$ and $(\tilde{f} : \exists^{paramater})$ be a neutrosophic soft set over $(\tilde{X}\ )$ Then $(\tilde{f} : \exists^{paramater})$ is supposed to be a $\alpha - open$ if. $(\tilde{f} : \exists^{paramater}) \subseteq int(cl(int((\tilde{f} : \exists^{paramater}))))$ and NS $\alpha - close$ if $(\tilde{f} : \exists^{paramater}) \supseteq cl(int(cl((\tilde{f} : \exists^{paramater}))))$

Definition 4.8. [8] Let NS be the family of all NS over father set $(\tilde{X}\ )$ and $x \propto (\tilde{X})$ The NS $x_{(a,b,c)}$ is supposed to be a neutrosophic point, for $0 < a, b, c \leq 1$, and is defined as follows:

$$x_{(a,b,c)^y} = \left\lfloor \begin{cases} (a,b,c) \text{ provided } y = x\} \\ (0,0,c) \text{ provided } y \neq x\} \end{cases} \right\rfloor \tag{1}$$

Example 4.9. Suppose that $(\tilde{X}) = \{x_1, x_2\}$ Then N set $A = \{< x_1, 0.1, 0.3, 0.5 >, < x_2, 0.5, 0.4, 0.7 >\}$ is the union of N points $x_{1_{(0.1,0.3,0.5)}}$ and $x_{2_{(0.5,0.4,0.7)}}$ Now we define the concept of NSpoints for NSsets. .

Definition 4.10. Let $NSS(\langle\tilde{X}\rangle)$be the family of all N soft sets over the father set $(\tilde{X})$ Then NSS $(X_{(a,b,c)})^e$ is called a NSpoint, for every $x \propto \langle\tilde{X}\rangle, 0 \prec \{a, b, c \preceq 1\}, e \propto \daleth^{parameter}$, and is defined as follows:

$$x^e_{(a,b,c)^{(e)^y}} = \begin{pmatrix} (a,b,c) \text{ provided } e = e \wedge y = x\} \\ (a,b,c) \text{ provided } e \neq e \wedge y \neq x\} \end{pmatrix}. \tag{2}$$

Definition 4.11. Suppose that the father set $(\tilde{X})$ is assumed to be $(\tilde{X}) = \{x_1, x_2\}$ and the set of conditions by $\daleth^{parameter} = \{e_1, e_2\}$ Let us consider NSS $(\tilde{f}, \daleth^{paramater})$ over the father set $(\tilde{X})$ as follows:

$$(\tilde{f}, \daleth^{paramater}) = \left\lfloor \left\{ \begin{cases} (a,b,c) \text{ provided } y = x\} \\ (0,0,c) \text{ provided } y \neq x\} \end{cases} \right\} \right\rfloor \tag{3}$$

It is clear that $(\tilde{f}, \daleth^{paramater})$ is the union of its NSpoints $X^{e_{1_{1(0.3,0.7,0.6)}}}$ $X^{e_{2_{1(0.4,0.6,0.8)}}}$, $X^{e_{1_{2(0.4,0.3,0.8)}}}{}^{AND}$ $X^{e_{1_{2(0.3,0.7,0.2)}}}$.

$$X^{e_{1_{1(0.3,0.7,0.6)}}} = \left\lfloor \begin{cases} e_1 = \{< x_1, 0.3, 0.7, 0.6 >, < X_2, 0, 0, 1 >\} \\ e_2 = \{< x_1, , 0, 0, 1 >, < X_2, 0, 0, 1 >\} \end{cases} \right\rfloor \tag{4}$$

$$X^{e_{2_{1(0.4,0.6,0.8)}}} = \left\lfloor \begin{cases} e_1 = \{< x_1, 0, 0, 1 >, < X_2, 0, 0, 1 >\} \\ e_2 = \{< x_1, 0.4, 0.6, 0.8 >, < X_2, 0, 0, 1 >\} \end{cases} \right\rfloor \tag{5}$$

$$X^{e_{2_{2(0.3,0.7,0.2)}}} = \left\lfloor \begin{cases} e_1 = \{< x_1, 0, 0, 1 >, < X_2, 0, 0, 1 >\} \\ e_2 = \{< x_1, 0, 0, 1 >, < X_2, 0.3, 0.7, 0.2 >\} \end{cases} \right\rfloor \tag{6}$$

Definition 4.12. Let $(\tilde{F}, \daleth^{parameter})$ be a NSS over the father set $< \tilde{x} >$. we say that $x^e_{(a,b,c)} \propto (\tilde{F}, \daleth^{parameter})$ read as belonging to the NSS $(\tilde{F}, \daleth^{parameter})$ whenever $a \preceq T_{\tilde{f}(x)^{(x)}}, b \preceq I_{\tilde{f}(x)^{(x)}}, c \succeq F_{\tilde{f}(x)^{(x)}}$

Definition 4.13. $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over $(\tilde{X})$ $(\tilde{f}, \daleth^{parameter})$ be a NSS over $(\tilde{X})$ NSS $(\tilde{f}, \daleth^{parameter})$ in $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is called a Nnbhd of the NS point $x^e_{(a,b,c)} \propto (\tilde{f}, \daleth^{parameter})$ if $\exists$ a NS α open set $(\tilde{g}, \daleth^{parameter})$ such that $x^e_{(a,b,c)} \propto (\tilde{g}, \daleth^{parameter}) \sqsubset (\tilde{f}, \daleth^{parameter})$

Theorem 4.14. *$(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NSTS over and $(\tilde{f}, \exists^{parameter})$ Then $(\tilde{f}, \exists^{parameter})$ is a NS α open set $\Leftrightarrow$ $(\tilde{f}, \exists^{parameter})$ is a NS nbhd of its NS points.*

Proof. . Let $(\tilde{f}, \exists^{parameter})$ be a NS open set and $x^e_{(a,b,c)} \propto (\tilde{f}, \exists^{parameter}) \sqsubset$ is a NS nbhd of $x^e_{(a,b,c)}$. conversely, let $(\tilde{f}, \exists^{parameter})$ be a NS nbhd of its NS points. Let $x^e_{(a,b,c)} \propto (\tilde{f}, \exists^{parameter})$ since $(\tilde{f}, \exists^{parameter})$ is a NS nbhd of the NS point$x^e_{(a,b,c)}$ $\exists$ $(\tilde{g}, \exists^{parameter})$ $\propto$ $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ s.t. $x^e_{(a,b,c)} \propto (\tilde{g}, \exists^{parameter}) \sqsubset (\tilde{f}, \exists^{parameter})$ Since $(\tilde{f}, \exists^{parameter})$ $= \sqcup x^e_{(a,b,c)}$ such that $x^e_{(a,b,c)} \propto (\tilde{f}, \exists^{parameter})$ it follows
That $(\tilde{f}, \exists^{parameter})$ is a union of NS α open sets and hence $(\tilde{f}, \exists^{parameter})$ is a NS α open set. The nbhd system of a NS point $x^e_{(a,b,c)}$, denoted by $\sqcup\ x^e_{(a,b,c)}\ , \exists^{parameter})$, is the family of all its nbhds. $\square$

Theorem 4.15. *The nbhd system $\sqcup\ x^e_{(a,b,c)}\ , \exists^{parameter})$ at $x^e_{(a,b,c)}$ in a NSTS $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ has the following properties:*

$$\left\{\begin{array}{c} (1) if (\tilde{f}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}), then x^e_{(a,b,c)}\ , \exists^{parameter}) \\ (2) if (\tilde{f}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}), (\tilde{f}, \exists^{parameter}) \sqsubset (\tilde{h}, \exists^{parameter}) \\ then (\tilde{h}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}) : \\ (3) if (\tilde{f}, \exists^{parameter}), (\tilde{g}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}) then \\ (\tilde{f}, \exists^{parameter}) \sqcap (\tilde{g}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}) \\ (4) if (\tilde{f}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}) then \\ \exists if (\tilde{g}, \exists^{parameter}) \propto \sqcup x^e_{(a,b,c)}, \exists^{parameter}) s.t \\ if (\tilde{g}, \exists^{parameter}) \propto \sqcup x^{e'}_{(a',b',c')}, \exists^{parameter}), foreach \\ x^{e'}_{(a',b',c')}, \exists^{parameter})(\tilde{f}, \exists^{parameter}) \sqcap (\tilde{g}, \exists^{parameter}) \propto x^e_{(a,b,c)}, \exists^{parameter}) \end{array}\right. \tag{7}$$

Proof. The proof of 1),2),and 3) is obvious from Definition 3.13.
if $(\tilde{f}, \exists^{parameter}) \propto \cup x^e_{(a,b,c)}, \exists^{parameter})$ then $\exists$ NS α open set $(\tilde{g}, \exists^{parameter})$ s.t $x^e_{(a,b,c)}$, $\exists^{parameter})$ $\propto$ $(\tilde{g}, \exists^{parameter})$ $\sqsubset$ $(\tilde{f}, \exists^{parameter})$ From Proposition 3.14,$(\tilde{g}, \exists^{parameter})$ $\propto$ $\cup x^e_{(a,b,c)}, \exists^{parameter})$ so for each $x^{e'}_{(a',b',c')}, \propto (\tilde{g}, \exists^{parameter}), (\tilde{g}, \exists^{parameter}) \propto (\tilde{g}, \exists^{parameter}) \propto$ $\cup x^{e'}_{(a',b',c')}, \exists^{parameter})$ is obtained. $\square$

Definition 4.16. Let $x^e_{(a,b,c)}$ and $x^{e'}_{(a',b',c')}$ be two NSpoints. For the NSpoints Over father set $(\tilde{X})$ we say that the NSpoints are distinct points $x^e_{(a,b,c)} \sqcap x^{e'}_{(a',b',c')} = 0, (\tilde{x}, \exists^{parameter})$ It is clear that $x^e_{(a,b,c)}$ and $x^{e'}_{(a',b',c')}$ are distinct NS points if and only if $x \succ y$ or $X \prec Y$ or $e' \succ e$ or $e' \prec e$

5 Neutrosophic soft α separation axioms

In this section, we define neutrosophical soft α separation in neutrosophical soft topological space with respect to NS points.

Definition 5.1. Let $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NSTS over$(\tilde{X})$, and

$$\begin{cases} x^{e}_{(a,b,c)} \succ x^{e'}_{(a',b',c')} \\ \quad\quad or \\ x^{e}_{(a,b,c)} \prec x^{e'}_{(a',b',c')} \end{cases} \tag{8}$$

NSpoints. If there exist NS α-open sets $(\tilde{f}, \exists^{parameter})$ and $(\tilde{g}, \exists^{parameter})$ such that $x^{e}_{(a,b,c)} \propto (\tilde{f}, \exists^{parameter})$ $x^{e}_{(a,b,c)} \sqcap (\tilde{f}, \exists^{parameter}) = 0_{(\tilde{X},, \exists^{parameter})}$ or $y^{e'}_{(a',b',c')} \propto (\tilde{g}, \exists^{parameter})$, $y^{e'}_{(a',b',c')} \sqcap (\tilde{g}, \exists^{parameter}) = 0_{(\tilde{X},, \exists^{parameter})}$ Then $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ is called a NS α_0

Definition 5.2. Let $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NSTS over$(\tilde{X})$, and

$$\begin{cases} x^{e}_{(a,b,c)} \succ x^{e'}_{(a',b',c')} \\ \quad\quad or \\ x^{e}_{(a,b,c)} \prec x^{e'}_{(a',b',c')} \end{cases} \tag{9}$$

NSpoints. If there exist NS α-open sets $(\tilde{f}, \exists^{parameter})$ and $(\tilde{g}, \exists^{parameter})$ such that $x^{e}_{(a,b,c)} \propto (\tilde{f}, \exists^{parameter})$ $x^{e}_{(a,b,c)} \sqcap (\tilde{f}, \exists^{parameter}) = 0_{(\tilde{X},, \exists^{parameter})}$ or $y^{e'}_{(a',b',c')} \propto (\tilde{g}, \exists^{parameter})$, $y^{e'}_{(a',b',c')} \sqcap (\tilde{g}, \exists^{parameter}) = 0_{(\tilde{X},, \exists^{parameter})}$ Then $(< \tilde{X} >, \tau, \exists^{parameter})$ is called a NS α_1

Definition 5.3. Let $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NSTS over$(\tilde{X})$, and

$$\begin{cases} x^{e}_{(a,b,c)} \succ x^{e'}_{(a',b',c')} \\ \quad\quad or \\ x^{e}_{(a,b,c)} \prec x^{e'}_{(a',b',c')} \end{cases} \tag{10}$$

NSpoints. If there exist NS α-open sets $(\tilde{f}, \exists^{parameter})$ and $(\tilde{g}, \exists^{parameter})$ such that $x^{e}_{(a,b,c)} \propto (\tilde{f}, \exists^{parameter})$ $x^{e}_{(a,b,c)} \sqcap (\tilde{f}, \exists^{parameter}) = 0_{(\tilde{X},, \exists^{parameter})}$ or $y^{e'}_{(a',b',c')} \propto (\tilde{g}, \exists^{parameter})$, $y^{e'}_{(a',b',c')} \sqcap (\tilde{g}, \exists^{parameter}) = 0_{(\tilde{X},, \exists^{parameter})}$ Then $(< \tilde{X} >, \tau, \exists^{parameter})$ is called a NS α_2

Example 5.4. Suppose that the father set $(\tilde{X})$ is assumed to be $(\tilde{X}) = \{x_1, x_2\}$ and the set of conditions by $\exists^{parameter} = \{e_1, e_2\}$.Let us consider NSset $(\tilde{f}, \exists^{parameter})$ over the father set $(\tilde{X})$ and $x^{e_{1_1(0.1,0.4,0.7)}}$, $x^{e_{2_1(0.2,0.5,0.6)}}$, $x^{e_{1_2(0.3,0.3,0.5)}}$ and $x^{e_{1_2(0.4,0.4,0.4)}}$ NSpoints. Then the family
$\tau = \{0_{(\tilde{X},, \exists^{parameter})}, 1_{(\tilde{X},, \exists^{parameter})}, (\tilde{f}_1, \exists^{parameter}),(\tilde{f}_2, \exists^{parameter}),(\tilde{f}_3, \exists^{parameter}),(\tilde{f}_4, \exists^{parameter}), (\tilde{f}_5, \exists^{parameter}),(\tilde{f}_6, \exists^{parameter}),(\tilde{f}_7, \exists^{parameter}),(\tilde{f}_8, \exists^{parameter})\}$.

where
$(\tilde{f}_1, \exists^{parameter}) = x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$ $(\tilde{f}_2, \exists^{parameter}) = x^{e_{2_1}}{}_{(0.2,0.5,0.6)}$, $(\tilde{f}_3, \exists^{parameter}) = x^{e_{1_2}}{}_{(0.3,0.3,0.5)}$, $(\tilde{f}_4, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_2, \exists^{parameter})$, $(\tilde{f}_5, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter})$, $(\tilde{f}_6, \exists^{parameter}) = (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter})$, $(\tilde{f}_7, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter})$,
$(\tilde{f}_8, \exists^{parameter})\} = \{x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$, $x^{e_{2_1}}{}_{(0.2,0.5,0.6)}$, $x^{e_{1_2}}{}_{(0.3,0.3,0.5)}$ and $x^{e_{1_2}}{}_{(0.4,0.4,0.4)}$ is a NSTSover the father set $(\tilde{X})$. Thus $(< \tilde{X} >, \tau, \exists^{parameter})$ be a NSTSover the father set $(\tilde{X})$. Also $(< \tilde{X} >, \tau, \exists^{parameter})$ is NS α_0 structure but it is not NS α_1 because for NSpoints $x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$,$x^{e_{2_2}}{}_{(0.4,0.4,0.4)}$,$(< \tilde{X} >, \tau, \exists^{parameter})$ not soft neutrosophic α_1

Example 5.5. Suppose that the father set $(\tilde{X})$ is assumed to be $(\tilde{X}) = \{x_1, x_2\}$ and the set of conditions by $\exists^{parameter} = \{e_1, e_2\}$.Let us consider NSset $(\tilde{f}, \exists^{parameter})$ over the father set $(\tilde{X})$ and $x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$, $x^{e_{2_1}}{}_{(0.2,0.5,0.6)}$, $x^{e_{1_2}}{}_{(0.3,0.3,0.5)}$ and $x^{e_{1_2}}{}_{(0.4,0.4,0.4)}$ NSpoints. Then the family
$\tau = \{0_{(\tilde{X},, \exists^{parameter})}, 1_{(\tilde{X},, \exists^{parameter})}$, $(\tilde{f}_1, \exists^{parameter})$,$(\tilde{f}_2, \exists^{parameter})$,$(\tilde{f}_3, \exists^{parameter})$,$(\tilde{f}_4, \exists^{parameter})$, $(\tilde{f}_5, \exists^{parameter})$,$(\tilde{f}_6, \exists^{parameter})$,$(\tilde{f}_7, \exists^{parameter})$,$(\tilde{f}_8, \exists^{parameter})\}$.....$(\tilde{f}_{15}, \exists^{parameter})$.
where
$(\tilde{f}_1, \exists^{parameter}) = x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$ $(\tilde{f}_2, \exists^{parameter}) = x^{e_{2_1}}{}_{(0.2,0.5,0.6)}$, $(\tilde{f}_3, \exists^{parameter}) = x^{e_{1_2}}{}_{(0.3,0.3,0.5)}$, $(\tilde{f}_4, \exists^{parameter})$ $x^{e_{2_2}}{}_{(0.4,0.4,0.4)}$ $(\tilde{f}_3, \exists^{parameter}) = (\tilde{f}_3, \exists^{parameter}) \cup (\tilde{f}_2, \exists^{parameter})$, $(\tilde{f}_6, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter})$, $(\tilde{f}_7, \exists^{parameter}) = (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_4, \exists^{parameter})$, $(\tilde{f}_8, \exists^{parameter}) = (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter})$, $(\tilde{f}_9, \exists^{parameter}) = (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_4, \exists^{parameter})$, $(\tilde{f}_{10}, \exists^{parameter}) = (\tilde{f}_3, \exists^{parameter}) \cup (\tilde{f}_4, \exists^{parameter})$, $(\tilde{f}_{11}, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_2, \exists^{parameter})$, $(\tilde{f}_3, \exists^{parameter})$, $(\tilde{f}_{12}, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_4, \exists^{parameter})$, $(\tilde{f}_{13}, \exists^{parameter}) = (\tilde{f}_2, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter}) \cup (\tilde{f}_4, \exists^{parameter})$, $(\tilde{f}_{14}, \exists^{parameter}) = (\tilde{f}_1, \exists^{parameter}) \cup (\tilde{f}_3, \exists^{parameter}) \cup (\tilde{f}_4, \exists^{parameter})$,
$(\tilde{f}_{15}, \exists^{parameter}) = \{x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$, $x^{e_{2_1}}{}_{(0.2,0.5,0.6)}$, $x^{e_{1_2}}{}_{(0.3,0.3,0.5)}$ and $x^{e_{1_2}}{}_{(0.4,0.4,0.4)}$ is a NSTSover the father set $(\tilde{X})$. Thus $(< \tilde{X} >, \tau, \exists^{parameter})$ be a NSTSover the father set $(\tilde{X})$. Also $(< \tilde{X} >, \tau, \exists^{parameter})$ is NS α_1 because for NSpoints $x^{e_{1_1}}{}_{(0.1,0.4,0.7)}$,$x^{e_{2_2}}{}_{(0.4,0.4,0.4)}$,$(< \tilde{X} >, \tau, \exists^{parameter})$ not soft neutrosophic α_2

Theorem 5.6. *Let $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NSTS over the father set $(\tilde{X})$.Then $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NST_1 structure iffeach NSpointis a NS $\alpha -$ closed set.*

Proof. Let $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a NSTS over the father set $(\tilde{X})$. $(x^e_{(a,b,c)}, \exists^{parameter})$ be an arbitrary NSpoint.We establish $(x^e_{(a,b,c)}, \exists^{parameter})$ parameter is a NS $\alpha - open$ set. Let $(y^{e'}_{(a',b',c')}, \exists^{parameter}) \propto (x^e_{(a,b,c)}, \exists^{parameter})$. Then either $(y^{e'}_{(a',b',c')}, \exists^{parameter}) \succ (x^e_{(a,b,c)}, \exists^{parameter})$ or $(y^{e'}_{(a',b',c')}, \exists^{parameter}) \prec (x^e_{(a,b,c)}, \exists^{parameter})$
or $(y^{e'}_{(a',b',c')}, \exists^{parameter}) \succ\succ (x^e_{(a,b,c)}, \exists^{parameter})$ or $(y^{e'}_{(a',b',c')}, \exists^{parameter}) \prec\prec (x^e_{(a,b,c)}, \exists^{parameter})$

. This means that $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ and $(x^{e}_{(a,b,c)}, \daleth^{parameter})$ are two are distinct NS-points. Thus $x \succ y$ or $X \prec Y$ or $e' \succ e$ or $e' \prec e$,$x \succ\succ y$ or $X \prec\prec Y$ or $e' \succ\succ e$ or $e' \prec\prec e$ Since $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NST_1 structure, ?a NS $\alpha - open$ set $(\tilde{g}, \daleth^{parameter})$ so that $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto (\tilde{g}, \daleth^{parameter})$ and $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \sqcap (\tilde{g}, \daleth^{parameter}) = 0_{(\tilde{X}_{,}, \daleth^{parameter})}$ since $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \sqcap (\tilde{g}, \daleth^{parameter}) = 0_{(\tilde{X}_{,}, \daleth^{parameter})}$ so
$(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto (\tilde{g}, \daleth^{parameter}) \subset (x^{e}_{(a,b,c)}, \daleth^{parameter})$ Thus $(x^{e}_{(a,b,c)}, \daleth^{parameter})$ is a NS $\alpha - open$ set, i.e.$(x^{e}_{(a,b,c)}, \daleth^{parameter})$ is a NS $\alpha - closed$ set. Suppose that each NSpoint $(x^{e}_{(a,b,c)}, \daleth^{parameter})$is a NS$\alpha - closed$ set.Then $(x^{e}_{(a,b,c)}, \daleth^{parameter})^C$is a NS $\alpha - open$ set. Let $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \cap (y^{e'}_{(a',b',c')}, \daleth^{parameter}) = 0_{(\tilde{X}_{,}, \daleth^{parameter})}$
Thus
$(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto (x^{e}_{(a,b,c)}, \daleth^{parameter})^C$ and
$(x^{e}_{(a,b,c)}, \daleth^{parameter}) \cap (x^{e}_{(a,b,c)}, \daleth^{parameter})^C = 0_{(\tilde{X}_{,}, \daleth^{parameter})}$ So $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a $NS-\alpha_1$ space. □

Theorem 5.7. *Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over the father set $(\tilde{X})$.Then $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NST_2 spaceiff for distinct NSpoints . $(x^{e}_{(a,b,c)}, \daleth^{parameter})$ and $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$, there exists a NS $\alpha - open$ set $(\tilde{f}\daleth^{parameter})$ containing $\exists$ but not $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ s.t.$(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \ni \overline{(\tilde{f}\daleth^{parameter})}$*

Proof. $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \succ (y^{e'}_{(a',b',c')}, \daleth^{parameter})$ be two NSpoints in NS_2 space. Then $\exists$ disjoint NS α open sets $(\tilde{f}\daleth^{parameter})$ and $(\tilde{g}\daleth^{parameter})$ such $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{f}\daleth^{parameter})$
and
$(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto (\tilde{g}\daleth^{parameter})$.
since
$(x^{e}_{(a,b,c)}, \daleth^{parameter}) \sqcap (y^{e'}_{(a',b',c')}, \daleth^{parameter}) = 0_{(\tilde{X}_{,}, \daleth^{parameter})}$
and
$(\tilde{f}, \daleth^{parameter}) \sqcap \overline{(\tilde{g}, \daleth^{parameter}) = 0_{(\tilde{X}_{,}, \daleth^{parameter})}}$ $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \ni (\tilde{f}, \daleth^{parameter}) \Rightarrow (y^{e'}_{(a',b',c')}, \daleth^{parameter}) \ni \overline{(\tilde{f}\daleth^{parameter})}$. Next suppose that, $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \succ (y^{e'}_{(a',b',c')}, \daleth^{parameter})$, $\exists$ a NS α open set $(\tilde{f}, \daleth^{parameter})$ containing $(x^{e}_{(a,b,c)}, \daleth^{parameter})$ but not $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ s.t. $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \ni \overline{(\tilde{f}\daleth^{parameter})^c}$ that is $(\tilde{f}, \daleth^{parameter})$ and $\overline{(\tilde{f}\daleth^{parameter})^c}$ are mutually exclusive NS α open sets supposing $(x^{e}_{(a,b,c)}, \daleth^{parameter})$ and $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ in turn. □

Theorem 5.8. *Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over the father set $(\tilde{X})$ Then $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is NS_1 space if every NS point $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{f}, \daleth^{parameter}) \propto (\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$. If there exists a NS α open set $(\tilde{g}, \daleth^{parameter})$ s.t. $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \propto (\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ a $NS_2 space$.*

Proof. Suppose$(x^e_{(a,b,c)}, \daleth^{parameter})\sqcap$ $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) = 0_{(\tilde{X},, \daleth^{parameter})}$. Since $(\langle\tilde{x}\rangle, \tau,$ $\daleth^{parameter})$ is NS_1space. $(x^e_{(a,b,c)}, \daleth^{parameter})$ and $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ are NS $\alpha - closed$ sets in $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ Then $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto ((y^{e'}_{(a',b',c')}, \daleth^{parameter}))^c \propto (\langle\tilde{x}\rangle, \tau,$ $\daleth^{parameter})$. Thus $\exists$ a NS $(\tilde{g}, \daleth^{parameter}) \propto (\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ s.t. $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto$ $(\tilde{g}, \daleth^{parameter}) \subset \overline{(\tilde{g}, \daleth^{parameter})} \subset ((y^{e'}_{(a',b',c')}, \daleth^{parameter}))^c$. So we have $((y^{e'}_{(a',b',c')}, \daleth^{parameter}))^c$ $\propto (\tilde{g}, \daleth^{parameter})$ and $(((\tilde{g}, \daleth^{parameter})\sqcap((\tilde{g}, \daleth^{parameter})^c = 0_{(\tilde{X},, \daleth^{parameter})}$,i.e. $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is a NS_2 space. . □

Definition 5.9. . Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over the father set $(\tilde{X})$ $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NS $\alpha - closed$ set and $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{f}, \daleth^{parameter}) = 0_{(\tilde{X},, \daleth^{parameter})}$. If $\exists$ NS α open sets $((\tilde{g}_1, \daleth^{parameter})$ and$((\tilde{g}_2, \daleth^{parameter})$ s.t. $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto ((\tilde{g}_2, \daleth^{parameter}),$ $(\tilde{f}, \daleth^{parameter}) \subset ((\tilde{g}_2, \daleth^{parameter})$ and $(x^e_{(a,b,c)}, \daleth^{parameter})\sqcap((\tilde{g}_1, \daleth^{parameter}) = 0_{(\tilde{X},, \daleth^{parameter})}$, then $(< \tilde{X} >, \tau, \daleth^{parameter})$ is called a NS $\alpha - regular$ space. $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is said to be NS_3 space, if is both a a NS $\alpha - regular$ and NS_1 space.

Theorem 5.10. *Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over the father set $(\tilde{X})$ $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is soft α space iff for every $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{f}, \daleth^{parameter})$,i.e, $(\tilde{f}, \daleth^{parameter}) \propto (<$ $\tilde{X} >, \tau, \daleth^{parameter})$ s.t. $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{g}, \daleth^{parameter}) \subset \overline{(\tilde{g}, \daleth^{parameter})}, (\tilde{f}, \daleth^{parameter})$.*

Proof. Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is $NS - \alpha_3$ space and $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{f}, \daleth^{parameter}) \propto$ $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$. Since $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is$NS - \alpha_3$ space for the NS point $(x^e_{(a,b,c)},$ $\daleth^{parameter})$ and a NS $\alpha - closed$ set$(\tilde{f}, \daleth^{parameter})^c$ $\exists(\tilde{g}_1, \daleth^{parameter})$ and $(\tilde{g}_2, \daleth^{parameter})$ s.t. $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{g}_1, \daleth^{parameter}), (\tilde{f}, \daleth^{parameter})^c \subset (\tilde{g}_2, \daleth^{parameter})$ and $(\tilde{g}_1, \daleth^{parameter})\sqcap$ $(\tilde{g}_2, \daleth^{parameter}) = 0_{(\tilde{X},, \daleth^{parameter})}$ Conversely, let $(x^e_{(a,b,c)}, \daleth^{parameter}) \cap (\tilde{H}, \daleth^{parameter}) =$ $0_{(\tilde{X},, \daleth^{parameter})}$ and $(\tilde{H}, \daleth^{parameter})$ be a NS $\alpha - closed$ set. $(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{H}, \daleth^{parameter})^c$ and from the condition of the theorem, we have

$(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{g}, \daleth^{parameter}) \subset \overline{(\tilde{g}, \daleth^{parameter})}(\tilde{H}, \daleth^{parameter})^c$.Thus

$(x^e_{(a,b,c)}, \daleth^{parameter}) \propto (\tilde{g}, \daleth^{parameter}) \subset (\tilde{H}, \daleth^{parameter})\overline{(}\tilde{g}, \daleth^{parameter})^c$

and

$(\tilde{g}, \daleth^{parameter}) \cap (\tilde{g}, \daleth^{parameter})^c = 0_{(\tilde{X},, \daleth^{parameter})}$. So $(< \tilde{X} >, \tau, \daleth^{parameter})$ is $NS - \alpha_3$ space. □

Definition 5.11. Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over the father set $(\tilde{X})$ This space is a NS $\alpha - normal$ space, if for every pair of disjoint a NS α closed sets $(\tilde{f}_1, \daleth^{parameter})$ and $(\tilde{f}_2, \daleth^{parameter})$, disjoint a NS $\alpha - open$ sets $((\tilde{g}_1, \daleth^{parameter})$ and $(\tilde{g}_2, \daleth^{parameter})$ s.t.$(\tilde{f}_1, \daleth^{parameter}) \subset (\tilde{g}_1, \daleth^{parameter})$ and $(\tilde{f}_2, \daleth^{parameter}) \subset (\tilde{g}_2, \daleth^{parameter})$. $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ is said to be a NS α_4 space if it is both a NS $\alpha - normal$ and NS α_4 space.

Theorem 5.12. *Let $(\langle\tilde{x}\rangle, \tau, \daleth^{parameter})$ be a NSTS over the father set $(\tilde{X})$ This space is a $NS - \alpha_4$ space $\Leftrightarrow$ for each NS $\alpha - closed$ set $(\tilde{f}, \daleth^{parameter})$and NS $\alpha - open$ set*

$(\tilde{g}, \exists^{parameter})$ *with* $(\tilde{f}, \exists^{parameter}) \subset \overline{(\tilde{g}, \exists^{parameter})}$, $\exists$ *a NS* $\alpha-$*open set* $(\tilde{D}, \exists^{parameter})$*s.t.* $(\tilde{f}, \exists^{parameter}) \subset (\tilde{D}, \exists^{parameter}) \subset \overline{(\tilde{D}, \exists^{parameter})} \subset (\tilde{g}, \exists^{parameter})$.

Proof. Let $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a $NS-\alpha_4$ over the father set $(\tilde{X})$. and let $(\tilde{f}, \exists^{parameter}) \subset (\tilde{g}, \exists^{parameter})$. Then $(\tilde{g}, \exists^{parameter})^c$ is a NS $\alpha-$*closed* set and $(\tilde{f}, \exists^{parameter}) \cap (\tilde{g}, \exists^{parameter}) = 0_{(\langle\tilde{x}\rangle, \exists^{parameter})}$. Since $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a $NS-\alpha_4$ space,$\exists$ NS α open sets $(\tilde{D}_1, \exists^{parameter})$ and $(\tilde{D}_2, \exists^{parameter})$ s.t. $(\tilde{f}, \exists^{parameter}) \subset (\tilde{D}_1, \exists^{parameter})$,$(\tilde{g}, \exists^{parameter})^c \subset (\tilde{D}_2, \exists^{parameter})$ *and*$(\tilde{D}_1, \exists^{parameter}) \cap (\tilde{D}_2, \exists^{parameter}) = 0_{(\langle\tilde{x}\rangle, \exists^{parameter})}$. Thus $(\tilde{f}, \exists^{parameter}) \subset \overline{(\tilde{D}_1, \exists^{parameter})} \subset (\tilde{D}_2, \exists^{parameter})^c \subset (\tilde{g}, \exists^{parameter})$, $(\tilde{D}_2, \exists^{parameter})^c$ is a NS $b-$*closed* set and $\overline{(\tilde{D}_1, \exists^{parameter})} \subset (\tilde{D}_2, \exists^{parameter})^c$. So $(\tilde{f}, \exists^{parameter}) \subset (\tilde{D}_1, \exists^{parameter}) \subset \overline{(\tilde{D}_1, \exists^{parameter})} \subset (\tilde{g}, \exists^{parameter})$
Conversely, let $(\tilde{f}_1, \exists^{parameter})$ and $(\tilde{f}_2, \exists^{parameter})$, disjoint a NS $\alpha-$*closed*. Then$(\tilde{f}_1, \exists^{parameter}) \subset (\tilde{f}_2, \exists^{parameter})^c$. From the condition of theorem, there exists a NS $\alpha-$*open* set $(\tilde{D}, \exists^{parameter})$ s.t. $(\tilde{f}_1, \exists^{parameter}) \subset (\tilde{D}, \exists^{parameter}) \subset \overline{(\tilde{D}_1, \exists^{parameter})} \subset (\tilde{f}_2, \exists^{parameter})^c$ Thus $(\tilde{D}, \exists^{parameter})$ and $(\tilde{D}, \exists^{parameter})^c$ are NS ? -open$\alpha - open$ sets and$(\tilde{f}_1, \exists^{parameter}) \subset (\tilde{f}_2, \exists^{parameter})^c$, $(\tilde{f}_2, \exists^{parameter}) \subset \overline{(\tilde{D}, \exists^{parameter})}$ and $(\tilde{D}, \exists^{parameter})$ and $((\tilde{D}, \exists^{parameter}) = 0_{(\langle\tilde{x}\rangle, \exists^{parameter})}$. $(\langle\tilde{x}\rangle, \tau, \exists^{parameter})$ be a $NS - \alpha_4$ space. □

6 Neutrosophic soft Countability:

Theorem 6.1. *Let* $(x^e_{(a,b,c)}, \exists^{parameter})$ *be a point in a first NS countable space* $\langle x^{crip}, \Im, \partial\rangle$ *and let* $\{\langle W, \partial\rangle_i : i = 1, 2, 3....\}$ *generates a NS countable* $\alpha - open$ *base about the point* $(x^e_{(a,b,c)}, \exists^{parameter})$, *then there exists an infinite soft sub-sequence* $\{\langle V, \partial\rangle_i : i = 1, 2, 3....\}$ *of the NS sequence* $\{\langle W, \partial\rangle_i : i = 1, 2, 3....\}$,*such that* (i) *for any NS* $\alpha - open$ *set* $\langle U, \partial\rangle$,*containing* $(x^e_{(a,b,c)}, \exists^{parameter})$,$\exists$ *a suffix* m *such that* $\{\langle V, \partial\rangle_i \Subset \langle U, \partial\rangle$ *for all* $i \succcurlyeq m$; *and* (ii) *if* $\langle x^{crip}, \Im, \partial\rangle$ *be, in particular , a NS* $\alpha_1 - space$, *then* $\cap\{\langle V, \partial\rangle_i : i = 1, 2, 3....\} = \{(x^e_{(a,b,c)}, \exists^{parameter})\}$.

Proof. Given $\langle W, \partial\rangle_1 \widetilde{\cap} \langle W, \partial\rangle_2 \widetilde{\cap} \langle W, \partial\rangle_3 \widetilde{\cap} .... \langle W, \partial\rangle_k$ is NSopen sets, contains the point$(x^e_{(a,b,c)}, \exists^{parameter})$ As the NS sets $\{\langle W, \partial\rangle_i : i = 1, 2, 3....\}$ Forms NS $\alpha-$*open* base about $(x^e_{(a,b,c)}, \exists^{parameter})$ $\exists$ one among theNSsets $\{\langle W, \partial\rangle_i : i = 1, 2, 3....\}$, which we shall denote by $\langle V, \partial\rangle_k$, such that $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle V, \partial\rangle_k \Subset \langle W, \partial\rangle_1 \widetilde{\cap} \langle W, \partial\rangle_2 \widetilde{\cap} \langle W, \partial\rangle_3 \widetilde{\cap} .... \langle W, \partial\rangle_k$ for $k = 1, 2, 3.......$ The NSsequence $\{\langle V, \partial\rangle_i : i = 1, 2, 3....\}$ Thus obtained, has the required properties. In fact, if $\langle U, \partial\rangle$ is any NS $\alpha-$*open* set, containing $(x^e_{(a,b,c)}, \exists^{parameter})$ then $\exists$ a NS set $\langle W, \partial\rangle_m$ say, belonging to the family $\{\langle W, \partial\rangle_i : i = 1, 2, 3....\}$, s.t.$(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle W, \partial\rangle_m \langle U, \partial\rangle$. Also, since$\langle V, \partial\rangle_i \Subset \langle W, \partial\rangle_m$,for all $i \succcurlyeq m.Next$, let $\langle x^{crip}, \Im, \partial\rangle$ be NS $\alpha_1 - space$ and let$\cap\{\langle V, \partial\rangle_i : i = 1, 2, 3....\} = \{\langle M, \partial\rangle$ As$\{(x^e_{(a,b,c)}, \exists^{parameter})\}$ is contained in each $\{\langle V, \partial\rangle_i : i = 1, 2, 3....\}$ that is $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle V, \partial\rangle_1$, $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle V, \partial\rangle_2$, $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle V, \partial\rangle_3$, $(x^e_{(a,b,c)}, \exists^{parameter}) \langle V, \partial\rangle_4 ......(x^e_{(a,b,c)}, \exists^{parameter}) \propto$

$\langle V, \partial\rangle_n$ it follows that $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle M, \partial\rangle$, Let $(y^{e'}_{(a',b',c')}), \exists^{parameter})$ be any point of x^{crip} from x that is $(x^e_{(a,b,c)}, \exists^{parameter}) \succ (y^{e'}_{(a',b',c')}), \exists^{parameter})$ or$(x^e_{(a,b,c)}, \exists^{parameter}) \prec (y^{e'}_{(a',b',c')}, \exists^{parameter})$ By definition of NS $\alpha_1-space$,$\exists$ NS $\alpha-open$ set$\langle U, \partial\rangle$,s.t. $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle U, \partial\rangle$ and $(y^{e'}_{(a',b',c')}), \exists^{parameter}) \ni \langle U, \partial\rangle$ .There exists a suffix m ,s.t. $\langle V, \partial\rangle_1 \Subset \langle U, \partial\rangle, \langle V, \partial\rangle_2 \Subset \langle U, \partial\rangle, \langle V, \partial\rangle_3 \Subset \langle U, \partial\rangle, \langle V, \partial\rangle_4, \langle U, \partial\rangle ......\langle V, \partial\rangle_i \Subset \langle U, \partial\rangle$ for all $i \succeq m$. consequently,$(y^{e'}_{(a',b',c')}, \exists^{parameter}$ $\langle V, \partial\rangle_i$ for all $i \succeq m$; hence$(y^{e'}_{(a',b',c')}, \exists^{parameter}) \ni \langle M, \partial\rangle_i$. Thus $\langle M, \partial\rangle_i$ cosists of the point $(x^e_{(a,b,c)}, \exists^{parameter})$ only. □

Theorem 6.2. *Soft second NS countability is first NS countability.*

Proof. Let $\langle x^{crip}, \Im, \partial\rangle$ be a soft 2nd NS countable space. Then this situation permit that there live a NS countable base $\Re$ for $\langle x^{crip}, \Im, \partial\rangle$ In order to justify that $\langle x^{crip}, \Im, \partial\rangle$ isNS, we proceed as, let$(x^e_{(a,b,c)}, \exists^{parameter}) \propto X^{crip}$ be arbitrary point. Let us assemble those members of $\Re$ which absorbs $(x^e_{(a,b,c)}, \exists^{parameter})$ and named as $\Re^{(x^e_{(a,b,c)}, \exists^{parameter})}$. If $\langle \aleph, \partial\rangle$ is soft $n-hood$ of$(x^e_{(a,b,c)}, \exists^{parameter})$, then this permit $\exists$ NS open set$\langle \wp, \partial\rangle$ arresting $(x^e_{(a,b,c)}, \exists^{parameter})$ in $\Re$ and so in $\Re^{(x^e_{(a,b,c)}, \exists^{parameter})}$.s.t. $(x^e_{(a,b,c)}, \exists^{parameter}) \propto \langle \wp, \partial\rangle \Subset \langle \aleph, \partial\rangle$. This justifies that is a NS local base at $(x^e_{(a,b,c)}, \exists^{parameter})$.One step more,$\Re^{(x^e_{(a,b,c)}, \exists^{parameter})}$ being a sub-family of a NScountable family $\Re$, it is therefore NS countable. Thus every crisp point of x^{crip} supposes a countable NS local base. This leads us to say $\langle x^{crip}, \Im, \partial\rangle$ is soft first N countable. □

Theorem 6.3. *A NS α countable space in which every NSconvergent sequence has a unique soft limit is a $NS\alpha_2$ space.*

Proof. Let $\langle x^{crip}, \Im, \partial\rangle$ be $NS\alpha_2$ space and let $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle$ be a soft convergent sequence in $\langle x^{crip}, \Im, \partial\rangle$ .We prove that the limit of this sequence is unique. We prove this result by contradiction. Suppose $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle$ converges to two soft points $\tilde{I}$ and $\tilde{m}$ such that $\tilde{I} \neq \tilde{m}$. Then by trichotomy law either $\tilde{I} < \tilde{m}$ or $\tilde{I} > \tilde{m}$. Since the possess the $NS\alpha_2$f characteristics, there must happen two NS α open sets $\langle \wp, \partial\rangle$ and $\langle \rho, \partial\rangle$ such that $\langle \wp, \partial\rangle \widetilde{\cap} \langle \rho, \partial\rangle = 0_{(\langle \widetilde{x}\rangle, \partial^{parameter})}$.That is, the possibility of one rules out the possibility of other.Now, $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle$ converges to $\tilde{I}$ so $\exists$ an interger n_1 s.t.$\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle \propto \langle \wp, \partial\rangle \forall n \geq n_1$. Also, $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle$ converges to $\tilde{m}$ so there exists an interger n_2 such that$\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle \propto \langle \rho, \partial\rangle \forall n \geq n_1$. We are interested to discuss the maximum possibility, fot that we must suppose maximum of both the integers which will enable us to discuss the soft sequence for single soft number. Max$(n_1, n_2) = n_0$. Which leads to the situation $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle \propto \langle \wp, \partial\rangle \forall n \geq n_0$ and $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle \propto \langle \rho, \partial\rangle \forall n \geq n_0$. This implies that $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle \propto \langle \wp, \partial\rangle$ and $\langle (x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n\rangle \propto \langle \rho, \partial\rangle, \forall n \geq n_0$.This guarantees that $(x^e_{(a,b,c)}, \widetilde{\exists^{parameter}})_n \propto$

$(\langle\wp,\partial\rangle\widetilde{\cap}\langle\rho,\partial\rangle)\forall n \geq n_0$ .Which beautifully contradict the fact that $\langle\wp,\partial\rangle\widetilde{\cap}\langle\rho,\partial\rangle = 0_{(\langle\widetilde{x}\rangle,\partial^{parameter})}$. Hence, the limit of the NSsequence must be unique. Let $\langle x^{crip},\Im,\partial\rangle$ be $NS\alpha_2$ be the NS first axiom space in which every NSconvergent sequence has a unique soft limit. We prove the space is NS_2 space, suppose if possible $\langle x^{crip},\Im,\partial\rangle$ be $NS\alpha_2$ is not NS_2 space. Then there must exists two points $(x^e_{(a,b,c)},\daleth^{parameter}) \neq (y^{e'}_{(a',b',c')},\daleth^{parameter})$,that is $(x^e_{(a,b,c)},\daleth^{parameter}) \succ (y^{e'}_{(a',b',c')},\daleth^{parameter})$ or $(x^e_{(a,b,c)},\daleth^{parameter}) \prec (y^{e'}_{(a',b',c')},\daleth^{parameter})$ such that every soft set containinig $(x^e_{(a,b,c)},\daleth^{parameter})$ has a non-empty intersection with every NS α open set containing $(y^{e'}_{(a',b',c')},\daleth^{parameter})$. Since the space is NSfirst countable, there exist nested monotone decreasing NS local bases at x and $(y^{e'}_{(a',b',c')},\daleth^{parameter})$. Let $\Re(x^e_{(a,b,c)},\daleth^{parameter}) = \langle B_1(x^e_{(a,b,c)},\daleth^{parameter}),......B_n((x^e_{(a,b,c)},\daleth^{parameter}))$ $n \propto N\rangle$ and
$\Re(y^{e'}_{(a',b',c')},\daleth^{parameter}) = \langle B_1(y^{e'}_{(a',b',c')},\daleth^{parameter}),......B_n(y^{e'}_{(a',b',c')},\daleth^{parameter}) : n \propto N\rangle$ be the nested NS local bases at$(x^e_{(a,b,c)},\daleth^{parameter})$ and $(y^{e'}_{(a',b',c')},\daleth^{parameter})$. respectively.Then, we nust have $B_n(x^e_{(a,b,c)},\daleth^{parameter})\widetilde{\cap}B_n((y^{e'}_{(a',b',c')},\daleth^{parameter})) \neq \widetilde{O}\forall n \propto N$ and so $\exists(x^e_{(a,b,c)},\daleth^{parameter})_n \propto \langle x^{crip},\Im,\partial\rangle$ s.t. $(x^e_{(a,b,c)},\daleth^{parameter}) \propto B_n(x^e_{(a,b,c)},\daleth^{parameter})\widetilde{\cap}B_n((y^{e'}_{(a',b',c')}$ N. Therefore,
$(x^e_{(a,b,c)},\daleth^{parameter})_n \propto B_n(x)$ and $(x^e_{(a,b,c)},\daleth^{parameter})_n \propto B_n((y^{e'}_{(a',b',c')},\daleth^{parameter}))\forall n \propto N$.Let $\langle\wp,\partial\rangle$ and $\langle\rho,\partial\rangle$ be arbitrary NS α open set such that $(x^e_{(a,b,c)},\daleth^{parameter}) \propto \langle\wp,\partial\rangle$ and$(y^{e'}_{(a',b',c')},\daleth^{parameter}) \propto \langle\rho,\partial\rangle$. Then by definition of soft nested base, there exists an integer n_o such that $B_n((x^e_{(a,b,c)},\daleth^{parameter})) \Subset \langle\wp,\partial\rangle$ and $B_n((y^{e'}_{(a',b',c')},\daleth^{parameter})) \Subset \langle\rho,\partial\rangle n > n_0$. This means that $(x^e_{(a,b,c)},\daleth^{parameter})_n \propto \langle\wp,\partial\rangle$ and $(y^{e'}_{(a',b',c')},\daleth^{parameter})_n \propto \langle\rho,\partial\rangle n > n_0$. It allows that $(x^e_{(a,b,c)},\daleth^{parameter})_n \to (x^e_{(a,b,c)},\daleth^{parameter})$ and $(y^{e'}_{(a',b',c')},\daleth^{parameter})_n \to (y^{e'}_{(a',b',c')},\daleth^{parameter}$. But this is contradiction to the given that every soft convergent sequence in $\langle x^{crip},\Im,\partial\rangle$ has unique soft limit. Hence $\langle x^{crip},\Im,\partial\rangle$ is NS α_2 space. □

Theorem 6.4. *The cardinality of all NSopen sets in a neutrosophic second countable space is at most equal to C (the power of the continuum).*

Proof. Let $\langle x^{crip},\Im,\partial\rangle$ be NSSTsuch that is NSsecond countable space. Let $\langle\zeta,\partial\rangle$ be any soft set of $\langle x^{crip},\Im,\partial\rangle$. Then $\langle\zeta,\partial\rangle$ is the soft union of a certain soft sub-collection of the NScountable collection $\{\langle W,\partial\rangle_i : i = 1,2,3....\}$. Hence the cardinality of the set of all NS? open sets in $\langle x^{crip},\Im,\partial\rangle$ is not greater than the cardinality of the soft set of all soft sub-collections of the NScountable collection $\{\langle W,\partial\rangle_i : i = 1,2,3....\}$. thus the cardinality of $\langle x^{crip},\Im,\partial\rangle \prec C$. □

Theorem 6.5. *Any collection of mutually exclusive NSopen sets in a NSsecond countable space is at most NScountable.*

Proof. $\langle x^{crip},\Im,\partial\rangle$ be NSSTsuch that it is NSsecond countable. Let $\langle\sqsupset,\partial\rangle$ signifies any collection of mutually exclusive NS α open sets in $\langle x^{crip},\Im,\partial\rangle$. . Let $\langle U,\partial\rangle \propto \langle\sqsupset,\partial\rangle$, then $\exists$ at least one soft set $\langle W, \partial \rangle_m$,belonging to the collection $\{\langle W, \partial \rangle_i : i = 1, 2, 3....\}$ such that$\langle W, \partial \rangle_m \Subset \langle U, \partial \rangle$. Let n be the smallest suffix for which $\langle W, \partial \rangle_n \Subset \langle U, \partial \rangle$. Since the soft sets $\langle U, \partial \rangle$ in $\langle \beth, \partial \rangle$ are mutually disjoint, it follows that, for different soft sets $\langle U, \partial \rangle \propto \langle \beth, \partial \rangle$,there correspond soft sets$\langle W, \partial \rangle_n$ with different suffices n. Hence the soft sets in $\langle \zeta, \partial \rangle$ are in a (1,1)-correspondence with a soft sub-collection of the NScountable collection $\{\langle W, \partial \rangle_i : i = 1, 2, 3....\}$; consequently,the cardinality of $\langle \beth, \partial \rangle$ is less than or equal to d. $\square$

Theorem 6.6. *Let $\langle x^{crip}, \Im, \partial \rangle$ be NSST such that it is α_2 space.Then this space posses an infinite soft sequence of non NS α open sets which are mutually exclusive*

Proof. Given $\langle x^{crip}, \Im, \partial \rangle$ be NSST.If the space $\langle x^{crip}, \Im, \partial \rangle$ is soft discrete, then definitely every one-pointic soft set is NS α open ; and these NS α open sets constitute an in-finite soft set of mutually dis-joint non-null NS α open subsets of $\langle x^{crip}, \Im, \partial \rangle$,and an infinite soft sequence can be picked from this infinite soft set. Next, $\langle x^{crip}, \Im, \partial \rangle$ be not a soft discrete space; then ?a point $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ in x^{crip} which is not a limiting point of x^{crip}. Let $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1$ be a poit in x^{crip} which may be at least one of the possibilities $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \succ (x^{e}_{(a,b,c)}, \daleth^{parameter})_1$ or $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1 \prec (y^{e'}_{(a',b',c')}, \daleth^{parameter})$ then by NS α_2 space,$\exists$ two NS α open sets $\langle U_1, \partial \rangle$ and $\langle V_1, \partial \rangle$ such that $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1 \propto \langle U_1, \partial \rangle, q \propto \langle V_1, \partial \rangle$ and $\langle U_1, \partial \rangle \widetilde{\cap} \langle V_1, \partial \rangle = 0_{(\langle \tilde{X} \rangle,, \daleth^{parameter})}$. Again, since $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto \langle V_1, \partial \rangle$ and q is limiting point of x^{crip} $\exists$ a point $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \propto \langle V_1, \partial \rangle$ which behaves as $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \succ q$ or $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \prec (y^{e'}_{(a',b',c')}, \daleth^{parameter})$. This implies that these are points of NS α_2 space. So by definition $\exists$ two NS α open sets $\langle U_2, \partial \rangle$ and $\langle V_2, \partial \rangle$ such that $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \propto \langle U_2, \partial \rangle$,$(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto \langle V_2, \partial \rangle$ and $\langle U_1, \partial \rangle \widetilde{\cap} \langle V_1, \partial \rangle = o_{(\langle \widetilde{x} \rangle, \partial^{\widetilde{parameter}})}$ Again, since $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto \langle V_1, \partial \rangle$ and q is limiting point of x^{crip} $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \propto \langle V_1, \partial \rangle$ which behaves as $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \succ q$ or $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \prec (y^{e'}_{(a',b',c')}, \daleth^{parameter})$. This implies that these are points of NS α_2 space. So by definition $\exists$ two NS α open sets $\langle U_2, \partial \rangle$ and $\langle V_2, \partial \rangle$ such that $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \propto \langle U_2, \partial \rangle, (y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto \langle V_2, \partial \rangle$ and $\langle U_2, \partial \rangle \widetilde{\cap} \langle V_2, \partial \rangle = o_{(\langle \widetilde{x} \rangle, \partial^{\widetilde{parameter}})}$ as$\langle U_2, \partial \rangle \Subset \langle V_1, \partial \rangle$ and $\langle V_2, \partial \rangle \Subset \langle V_1, \partial \rangle$, for otherwise we could pick $\langle U_2, \partial \rangle \widetilde{\cap} \langle V_1, \partial \rangle$ and $\langle V_2, \partial \rangle \widetilde{\cap} \langle V_1, \partial \rangle$ in place of $\langle U_2, \partial \rangle$ and $\langle V_2, \partial \rangle$ respectively. Keeping continue the above process repeatedly and determing the points $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1, (x^{e}_{(a,b,c)}, \daleth^{parameter})_2, , , (x^{e}_{(a,b,c)}, \daleth^{parameter})_n$ which may behave as

$(x^{e}_{(a,b,c)}, \daleth^{parameter})_1 \succ, (x^{e}_{(a,b,c)}, \daleth^{parameter})_2 \succ, (x^{e}_{(a,b,c)}, \daleth^{parameter})_3 \succ, (x^{e}_{(a,b,c)}, \daleth^{parameter})_4 \succ$ $, (x^{e}_{(a,b,c)},$
$\daleth^{parameter})_5 \succ, (x^{e}_{(a,b,c)}, \daleth^{parameter})_6 \succ (x^{e}_{(a,b,c)}, \daleth^{parameter})_7 \succ ... \succ (x^{e}_{(a,b,c)}, \daleth^{parameter})_n$

or

$(x^e_{(a,b,c)}, \daleth^{parameter})_1 \prec, (x^e_{(a,b,c)}, \daleth^{parameter})_2 \prec, (x^e_{(a,b,c)}, \daleth^{parameter})_3 \prec, (x^e_{(a,b,c)}, \daleth^{parameter})_4 \prec$
$, (x^e_{(a,b,c)},$
$\daleth^{parameter})_5 \prec, (x^e_{(a,b,c)}, \daleth^{parameter})_6 \prec (x^e_{(a,b,c)}, \daleth^{parameter})_7 \prec ... \prec (x^e_{(a,b,c)}, \daleth^{parameter})_n$ and accordingly the NSsoft open sets
$\langle U_1, \partial\rangle, \langle U_2, \partial\rangle, \langle U_3, \partial\rangle, \langle U_4, \partial\rangle, \langle U_5, \partial\rangle......\langle U_n, \partial\rangle$ and $\langle V_1, \partial\rangle, \langle V_2, \partial\rangle, \langle V_3, \partial\rangle, \langle V_4, \partial\rangle, \langle V_5, \partial\rangle.....\langle V_n, \partial\rangle$ such that for $k = 1, 2, 3, 4, ., n, (x^e_{(a,b,c)}, \daleth^{parameter})_k \propto \langle U_k, \partial\rangle, \Subset \langle V_{k-1}, \partial\rangle, (y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto$ $\langle V_k, \partial\rangle \Subset \langle V_{k-1}, \partial\rangle$ and $\langle U_k, \partial\rangle \widetilde{\cap} \langle V_k, \partial\rangle = o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}$ Now, since $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$ is a limiting point of X^{crip} and $\langle V_n, \partial\rangle$ NS α open set containing $(y^{e'}_{(a',b',c')}, \daleth^{parameter})$, there exists a point $(x^e_{(a,b,c)}, \daleth^{parameter})_{k+1}$ in X^{crip}, different from q and lying within $\langle V_n, \partial\rangle$. Then by NS α_2 space, $\exists$ two NSopen sets $\langle U_{n+1}, \partial\rangle$ and $\langle V_{n+1}, \partial\rangle$ satisfying the conditions $(x^e_{(a,b,c)}, \daleth^{parameter})_{k+1} \propto \langle U_{n+1}, \partial\rangle$, $(y^{e'}_{(a',b',c')}, \daleth^{parameter}) \propto \langle V_{n+1}, \partial\rangle$ and $\langle U_{n+1}, \partial\rangle \widetilde{\cap} \langle V_{n+1}, \partial\rangle = o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}$. As above it may be so disciplinized as $\langle U_{n+1}, \partial\rangle \Subset$ $\langle V_n, \partial\rangle$ and $\langle V_{n+1}, \partial\rangle \Subset \langle V_n, \partial\rangle$.The infinite soft sequence of NS α open sets $\langle U_1, \partial\rangle, \langle U_2, \partial\rangle, \langle U_3, \partial\rangle, \langle U_4, \partial$ thus defined by induction, the presence of one rules out the possibility of other. Since $\langle U_{n+1}, \partial\rangle \Subset \langle V_n, \partial\rangle, \langle U_k, \partial\rangle \widetilde{\cap} \langle V_k, \partial\rangle = o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}$ and $\langle V_n, \partial\rangle \Subset \langle V_{n+1}, \partial\rangle$ for $n =$ $1, 2, 3....$ □

Theorem 6.7. *Let $\langle x^{crip}, \Im, \partial\rangle$ be NSSTsuch that it is NSsecond countabley NS α_2 space. Then set of all NS α open sets has the cardinalityC.*

Proof. Let $\langle x^{crip}, \Im, \partial\rangle$ be second countable NS α_2.Then, by theorem 2.6, there exists in NSST $\langle x^{crip}, \Im, \partial\rangle$ an infinite soft sequence of NS α open sets $\langle \zeta, \partial\rangle_1, \langle \zeta, \partial\rangle_2, \langle \zeta, \partial\rangle_3, \langle \zeta, \partial\rangle_4 \langle \zeta, \partial\rangle_5, ..., \langle \zeta, \partial$
$o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}, \langle \zeta, \partial\rangle_2 \neq o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}, \langle \zeta, \partial\rangle_3 \neq o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}, \langle \zeta, \partial\rangle_4 \neq o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}, ...\langle \zeta, \partial\rangle_n \neq$
$o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}.......$
$\langle \zeta, \partial\rangle_1 \widetilde{\cap} \langle \zeta, \partial\rangle_2 \widetilde{\cap} \langle \zeta, \partial\rangle_3 \widetilde{\cap} \langle \zeta, \partial\rangle_4 \widetilde{\cap}.....\widetilde{\cap} \langle \zeta, \partial\rangle_n \widetilde{\cap}... = o_{(\widetilde{\langle x\rangle}, \widetilde{\partial^{parameter}})}$. sub-sequences of the sequences
$\langle \zeta, \partial\rangle_1, \langle \zeta, \partial\rangle_2, \langle \zeta, \partial\rangle_3, \langle \zeta, \partial\rangle_4 \langle \zeta, \partial\rangle_5, ..., \langle \zeta, \partial\rangle_n$ nwill determine, as their unions, different NS α open sets. But since the soft set of all soft sub-sets of a countable soft set has the cardinality C, it follows that soft set of all the NS α open sets in $\langle x^{crip}, \Im, \partial\rangle$ has the cardinality$\succeq C$. Again by the theorem 2.5, the cardinality of the soft set of all NS open sets in $\langle x^{crip}, \Im, \partial\rangle \succeq C$. consequently, the cardinality of all NS α open sets in $\langle x^{crip}, \Im, \partial\rangle$ is exactly equal to C. □

Theorem 6.8. *Let $\langle x^{crip}, \Im, \partial\rangle$ be NSSTsuch that it is NS second countable α_1 space the the soft set of all NSpoints in this space has the cardinality at most equal to C(i.e, posses the power of the continuum at most).*

Proof. Given that $\langle x^{crip}, \Im, \partial\rangle$ is NS second countable T_1 space, for which the NS sets $\langle W, \partial\rangle_1, \langle W, \partial\rangle_2, \langle W, \partial\rangle_3, \langle W, \partial\rangle_4 \langle W, \partial\rangle_5, .$ Forms a soft countable NS α open base of

$\langle x^{crip}, \Im, \partial\rangle$. Then, for any given point $(x^e_{(a,b,c)}, \dashv^{parameter}) \propto x^{crip}, \langle W, \partial\rangle_{\zeta 1}, \langle W, \partial\rangle_{\zeta 2}, \langle W, \partial\rangle_{\zeta 3}, \langle W, \partial\rangle_{\zeta 4} \langle$ generate countable NS open base about the point ζ in NSST $\langle x^{crip}, \Im, \partial\rangle$, where $\zeta i, i = 1, 2, 3, ...,$ is a soft sub-sequence of the soft sequence $\langle W, \partial\rangle_n : n = 1, 2, 3, ...,$ consisting of all those $\langle W, \partial\rangle_4$ which contain the point ζ .Since a second NS countable space is necessary first NS countable. So corresponding to the point ζ , there exists an infinite soft sequence of NS open sets $\{\langle \ell, \partial\rangle_n : i = i, 2, 3, \}$ which is a soft sub-sequence of the soft sequence. $\{\langle W, \partial\rangle_{pn} : n = 1, 2, 3, ...\}$,and therefore also a soft sub-sequence of the soft sequence $\{\langle W, \partial\rangle_n : n = 1, 2, 3, ...\}$, such that $\widetilde{\cap}\{\langle \ell, \partial\rangle_i : i = i, 2, 3, \} = \{(x^e_{(a,b,c)}, \dashv^{parameter})\}$. Thus to each point $(x^e_{(a,b,c)}, \dashv^{parameter})$ in x^{crip},there corresponds a soft sub-sequence $\{\langle \ell, \partial\rangle_i : i = i, 2, 3, \}$ of the soft sequence $\{\langle W, \partial\rangle_n : n = 1, 2, 3, ...\}$; and two different points there correspond two such different soft sub-spaces $\{\langle \ell, \partial\rangle_i : i = i, 2, 3, \}$. Hence the soft set of all points in x^{crip} ,i.e. the crisp set x^{crip} ,has the same cardinality as that of a certain soft sub-collection of the soft collection of all soft sub-space of the sequence $\{\langle W, \partial\rangle_n : n = 1, 2, 3, ...\}$. Thus the cardinality of x^{crip} less than or equal to C. In other words, the crisp set x^{crip} has the power of the continuum at most. □

Theorem 6.9. *Let $\langle x^{crip}, \Im, \partial\rangle$ be NSST such that it is NS second countable. Every soft uncountable soft subsets contains a point of condensation.*

Proof. Since $\langle x^{crip}, \Im, \partial\rangle$ is NS second countable space and $\{\langle W, \partial\rangle_i : i = 1, 2, 3, ...\}$ be a soft countable NS α open base of $\langle x^{crip}, \Im, \partial\rangle$. Let $\langle \wp, \partial\rangle$ be a NS sub-set of x^{crip} such that $\langle \wp, \partial\rangle$ does not contain any point of condensation. For each point $(x^e_{(a,b,c)}, \dashv^{parameter}) \propto \langle \wp, \partial\rangle, (x^e_{(a,b,c)}, \dashv^{parameter})$ is not point of condensation of $\langle \wp, \partial\rangle$. Hence there exists NS α open set $\langle \zeta, \partial\rangle$ containing $(x^e_{(a,b,c)}, \dashv^{parameter})$ such that $\langle \zeta, \partial\rangle \widetilde{\cap} \langle \wp, \partial\rangle$ is soft countable at most. $\exists$ a suffix n $((x^e_{(a,b,c)}, \dashv^{parameter}))$, such that $(x^e_{(a,b,c)}, \dashv^{parameter}) \propto \langle W, \partial\rangle_{n(p)} \Subset \langle \zeta, \partial\rangle$;and then $\langle \wp, \partial\rangle \widetilde{\cap} \langle W, \partial\rangle_{n(p)}$ is also NS countable at most. But we can express $\langle \wp, \partial\rangle$ in the form $\langle \wp, \partial\rangle = \widetilde{\cap}\{(x^e_{(a,b,c)}, \dashv^{parameter}) : (x^e_{(a,b,c)}, \dashv^{parameter}) \propto \langle \wp, \partial\rangle \widetilde{\cup} \{\langle \wp, \partial\rangle \widetilde{\cap} \langle W, \partial\rangle_{n((x^e_{(a,b,c)}, \dashv^{parameter}}$ $(x^e_{(a,b,c)}, \dashv^{parameter}) \propto \langle \wp, \partial\rangle\}$ be at most a NS countable number of different suffices. So, $\langle \wp, \partial\rangle$ is at most a NS union of NS uncountable soft sub-set; that is $\langle \wp, \partial\rangle$ is at most a NS countable soft sub-set of $\widetilde{X}$. Consequently, if $\langle \wp, \partial\rangle$ is soft uncountable soft subset of, then it must possess a point of condensation. □

Theorem 6.10. *$\langle x^{crip}, \Im, \partial\rangle$ be NSST such that it is NS second countable. If $\langle \Psi, \partial\rangle$ is uncountable NS sub-set of $\langle x^{crip}, \Im, \partial\rangle$, then the soft sub-set $\langle \varpi, \partial\rangle$, consisting of all those N points of $\langle \Psi, \partial\rangle$ which are not points of condensation of $\langle \Psi, \partial\rangle$, is at most NS countable.*

Proof. Since $\langle x^{crip}, \Im, \partial\rangle$ is NS second countable space, and $\{\langle W, \partial\rangle_i : i = 1, 2, 3, ...\}$ be a countable NS? open base of $\langle x^{crip}, \Im, \partial\rangle$. Let $\{\langle V, \partial\rangle_i : i = 1, 2, 3, ...\}$ be a soft sub-

sequence of the soft sequence $\{\langle W,\partial\rangle_i : i=1,2,3,...\}$. consisting of all those soft sets $\langle W,\partial\rangle_j$ for which $\langle W,\partial\rangle_j \widetilde{\cap} \langle \varpi,\partial\rangle$ is at most soft countable. Then $\{\langle V,\partial\rangle_i\}\widetilde{\cap}\langle\varpi,\partial\rangle : i = 1,2,3,...$is at most NS countable .We shall establish that $\langle\varpi,\partial = \langle\Psi,\partial\rangle\rangle\widetilde{\cap}(\langle V,\partial\rangle_1\widetilde{\cup}\langle V,\partial\rangle_2\widetilde{\cup},....)$. $(x^e_{(a,b,c)},\daleth^{parameter}) \propto \langle\Psi,\partial\rangle$,there is not a point of condensation of $\langle\Psi,\partial\rangle$;hence there exists a soft nbd $\langle U,\partial\rangle$ of $(x^e_{(a,b,c)},\daleth^{parameter})$, such that $\langle U,\partial\rangle\widetilde{\cap}\langle\Psi,\partial\rangle$ is at most NS countable.Also,$\exists$ a soft set $\langle\Psi,\partial\rangle_j$, belonging to the soft sequence $\{\langle W,\partial\rangle_i : i = 1,2,3,...\}$,satisfying $(x^e_{(a,b,c)},\daleth^{parameter}) \propto \langle W,\partial\rangle_j \Subset \langle U,\partial\rangle$. Then $\langle W,\partial\rangle_j\widetilde{\cap}\langle\Psi,\partial\rangle$ is at most NS countable, and so $\langle W,\partial\rangle_j$ must be one of the soft sets $\langle V,\partial\rangle_j$ and therefore $(x^e_{(a,b,c)},\daleth^{parameter}) \propto \langle V,\partial\rangle_i$. Again, since $(x^e_{(a,b,c)},\daleth^{parameter}) \propto \langle\Psi,\partial\rangle$, it follows that $(x^e_{(a,b,c)},\daleth^{parameter}) = \langle\Psi,\partial\rangle\rangle\widetilde{\cap}(\langle V,\partial\rangle_1\widetilde{\cup}\langle V,\partial\rangle_2\widetilde{\cup},....)$.. Next, let
$(y^{e'}_{(a',b',c')},\daleth^{parameter}) = \langle\Psi,\partial\rangle\rangle\widetilde{\cap}(\langle V,\partial\rangle_1\widetilde{\cup}\langle V,\partial\rangle_2\widetilde{\cup},....)$. then
$(y^{e'}_{(a',b',c')},\daleth^{parameter})$ belomgs to some $\langle V,\partial\rangle_i$. Now, as $\langle V,\partial\rangle_i$ is a soft nbd of
$(y^{e'}_{(a',b',c')},\daleth^{parameter})$,and $\langle V,\partial\rangle_i\widetilde{\cap}\langle\Psi,\partial\rangle$ is at most NS countable.
$(y^{e'}_{(a',b',c')},\daleth^{parameter})$ can not be a point of condensation of $\langle\Psi,\partial\rangle$. Also, as
$(y^{e'}_{(a',b',c')},\daleth^{parameter}) \propto \langle\Psi,\partial\rangle$. It follows that $(y^{e'}_{(a',b',c')},\daleth^{parameter}) \propto \langle\varpi,\partial\rangle$.Thus
$\langle\varpi,\partial = \langle\Psi,\partial\rangle\rangle\widetilde{\cap}(\langle V,\partial\rangle_1\widetilde{\cup}\langle V,\partial\rangle_2\widetilde{\cup},....) = \langle\varpi,\partial = \langle\Psi,\partial\rangle\rangle\widetilde{\cap}(\langle V,\partial\rangle_1\widetilde{\cup}\langle V,\partial\rangle_2\widetilde{\cup},....)$.
and since each $\langle\Psi,\partial\rangle\widetilde{\cup}\langle V,\partial\rangle_1$ is at most NS countable. It follows that $\langle\varpi,\partial\rangle$ is also at most NS countable. $\square$

7 Neutrosophic soft Monotonous Functions

Theorem 7.1. *Let $\langle\chi^{crip},\Im,\partial\rangle$ be NSST such that it is NS α_2 space and $\langle Y^{crip},\Im,\partial\rangle$ be anyNSST. Let$\langle f,\partial\rangle$: $\langle\chi^{crip},\Im,\partial\rangle \to \langle\chi^{crip},\Im,\partial\rangle$ be a soft fuction such that it is soft monotone and continuous. Then $\langle\chi^{crip},\Im,\partial\rangle$ is also of characteristics of NS α_2.*

Proof. Suppose $(x^e_{(a,b,c)},\daleth^{parameter})_1$, $(x^e_{(a,b,c)},\daleth^{parameter})_2 \ltimes \chi^{crip}$ s.t. either $(x^e_{(a,b,c)},\daleth^{parameter})_1$ $> (x^e_{(a,b,c)},\daleth^{parameter})_2$ or $(x^e_{(a,b,c)},\daleth^{parameter})_1 < (x^e_{(a,b,c)},\daleth^{parameter})_2$. Since $\langle f,\partial\rangle$ is soft monotone. Let us suppose the monotonically increasing case. So, $(x^e_{(a,b,c)},\daleth^{parameter})_1 >$ $f_(x^e_{(a,b,c)},\daleth^{parameter})_2$ or $(x^e_{(a,b,c)},\daleth^{parameter})_1 < f_(x^e_{(a,b,c)},\daleth^{parameter})_2$ implies that $f_(x^e_{(a,b,c)},\daleth^{parameter})_1$ $> f(x_2)$ or $f_(x^e_{(a,b,c)},\daleth^{parameter})_1 < (x^e_{(a,b,c)},\daleth^{parameter})_2$ respectively. Suppose $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_1$,$(y^{e/}_{(a$ $\ltimes Y^{crip}$ such that $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_1$ $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_2$ or $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_1$ $< (y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_2$ So, $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_1 > (y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_2$ or $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_1$ $< (y^{e/}_{(a/,b/,c/)},\daleth^{parameter})_2$ respectively such that $(y^{e/}_{(a/,b/,c/)},\daleth^{parameter}) = f_{(x^e_{(a,b,c)},\daleth^{parameter})}1$,$(y^{e/}_{(a/,b/,c/)},\daleth$ $= f_{(x^e_{(a,b,c)},\daleth^{parameter})}2$.
Since $\langle\chi^{crip},\Im,\partial\rangle$ is NS α_2 space so there exists mutually disjoint NS α open sets $\langle k_1,\partial\rangle$ and $\langle k_2,\partial\rangle$, $\ltimes$ $\langle\chi^{crip},\Im,\partial\rangle \Rightarrow f(\langle k_1,\partial\rangle)$ and $\langle k_2,\partial\rangle$, $\ltimes$ $\langle\chi^{crip},\Im,\partial\rangle$. We claim that $f(\langle k_1,\partial\rangle)\widetilde{\cap}f(\langle k_2,\partial\rangle) = 0_{(\langle\widetilde{\chi}\rangle,\partial parameter)}$. Otherwise $\exists$ $(t^{e//}_{(a//,b//,c//)},\daleth^{parameter})_1$ $\ltimes$

$f|(\langle k_1, \partial\rangle)\widetilde{\cap}f(\langle k_2, \partial\rangle) \Rightarrow (t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_1\, f(\langle k_1, \partial\rangle)$ and $(t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_1$ $\ltimes f(\langle k_2, \partial\rangle)$, $(t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_1 \ltimes f(\langle k_1, \partial\rangle)$, f is soft one-one $\exists\ (t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_2$ $\ltimes f(\langle k_1, \partial\rangle)$ s.t. $(t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_1 = f((t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_2)$, $(t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_1$ $\ltimes f(\langle k_2, \partial\rangle) \Rightarrow \exists\ (t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_3 \ltimes f(\langle k_2, \partial\rangle)$ s.t. $(t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_1 =$ $f((t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_3) \Rightarrow (t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_2 \ltimes f(\langle k_1, \partial\rangle)$, $(t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_2$ $\ltimes f(\langle k_2, \partial\rangle) \Rightarrow (t^{e//}_{(a//,b//,c//)}, \daleth^{parameter})_2 \ltimes f(\langle k_1, \partial\rangle)\ \widetilde{\cap}f(\langle k_2, \partial\rangle)$. This is contradiction because $\langle k_1, \partial\rangle\widetilde{\cap}\langle k_2, \partial\rangle = 0_{(\langle\widetilde{\chi}\rangle,\partial parameter)}$. Therefore $f(\langle k_1, \partial\rangle)\widetilde{\cap}f(\langle k_2, \partial\rangle) = 0_{(\langle\widetilde{\chi}\rangle,\partial parameter)}$. Finally, $(x^e_{(a,b,c)}, \daleth^{parameter})_1 > (x^e_{(a,b,c)}, \daleth^{parameter})_2$ or $(x^e_{(a,b,c)}, \daleth^{parameter})_1 < (x^e_{(a,b,c)}, \daleth^{parameter})_2$ $\Rightarrow (x^e_{(a,b,c)}, \daleth^{parameter})_1 \neq (x^e_{(a,b,c)}, \daleth^{parameter})_2 \Rightarrow f((x^e_{(a,b,c)}, \daleth^{parameter})_1) > f((x^e_{(a,b,c)}, \daleth^{parameter})_2)$ or $f((x^e_{(a,b,c)}, \daleth^{parameter})_1) < f((x^e_{(a,b,c)}, \daleth^{parameter})_2) \Rightarrow f((x^e_{(a,b,c)}, \daleth^{parameter})_1) \neq f((x^e_{(a,b,c)}, \daleth^{parameter})_2)$. Given a pair of points $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1$, $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2 \ltimes Y^{crip} \ni (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1$ $\neq (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ We are able to find out mutually exclusive NS α open sets $f(\langle k_1, \partial\rangle)$, $f(\langle k_2, \partial\rangle) \ltimes \langle Y^{crip}, \Im, \partial\rangle$ s.t. $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 \ltimes f(\langle k_1, \partial\rangle)$, $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ $\ltimes f(\langle k_2, \partial\rangle)$. This proves that $\langle Y^{crip}, \Im, \partial\rangle$ is NS $\alpha_{(2)}$ space. □

Theorem 7.2. *Let $\langle \chi^{crip}, \Im, \partial\rangle$ be NSST and $\langle Y^{crip}, \Im, \partial\rangle$ be an-other NSST which satisfies one more condition of NS α_2. Let$\langle f, \partial\rangle$: $\langle \chi^{crip}, \Im, \partial\rangle \to \langle Y^{crip}, \Im, \partial\rangle$ be a soft function such that it is soft monotone and continuous. Then $\langle \chi^{crip}, \Im, \partial\rangle$ is also of characteristics of NS α_2.*

Proof. Suppose $(x^e_{(a,b,c)}, \daleth^{parameter})_1$, $(x^e_{(a,b,c)}, \daleth^{parameter})_2 \ltimes \chi^{crip}$ s.t. either $(x^e_{(a,b,c)}, \daleth^{parameter})_1$ $> (x^e_{(a,b,c)}, \daleth^{parameter})_2$ or $(x^e_{(a,b,c)}, \daleth^{parameter})_1 < (x^e_{(a,b,c)}, \daleth^{parameter})_2$. Let us suppose the monotonically increasing case. So, $(x^e_{(a,b,c)}, \daleth^{parameter})_1 > f_(x^e_{(a,b,c)}, \daleth^{parameter})_2$ or $(x^e_{(a,b,c)}, \daleth^{parameter})_1 < f_(x^e_{(a,b,c)}, \daleth^{parameter})_2$ implies that $f((x^e_{(a,b,c)}, \daleth^{parameter})_1) > f((x^e_{(a,b,c)}, \daleth^{parameter})_2)$ or $f((x^e_{(a,b,c)}, \daleth^{parameter})_1) < f((x^e_{(a,b,c)}, \daleth^{parameter})_2)$ respectively. Suppose $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1$,$(y^{e/}_{(a/,b/}$ $\ltimes Y^{crip}$ such that $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ or $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1$ $< (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ So, $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ or $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1$ $< (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ respectively such that
$(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 = f((x^e_{(a,b,c)}, \daleth^{parameter})_1)$,$(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2 = f((x^e_{(a,b,c)}, \daleth^{parameter})_2)$ s.t. $(x^e_{(a,b,c)}, \daleth^{parameter})_1 = f^{-1}(y^1)$ and $(x^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 = f^{-1}((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2)$. Since $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1$, $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2 \ltimes Y^{crip}$ but $\langle Y^{crip}, \Im, \partial\rangle$ is NS α_2 space. So according to definition $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ or $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 < (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$. There definitely exists NS open sets $\langle k_1, \partial\rangle$ and $\langle k_2, \partial\rangle \ltimes Y^{crip}, \Im, \partial\rangle$ such that $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 \ltimes \langle k_1, \partial\rangle$ and $(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ $\ltimes \langle k_2, \partial\rangle$ and these two NS open sets are guaranteedly mutually exclusive because the possibility of one rules out the possibility of other. Since $f^{-1}(\langle k_1, \partial\rangle)$ And $f^{-1}(\langle k_2, \partial\rangle)$

are NS open in $\langle \chi^{crip}, \Im, \partial \rangle$. Now $f^{-1}(\langle k_1, \partial \rangle) \widetilde{\cap} f^{-1}(\langle k_1, \partial \rangle) = f^{-1}(\langle k_1, \partial \rangle) \widetilde{\cap} f^{-1}(\langle k_2, \partial \rangle)$ $= f^{-1}(\widetilde{\phi}) = 0_{(\langle \widetilde{\chi} \rangle, \partial \widetilde{parameter})}$ and $(y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})_1 \ltimes \langle k_1, \partial \rangle \Rightarrow f^{-1}((y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})_1)$ $\ltimes f^{-1}(\langle k_1, \partial \rangle) \Rightarrow (x^e_{(a,b,c)}, \dashv^{parameter})_1 \ltimes (\langle k_1, \partial \rangle)$, $(y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})_2 \ltimes \langle k_2, \partial \rangle \Rightarrow$ $f^{-1}((y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})_2) \ltimes f^{-1}(\langle k_2, \partial \rangle)$ implies that $(x^e_{(a,b,c)}, \dashv^{parameter})_2 \ltimes (\langle k_2, \partial \rangle)$. We see that it has been shown for every pair of distinct points of χ^{crip}, there suppose disjoint NS α open sets namely, $f^{-1}(\langle k_1, \partial \rangle)$ and $f^{-1}(\langle k_2, \partial \rangle)$ belong to $\chi^{crip}, \Im, \partial \rangle$ such that $(x^e_{(a,b,c)}, \dashv^{parameter})_1 \ltimes f^{-1}(\langle k_1, \partial \rangle)$ and $(x^e_{(a,b,c)}, \dashv^{parameter})_2 \ltimes f^{-1}(\langle k_2, \partial \rangle)$. Accordingly, NSST is NS α_2 space. □

Theorem 7.3. *Let $\langle \chi^{crip}, \Im, \partial \rangle$ be NSST and $\langle Y^{crip}, \Im, \partial \rangle$ be an-other NSST. Let$\langle f, \partial \rangle$: $\langle \chi^{crip}, \Im, \partial \rangle \to \langle Y^{crip}, \Im, \partial \rangle$ be a soft mapping such that it is continuous mapping. Let $\langle Y^{crip}, \Im, \partial \rangle$ is NS α_2 space then it is guarateed that*
$\{((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})) : f((x^e_{(a,b,c)}, \dashv^{parameter}) = f(y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))\}$
is a NS closed sub-set of $\langle \chi^{crip}, \Im, \partial \rangle \times \langle \chi^{crip}, \Im, \partial \rangle$.

Proof. Let $\langle \chi^{crip}, \Im, \partial \rangle$ be NSST and $\langle Y^{crip}, \Im, \partial \rangle$ be an-other NSST. Let$\langle f, \partial \rangle$: $\langle \chi^{crip}, \Im, \partial \rangle$ $\to \langle Y^{crip}, \Im, \partial \rangle$ be a soft mapping such that it is continuous mapping. Let $\langle Y^{crip}, \Im, \partial \rangle$ is NS α_2 space then we will prove that
$\{((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})) : f((x^e_{(a,b,c)}, \dashv^{parameter}) = f(y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))\}$
is a NS closed sub-set of $\langle \chi^{crip}, \Im, \partial \rangle \times \langle \chi^{crip}, \Im, \partial \rangle$. Equavilintly, we will prove that
$\{((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})) : f((x^e_{(a,b,c)}, \dashv^{parameter}) = f((y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))\}^c$
is a NS closed sub-set of $\langle \chi^{crip}, \Im, \partial \rangle \times \langle \chi^{crip}, \Im, \partial \rangle$. Let $((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))$
$\ltimes$
$\{((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))$ with
$((x^e_{(a,b,c)}, \dashv^{parameter}) > (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})) : f((x^e_{(a,b,c)}, \dashv^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))\}^c$
or
$((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})) \ltimes \{((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))$
with
$((x^e_{(a,b,c)}, \dashv^{parameter}) < (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})) : f((x^e_{(a,b,c)}, \dashv^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))\}^c$.
Then $f((x^e_{(a,b,c)}, \dashv^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))$ or $< f((x^e_{(a,b,c)}, \dashv^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \dashv^{para$ accordingly. Since, $\langle Y^{crip}, \Im, \partial \rangle$ is NS α_2 space. Definitely, $f((x^e_{(a,b,c)}, \dashv^{parameter})), f((y^{e/}_{(a/,b/,c/)}, \dashv^{paramete$ are point of $\langle Y^{crip}, \Im, \partial \rangle$, there exists NS α open sets $\langle G, \partial \rangle, \langle K, \partial \rangle \ltimes \langle Y^{crip}, \Im, \partial \rangle$ such that
$f((x^e_{(a,b,c)}, \dashv^{parameter})) \ltimes \langle G, \partial \rangle and f((x^e_{(a,b,c)}, \dashv^{parameter})) \ltimes \langle K, \partial \rangle$ provided $\langle G, \partial \rangle \widetilde{\cap} \langle K, \partial \rangle$ $= 0_{(\langle \widetilde{\chi} \rangle, \dashv \widetilde{parameter})_Y}$. Since, $\langle G, \partial \rangle$ is soft continuous. $f^{-1}(\langle G, \partial \rangle)$ and $f^{-1}(\langle K, \partial \rangle)$ are NS α open sets in $\langle \chi^{crip}, \Im, \partial \rangle$ supposing $(x^e_{(a,b,c)}, \dashv^{parameter})$ and $(y^{e/}_{(a/,b/,c/)}, \dashv^{parameter})$ respectively and so $f^{-1}(\langle G, \partial \rangle) \times f^{-1}(\langle H, \partial \rangle)$ is basic NS open set in $\langle \chi^{crip}, \Im, \partial \rangle \times \langle \chi^{crip}, \Im, \partial \rangle$ containing $((x^e_{(a,b,c)}, \dashv^{parameter}), (y^{e/}_{(a/,b/,c/)}, \dashv^{parameter}))$. Since $\langle G, \partial \rangle \widetilde{\cap} \langle K, \partial \rangle$

$= 0_{(\langle\widetilde{\chi}\rangle, \daleth \widetilde{parameter})_Y}$. It is clear by the definition of
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) : f((x^{e}_{(a,b,c)}, \daleth^{parameter}) = f(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}$
that
$\{f^{-1}(\langle G, \partial\rangle)$ *and* $f^{-1}(\langle K, \partial\rangle)\} \widetilde{\cap}$
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) : f(x) = f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\} = 0_{(\langle\widetilde{\chi}\rangle, \daleth \widetilde{parameter})}$,
that is
$f^{-1}(\langle G, \partial\rangle) \times f^{-1}(\langle K, \partial\rangle) \Subset$
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) : f((x^{e}{}_{(a/,b/,c/)}, \daleth^{parameter})) = f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$.
Hence,
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) : f((x^{e}{}_{(a/,b/,c/)}, \daleth^{parameter})) = f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$
implies that
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) : f((x^{e}{}_{(a/,b/,c/)}, \daleth^{parameter})) = f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}$
is NS α closed. □

Theorem 7.4. *Let* $\langle\chi^{crip}, \Im, \partial\rangle$ *be NSST and* $\langle Y^{crip}, \Im, \partial\rangle$ *be an-other NSST. Let*$\langle f, \partial\rangle$*:* $\langle\chi^{crip}, \Im, \partial\rangle \to \langle Y^{crip}, \Im, \partial\rangle$ *be a NS open mapping such that it is onto. If the soft set* $\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) : f((x^{e}_{(a,b,c)}, \daleth^{parameter}) = f(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}$ *is a NS closed in* $\langle\chi^{crip}, \Im, \partial\rangle \times \langle\chi^{crip}, \Im, \partial\rangle$*, then* $\langle\chi^{crip}, \Im, \partial\rangle$ *will behave as NS* α_2 *space.*

Proof. Suppose $f((x^{e}_{(a,b,c)}, \daleth^{parameter}) = f(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ be two points of Y^{crip} such that either $f((x^{e}_{(a,b,c)}, \daleth^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ or $f((x^{e}_{(a,b,c)}, \daleth^{parameter})) < f(y)$. Then $((x^{e}_{(a,b,c)}, \daleth^{parameter}))$,$((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ not belong to $\{(x, (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with
$(x^{e}_{(a,b,c)}, \daleth^{parameter}) > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) : f((x^{e}_{(a,b,c)}, \daleth^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}$
or
$((x^{e}_{(a,b,c)}, \daleth^{parameter}))$,$((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ not belong to $\{(x, (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with
$(x^{e}_{(a,b,c)}, \daleth^{parameter}) < (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) : f((x^{e}_{(a,b,c)}, \daleth^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}$
or $((x^{e}_{(a,b,c)}, \daleth^{parameter}))$,$((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$, that is $((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\ltimes$
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with $(x^{e}_{(a,b,c)}, \daleth^{parameter}) > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})$:
$f((x^{e}_{(a,b,c)}, \daleth^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$ or $((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\ltimes$
$\{((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with $(x^{e}_{(a,b,c)}, \daleth^{parameter}) < (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})$:
$f((x^{e}_{(a,b,c)}, \daleth^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$. Since $((x^{e}_{(a,b,c)}, \daleth^{parameter}), y)\ltimes$
$\{(x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})$ with $(x^{e}_{(a,b,c)}, \daleth^{parameter}) > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})$:

$f((x^{e}_{(a,b,c)}, \daleth^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$ or $((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\ltimes$
$\{(x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})$ with $(x^{e}_{(a,b,c)}, \daleth^{parameter}) < (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})$:

$f((x^e_{(a,b,c)}, \daleth^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$ is soft in $\langle \chi^{crip}, \Im, \partial\rangle \times \langle \chi^{crip}, \Im, \partial\rangle$, then $\exists$ NS open sets $\langle G, \partial\rangle and \langle K, \partial\rangle$ in $\langle \chi^{crip}, \Im, \partial\rangle$ s.t. $((x^e_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\ltimes$ $\langle G, \partial\rangle \times \langle K, \partial\rangle \Subset \{((x^e_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with $(x^e_{(a,b,c)}, \daleth^{parameter}) > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) : f((x^e_{(a,b,c)}, \daleth^{parameter})) > f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$ or $((x^e_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\ltimes \langle G, \partial\rangle \times \langle K, \partial\rangle \Subset$ $\{((x^e_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with $(x^e_{(a,b,c)}, \daleth^{parameter}) < (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) : f((x^e_{(a,b,c)}, \daleth^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\}^c$. Then, since f is NS α open, $f(\langle G, \partial\rangle) and f(\langle K, \partial\rangle)$ are NS α open sets in $\langle Y^{crip}, \Im, \partial\rangle$ containing $f((x^e_{(a,b,c)}, \daleth^{parameter}))$ and $((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ respectively,and $f(\langle G, \partial\rangle)\widetilde{\cap} f(\langle K, \partial\rangle)$ $= 0_{(\langle\widetilde{\chi}\rangle, \widetilde{\daleth parameter})}$ otherwise $f(\langle G, \partial\rangle) \times f(\langle K, \partial\rangle)\ \widetilde{\cap}$ $\{((x^e_{(a,b,c)}, \daleth^{parameter})), with (x^e_{(a,b,c)}, \daleth^{parameter}) > (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) : f((x^e_{(a,b,c)}, \daleth^{parameter})) >$ $f(y)\}$ or $(((x^e_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})))$ not belong to $\{((x^e_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ with $(x^e_{(a,b,c)}, \daleth^{parameter}) < (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) :$ $f((x^e_{(a,b,c)}, \daleth^{parameter})) < f((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))\} = 0_{(\langle\widetilde{\chi}\rangle, \widetilde{\daleth parameter})}$. It follows that $\langle Y^{crip}, \Im, \partial\rangle$ is NS α_2 space. □

Theorem 7.5. *Let $\langle \chi^{crip}, \Im, \partial\rangle$ be a NS second countable space then it is guaranteed that every family of non-empty dis-joint NS α open subsets of a NS second countable space $\langle \chi^{crip}, \Im, \partial\rangle$ is NS countable.*

Proof. Given that $\langle \chi^{crip}, \Im, \partial\rangle$ be a NS second countable space. Then, $\exists$ a NS countable base $\beta = \langle B^1, B^2, B^3, B^4, ..B^n : n \ltimes \mathbb{N}\rangle$ for $\langle \chi^{crip}, \Im, \partial\rangle$. Let $\langle \mathcal{C}, \partial\rangle$ be a family of non-vacuous mutually exclusive NS α open sub-sets of $\langle \chi^{crip}, \Im, \partial\rangle$. Then, for each $\langle f, \partial\rangle$ of in $\langle \mathcal{C}, \partial\rangle$ a soft $B^n \ltimes \beta$ in such a way that $B^n \Subset \langle f, \partial\rangle$. Let us attach with $\langle f, \partial\rangle$, the smallest positive interger n such that $B^n \Subset \langle f, \partial\rangle$. Since the candidates of $\langle \mathcal{C}, \partial\rangle$ are mutully exclusive because of this behaviour distinct candidates will be associated with distinct positive integers. Now, if we put the elements of $\langle \mathcal{C}, \partial\rangle$ in order so that the order is increasing relative to the positive integers associated with them, we obtain a sequence of of candidates of $\langle \mathcal{C}, \partial\rangle$. This verifies that $\langle \mathcal{C}, \partial\rangle$ is NS countable. □

Theorem 7.6. *Let $\langle \chi^{crip}, \Im, \partial\rangle$ be a NS second countable space and let $\langle f, \partial\rangle$ be NS uncountable sub set of $\langle \chi^{crip}, \Im, \partial\rangle$. Then, at least one point of $\langle f, \partial\rangle$ is a soft limit point of $\langle f, \partial\rangle$.*

Proof. Let $\beta = \langle B^1, B^2, B^3, B^4, ..B^n : n \ltimes \mathbb{N}\rangle$ for $\langle \chi^{crip}, \Im, \partial\rangle$. Let, if possible, no point of $\langle f, \partial\rangle$ be a soft limit point of $\langle f, \partial\rangle$. Then, for each $(x^e_{(a,b,c)}, \daleth^{parameter})\ltimes \langle f, \partial\rangle\ \exists\ NS\ \alpha$ open set $\langle \rho, \partial\rangle_{(x^e_{(a,b,c)}, \daleth^{parameter})}$ such that $(x^e_{(a,b,c)}, \daleth^{parameter})\ltimes \langle \rho, \partial\rangle_{(x^e_{(a,b,c)}, \daleth^{parameter})}$ and $\langle \rho, \partial\rangle_{(x^e_{(a,b,c)}, \daleth^{parameter})} \widetilde{\cap} \langle f, \partial\rangle = \{(x^e_{(a,b,c)}, \daleth^{parameter})\}$. Since β is soft base $\exists\ B_{n(x^e_{(a,b,c)}, \daleth^{parameter})}$

$\ltimes\beta$ such that $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \ltimes B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})} \Subset \langle \rho, \partial \rangle_{(x^{e}_{(a,b,c)}, \daleth^{parameter})}$.
Therefore $B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})} \widetilde{\cap} \langle f, \partial \rangle \Subset \langle \rho, \partial \rangle_{(x^{e}_{(a,b,c)}, \daleth^{parameter})} \widetilde{\cap} \langle f, \partial \rangle = \{(x^{e}_{(a,b,c)}, \daleth^{parameter})\}$.
More-over, if $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1$ and $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2$ be any two NS points such that $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1 \neq (x^{e}_{(a,b,c)}, \daleth^{parameter})_2$ which means either $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1 > (x^{e}_{(a,b,c)}, \daleth^{parameter})_2$ or $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1$ ¡ $(x^{e}_{(a,b,c)}, \daleth^{parameter})_2$ then $\exists\, B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_1}$ and $B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_2}$ in β such that $B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_1} \widetilde{\cap} \langle f, \partial \rangle = \{(x^{e}_{(a,b,c)}, \daleth^{parameter})_1\}$ and $B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_2} \widetilde{\cap} \langle f, \partial \rangle = \{(x^{e}_{(a,b,c)}, \daleth^{parameter})_2\}$. Now, $(x^{e}_{(a,b,c)}, \daleth^{parameter})_1 \neq (x^{e}_{(a,b,c)}, \daleth^{parameter})_2$ which guarantees that $\{(x^{e}_{(a,b,c)}, \daleth^{parameter})_1\} \neq \{(x^{e}_{(a,b,c)}, \daleth^{parameter})_2\}$ which $\Rightarrow B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_1} \widetilde{\cap} \langle f, \partial \rangle \neq B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_2} \widetilde{\cap} \langle f, \partial \rangle \Rightarrow B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_1}$ $\neq B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})_2}$. Thus, $\exists$ a one to one soft correspondence of $\langle f, \partial \rangle$ on to $B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})} : (x^{e}_{(a,b,c)}, \daleth^{parameter}) \ltimes \langle f, \partial \rangle$. Now, $\langle f, \partial \rangle$ being NS uncountable,it follows that $\{B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})} : (x^{e}_{(a,b,c)}, \daleth^{parameter}) \ltimes \langle f, \partial \rangle\}$ is NS uncountable.But,this is purely a contradiction, since $\{B_{n(x^{e}_{(a,b,c)}, \daleth^{parameter})} : (x^{e}_{(a,b,c)}, \daleth^{parameter}) \ltimes \langle f, \partial \rangle\}$ being a NS sub-family of the NS countable collection β. This contradiction is taking birth that on point of $\langle f, \partial \rangle$ is a soft limit point of $\langle f, \partial \rangle$, so at least one point of $\langle f, \partial \rangle$ is a soft limit point of $\langle f, \partial \rangle$. □

8 Neutrosophic soft Product Spaces

Theorem 8.1. *Let $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\chi^{crip^2}, \Im, \mathbb{A})$ be two NS second countable NST Spaces then their product, that is $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \Im, \mathbb{A})$ is also NS second countable NST Space.*

Proof. To prove $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \Im, \mathbb{A})$ is NS second countable NST Spaces. Our assumption $\Rightarrow$ $\exists$ countable NS bases $B^1 = \{B^i : i \in N\}$ and $B^2 = \{\mathcal{C}^i : i \in N\}$ for $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\chi^{crip^2}, \Im, \mathbb{A})$ respectively. $B = \langle \Psi_1, \mathbb{A} \rangle * \langle \Psi_2, \mathbb{A} \rangle$; such that $\langle \Psi_1, \mathbb{A} \rangle$ and $\langle \Psi_2, \mathbb{A} \rangle$ are NS open s.t. $\langle \Psi_1, \mathbb{A} \rangle \in \{(\chi^{crip^1}, \Im, \mathbb{A}), \langle \Psi_2, \mathbb{A} \rangle \in (\chi^{crip^2}, \Im, \mathbb{A})\}$ is a soft base for neutrosophic soft product topology(NSPT) $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \Im, \mathbb{A})$.
Write $\mathcal{C} = \{B^i * \mathcal{C}^j; i, j \in N\} = B^1 * B^2$ is soft countable this implies that is NS countable. By definition of soft base B, and $((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ $\in (\chi^{crip^1}, \Im, \mathbb{A})$
$*(\chi^{crip^2}, \Im, \mathbb{A}) \Rightarrow$ NS α open sets $\langle G, \mathbb{A} \rangle, \langle H, \mathbb{A} \rangle \in B$ s.t. $((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}))$ $\in \langle G, \mathbb{A} \rangle, *\langle H, \mathbb{A} \rangle \widetilde{\subset} N \Rightarrow (x^{e}_{(a,b,c)}, \daleth^{parameter}) \in \langle G, \mathbb{A} \rangle \in (\chi^{crip^1}, \Im, \mathbb{A}), y \in \langle H, \mathbb{A} \rangle \in (\chi^{crip^2}, \Im, \mathbb{A})$ $\Rightarrow B^i \in B^1, \mathcal{C}^j \in B^2$ such that $(x^{e}_{(a,b,c)}, \daleth^{parameter}) \in B^i \widetilde{\subset} \langle G, \mathbb{A} \rangle, (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter}) \in \mathcal{C}^j \widetilde{\subset} \langle H, \mathbb{A} \rangle$ $\Rightarrow ((x^{e}_{(a,b,c)}, \daleth^{parameter}), (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})) \in B^i * \mathcal{C}^j \widetilde{\subset} \langle G, \mathbb{A} \rangle * \langle H, \mathbb{A} \rangle \widetilde{\subset} N$. By definition this proves that $\mathcal{C}$ is s.t. base for the neutrosophic soft product topology(NSPT) $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \Im, \mathbb{A})$. Also $\mathcal{C}$ has been shown to be NS countable. Hence,

$(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \Im, \mathbb{A})$ NS second countable relative to NS α open set. $\square$

Theorem 8.2. *Let $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\chi^{crip^2}, \tau, \mathbb{A})$ be two NST Spaces on the crisp sets $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\chi^{crip^2}, \Im, \mathbb{A})$ respectively. The collection $B = \langle G_1, \mathbb{A}\rangle * \langle G_2, \mathbb{A}\rangle : \langle G_1, \mathbb{A}\rangle \in \Im$, $\langle G_2, \mathbb{A}\rangle \in \tau$ is a s.t. base for some (NSPT) $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \tau, \mathbb{A})$ such that $\langle G_1, \mathbb{A}\rangle, and \langle G_1, \mathbb{A}\rangle$ are NS α open sets in their corresponding NST Spaces.*

Proof. Let $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\chi^{crip^2}, \tau, \mathbb{A})$ be two NST Spaces on the crisp sets $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\chi^{crip^2}, \tau, \mathbb{A})$ respectively. Suppose $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \tau, \mathbb{A})$ be the (NSPT). $B = \{\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle : \langle u, \mathbb{A}\rangle \in (\chi^{crip^1}, \Im, \mathbb{A}), \langle u_2, \mathbb{A}\rangle \in (\chi^{crip^2}, \tau, \mathbb{A})\}$ where $\langle u_1, \mathbb{A}\rangle$ is NS α open in $(\chi^{crip^1}, \Im, \mathbb{A})$ and $\langle u_2, \mathbb{A}\rangle$ is NS α open in $(\chi^{crip^2}, \tau, \mathbb{A})$. We need to prove B is a NS base for some NST on $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \tau, \mathbb{A})$. To prove that $\cup \{B : B \in \beta\}$ $= (\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \tau, \mathbb{A})$. Clearly, $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \tau, \mathbb{A}) \in \beta$. This implies that $(\chi^{crip^1}, \Im, \mathbb{A}) * (\chi^{crip^2}, \tau, \mathbb{A}) = \cup \{B : B \in \beta\}$. Next let $\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle$, $\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle \in B$ where $\langle \Im_1, \mathbb{A}\rangle$ is NS α open in $(\chi^{crip^1}, \Im, \mathbb{A})$ and $\langle \Im_2, \mathbb{A}\rangle$ is NS α open in $(\chi^{crip^2}, \tau, \mathbb{A})$ and suppose $x_1, x_2 \in (\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle) \widetilde{\cap} (\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle) \in B$. To prove that $\exists$ $(\langle \Im_1, \mathbb{A}\rangle)$ NS α open in $(\chi^{crip^1}, \Im, \mathbb{A})$ and $(\langle \Im_2, \mathbb{A}\rangle)$ is NS α open in $(\chi^{crip^2}, \tau, \mathbb{A})$ s.t. $\langle f_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle \in B$ s.t. $((x^e_{(a,b,c)}, \exists^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2)$ $\in \langle f_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle$
$\widetilde{\subset}(\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle)\widetilde{\cap}(\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle)$ that is $((x^e_{(a,b,c)}, \exists^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2)$ $\in (\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle)\widetilde{\cap}(\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle) \Rightarrow ((x^e_{(a,b,c)}, \exists^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2)$
$\in$
$(\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle)$, $((x^e_{(a,b,c)}, \exists^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2) \in (\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle)$
$\Rightarrow (x^e_{(a,b,c)}, \exists^{parameter})_1 \in \langle u_1, \mathbb{A}\rangle$,$(x^e_{(a,b,c)}, \exists^{parameter})_2 \in \langle u_2, \mathbb{A}\rangle$,$(x^e_{(a,b,c)}, \exists^{parameter})_1 \in \langle \Im_1, \mathbb{A}\rangle$, $(x^e_{(a,b,c)}, \exists^{parameter})_2 \in \langle \Im_2, \mathbb{A}\rangle \Rightarrow (x^e_{(a,b,c)}, \exists^{parameter})_1 \in (\langle u_1, \mathbb{A}\rangle\widetilde{\cap}\langle \Im_1, \mathbb{A}\rangle)$, $(x^e_{(a,b,c)}, \exists^{parameter})_2$ $\in (\langle u_2, \mathbb{A}\rangle\widetilde{\cap}\langle \Im_2, \mathbb{A}\rangle) \Rightarrow (x^e_{(a,b,c)}, \exists^{parameter})_1 \in (\langle f_1, \mathbb{A}\rangle)$,$(x^e_{(a,b,c)}, \exists^{parameter})_2 \in (\langle f_2, \mathbb{A}\rangle)$.
On taking $(\langle f_1, \mathbb{A}\rangle) = (\langle u_1, \mathbb{A}\rangle) \widetilde{\cap} \langle \Im_1, \mathbb{A}\rangle$,$\langle \Im_2, \mathbb{A}\rangle = (\langle u_2, \mathbb{A}\rangle) \widetilde{\cap} \langle \Im_2, \mathbb{A}\rangle$
$\Rightarrow ((x^e_{(a,b,c)}, \exists^{parameter})_1, (x^e_{(a,b,c)}, \exists^{parameter})_2) \in \langle f_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle$,$\langle u_1, \mathbb{A}\rangle * \langle u_2, \mathbb{A}\rangle \in B$, $\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle \in \beta \Rightarrow \langle u_1, \mathbb{A}\rangle \in (\chi^{crip^1}, \Im, \mathbb{A})$,$\langle u_2, \mathbb{A}\rangle \in (\chi^{crip^2}, \tau, \mathbb{A})$;$\langle \Im_1, \mathbb{A}\rangle \in (\chi^{crip^1}, \Im, \mathbb{A})$,$\langle \Im_2, \mathbb{A}\rangle$ $\in (\chi^{crip^2}, \tau, \mathbb{A}) \Rightarrow \langle u_1, \mathbb{A}\rangle \widetilde{\cap} \langle \Im_1, \mathbb{A}\rangle \in (\chi^{crip^1}, \Im, \mathbb{A})$,$\langle u_2, \mathbb{A}\rangle \widetilde{\cap} \langle \Im_2, \mathbb{A}\rangle \in (\chi^{crip^2}, \tau, \mathbb{A})$ $\langle f_1, \mathbb{A}\rangle \in (\chi^{crip^1}, \Im, \mathbb{A})$,$\langle f_2, \mathbb{A}\rangle \in (\chi^{crip^2}, \tau, \mathbb{A}) \Rightarrow \langle f_1, \mathbb{A}\rangle * \langle f_2, \mathbb{A}\rangle \in \beta$. It revealed that $\exists$ $\langle f_1, \mathbb{A}\rangle * \langle f_2, \mathbb{A}\rangle \in \beta$ s.t.
$((x^e_{(a,b,c)}, \exists^{parameter})_1, (x^e_{(a,b,c)}, \exists^{parameter})_2) \in \langle f_1, \mathbb{A}\rangle * \langle f_2, \mathbb{A}\rangle$. That need to be proven $\langle f_1, \mathbb{A}\rangle * \langle f_2, \mathbb{A}\rangle \widetilde{\subset} (((\langle u_1, \mathbb{A}\rangle) * (\langle u_2, \mathbb{A}\rangle)) \widetilde{\cap} (\langle \Im_1, \mathbb{A}\rangle * \langle \Im_2, \mathbb{A}\rangle)$.
Let $((x^e_{(a,b,c)}, \exists^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2) \in \langle f_1, \mathbb{A}\rangle * \langle f_2, \mathbb{A}\rangle$ be arbitrary matually exclusive points. $((y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2) \in (\langle f_1, \mathbb{A}\rangle * \langle f_2, \mathbb{A}\rangle)$
$\Rightarrow (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_1 \in \langle f_1, \mathbb{A}\rangle$,$(y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2 \in \langle f_2, \mathbb{A}\rangle \Rightarrow (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_1$ $\in (\langle u_1, \mathbb{A}\rangle\widetilde{\cap}\langle \Im_1, \mathbb{A}\rangle)$,$(y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_2 \in (\langle u_2, \mathbb{A}\rangle\widetilde{\cap}\langle \Im_2, \mathbb{A}\rangle) \Rightarrow (y^{e/}_{(a/,b/,c/)}, \exists^{parameter})_1$

$\in \langle u_1, \mathbb{A}\rangle$,$(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1 \in \langle \Im_1, \mathbb{A}\rangle$;$(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2 \in \langle u_2, \mathbb{A}\rangle$,$(y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2$ $\in \langle \Im_2, \mathbb{A}\rangle \Rightarrow ((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2) \in ((\langle u_1, \mathbb{A}\rangle) * (\langle u_2, \mathbb{A}\rangle))$, $((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2) \in ((\langle \Im_1, \mathbb{A}\rangle) * (\langle \Im_2, \mathbb{A}\rangle))$ $\Rightarrow ((y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_1, (y^{e/}_{(a/,b/,c/)}, \daleth^{parameter})_2) \in ((\langle u_1, \mathbb{A}\rangle) * (\langle u_2, \mathbb{A}\rangle)) \widetilde{\cap} ((\langle \Im_1, \mathbb{A}\rangle) * (\langle \Im_2, \mathbb{A}\rangle))$ $\Rightarrow ((\langle f_1, \mathbb{A}\rangle) * (\langle f_2, \mathbb{A}\rangle)) \widetilde{\subset} ((\langle u_1, \mathbb{A}\rangle) * (\langle u_2, \mathbb{A}\rangle)) \widetilde{\cap} ((\langle \Im_1, \mathbb{A}\rangle) * (\langle \Im_2, \mathbb{A}\rangle))$. Thus β is a NS base. □

Theorem 8.3. *Let $(x^{crip^1}, \Im, A)$ and (x^{crip^2}, τ, A) be two nstas spaces on the crisp sets x^{crip^1} and x^{crip^2} respectively. that is $(x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A)$be the NS product space, Then the product space$\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ is NS regular space iff each NS co-ordinate space $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$ is NS regular space.*

Proof. Suppose each co-ordinate space $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$ is NS regular space. Let $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle = \langle\langle x, A\rangle^{\partial,\partial:\triangle}\rangle$ be any point of the NS space $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ and $\angle, g, A\rangle$ be any NS α open in $\times\{((x^{crip^1}, \Im, A)* (x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ s.t. $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle \in \langle, g, A\rangle$ Then $\exists$ a NS α open set $\langle h, A\rangle$ in $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ such that $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle \in \langle h, A\rangle \widetilde{\subseteq} \langle, g, A\rangle$. Let,$\times\{(\langle, g, A\rangle^{\partial:\partial\in\triangle}\}$ is the NS product space such that $\langle, g, A\rangle^{\partial}$ is NS α open in $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$. Since each $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$. Since each $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \widehat{\tau_1}, \widehat{\tau_2}, A))^{\partial}\}$ is NS regular and $\langle, g, A\rangle$ is NS α open in $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$ containing $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle^{\partial}$ NS α open set $(\langle, h, A\rangle)^{\partial}$ in $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial}\}$ s.t. $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle^{\partial} \in (\langle, K, A\rangle)^{\partial}$ and $\overline{(\langle, h, A\rangle)^{\partial}} \widetilde{\subseteq} \langle, g, A\rangle^{\partial}$ Let $\{(\langle h, A\rangle)^{\partial:\partial\in\triangle}\}$, then $\{(\langle h, A\rangle)^{\partial:\partial\in\triangle}\}$ is NSopen set in $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial}\}$ and contains $\langle x, A\rangle$. Also, $\times\overline{(\langle h, A\rangle)^{\partial::\partial\in\triangle}} = \times\{(\langle h, A\rangle)^{\partial:\partial\in\triangle} : \partial \in\triangle\}$ Further, since $\overline{(\langle, h, A\rangle)^{\partial}} \widetilde{\subseteq} \langle, g, A\rangle^{\partial}$ for each ∂ ,we have $\times\{(\langle h, A\rangle)^{\partial:\partial\in\triangle} : \partial \in\triangle \}\widetilde{\subseteq}\{(\langle g, A\rangle)^{\partial:\partial\in\triangle}\}$. this shows that for every soft point $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle \in \times\{((x^{crip^1}, \Im, A)* (x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$
and every NS α open set $\langle, g, A\rangle$ containing $\langle x, A\rangle$ there exists NS α open set $\times\{(\langle h, A\rangle)^{\partial:\partial\in\triangle} : \partial \in \times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ such that $\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle^{\partial} \in (\langle, h, A\rangle)^{\partial}$ and
$\times\overline{(\langle h, A\rangle)^{\partial::\partial\in\triangle}} \widetilde{\subseteq}\{(\langle g, A\rangle)^{\partial:\partial\in\triangle}\}$. Hence, $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ is NS regular. Conversely, let the non-empty NS product space $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ be NS regular and let $\times(\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\})^{\beta}$ be an arbitrary soft co-ordinate space. Then, we must show that is NS regular. Let $((x^e_{(a,b,c)}, \daleth^{parameter}), A)^{\beta}$ be any soft point of $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ and let $\langle, g, A\rangle$ be any NS open in $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ such that $((x^e_{(a,b,c)}, \daleth^{parameter}), A)^{\beta} \in \langle, g, A\rangle$. now, choose, soft element
$\langle (x^e_{(a,b,c)}, \daleth^{parameter}), A\rangle$ in $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\triangle}\}$ whose βth co-ordinate

in $\langle x, A\rangle$.
Let $\langle, g, A\rangle = \pi^{\beta-1}((x^e_{(a,b,c)}, \widetilde{\exists^{parameter}}), A)^i$. Then, $((x^e_{(a,b,c)}, \exists^{parameter}), A)^\beta \in \langle, g, A\rangle$ and by soft continuity of $\pi^{\beta-1}$, $\langle, g, A\rangle$ is NS soft open in $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}$. Since, $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}$ is NS regular space so $\exists$ basic NS open set $\times\{(\langle h, A\rangle)^{\partial:\partial\in\Delta}\}$, Where each $(\langle, h, A\rangle)^\partial$ is NSopen in $\times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}$ s.t. $((x^e_{(a,b,c)}, \exists^{parameter}), A) \in \times\{(\langle h, A\rangle)^{\partial:\partial\in\Delta}\}$ and $\overline{\{(\langle h, A\rangle)^{\partial:\partial\in\Delta}\}}\widetilde{\subseteq}\langle g, A\rangle^\beta$ This shows that $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}$ is NS regular and hence, each co-ordinate space is NS regular. □

Theorem 8.4. *Let $(x^{crip^1}, \Im, A)$ and (x^{crip^2}, τ, A) be two nstas spaces on the crisp sets x^{crip^1} and x^{crip^2} respectively. that is $(x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A)$be the NS product space, Then the product space $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}$ is NS completely regular space iff each soft co-ordinate space $\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$ is soft completely NS regular space.*

Proof. Let each soft co-ordinate space $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial}\}$ is NS completely regular. Then, we must show that the NS product space $\times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}$. Let $(\aleph, A)$ be $\widetilde{\text{any}}$ member of the usual $\widetilde{\text{soft}}$ sub-base for the NS product topology and let $((x^e_{(a,b,c)}, \widetilde{\exists^{parameter}}), A) = \langle((x^e_{(a,b,c)}, \widetilde{\exists^{parameter}}), A)^{\partial:\partial\in\Delta}\rangle$ be any soft point in $(\aleph, A)$. Then $(\aleph, A) = \pi^{\beta-1}(\langle g, A\rangle^\beta)$ is NS α open in $\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\beta}\}$ and contains $\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle^\beta)$. Since, $\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\beta}\}$ is NS completely regular $\Rightarrow$ soft mapping $f : \{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\beta}\} \to [\tilde{0}, \tilde{1}]$ such that $f(\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle^\beta) = 0_{\widetilde{\widetilde{(\langle\widetilde{x}\rangle, \exists^{parameter})}}}$ and $f(\tilde{y}, A) = \tilde{1}\forall y \in \{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\beta}\} - \langle g, A\rangle^\beta$. . Since π^β is NS continuous and f is soft continuous, so the NS composite mapping $(fo\pi^\beta : \times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\beta}\widetilde{\} \to} [\tilde{0}, \tilde{1}]$ is NS continuous. Now, if $\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle^\beta) \in (\aleph, A)$ then $((x^e_{(a,b,c)}, \exists^{parameter}), A) \in \pi^{\beta^{-1}}(\langle g, A\rangle^\beta)$ implies that $\pi^\beta(\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle) \in \langle g, A\rangle^\beta \Rightarrow f[\pi^\beta(((x^e_{(a,b,c)}, \exists^{parameter}), A))] = f(((x^e_{(a,b,c)}, \exists^{parameter}), A)) = 0_{(\langle\widetilde{x}\rangle, \widetilde{\exists^{parameter}})} \Rightarrow (fo\pi^\beta)(\langle x, A\rangle) = 0_{(\langle\widetilde{x}\rangle, \widetilde{\exists^{parameter}})}$ Again, if $\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle \in \times\{((x^{crip^1}, \Im, A)*(x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\}-(\aleph, A)$ Then, $\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle \in \times\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\} - (\aleph, A)$ implies that
$\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle \in \pi^{\beta^{-1}} \times \{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\} - \pi^{\beta^{-1}}(\langle g, A\rangle)$ implies that
$\langle(x^e_{(a,b,c)}, \exists^{parameter}), A\rangle \in \pi^{\beta^{-1}} \times (\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\} - \langle g, A\rangle)$ implies that
$\pi^{\beta^{-1}}((x^e_{(a,b,c)}, \exists^{parameter}), A) \in \pi^{\beta^{-1}} \times (\{((x^{crip^1}, \Im, A) * (x^{crip^2}, \tau, A))^{\partial:\partial\in\Delta}\} - \langle g, A\rangle) \Rightarrow$
$f[\pi^\beta(((x^e_{(a,b,c)}, \exists^{parameter}), A))] = 1_{(\langle\widetilde{x}\rangle, \widetilde{\exists^{parameter}})} \Rightarrow (fo\pi^\beta)(x^e_{(a,b,c)}, \exists^{parameter}), A) = 1_{(\langle\widetilde{x}\rangle, \widetilde{\exists^{parameter}})}$

thus .

$$f[\pi^{\beta}(\langle x,A\rangle)] = \begin{cases} 0_{(\langle\widetilde{x}\rangle,\widetilde{\dashv^{parameter}})} = if\langle x,A\rangle \in \langle g,A\rangle \\ 0_{(\langle\widetilde{x}\rangle,\widetilde{\dashv^{parameter}})} = ((x^{e}_{(a,b,c)},\dashv^{parameter}),A) \\ hance \times\{((x^{crip^1},\Im,A)* \in \times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\partial:\partial\in\triangle}\} - (\aleph,A) \end{cases} \tag{11}$$

$(x^{crip^2},\tau,A))^{\partial:\partial\in\triangle}\}$ is soft $\Rightarrow$-regular. Conversely, let the soft product space $\times\{((x^{crip^1},\Im,A)* \in \times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\partial:\partial\in\triangle}\}$ be soft NScompletely regular and let $\times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\beta}\}$ be arbitrary soft co-ordinate space. Then, keeping continue on the lines that are traced in the second part of the proof of the Theorem 3.6, we can show that $\times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\beta}\}$ is the NS homeomorphic image of a NS sub-space of $\times\{((x^{crip^1},\Im,A)* \in \times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\partial:\partial\in\triangle}\}$. Now, every NS sub-space of a NS completely NS regular space being NS completely regular and NS image of a NS completely regular space being NS completely regular, it follows that $\times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\beta}\}$ is NS completely regular. Hence, each co-ordinate space of $\times\{((x^{crip^1},\Im,A)* \in \times\{((x^{crip^1},\Im,A)*(x^{crip^2},\tau,A))^{\partial:\partial\in\triangle}\}$ is NS completely regular.

□

9 conclusion:

The concept of a neutrosophic set was introduced by Smarandache. This theory is a straight forward generalization of classical set theory, fuzzy set theory, intuitionistic fuzzy set theory etc. This set attracted the attention of mathematicians more because of the strange characteristics. This set supposes three possibilities at a time that is truth membership function, an indeterminacy membership function, and a falsity membership function. When separation in neutrosophic soft topological spaces among points are measured then in that case the concept of separation axioms come in action. Neutrosophic soft separation axioms are the most important and interesting concepts via neutrosophic soft topology. We have introduced the concept of generalized neutrosophic soft separation axioms in neutrosophic soft topological spaces with respect to soft points. Later on the important results are discussed related to these newly defined concepts with respect to soft points. The concept of neutrosophic soft ?-separation axioms of neutrosophic soft topological spaces is used in different results with respect to soft points. Results on neutrosophic Countability, convergence of sequences in Hausdorff spaces, neutrosophic monotonous functions and neutrosophic soft product spaces with respect to soft points are addressed...

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