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NEUTROSOPHIC SOFT B-OPEN SET

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ABSTRACT. Smarandache introduced the idea of neutrosophic sets. Neutrosophic soft sets deals with uncertain data. This paper defines the notion of neutrosophic soft b-open sets. The relationship between neutrosophic soft b-open set and other neutrosophic soft open sets are discussed. Also the properties of neutrosophic soft b-open are investigated.

1. INTRODUCTION

Everything on the planet including uncertainties are a significant issue in numerous fields of genuine, for example, monetary, designing, environment, social sciences, clinical sciences and business the executives. The complications and problems of traditional mathematical modeling could be triggered by unpredictable estimates in those areas. Many methods have been used by researchers to prevent problems in struggling with uncertainties. Some of the tools are Fuzzy sets [1], Rough sets [2] and Intuitionstic fuzzy sets [3]. Smarandache [4] characterized the thought of neutrosophic set which is a scientific device for managing issues including imprecise and indeterminant information. Molodstov [5] acquainted idea of delicate sets with take care of confounded issues and different sorts of uncertainities. Maji [6] combined the

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concept of soft set and neutrosophic set together by introducing a new concept called neutrosophic soft set. In [7] neutrosophic soft set was applied in making decision. Several researchers [8–12] applied in various mathematical systems the concept of neutrosophic soft sets. Bera [13] introduced neutrosophic soft topological spaces. Iswaraya et.al. [14] studied the concept of neutrosophic semi-open sets[NSO] and neutrosophic semi-closed sets[NSC]. Arokiarani et.al. [15] defined neutrosophic semi-open (resp. pre-open and α-open) functions and investigated their relations. Rao et.al. [16] introduced neutrosophic pre-open sets. Evanzalin et.al. [17] defined neutrosophic b-open sets. Evanzalin et.al. [18] defined neutrosophic soft region in neutrosophic soft topological space. In this paper, a new class of generalized neutrosophic soft open sets in neutrosophic soft topological spaces, called neutrosophic soft b-open sets, is introduced and studied. Then discussed the relationships among other neutrosophic soft sets. Also the properties of neutrosophic soft b-open are investigated.

2. Preliminaries

Definition 2.1. [5]. *A pair (F,E) is called a soft set over X where $F:E\to P(X)$ is a mapping where $P(X)$ is a power set of X. We denote (F,E) by $\widetilde{F}$. We write $\widetilde{F}=\{(e,F(e)):e\in E\}$.*

Definition 2.2. [4]. *A neutrosophic set (NS) A on the universe of discourse X is defined as: $A=\{<x,T_A(x),I_A(x),F_A(x)>:x\in X\}$ where $T,I,F:X\longrightarrow\ ]^-0,1^+[$ and $-0\leq T_A(x)+I_A(x)+F_A(x)\leq 3^+$*

Definition 2.3. [6]. *Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic soft set(NSS) of X. Then the pair (F,E) is called a NSS over X where $F:E\to P(X)$ is a mapping. We denote the neutrosophic soft set (F,E) by $\widetilde{F}_N$.*

In other words, the neutrosophic soft set $\widetilde{F}_N$ is a parameterized family of some elements of the set $P(X)$ and therefore can be written as a set of ordered pair: $\widetilde{F}_N=\{(e,\{<x,T_{F_{N(e)}}(x),I_{F_{N(e)}}(x),F_{F_{N(e)}}(x)>:x\in X\})e\in E\}$

Definition 2.4. [13]. *The complement of the neutrosophic soft set $\widetilde{F}_N$ is denoted by $(\widetilde{F}_N)^c$ and is defined by $(\widetilde{F}_N)^c=\{(e,\{<x,F_{F_{N(e)}}(x),\ I_{F_{N(e)}}(x),T_{F_{N(e)}}(x)>:x\in X\}:e\in E\}$*

Definition 2.5. [13]. *For any two neutrosophic soft sets $\widetilde{F}_N$ and $\widetilde{G}_N$ over X, $\widetilde{F}_N$ is a neutrosophic soft subset of $\widetilde{G}_N$ if $T_{F_{N(e)}}(x) \leq T_{G_{N(e)}}(x)$; $I_{F_{N(e)}}(x) \leq I_{G_{N(e)}}(x)$; $F_{F_{N(e)}}(x) \geq F_{G_{N(e)}}(x)$; for all $e \in E$ and $x \in X$.*

Definition 2.6. [13]. *A neutrosophic soft set $\widetilde{F}_N$ over X is said to be null neutrosophic soft set if $T_{F_{N(e)}}(x) = 0$; $I_{F_{N(e)}}(x) = 0$; $F_{F_{N(e)}}(x) = 1$; for all $e \in E$ and $x \in X$. It is denoted by $\widetilde{\Phi}_N$.*

Definition 2.7. [13]. *A neutrosophic soft set $\widetilde{F}_N$ over X is said to be absolute neutrosophic soft set if $T_{F_{N(e)}}(x) = 1$; $I_{F_{N(e)}}(x) = 1$; $F_{F_{N(e)}}(x) = 0$; for all $e \in E$ and $x \in X$. It is denoted by $\widetilde{X}_N$*

Definition 2.8. [13]. *The union of two neutrosophic soft sets $\widetilde{F}_N$ and $\widetilde{G}_N$ is denoted by $\widetilde{F}_N \cup \widetilde{G}_N$ and is defined by $\widetilde{H}_N = \widetilde{F}_N \cup \widetilde{G}_N$, where the truth-membership, indeterminacy-membership and falsity membership of $\widetilde{H}_N$ are as follows*

$$T_{H_{N(e)}}(x) = \begin{cases} T_{F_{N(e)}}(x) & \textit{if } e \in A - B \\ T_{G_{N(e)}}(x) & \textit{if } e \in B - A \\ max\{T_{F_{N(e)}}(x), T_{G_{N(e)}}(x)\} & \textit{if } e \in A \cap B \end{cases},$$

$$I_{H_{N(e)}}(x) = \begin{cases} I_{F_{N(e)}}(x) & \textit{if } e \in A - B \\ I_{G_{N(e)}}(x) & \textit{if } e \in B - A \\ \frac{I_{F_{N(e)}}(x) + I_{G_{N(e)}}(x)\}}{2} & \textit{if } e \in A \cap B \end{cases},$$

$$F_{H_{N(e)}}(x) = \begin{cases} F_{F_{N(e)}}(x) & \textit{if } e \in A - B \\ F_{G_{N(e)}}(x) & \textit{if } e \in B - A \\ min\{F_{F_{N(e)}}(x), F_{G_{N(e)}}(x)\} & \textit{if } e \in A \cap B \end{cases}.$$

Definition 2.9. [13]. *The intersection of two neutrosophic soft sets $\widetilde{F}_N$ and $\widetilde{G}_N$ is denoted by $\widetilde{F}_N \cap \widetilde{G}_N$ and is defined by $\widetilde{H}_N = \widetilde{F}_N \cap \widetilde{G}_N$, where the truth-membership, indeterminacy-membership and falsity membership of $\widetilde{H}_N$ are as follows*

$$T_{H_{N(e)}}(x) = min\{T_{F_{N(e)}}(x), T_{G_{N(e)}}(x)\},$$

$$I_{H_{N(e)}}(x) = \frac{I_{F_{N(e)}}(x) + I_{G_{N(e)}}(x)\}}{2},$$

$$F_{H_{N(e)}}(x) = max\{F_{F_{N(e)}}(x), F_{G_{N(e)}}(x)\}.$$

Definition 2.10. [13]. *Let $NSS(X, E)$ be the family of all neutrosophic soft sets over X and $\widetilde{\tau}_N \subset NSS(X, E)$. Then $\widetilde{\tau}_N$ is called neutrosophic soft topology on (X, E) if the following conditions are satisfied:*

(i) $\widetilde{\Phi}_N$, $\widetilde{X}_N \in \widetilde{\tau}_N$
(ii) $\widetilde{\tau}_N$ *is closed under arbitrary union.*
(iii) $\widetilde{\tau}_N$ *is closed under finite intersection.*

Then the triplet $(X, \widetilde{\tau}_N, E)$ is called neutrosophic soft topological space. The members of $\widetilde{\tau}_N$ are called neutrosophic soft open sets in $(X, \widetilde{\tau}_N, E)$. A neutrosophic soft set $\widetilde{F}_N$ in $NSS(X, E)$ is soft closed in $(X, \widetilde{\tau}_N, E)$ if its complement $(\widetilde{F}_N)^c$ is neutrosophic soft open set in $(X, \widetilde{\tau}_N, E)$.

The neutrosophic soft closure of $\widetilde{F}_N$ is the neutrosophic soft set,
$N\tilde{s}cl(\widetilde{F}_N) = \cap\{\widetilde{G}_N : \widetilde{G}_N$ is neutrosophic soft closed and $\widetilde{F}_N \subseteq \widetilde{G}_N\}$.

The neutrosophic soft interior of $\widetilde{F}_N$ is the neutrosophic soft set,
$N\tilde{s}int(\widetilde{F}_N) = \cup\{\widetilde{O}_N : \widetilde{O}_N$ is neutrosophic soft closed and $\widetilde{O}_N \subseteq \widetilde{F}_N\}$.

It is easy to see that $\widetilde{F}_N$ is neutrosophic soft open if and only if
$\widetilde{F}_N = N\tilde{s}int(\widetilde{F}_N)$ and neutrosophic soft closed if and only if $\widetilde{F}_N = N\tilde{s}cl(\widetilde{F}_N)$.

Definition 2.11. [18]. *Let $(X, \widetilde{\tau}_N, E)$ be a neutrosophic soft topological space and $\widetilde{F}_N$ be a neutrosophic soft open set in (X, E), then $\widetilde{F}_N$ is called:*

(i) *Neutrosophic soft α-open iff $\widetilde{F}_N \subseteq N\tilde{s}int(N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N)))$,*
(ii) *Neutrosophic soft pre-open iff $\widetilde{F}_N \subseteq N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))$,*
(iii) *Neutrosophic soft semi-open iff $\widetilde{F}_N \subseteq N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))$,*
(iv) *Neutrosophic soft β-open iff $\widetilde{F}_N \subseteq N\tilde{s}cl(N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)))$,*
(v) *Neutrosophic soft regular-open iff $\widetilde{F}_N = N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))$.*

Definition 2.12. [18]. *Let $(X, \widetilde{\tau}_N, E)$ be a neutrosophic soft topological space and $\widetilde{F}_N \in NSS(X, E)$, then $\widetilde{F}_N$ is called:*

(i) *Neutrosophic soft α-closed iff $N\tilde{s}cl(N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))) \subseteq \widetilde{F}_N$,*
(ii) *Neutrosophic soft pre-closed iff $N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N)) \subseteq \widetilde{F}_N$,*
(iii) *Neutrosophic soft semi-clsed iff $N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \subseteq \widetilde{F}_N$,*
(iv) *Neutrosophic soft β-closed iff $N\tilde{s}int(N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))) \subseteq \widetilde{F}_N$,*
(v) *Neutrosophic soft regular-closed iff $\widetilde{F}_N = N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))$.*

3. NEUTROSOPHIC SOFT B-OPEN SETS

Definition 3.1. *A NSS $\widetilde{F}_N$ in a NSTS X is called*

(i) *neutrosophic soft b-open (NSBO) set iff $\widetilde{F}_N \subseteq N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \cup N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))$,*
(ii) *neutrosophic soft b-closed (NSBC) set iff $\widetilde{F}_N \supseteq N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \cap N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))$.*

It is obvious that $NSPO(X) \cup NSSO(X) \subseteq NSBO(X)$. The inclusion cannot be replaced with equalities.

Theorem 3.1. *For a NSS $\widetilde{F}_N$ in a NSTS X*

(i) *$\widetilde{F}_N$ is a NSBO set iff $(\widetilde{F}_N)^c$ is a NSBC set.*
(ii) *$\widetilde{F}_N$ is a NSBC set iff $(\widetilde{F}_N)^c$ is a NSBO set.*

Proof. Obvious from the definition. □

Definition 3.2. *Let $(X, \tilde{\tau}_N, E)$ be a NSTS and $\widetilde{F}_N$ be a NSS over X.*

(i) *Neutrosophic soft b-interior of $\widetilde{F}_N$ briefly [$N\tilde{s}bint(\widetilde{F}_N)$] is the union of all neutrosophic soft b-open sets of X contained in $\widetilde{F}_N$. That is, $N\tilde{s}bint(\widetilde{F}_N) = \cup\{\widetilde{G}_N : \widetilde{G}_N$ is a NSBO set in X and $\widetilde{G}_N \subseteq \widetilde{F}_N\}$.*
(ii) *Neutrosophic soft b-closure of $\widetilde{U}_N$ briefly [$N\tilde{s}bcl(\widetilde{F}_N)$] is the intersection of all neutrosophic soft b-closed sets of X contained in $\widetilde{F}_N$. That is, $N\tilde{s}bcl(\widetilde{F}_N) = \cap\{\widetilde{H}_N : \widetilde{H}_N$ is a NSBC set in X and $\widetilde{H}_N \supseteq \widetilde{F}_N\}$.*

Clearly $N\tilde{s}bcl(\widetilde{F}_N)$ is the smallest neutrosophic soft b-closed set over X which contains $\widetilde{F}_N$ and $N\tilde{s}bint(\widetilde{F}_N)$ is the largest neutrosophic soft b-open set over X which is contained in $\widetilde{F}_N$.

Theorem 3.2. *Let $\widetilde{F}_N$ be a NSS in a NSTS X. Then,*

(i) $(N\tilde{s}bint(\widetilde{F}_N))^c = N\tilde{s}bcl((\widetilde{F}_N)^c)$
(ii) $(N\tilde{s}bcl(\widetilde{F}_N))^c = N\tilde{s}bint((\widetilde{F}_N)^c)$

Proof. (i) Let $\widetilde{F}_N$ be a NSS in NSTS. Now $N\tilde{s}bint(\widetilde{F}_N) = \cup\{\widetilde{D}_N : \widetilde{D}_N$ is a NSBO set in X and $\widetilde{D}_N \subseteq \widetilde{F}_N\}$. Then $(N\tilde{s}bint(\widetilde{F}_N))^c = [\cup\{\widetilde{D}_N : \widetilde{D}_N$ is a NSBO set in Z and $\widetilde{D}_N \subseteq \widetilde{F}_N\}]^c = \cap\{(\widetilde{D}_N)^c : (\widetilde{D}_N)^c$ is a NSBC set in X and $(\widetilde{D}_N)^c \subseteq (\widetilde{F}_N)^c\}$. Replacing $(\widetilde{D}_N)^c$ by $\widetilde{M}_N$, we get $NSbint(\widetilde{F}_N)^c = \cap\{\widetilde{M}_N : \widetilde{M}_N$ is a NSBC set in X and $\widetilde{M}_N \supseteq (\widetilde{F}_N)^c\}$, $(N\tilde{s}bint(\widetilde{F}_N))^c = N\tilde{s}bcl((\widetilde{F}_N))^c)$. This proves (i).
Analogously (ii) can be proved. □

Lemma 3.1. *In a NSTS we have the following*

(i) *Every neutrosophic soft regular open set is neutrosophic soft open.*
(ii) *Every neutosophic soft open set is neutrosophic soft α-open.*
(iii) *Every neutrosophic soft α-open set is both neutrosophic soft semi-open and neutrosophic soft pre-open.*
(iv) *Every neutrosophic soft semi-open set and every neutrosophic soft pre-open set is neutrosophic soft b-open.*

Theorem 3.3. *In a neutrosophic soft topological space X*

(i) *Every neutrosophic soft pre-open set is a neutrosophic soft b-open set.*
(ii) *Every neutrosophic soft semi-open set is a neutrosophic soft b-open set.*

Proof. (i) Let $\widetilde{F}_N$ be a NSPO set in a NSTS X. Then $\widetilde{F}_N \subseteq N\tilde{s}int\ N\tilde{s}cl(\widetilde{F}_N)$ which implies $\widetilde{F}_N \subseteq N\tilde{s}int\ N\tilde{s}cl(\widetilde{F}_N) \cup N\tilde{s}int\widetilde{F}_N \subseteq N\tilde{s}int\ N\tilde{s}cl\widetilde{F}_N \cup N\tilde{s}cl\ N\tilde{s}int\widetilde{F}_N$. Thus $\widetilde{F}_N$ is a NSBO set.
(ii) Let $\widetilde{F}_N$ be a NSSO set in a NSTS X. Then $\widetilde{F}_N \subseteq N\tilde{s}cl\ N\tilde{s}int(\widetilde{F}_N)$ which implies $\widetilde{F}_N \subseteq N\tilde{s}clN\tilde{s}int(\widetilde{F}_N) \cup N\tilde{s}int\widetilde{F}_N \subseteq N\tilde{s}cl\ N\tilde{s}int\widetilde{F}_N \cup N\tilde{s}int\ N\tilde{s}cl\widetilde{F}_N$. Thus $\widetilde{F}_N$ is a NSBO set. □

Theorem 3.4. *Let $\widetilde{F}_N$ be a NSS in NSTS $(X, \widetilde{\tau}, E)$. Then*

(i) $N\tilde{s}scl\widetilde{F}_N = \widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$ *and* $N\tilde{s}sint\widetilde{F}_N = \widetilde{F}_N \cap N\tilde{s}clN\tilde{s}int\widetilde{F}_N$.
(ii) $N\tilde{s}pcl\widetilde{F}_N = \widetilde{F}_N \cup N\tilde{s}clN\tilde{s}int\widetilde{F}_N$ *and* $N\tilde{s}pint\widetilde{F}_N = \widetilde{F}_N \cap N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$.

Proof. (i) $N\tilde{s}scl\widetilde{F}_N \supseteq N\tilde{s}intN\tilde{s}clN\tilde{s}scl\widetilde{F}_N \supseteq N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$, and $\widetilde{F}_N \cup N\tilde{s}scl\widetilde{F}_N = N\tilde{s}scl\widetilde{F}_N \supseteq \widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$. So,

$$\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N \subseteq N\tilde{s}scl\widetilde{F}_N . \tag{3.1}$$

Also $\widetilde{F}_N \subseteq N\tilde{s}scl\widetilde{F}_N$, $N\tilde{s}intN\tilde{s}cl\widetilde{F}_N \subseteq N\tilde{s}intN\tilde{s}clN\tilde{s}scl\widetilde{F}_N \subseteq N\tilde{s}scl\widetilde{F}_N$.

$$\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N \subseteq N\tilde{s}scl\widetilde{F}_N \cup \widetilde{F}_N \subseteq N\tilde{s}scl\widetilde{F}_N . \tag{3.2}$$

From (3.1) and (3.2), $N\tilde{s}scl\widetilde{F}_N = \widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$.
$N\tilde{s}sint\widetilde{F}_N = \widetilde{F}_N \cap N\tilde{s}clN\tilde{s}int\widetilde{F}_N$ can be proved by taking the complement of $N\tilde{s}scl\widetilde{F}_N = \widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$. This proves (i).
The proof for (ii) is analogous. □

Theorem 3.5. *Let $\widetilde{F}_N$ be a NSS in NSTS. Then:*

(i) $N\tilde{s}bcl\widetilde{F}_N = N\tilde{s}scl\widetilde{F}_N \cap N\tilde{s}pcl\widetilde{F}_N$,
(ii) $N\tilde{s}bint\widetilde{F}_N = N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}pint\widetilde{F}_N$.

Proof. (i) Since $N\tilde{s}bcl\widetilde{F}_N$ is a NSBO set. We have

$N\tilde{s}bcl\widetilde{F}_N \supseteq N\tilde{s}intN\tilde{s}cl(N\tilde{s}bcl\widetilde{F}_N)\cap N\tilde{s}clN\tilde{s}int(N\tilde{s}bcl\widetilde{F}_N) \supseteq N\tilde{s}intN\tilde{s}cl\widetilde{F}_N\cap N\tilde{s}clN\tilde{s}int\widetilde{F}_N$

and also $N\tilde{s}bcl\widetilde{F}_N \supseteq \widetilde{F}_N\cup N\tilde{s}intN\tilde{s}cl\widetilde{F}_N\cap N\tilde{s}clN\tilde{s}int\widetilde{F}_N = N\tilde{s}scl\widetilde{F}_N\cap N\tilde{s}pcl\widetilde{F}_N$.

The reverse inclusion is clear. Therefore $N\tilde{s}bcl\widetilde{F}_N = N\tilde{s}scl\widetilde{F}_N \cap N\tilde{s}pcl\widetilde{F}_N$.

Analogously (ii) can be proved. □

Theorem 3.6. *Let U be a NSS in NSTS. Then:*

(i) $N\tilde{s}sclN\tilde{s}sint\widetilde{F}_N = N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}clN\tilde{s}int\widetilde{F}_N$,

(ii) $N\tilde{s}sintN\tilde{s}scl\widetilde{F}_N = N\tilde{s}scl\widetilde{F}_N \cap N\tilde{s}clN\tilde{s}intN\tilde{s}cl\widetilde{F}_N$.

Proof. We have:

$$\begin{aligned} N\tilde{s}sclN\tilde{s}sint\widetilde{F}_N &= N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl(N\tilde{s}sint\widetilde{F}_N) \\ &= N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}int(N\tilde{s}cl[\widetilde{F}_N \cap N\tilde{s}clN\tilde{s}int\widetilde{F}_N]) \\ &\subseteq N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}int[N\tilde{s}cl\widetilde{F}_N \cap N\tilde{s}cl(N\tilde{s}int\widetilde{F}_N)] \\ &= N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}int[N\tilde{s}cl(N\tilde{s}int\widetilde{F}_N)] . \end{aligned}$$

To establish the opposite inclusion we observe that:

$$\begin{aligned} N\tilde{s}scl(N\tilde{s}sint\widetilde{F}_N) &= N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl(N\tilde{s}sint\widetilde{F}_N) \\ &\supseteq N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl(N\tilde{s}int\widetilde{F}_N) \end{aligned}$$

Therefore we have $N\tilde{s}sclN\tilde{s}sint\widetilde{F}_N = N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}clN\tilde{s}int\widetilde{F}_N$.

This proves (i), and the proof for (ii) is analogous. □

Theorem 3.7. *Let $\widetilde{F}_N$ be a NSS in NSTS. Then:*

(i) $N\tilde{s}pclN\tilde{s}pint\widetilde{F}_N = N\tilde{s}pint\widetilde{F}_N \cup N\tilde{s}clN\tilde{s}int\widetilde{F}_N$,

(ii) $N\tilde{s}pintN\tilde{s}pcl\widetilde{F}_N = N\tilde{s}pcl\widetilde{F}_N \cap N\tilde{s}intN\tilde{s}cl\widetilde{F}_N$.

Proof. We have:

$$\begin{aligned} N\tilde{s}pclN\tilde{s}pint\widetilde{F}_N &= N\tilde{s}pint\widetilde{F}_N \cup N\tilde{s}clN\tilde{s}int(N\tilde{s}pint\widetilde{F}_N) \\ &= N\tilde{s}pint\widetilde{F}_N \cup N\tilde{s}clN\tilde{s}int[\widetilde{F}_N \cap N\tilde{s}intN\tilde{s}cl\widetilde{F}_N]) \\ &= N\tilde{s}pint\widetilde{F}_N \cup N\tilde{s}cl[N\tilde{s}int\widetilde{F}_N \cap N\tilde{s}int(N\tilde{s}intN\tilde{s}cl\widetilde{F}_N)] \\ &= N\tilde{s}pint\widetilde{F}_N \cup N\tilde{s}cl[N\tilde{s}int\widetilde{F}_N] . \end{aligned}$$

To establish the opposite inclusion we observe that,

$$\begin{aligned} N\tilde{s}scl(N\tilde{s}sint\widetilde{F}_N) &= N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl(N\tilde{s}sint\widetilde{F}_N) \\ &\supseteq N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}cl(N\tilde{s}int\widetilde{F}_N)\,. \end{aligned}$$

Therefore we have $N\tilde{s}sclN\tilde{s}sint\widetilde{F}_N = N\tilde{s}sint\widetilde{F}_N \cup N\tilde{s}intN\tilde{s}clN\tilde{s}int\widetilde{F}_N$.

This proves (i), and analogously (ii) can be proved. □

Theorem 3.8. *In a NSTS X, every NSBO (NSBC) set is neutrosophic soft β-open(neutrosophic soft β-closed) set.*

Proof. Let $\widetilde{F}_N$ be a NSBO set in NSTS. Then:

$$\begin{aligned} \widetilde{F}_N &\subseteq N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N)) \cup N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \\ &\subseteq N\tilde{s}cl(N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))) \cup N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \\ &\subseteq N\tilde{s}cl(N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)))\,. \end{aligned}$$

Thus $\widetilde{F}_N$ is neutrosophic soft β-open set. The converse is not true. □

Theorem 3.9. *In a NSTS X*

(i) *An arbitrary union of NSBO sets is a NSBO set.*
(ii) *An arbitrary intersection of NSBC sets is a NSBC set.*

Proof. (i) Let $(\widetilde{F}_N)_\alpha$ be a collection of NSBO sets in NSTS. Then for each α,

$$(\widetilde{F}_N)_\alpha \subseteq N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N)_\alpha) \cup N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)_\alpha)\,.$$

Now,

$$\begin{aligned} \cup(\widetilde{F}_N)_\alpha &\subseteq \cup[N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N)_\alpha) \cup N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)_\alpha)] \\ &\subseteq [N\tilde{s}cl(N\tilde{s}int(\cup(\widetilde{F}_N)_\alpha)) \cup N\tilde{s}int(N\tilde{s}cl(\cup(\widetilde{F}_N)_\alpha))]\,. \end{aligned}$$

Hence $\cup(\widetilde{F}_N)_\alpha$ is a NSBO set.

(ii) Similarly by taking complements. □

Theorem 3.10. *In NSTS X, $\widetilde{F}_N$ is NSBC (NSBO) set if and only if $\widetilde{F}_N = N\tilde{s}bcl\widetilde{F}_N$ ($\widetilde{F}_N = N\tilde{s}bint\widetilde{F}_N$).*

Proof. Suppose that

$$\widetilde{F}_N = N\tilde{s}bcl\widetilde{F}_N = \cap\{\widetilde{H}_N : \widetilde{H}_N \text{ is a NSBC set in } X \text{ and } \widetilde{H}_N \supseteq \widetilde{F}_N\}\,.$$

That implies $\widetilde{F}_N \in \cap\{\widetilde{H}_N : \widetilde{H}_N$ is a NSBC set in X and $\widetilde{H}_N \supseteq \widetilde{F}_N\}$. That implies $\widetilde{F}_N$ is NSBC set.
Conversely, suppose $\widetilde{F}_N$ is NSBC set in X. we take $\widetilde{F}_N \subseteq \widetilde{F}_N$ and $\widetilde{F}_N$ is NSBC set. Therefore $\widetilde{F}_N \in \cap\{\widetilde{H}_N : \widetilde{H}_N$ is a NSBC set in X and $\widetilde{H}_N \supseteq \widetilde{F}_N\}$. $\widetilde{F}_N \subseteq \widetilde{H}_N$ implies $\widetilde{F}_N \subseteq \cap\{\widetilde{H}_N : \widetilde{H}_N$ is a NSBC set in X and $\widetilde{H}_N \supseteq \widetilde{F}_N\} = N\tilde{s}bcl\widetilde{F}_N$.
For $\widetilde{F}_N = N\tilde{s}bint\widetilde{F}_N$ we apply neutrosophic soft interior. □

Theorem 3.11. *Let $(X, \widetilde{\tau}_N, E)$ be a neutrosophic soft topological space over X and $\widetilde{F}_N \in$ NSBC set. Then:*

(i) $N\tilde{s}bcl\widetilde{\Phi}_N = \Phi_N$,
(ii) $N\tilde{s}bint\widetilde{\Phi}_N = \Phi_N$,
(iii) $N\tilde{s}bcl\widetilde{F}_N$ *is a NSBC set*,
(iv) $N\tilde{s}bcl(N\tilde{s}bcl\widetilde{F}_N) = N\tilde{s}bcl\widetilde{F}_N$.

Proof. proof is obvious. □

Theorem 3.12. *In NSTS X, the following relation hold:*

(i) $N\tilde{s}bcl(\widetilde{F}_N \cup \widetilde{G}_N) \supseteq N\tilde{s}bcl\widetilde{F}_N \cup N\tilde{s}bcl\widetilde{G}_N$.
(ii) $N\tilde{s}bcl(\widetilde{F}_N \cap \widetilde{G}_N) \subseteq N\tilde{s}bcl\widetilde{F}_N \cap N\tilde{s}bcl\widetilde{G}_N$.

Proof. (i) $\widetilde{F}_N \subseteq \widetilde{F}_N \cup \widetilde{G}_N$ or $\widetilde{G}_N \subseteq \widetilde{F}_N \cup \widetilde{G}_N$. That implies

$$N\tilde{s}bcl(\widetilde{F}_N) \subseteq N\tilde{s}bcl[\widetilde{F}_N \cup \widetilde{G}_N],$$

or

$$N\tilde{s}bcl(\widetilde{G}_N) \subseteq N\tilde{s}bcl[\widetilde{F}_N \cup \widetilde{G}_N].$$

Thus $N\tilde{s}bcl(\widetilde{F}_N \cup \widetilde{G}_N) \supseteq N\tilde{s}bcl\widetilde{F}_N \cup N\tilde{s}bcl\widetilde{G}_N$.
(ii) Similar to that of (i). □

Theorem 3.13. *In NSTS X, the following relation hold:*

(i) $N\tilde{s}bint(\widetilde{F}_N \cup \widetilde{G}_N) \supseteq N\tilde{s}bint\widetilde{F}_N \cup N\tilde{s}bint\widetilde{G}_N$.
(ii) $N\tilde{s}bint(\widetilde{F}_N \cap \widetilde{G}_N) \subseteq N\tilde{s}bint\widetilde{F}_N \cap N\tilde{s}bint\widetilde{G}_N$.

Proof. (i) $\widetilde{F}_N \subseteq \widetilde{F}_N \cup \widetilde{G}_N$ or $\widetilde{G}_N \subseteq \widetilde{F}_N \cup \widetilde{G}_N$. That implies

$$N\tilde{s}bint(\widetilde{F}_N) \subseteq N\tilde{s}bint[\widetilde{F}_N \cup \widetilde{G}_N],$$

or

$$N\tilde{s}bint(\widetilde{G}_N) \subseteq N\tilde{s}bint[\widetilde{F}_N \cup \widetilde{G}_N].$$

Thus:

$$N\tilde{s}bint(\widetilde{F}_N \cup \widetilde{G}_N) \supseteq N\tilde{s}bint\widetilde{F}_N \cup N\tilde{s}bint\widetilde{G}_N\,.$$

(ii) Similar to that of (i). □

Theorem 3.14. *In NSTS X, If $\widetilde{G}_N$ is NSO set and $\widetilde{F}_N$ is NSBO set. Then $\widetilde{F}_N \cap \widetilde{G}_N$ is a NSBO set.*

Proof.

$$\begin{aligned}
\widetilde{G}_N \cap \widetilde{F}_N &\subseteq \widetilde{G}_N \cap [N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \cup N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))] \\
&= [\widetilde{G}_N \cap [N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))] \cup [\widetilde{G}_N \cap N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))] \\
&= [N\tilde{s}int(N\tilde{s}cl(\widetilde{G}_N)) \cap [N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))] \\
&\cup\ [N\tilde{s}cl(N\tilde{s}int(\widetilde{G}_N)) \cap N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))] \\
&= [N\tilde{s}intN\tilde{s}cl(\widetilde{G}_N \cap \widetilde{F}_N)] \cup [N\tilde{s}clN\tilde{s}int(\widetilde{G}_N \cap \widetilde{F}_N)]\,.
\end{aligned}$$

Hence $\widetilde{F}_N \cap \widetilde{G}_N$ is a NSBO set. □

Theorem 3.15. *In NSTS X, If $\widetilde{G}_N$ is neutrosophic soft α-open set and $\widetilde{F}_N$ is NSBO set. Then $\widetilde{F}_N \cap \widetilde{G}_N$ is a NSBO set.*

Proof.

$$\begin{aligned}
\widetilde{G}_N \cap \widetilde{F}_N &\subseteq N\tilde{s}int(N\tilde{s}cl(N\tilde{s}int\widetilde{G}_N))) \cap (N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N)) \cup N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))) \\
&= [N\tilde{s}int(N\tilde{s}cl(N\tilde{s}int\widetilde{G}_N))) \cap [N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))] \\
&\cup\ [N\tilde{s}int(N\tilde{s}cl(N\tilde{s}int\widetilde{G}_N))) \cap N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))]N\tilde{s}intN\tilde{s}cl\widetilde{F}_N \\
&\subseteq [N\tilde{s}int(N\tilde{s}cl\widetilde{G}_N)) \cap N\tilde{s}int(N\tilde{s}cl(\widetilde{F}_N))] \\
&\cup\ [N\tilde{s}cl(N\tilde{s}int\widetilde{G}_N))) \cap N\tilde{s}cl(N\tilde{s}int(\widetilde{F}_N))] \\
&\subseteq [N\tilde{s}intN\tilde{s}cl(\widetilde{G}_N \cap \widetilde{F}_N)] \cup [N\tilde{s}clN\tilde{s}int(\widetilde{G}_N \cap \widetilde{F}_N)]\,.
\end{aligned}$$

Hence $\widetilde{F}_N \cap \widetilde{G}_N$ is a NSBO set. □

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