

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2017.Doi Number

Neutrosophic soft sets forecasting model for multi-attribute time series

Hongjun Guan¹, Jie He¹, Shuang Guan², Aiwu Zhao^{3,*}

¹ School of management science and engineering, Shandong University of Finance and Economics, Jinan, 250014, China ²Courant Institute of Mathematical Sciences, New York University, USA ³School of management, Jiangsu University, Zhenjiang 212013, China

Corresponding author: Aiwu Zhao (e-mail: aiwuzh@ujs.edu.cn).

This work was supported in part by the China National Social Science Fund under Grant 15FGL018 and 17FJY001.

ABSTRACT Traditional time series forecasting models mainly assume a clear and definite functional relationship between historical values and current/future values of a dataset. In this paper, we extended current model by generating multi-attribute forecasting rules based on consideration of combining multiple related variables. In this model, neutrosophic soft sets (NSSs) are employed to represent historical statues of several closely related attributes in stock market such as volumes, stock market index and daily amplitudes. Specifically speaking, the status of up, equal and down in historical stock index can be represented by truth, indeterminacy and false respectively by Neutrosophic Sets (NSs) and NSSs can build mappings of different related attributes to NSs. The advantages of proposed model are: (1) Using NSSs to enclose different historical characteristics in time series to preserve inherent complexity of a dataset with mapping adequate features. (2) With existing researches of NSSs, it's efficient with using Euclidean distance to find the optimal rules thus the model can avoid incomplete of rules due to limited sample dataset. To evaluate the performance of the model, we explored the closing prices of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) as the major parameter we forecast and the stock amplitudes and volumes as other factors to facilitate the predicting of the TAIEX. To show the universality of the model, we applied the proposed model to forecast some other influential indexes as well.

INDEX TERMS fuzzy time series; neutrosophic soft set; Euclidean distance; forecasting model

I. INTRODUCTION

Predicting stock prices has been attractive to stock managers and financial analysts for a long time. The reason is stock market index would heavily influence stock investors when they make investment decisions, thus an accurate forecasting algorithm is eagerly desired. To achieve the algorithms, many scholars and financial analysts sought to generalize forecasting rules from historical data [1] and designed time series models with statistical tools. For example, regression analysis, moving averages, integrated moving average and autoregressive moving average models [2-5] have been widely used. Combining with complex network theory, many researchers implement nonlinear time series analysis by means of complex network methods [6-8]. Such methods obtain the characteristics of the transitions of the states in time series by analyzing the network's topological structure. With development in computer science, some novel models such as artificial neural networks [9], quantile regression [10, 11]

and random forest [12, 13] arose to handle upcoming problems that couldn't efficiently be solved by traditional statistical methods, such as missing data and nonlinear relationships. In the purpose of processing historical data with linguistic terms and random noises, Song and Chissom [14-16] came up with models in fuzzy time series method. After those, many researchers have improved the original fuzzy time series models. For example, Chen [17] and Huarng [18] improved forecasting accuracy by modifying and extending some attributes of fuzzy time series models with findings suggest length of intervals could be essential regarding model performance, etc. As a generation of fuzzy set theory [19], Atanassov [20] developed intuitionistic fuzzy set (IFS) for expressing and processing uncertainty in a better fashion. Recently, several scholars researched IFS regarding its similarity measures [21] and developed forecasting models [22, 23] based on IFS.

However, in stock index forecasting, many other inherent factors besides the historical data of the stock index itself

1

IEEE Access

can significantly interact with stock market performance such as volumes, daily amplitude and performances of foreign stock markets etc., it would be helpful to generalize those factors into one forecasting model with expectation of more complete rules and preserved complexity. Recent years, many studies [24-26, 5] presented fuzzy time series forecasting models with consideration of foreign markets to build rules for forecasting future trends. Extending discussion to other fields, in 2002, Chen and Hwang [27] presented a fuzzy time series method with multiple attributes to predict local temperature.

Regarding stock index forecasting, one efficient and accurate approach of conveniently comparing historical data is to divide them into three states of up, down and inconsistency. This idea coincided with neutrosophic sets (NSs) and their similarity comparisons. NS, firstly introduced by Smarandache [28], consists of three memberships: true, indeterminacy and false which could be matched to three states of stock index trends, respectively. Since introduced, NS has been developing robustly and introducing several extensions including single-valued NSs [29] and interval-valued NSs [30]. Practically speaking, NSs have been applying in various fields. For some selected example, many researchers applied NSs in urban construction, decision making and supplier evaluation [31-33]. Plus, similarity comparisons in NSs were also studied by scholars [34-36]. To deal with data with several attributes and uncertain items, Molodtsov [37] introduced theory of soft sets and a combined neutrosophic soft sets (NSSs) could take advantage of general mathematical tool proposed. NSSs have been extended to complex practical problems. For example, Peng et.al [38] presented singlevalued neutrosophic soft sets (SVNSSs) which can preserve more original information.

In this paper, we present a stock market forecasting model based on NSSs and fuzzy time series. The novelty and advantage of the proposed model are given by: 1. Using NSSs to take different historical features of a stock market into consideration helps the model preserving inherent complexity. 2. With existing researches of NSSs, we employed Euclidean distance measurements to find optimal rules and overcame difficulty of incomplete rules due to imperfect sampling data. In order to illustrate the prediction steps, we firstly select relating parameters of the stock market, such as closing price, volume and amplitude, and convert their historical training data from the original time series to the fluctuation time series. Secondly, we used NSSs to show the possibility of three trends. Then, the historical training data are fuzzified to form a fuzzy logic relationship based on NSSs. Finally, finding the most suitable logic rules for predicting its future through using Euclidean distance measurement on the logic relationships and historical data of NSSs obtained before.

The rest of the article is organized as follows: Section 2 provides basic notions and backgrounds of time series, NSs, NSSs, and Euclidean distances of NSSs. Section 3 proposes the forecasting model based on NSSs and fuzzy time series.

In Section 4, experimental results of the proposed method are compared with existing methods and we also use the proposed model to forecast TAIEX from 1998 to 2004 and several other influential indexes. The conclusions are discussed in Section 5.

II. PRELIMINARIES

2.1. Fuzzification

Definition 1 (Fuzzy Time Series). Let $L = \{l_1, l_2, ..., l_g\}$ be a linguistic set in the universe of discourse U; it can be defined by its membership function, $\mu_L: U \rightarrow [0, 1]$, where $\mu_L(u_i)$ denotes the grade of membership of $u_i, U = \{u_1, u_2, ..., u_l\}$.

Definition 2 (Fuzzy Fluctuation Time Series). The fluctuation trends of a stock market can be expressed by a linguistic set $L = \{l_1, l_2, l_3, l_4, l_5\} = \{\text{down, slightly down, equal, slightly up, up}\}$. The element l_i and its subscript i is strictly increasing, so the function can be defined as follows: $f: l_i = f(i)$.

Let F(t)(t = 1, 2, ..., T) be a time series of real numbers, where T is the number of the time series. G(t) is defined as a fluctuation time series, where G(t) = F(t) - F(t - 1). Each element of G(t) can be represented by a fuzzy set S(t)(t = 2, 3, ..., T) as defined in Definition 1. Then we call time series G(t) to be fuzzified into a fuzzy-fluctuation time series (FFTS) S(t).

Definition 3 (Fuzzy Fluctuation Logical Relationship). Let $S(t)(t = n + 1, n + 2, ..., T, n \ge 1)$ be a FFTS. If S(t) is determined by S(t - 1), S(t - 2), ..., S(t - n), then the fuzzy-fluctuation logical relationship is represented by:

$$S(t-1), S(t-2), \dots, S(t-n) \to S(t)$$
 (1)

In the same way, let $S(t)(t = n + 1, n + 2, ..., T, n \ge 1)$ be a three-factor FFTS. If the next status of S(t) is caused by the current status of $S_1(t)$, $S_2(t)$ and $S_3(t)$, the three-factor nth-order fuzzy-fluctuation is represented by:

$$((S_{1}(t-1), ..., S_{1}(t-n)), (S_{2}(t-1), ..., S_{2}(t-n)), (S_{3}(t-1), ..., S_{3}(t-n)))$$
(2)

$$\rightarrow S(t)$$

It is called the three-factor nth-order fuzzy-fluctuation logical relationship (FFLR) of the fuzzy-fluctuation time series, where $S_i(t - n), ..., S_i(t - 2), S_i(t - 1)(i = 1,2,3)$ is called the left-hand side (LHS) and S(t) is called the right-hand side (RHS) of the FFLR.

Example 1. Consider a three factor 2^{nd} -order fuzzy fluctuation logical relationship where t = 9, we may represent it as:

$$(S_1(7), S_1(8)), (S_2(7), S_2(8),), (S_3(7), S_3(8))$$
 (3)
 $\rightarrow S(9)$

2.2 Neutrosophic Soft Set

Definition 4 (Neutrosophic Set). Let X be a space of objects with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truthmembership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. If a neutrosophic set A consists of $T_A(x)$, $I_A(x)$, $F_A(x)$, they

^{2169-3536 (}c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

IEEEACCESS

can be defined by its membership function, while $T_A(x)$, $I_A(x)$, $F_A(x)$ are subsets of [0, 1], then A can be represent by:

$$A = \{ \langle T_A(x), I_A(x), F_A(x) \rangle | x \in X \}$$

$$(4)$$

Definition 5 (Neutrosophic Soft Set). Let U be a universe of discourse and E be a set of parameters. Let NS(U) denotes the set of all neutrosophic subsets of U and $A \subset E$, A pair $R_A = (N_{\{A\}}, E)$ is called a Neutrosophic Soft Set (NSS) over U, where $N_{\{A\}}$ is a mapping given by $N_{\{A\}}: E \to N_{\{A\}}$.

Example 2. Consider a random example. Let $E = \{e_1, e_2, e_3, \dots e_m\}$ be the set of parameters that reflecting the fluctuation of the stock market and $A = \{e_1, e_2, e_3\}$ be the set of parameters considered for forecasting problems. Assume:

$$N_{\{A\}}(e_1) = (0.60, 0.40, 0.80)$$
$$N_{\{A\}}(e_2) = (0.30, 0.50, 0.10)$$
$$N_{\{A\}}(e_3) = (0.30, 0.20, 0.30)$$

Here $N_{\{A\}}(e_1) = (0.6, 0.4, 0.8)$ expresses the upward, steady and downward trend of days with the parameter e_1 . Parameter e_1 expresses that the upward trend of these days using the degree of membership 0.6, indeterminacy 0.4 and falsity 0.8. Then the NSS $R_A = (N_{\{A\}}, E)$ is given by $R_A = (N_{\{A\}}, E) = (N_{\{A\}}, E)$

 $\begin{aligned} R_{A} &= \left(N_{\{A\}}, E\right) = \left(N_{A(e_{1})}, N_{A(e_{2})}, N_{A(e_{3})}\right) \\ &= \left((0.60, 0.40, 0.80), (0.30, 0.50, 0.10), (0.30, 0.20, 0.30)\right) \\ & 2.3 \ Generating \ Neutrosophic \ Soft \ Set \ of \ Logical \\ Relationship \end{aligned}$

Definition 6 (Conversion of Fuzzy Fluctuation Logical Relationship). Let $N_{A(t)}^{k}$ (k = 1,2,3) be the truthmembership, indeterminacy-membership and falsitymembership of a neutrosophic set A(t), respectively. The LHS of a nth-order FFLR S(t – n),..., S(t – 2), S(t – 1) can be generated by:

$$N_{A(t)}^{k} = \frac{\sum_{i=1..g, j=1..n} (w_{i,j} * c_{i,k})}{n}$$

= 1,2, ..., g; j = 1,2, ..., n; k = 1,2,3 (5)

where $w_{i,j} = 1$ if S(t - j) = i and 0, otherwise, $c_{i,k}$ represents the corresponding relationship between linguistic element li (li \in L) and the kth membership of a neutrosophic set A(t). Thus, the LHS of a nth-order FFLR S(t - n), ..., S(t - 2), S(t - 1) can be converted into a neutrosophic set($N_{A(t)}^{1}, N_{A(t)}^{2}, N_{A(t)}^{3}$).

Definition 7 (Neutrosophic Soft Set of Logical Relationship). Let $S(t)(t = n + 1, n + 2, ..., T, n \ge 1)$ be a FFTS, A(t) be the LHS of a neutrosophic soft matrices logical relationship and $E = \{e_1, e_2, e_3, ..., e_m\}$ be the set of parameters. The FFLRs with the similarity A(t) can be grouped into a FFLRG by putting all their RHSs together as on the RHS of the FFLRG. The RHSs of the FFRLG for A(t) can be represented by a Neutrosophic soft set of logical relationship(NSSLRs) as Definition 5.

$$\begin{pmatrix} N_{A(e_1)}, N_{A(e_2)}, N_{A(e_3)} \dots N_{A(e_m)} \end{pmatrix} \rightarrow \begin{pmatrix} N_{A(t)}^1, N_{A(t)}^2, N_{A(t)}^3 \end{pmatrix}$$
(6)

where $(N_{A(t)}^1, N_{A(t)}^2, N_{A(t)}^3)$ represent the downward, steady, upward probabilities of the RHSs of the FFRLG for A(t).

Example 3. With same data assumed in Example 2, let $S(t)(t = n + 1, n + 2, ..., T, n \ge 1)$ be a FFTS as definition 3. Then its NSSLR can be represent as:

 $((0.60, 0.40, 0.80), (0.30, 0.50, 0.10), (0.30, 0.20, 0.30)) \rightarrow (0.00, 1.00, 0.00)$

Here we assumed that the each pre-defined S(t) corresponds to the different NSs, such as $\{0, 0, 1\}$, $\{0, 0.5, 0.5\}$, $\{0, 1, 0\}$, $\{0.5, 0.5, 0\}$, and $\{1, 0, 0\}$.

2.4 Similarity Measurement

Definition 8 (Euclidean Distance). Let $A = \langle T_A, I_A, F_A \rangle$, $B = \langle T_B, I_B, F_B \rangle$ be respectively NSSs, the Euclidean distance between A and B can be defined as:

$$d(A, B) = \sqrt{\sum_{i=1}^{n} \frac{\Delta_{iT(x)}^{2} + \Delta_{iI(x)}^{2} + \Delta_{iF(x)}^{2}}{3}}$$
(7)

where $\begin{array}{l} \Delta_{iT(x)} = T_{A(e_i)} - T_{B(e_i)}, \\ \Delta_{iI(x)} = I_{A(e_i)} - I_{B(e_i)}, \\ \Delta_{iF(x)} = F_{A(e_i)} - F_{B(e_i)} \end{array}$

Through distance measurement, we define similarity between A and B as:

$$S(A, B) = \frac{1}{1 + d(A, B)}$$
 (8)

Example 4. In this way, let A = ((0.20, 0.50, 0.10), (0.00, 1.00, 0.00)) and B = ((0.10, 0.50, 0.10), (0.00, 1.00, 0.00)) be two NSSs. Then the distance between them can be represented as :

$$d(A,B) = \sqrt{\frac{(0.2 - 0.1)^2 + (0.5 - 0.5)^2 + (0.1 - 0.1)^2}{3}} + \sqrt{\frac{(0.0 - 0.0)^2 + (1.0 - 1.0)^2 + (0.0 - 0.0)^2}{3}} \approx 0.06$$

III. A NOVEL FORECASTING MODEL BASED ON NSSLRs

In order to construct an nth-order fuzzy time series model, we represent the fuzzy relation by NSSs. The data from January to October in one year are used as training time series and the data from November to December are used as testing dataset. The steps of the method based on NSSs are given below.

VOLUME XX, 2017

2169-3536 (c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information. This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI

Start Generate fluctuation time series: $G_i(t)=F_i(t)-F_i(t-1)$ (t=2,3,..., T,i=1,2,3), Define fuzzy sets: $u_{11} = (-\infty, -3l_i/2), u_{12} = [-3l_i/2, -l_i/2), u_{13} = [-l_i/2, l_i/2],$ • $u_{14} = (l_i / 2, 3l_i / 2], u_{15} = (3l_i / 2, +\infty)$ Historical Where l1,l2,l3: $\sum_{l=\frac{t-2}{T-l}}^{1} G(t)$ i=1,2,3 Training • Data $G_i(t) \in [-\infty, -3l_i/2)$ Fuzzify the FTS to FFTS: 2 $G_i(t) \in [-3l_i/2, -l_i/2]$ $S_i(t) = \begin{cases} 3 \end{cases}$ $G_i(t) \in [-li / 2, li / 2)$ $G_i(t) \in [l_i / 2, 3l_i / 2)$ $G_i(t) \in [3l_i / 2, +\infty)$ Establish the fuzzy-fluctuation logical relationships(FFLRs): $((S_1(t-1),...,S_1(t-n)), (S_2(t-1),...,S_2(t-n)), (S_3(t-1),...,S_3(t-n))) \rightarrow S_1(t)$ Convert FFLRs to NSSLRs, such as: $\left(N_{A(e_1)}, N_{A(e_2)}, \dots, N_{A(e_m)}\right) \rightarrow \left(N_{A(t)}^1, N_{A(t)}^2, N_{A(t)}^3\right)$ Use a NSS R_A to represent the historical statuses of current value Fi(t). • Find the similar sets A(k) in training data set based on similarity measures. According to the corresponding Foreca-NSSLR, obtain the probabilities of down, equal and up sting trends from R_A . Result Forecast the fluctuation value of future: $G'(t+1) = \left(T_{R_{A(t+1)}} - F_{R_{A(t+1)}}\right) \times l_1$ • F'(t+1) = G(t+1) + F(t)End

FIGURE 1. Flow chart of neutrosophic soft sets forecasting model for multi-attribute time series.

Step 1: Construct FFTS form the training data of two parameters. For each element $F_i(t)(t = 2,3,...T, i = 1,2,3)$ in the historical time series of two parameters, its fluctuation trend is defined by $G_i(t) = F_i(t) - F_i(t-1)(t = 2,3,...T, i = 1,2,3)$. $G_1(t)(t = 2,3,...T, i = 1,2,3)$

2,3,..*T*) can be fuzzified into a linguistic set {down, equal, up} depending on its range and orientation of the fluctuations. Thus, in the same way, we can also divide into 5 ranges such as {down, slightly down, equal, slightly up, up} $u_{11} = \left(-\infty, -\frac{3l_1}{2}\right), u_{12} =$

1

^{2169-3536 (}c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications/standards/publications/rights/index.html for more information.

IEEE Access

 $\left(-\frac{3l_1}{2},-\frac{l_1}{2}\right), u_{13} = \left(-\frac{l_1}{2},\frac{l_1}{2}\right) u_{14} = \left(\frac{l_1}{2},\frac{3l_1}{2}\right), u_{15} = \left(\frac{3l_1}{2},+\infty\right)$. Similarly $G_2(t)$ and $G_3(t)$ can also be divided into five parts, where $l_i(i = 1,2,3)$ are respectively defined as the whole mean of all elements in the fluctuation time series $G_i(t)(t = 2,3,..T, i = 1,2,3)$.

Step 2: According to definition 3, the time series of three-factor fuzzy fluctuation are determined. Each $S_i(t)(t > n)$ can be represented by the fuzzy volatility of the previous n days, which can be used to establish the nth order FFLRs.

Step 3: We use the NSS R_A to represent LHS of each FFLR, according to Definition 5 and 6. Each fuzzy number defined in step 1 represents a different magnitude of increase or decrease, so NSSs can be obtained by assigning weights to different states. Then, we can generate RHSs $R_{A(t)}$ for different LHSs described in definition 7. Thus, FFLRs of historical training data sets can be converted to NSSLRs.

The nth-order fuzzy-fluctuation trends of each point F(i) in the test dataset can be represented by a NSS $R_{A(i)}$. For each $R_{A(i)}$, compare $R_{A(i)}$ with $R_{A(t)}$ respectively and find the similar sets by using the similarity measure described in Definition 8.

Step 4: Choose the corresponding similar sets $R_{A(t)}$ as forecasting rule to forecast the fluctuation value G'(i + 1) of next point. Finally, we get the forecasting value by F'(i + 1) = F(i) + G'(i + 1)

IV. EMPIRICAL ANALYSIS

To measure the efficiency of the proposed model, in this part, we used TAIEX as our primary example due to TAIEX has been widely studied by previous researches and thus it's clear to compare our results with them. Also, to ensure the universality of our model, we employed Shanghai Stock Exchange Composite Index, NASDAQ Index and Shenzhen Index for comparisons with previous researches, respectively.

A. Forecasting TAIEX

The proposed method is applied to forecast the TAIEX1999, which is shown in Table2. These historical datasets of closing price, volume and amplitude are used as the training time series of three factors and the data from November to December are

used as the testing dataset. Volume reflects the current capital trading situation and stock amplitude refers to the range between the lowest price and the highest price in a certain period of time, which reflects the current stock market activity.

Step 1: First of all, we used the historical training data in TAIEX1999 to calculate the fluctuation trend. The intervals are determined by calculating the population means of the fluctuation numbers of the two training data sets. Then, the fluctuation time series of two factors can be converted into FFTS, respectively. For example, the whole means of the historical dataset of TAIEX1999 from January to October are 85 and 65. That is to say, l_1 =85.40, l_2 = 19724.09 and l_3 =65.87. For example, $F_1(1) = 6152.43$ and $F_1(2) = 6199.91$, $G_1(2) = F_1(2) - F_1(1) = 47.48$, $S_1(2) =$ 4, and $F_2(1) = 43900$, $F_2(2) = 63500$, $G_2(2) = F_2(2) - F_2(1)$ $S_2(2) = 4$, $F_3(1) = 30.97$, $F_3(2) =$ =19600, 47.60, $G_3(2) = F_3(2) - F_3(1) = 16.64$, $S_3(2) = 5$. In this way, the three-factor fuzzified fluctuation dataset are shown in Appendix Table A1, Table A2 and Table A3 respectively.

Step2: Considering the impact of the previous 5 days' historical data on future forecasting, we choose the previous 5 day to establish FFLRs. The 5-order FFLRs for the three-factor fuzzy fluctuation time series forecasting model are established based on the FFTS from 2nd January 1999 to 30th October 1999 shown in Table A1, Table A2and Table A3.

Step 3: Convert the LHSs of the FFLRs in Table A1 and Table A2 to NSSLRs. Due to different degrees of expression for each $S_i(t)$, we assumed that each predefined $S_i(t)$ in step 1 corresponds to different neutrosophic sets, such as {0, 0, 1}, {0, 0.5, 0.5}, {0, 1, 0}, {0.5, 0.5, 0}, and {1, 0, 0}. In this example, threshold similarity value is set to 0.90. Then, we can convert the RHSs of the corresponding FFLRs into neutrosophic sets. Details of conversion and grouping process are shown in Figure 2. We used a NSS to represent the RHSs group like Example 2 and Euclidean distances can be employed to obtain the most suitable NSSLRs. More detailed grouping and converting processes are shown in Fig2. The FFLRs of test dataset can be converted into NSSLRs as shown in Table 1.

VOLUME XX, 2017

2169-3536 (c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information. This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2897719, IEEE Access





FIGURE 2. Conversion and group process of fuzzy-fluctuation logical relationship (FFLRs)

TABLE 1

NSSLRS FROM 1 NOVEMBER 1999 TO 28 DECEMBER 1999.	
--	--

Date	NSSI Re
(YYYY/MM/DD)	NOOLAG
1999/11/1	$((0.20, 0.80, 0.00), (0.50, 0.50, 0.00), (0.00, 0.90, 0.10)) \rightarrow (0.25, 0.66, 0.09)$
1999/11/2	$((0.20, 0.80, 0.00), (0.40, 0.40, 0.20), (0.00, 0.80, 0.20)) \rightarrow (0.26, 0.65, 0.09)$
1999/11/3	$((0.20, 0.70, 0.10), (0.40, 0.40, 0.20), (0.10, 0.70, 0.20)) \rightarrow (0.2, 0.66, 0.14)$
1999/11/4	$((0.20, 0.50, 0.30), (0.40, 0.40, 0.20), (0.20, 0.70, 0.10)) \rightarrow (0.17, 0.63, 0.20)$
1999/11/5	$((0.20, 0.40, 0.40), (0.20, 0.60, 0.20), (0.20, 0.70, 0.10)) \rightarrow (0.19, 0.62, 0.19)$
1999/11/6	$((0.00, 0.60, 0.40), (0.00, 0.70, 0.30), (0.20, 0.60, 0.20)) \rightarrow (0.18, 0.53, 0.29)$
1999/11/8	$((0.00, 0.50, 0.50), (0.10, 0.80, 0.10), (0.30, 0.60, 0.10)) \rightarrow (0.23, 0.53, 0.25)$
1999/11/9	$((0.00, 0.60, 0.40), (0.10, 0.70, 0.20), (0.20, 0.70, 0.10)) \rightarrow (0.16, 0.53, 0.32)$
1999/11/10	$((0.00, 0.80, 0.20), (0.10, 0.70, 0.20), (0.10, 0.60, 0.30)) \rightarrow (0.12, 0.6, 0.29)$
1999/11/11	$((0.00, 0.90, 0.10), (0.20, 0.60, 0.20), (0.20, 0.50, 0.30)) \rightarrow (0.15, 0.65, 0.21)$
1999/11/15	$((0.20, 0.70, 0.10), (0.30, 0.60, 0.10), (0.20, 0.50, 0.30)) \rightarrow (0.11, 0.73, 0.16)$
1999/11/16	$((0.20, 0.80, 0.00), (0.20, 0.60, 0.20), (0.10, 0.60, 0.30)) \rightarrow (0.15, 0.63, 0.21)$
1999/11/17	$((0.30, 0.70, 0.00), (0.40, 0.50, 0.10), (0.10, 0.60, 0.30)) \rightarrow (0.24, 0.66, 0.10)$
1999/11/18	$((0.30, 0.70, 0.00), (0.40, 0.50, 0.10), (0.10, 0.80, 0.10)) \rightarrow (0.3, 0.64, 0.05)$
1999/11/19	$((0.40, 0.60, 0.00), (0.50, 0.40, 0.10), (0.10, 0.80, 0.10)) \rightarrow (0.33, 0.57, 0.10)$
1999/11/20	$((0.30, 0.70, 0.00), (0.50, 0.40, 0.10), (0.10, 0.90, 0.00)) \rightarrow (0.35, 0.59, 0.06)$
1999/11/22	$((0.50, 0.50, 0.00), (0.50, 0.50, 0.00), (0.10, 0.80, 0.10)) \rightarrow (0.29, 0.62, 0.10)$
1999/11/23	$((0.60, 0.40, 0.00), (0.40, 0.60, 0.00), (0.20, 0.70, 0.10)) \rightarrow (0.29, 0.57, 0.14)$
1999/11/24	$((0.60, 0.40, 0.00), (0.40, 0.50, 0.10), (0.20, 0.70, 0.10)) \rightarrow (0.28, 0.60, 0.13)$
1999/11/25	$((0.50, 0.40, 0.10), (0.20, 0.50, 0.30), (0.10, 0.80, 0.10)) \rightarrow (0.24, 0.54, 0.22)$
1999/11/26	$((0.40, 0.50, 0.10), (0.10, 0.60, 0.30), (0.10, 0.80, 0.10)) \rightarrow (0.20, 0.63, 0.17)$
1999/11/29	$((0.20, 0.50, 0.30), (0.30, 0.40, 0.30), (0.30, 0.70, 0.00)) \rightarrow (0.17, 0.63, 0.21)$
1999/11/30	$((0.20, 0.50, 0.30), (0.20, 0.30, 0.50), (0.20, 0.60, 0.20)) \rightarrow (0.21, 0.51, 0.28)$
1999/12/1	$((0.20, 0.40, 0.40), (0.20, 0.40, 0.40), (0.20, 0.60, 0.20)) \rightarrow (0.19, 0.56, 0.25)$
1999/12/2	$((0.30, 0.40, 0.30), (0.20, 0.40, 0.40), (0.20, 0.40, 0.40)) \rightarrow (0.31, 0.54, 0.15)$
1999/12/3	$((0.30, 0.40, 0.30), (0.30, 0.30, 0.40), (0.20, 0.40, 0.40)) \rightarrow (0.21, 0.59, 0.20)$
1999/12/4	$((0.40, 0.50, 0.10), (0.30, 0.30, 0.40), (0.00, 0.60, 0.40)) \rightarrow (0.23, 0.56, 0.21)$
1999/12/6	$((0.20, 0.70, 0.10), (0.40, 0.40, 0.20), (0.10, 0.70, 0.20)) \rightarrow (0.20, 0.66, 0.14)$
1999/12/7	$((0.20, 0.70, 0.10), (0.40, 0.30, 0.30), (0.10, 0.70, 0.20)) \rightarrow (0.20, 0.60, 0.20)$
1999/12/8	$((0.10, 0.70, 0.20), (0.40, 0.50, 0.10), (0.10, 0.90, 0.00)) \rightarrow (0.22, 0.62, 0.16)$
1999/12/9	$((0.10, 0.70, 0.20), (0.40, 0.50, 0.10), (0.10, 0.90, 0.00)) \rightarrow (0.22, 0.62, 0.16)$
1999/12/10	$((0.00, 0.70, 0.30), (0.20, 0.70, 0.10), (0.10, 0.90, 0.00)) \rightarrow (0.14, 0.56, 0.31)$
1999/12/13	$((0.00, 0.70, 0.30), (0.10, 0.70, 0.20), (0.00, 0.90, 0.10)) \rightarrow (0.14, 0.60, 0.26)$
1999/12/14	$((0.20, 0.60, 0.20), (0.10, 0.80, 0.10), (0.00, 0.90, 0.10)) \rightarrow (0.12, 0.67, 0.21)$
1999/12/15	$((0.20, 0.70, 0.10), (0.20, 0.70, 0.10), (0.00, 0.90, 0.10)) \rightarrow (0.17, 0.63, 0.20)$
1999/12/16	$((0.20, 0.70, 0.10), (0.10, 0.80, 0.10), (0.00, 0.90, 0.10)) \rightarrow (0.13, 0.61, 0.26)$
1999/12/17	$((0.20, 0.70, 0.10), (0.20, 0.70, 0.10), (0.20, 0.70, 0.10)) \rightarrow (0.13, 0.68, 0.19)$
1999/12/18	$((0.20, 0.70, 0.10), (0.20, 0.70, 0.10), (0.20, 0.70, 0.10)) \rightarrow (0.13, 0.68, 0.19)$
1999/12/20	$((0.10, 0.80, 0.10), (0.20, 0.60, 0.20), (0.20, 0.70, 0.10)) \rightarrow (0.14, 0.63, 0.23)$

VOLUME XX, 2017

2169-3536 (c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

Multidisciplinary : Ranid Review : Open Access Journal

1999/12/21	$((0.10, 0.80, 0.10), (0.10, 0.60, 0.30), (0.20, 0.70, 0.10)) \rightarrow (0.15, 0.61, 0.24)$
1999/12/22	$((0.30, 0.60, 0.10), (0.30, 0.40, 0.30), (0.30, 0.60, 0.10)) \rightarrow (0.23, 0.60, 0.18)$
1999/12/23	$((0.40, 0.60, 0.00), (0.30, 0.40, 0.30), (0.10, 0.70, 0.20)) \rightarrow (0.26, 0.58, 0.16)$
1999/12/24	$((0.50, 0.50, 0.00), (0.30, 0.30, 0.40), (0.10, 0.80, 0.10)) \rightarrow (0.26, 0.57, 0.17)$
1999/12/27	$((0.60, 0.40, 0.00), (0.50, 0.20, 0.30), (0.10, 0.70, 0.20)) \rightarrow (0.33, 0.63, 0.04)$
1999/12/28	$((0.80, 0.20, 0.00), (0.60, 0.20, 0.20), (0.30, 0.50, 0.20)) \rightarrow (0.38, 0.54, 0.08)$

Step 4: Based on the NSSLRs obtained in Step 3, we can forecast the test dataset from 1 November 1999 to 28 December. For example, as in Table 1, the forecasting values of the TAIEX on 1 November 1999 are calculated as follows.

Locate the NSSLRs with the highest similarity based on similarity measure as Definition 9, then $((0.20, 0.80, 0.00), (0.50, 0.50, 0.00), (0.00, 0.90, 0.10)) \rightarrow (0.25, 0.66, 0.09)$ can be obtained as the best rule to forecast its future. Respectively, we calculate the expected number of the NS (0.25, 0.66, 0.09), the expected number are:

E(i+1) = 0.25 - 0.09 = 0.16

The fluctuation from current value to next value can be obtained for forecasting by defuzzifying the fluctuation fuzzy number, shown as follows:

$$G'(i+1) = E(i+1) \times l_1 = 0.16 \times 85.40 = 13.66$$

Finally, the forecasted value can be obtained by current value and the fluctuation value:

$$F'(i + 1) = F(i) + G'(i + 1) = 7854.85 + 13.66 = 7868.51$$

According to the above steps, the results of the forecasting model are shown in Table 2 and Fig 3.



FIGURE 3. Forecasting results from 1 November1999 to 28 December 1999.

 TABLE 2

 FORECASTING RESULTS FROM 1 NOVEMBER 1999 TO 28 DECEMBER 1999.

Date	1	Г	(T 1)2	Date	1	F ((Environment Antonia)?	
(YYYY/MM/DD)	Actual	Forecast	(Forecast–Actual) ²	(YYYY/MM/DD)	Actual	Forecast	(Forecast–Actual) ²	
1999/11/1	7814.89	7868.51	2875.55	1999/12/1	7766.2	7715.75	2545.61	
1999/11/2	7721.59	7829.41	11624.76	1999/12/2	7806.26	7779.86	696.74	
1999/11/3	7580.09	7726.71	21498.61	1999/12/3	7933.17	7807.11	15890.11	
1999/11/4	7469.23	7577.53	11728.45	1999/12/4	7964.49	7934.88	876.87	
1999/11/5	7488.26	7469.23	362.14	1999/12/6	7894.46	7969.61	5648.13	
1999/11/6	7376.56	7478.87	10466.50	1999/12/7	7827.05	7894.46	4544.11	
1999/11/8	7401.49	7374.85	709.58	1999/12/8	7811.02	7832.17	447.49	
1999/11/9	7362.69	7387.83	631.81	1999/12/9	7738.84	7816.14	5975.92	
1999/11/10	7401.81	7348.17	2877.05	1999/12/10	7733.77	7724.32	89.27	
1999/11/11	7532.22	7396.69	18369.48	1999/12/13	7883.61	7723.52	25628.21	
1999/11/15	7545.03	7527.95	291.73	1999/12/14	7850.14	7875.92	664.81	
1999/11/16	7606.2	7539.91	4394.90	1999/12/15	7859.89	7847.58	151.59	
1999/11/17	7645.78	7618.16	763.08	1999/12/16	7739.76	7848.79	11887.08	
1999/11/18	7718.06	7667.13	2593.84	1999/12/17	7723.22	7734.64	130.32	
1999/11/19	7770.81	7737.70	1096.12	1999/12/18	7797.87	7718.10	6363.90	
1999/11/20	7900.34	7795.58	10975.43	1999/12/20	7782.94	7790.18	52.47	
1999/11/22	8052.31	7916.57	18426.38	1999/12/21	7934.26	7775.25	25282.94	
1999/11/23	8046.19	8065.12	358.35	1999/12/22	8002.76	7938.53	4125.49	
1999/11/24	7921.85	8059.00	18810.16	1999/12/23	8083.49	8011.30	5211.38	
1999/11/25	7904.53	7923.56	362.07	1999/12/24	8219.45	8091.18	16454.20	

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI

IEEEAccess

(11)

the

78.25.

denote

 $MAE = \frac{\sum_{t=1}^{n} |forecast(t) - actual(t)|}{n}$

forecasted, forecast(t) and actual(t)

Where n denotes the number of values to be

predicted value and actual value at time t_i

respectively. From Table 2, we can calculate the MSE,

RMSE and MAE are 10253.59, 101.26,

1999/11/26	7595.44	7907.09	97126.99	1999/12/27	8415.07	8244.22	29190.99	
1999/11/29	7823.9	7592.02	53766.50	1999/12/28	8448.84	8440.69	66.42	
1999/11/30	7720.87	7817.92	9419.08		RMSE		101.26	

In order to confirm the performance of the proposed method, we compare the difference between the forecasted values and the actual values. The performance can be evaluated using the mean squared error (MSE), root of the mean squared error (RMSE), mean absolute error (MAE) etc. These indicators are defined by Equations (9)–(11):



respectively.



FIGURE 4. The stock market fluctuation for TAIEX test dataset (1998–2006).

Due to different similarity measures can affect the prediction effect, we compared our proposed model to predict TAIEX, Hang Seng Index and Nikkei index from 1998 to 2006 and select 4 typical models to compare with proposed model which are shown in Figure 5. Among these chosen models, Chen's model [17] and Yu's model [39] are both typical fuzzy time series models, Wan's method [40] is a popular machine learning method and the last one [41] is

VOLUME XX, 2017

2169-3536 (c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications standards/publications/rights/index.html for more information.



based on rough set to forecast stock price. Table 4 shows the prediction errors of different prediction methods from 1999 to 2004. The validity of the model can be verified by comparing RMSEs of different methods and data sets of different years. The advantages of this method are NSSs effectively express the volatility degree of the previous n days of the stock market and convenient for computer calculation.

	I ABLE 5												
A COMPARISON OF RMS	ES AND AV	ERAGE RM	SE FOR DI	FFERENT	MEASURES	FOR FORI	ECASTINC	G THE TA	IEX1997	-2005.			
	1997	1998	1999	2000	2001	2002	2003	2004	2005	AVE.			
Hamming distance[1]	140.46	119.67	99.19	136.51	111.79	65.89	56.65	58.38	54.33	93.65			



TABLE 4

A COMPARISON OF RMSES FOR DIFFERENT METHODS FOR FORECASTING THE TAIEX1999-2004.	
	_

Metho	ds	1999	2000	2001	2002	2003	2004	Ave.
Huarng et al.'s method[43]	use NASDAQ	N/A	158.7	136.49	95.15	65.51	73.57	105.88
	use Dow Jones	N/A	165.80	138.25	93.73	72.95	73.49	108.84
Chen and Chang's model[44]	use NASDAQ	123.64	131.10	115.08	73.06	66.36	60.48	94.95
Chen and Chang's model[44]	use Dow Jones	101.97	148.85	113.70	79.81	64.08	82.32	98.46
Chan and Chan/and dal[45]	use NASDAQ	119.32	129.87	123.12	71.01	65.14	61.94	95.07
Chen and Chen's model[45]	use Dow Jones	Dow Jones 115.47 127.51 121.98 74,5	74,56	66.02	58.89	94.09		
Chen's Fuzzy Time S	eries Method[17]	120 176 148 101			74	84	102.83	
Yu and Huarng's	s method[46]	N/A	149.95	98.91	78.71	58.78	55.91	88.38
Cheng and Yang 's mode	l using rough set[41]	110.69	150.55	113.17	65.97	53.09	58.6	92.01
	Five-Day forecasting	101.24	128.71	112.19	66.13	55.82	54.74	86.47
The proposed method	Seven-Day forecasting	101.86	130.08	114.54	66.49	54.01	55.16	87.02
	Nine-Day forecasting	102.45	129.19	114.14	66.12	54.30	55.17	86.90

B. Forecast other indexes

In this section, the proposed method is used to predict the SHCECI, Shenzhen index and NASDAQ index which have important position in the financial industry. Daily closing price, volume and amplitude are selected as indicators to reflect the fluctuation of stock price. We use the three parameters data from January to October from 2004 to 2011 as the training data, establishing the NSSLRs and then predicted

1



these indexes from November to December. As is shown in Figure 6, we can see that forecasting of SHCECI, Shenzhen and NASDAQ stock market yields great results by using the proposed method.



Figure 6. The RMSEs of proposed method for forecasting SHSECI, Shenzhen and NASDAQ.

V. CONCLUSIONS

In this paper, a new financial forecasting model based on NSSs is proposed. The main contribution is the use of NSSs, which can preserve inherent complexity of the dataset by enclosing relevant parameters. Regarding efficiency, we calculated Euclidean distances between NSSs to measure the similarity effectively. The empirical analysis shows that this model can predict the stock market of different years well. Regarding future researches, in this paper, we enclosed three other factors into the model. In fact, there are many other factors that may improve model performance. For example, other stock market fluctuations can be thought of as an influencing factor. We will also consider applying this model to predict other time series, such as college enrollment, electricity consumption, etc. In addition, we may consider using other methods to compare similarities of historical data, such as information entropy.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation of China under grants 71471076 and 71704066. The authors are indebted to anonymous reviewers for their very insightful comments and constructive suggestions, which help ameliorate the quality of this paper.

REFERENCES

- Rıdvan Şahin, Ahmet Küçük. "On Similarity and Entropy of Neutrosophic Soft Sets," *Journal of Intelligent and Fuzzy Systems*, vol. 27, no. 5, pp. 2417-2430.
- [2] Bollerslev T. "Generalized autoregressive conditional heteroscedasticity," *Journal of Econometrics*, vol. 31, pp, 307-327, 1986.
- [3] Box G, Jenkins G. "Time Series Analysis: Forecasting and Control," Holden-Day, San Francisco, 1976.
- [4] Song Q, Chissom B S. "Fuzzy time series and its models," *Fuzzy Sets and Systems*, 1993, 54:269–277.
- [5] Guan S, Zhao A. "A Two-Factor Autoregressive Moving Average Model Based on Fuzzy Fluctuation Logical Relationships," *Symmetry*, 2018, 9:207.
- [6] Zou Y, Donner R V, Marwan N, Donges J F, Kurths J. "Complex network approaches to nonlinear time series analysis," Physics Reports, 2018, https://doi.org/10.1016/j.physrep.2018.10.005.
- [7] An S, Gao X, Jiang M, Sun X. "Multivariate financial time series in the light of complex network analysis," *Physica A*, 2018, 503: 1241–1255.
- [8] Bao J, Chen W, Shui Y S, et al. "Complexity analysis of traffic time series based on multifractality and complex network," 2017 4th International Conference on Transportation Information and Safety (ICTIS) IEEE, 2017.
- [9] Monfared S A, Enke D. "Volatility forecasting using a hybrid GJR-GARCH neural network model," *Procedia Computer Science*, 2014, 36:246–253.
- [10] Taylor J W. "A quantile regression neural network approach to estimating the conditional density of multiperiod returns," *Journal of Forecasting*, 2000, 19(4):299-311.
- [11] Kristjanpoller W, Minutolo M C. "Gold price volatility: A forecasting approach using the Artificial Neural Network–GARCH model," *Expert Systems with Applications*, 2015, 42(20):7245-7251.
- [12] Breiman L. "Random Forests," Mach. Learn, 2001, 45(1):5-32

^{2169-3536 (}c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

IEEE Access

- [13] Breiman L, Friedman J, Olshen R, Stone C. "Classification and Regression Trees," Belmont, Wadsworth, 1984
- [14] Song Q, Chissom B S. "Fuzzy time series and its models," *Fuzzy Sets and Systems*, 1993, 54:269–277
- [15] Huarng K. "Effective lengths of intervals to improve forecasting in fuzzy time series," *Fuzzy Sets and Systems*, 2001, 123:387–394.
- [16] Huarng K, Yu H K. " A type 2 fuzzy time series model for stock index forecasting," *Physica A*, 2005, 353:445– 462.
- [17] Chen S M. "Forecasting enrollments based on fuzzy time series," *Fuzzy Sets and System*, 1996, 81:311–319.
- [18] Huarng K, Yu H K. "An N-th order heuristic fuzzy time series model for TAIEX forecasting," *International Journal of Fuzzy Systems*, 2008, 5:247–253.
- [19] Zadeh L A. "FUZZY SETS," Information & Control, 1965, 8(3):338-353.
- [20] Atanassov K T. "Intuitionistic fuzzy sets," Fuzzy Sets & Systems, 1986, 20(1):87-96.
- [21] Song Y, Wang X, Quan W. "A new approach to construct similarity mearsure for intuitionistic fuzzy sets," *Soft Computing*, 2017, DOI:10.1007/s00500-017-2912-0
- [22] Fan X, Lei Y, Wang Y. "Adaptive partition intuitionistic fuzzy time series forecasting model," *Journal of Systems Engineering & Electronics*, 2017, 28(3):585-596.
- [23] Lu Y, Wang Y, Lei Y, et al. "Long-term intuitionistic fuzzy time series forecasting model based on vector quantisation and curve similarity measure," *IET Signal Processing*, 2016, 10(7).
- [24] Chen S M, Chen S W. "Fuzzy forecasting based on twofactors second-order fuzzy-trend logical relationship groups and the probabilities of trends of fuzzy logical relationships," *IEEE Trans, Cybern.* 2015, 45:405–417.
- [25] Chen S M Jian W S. "Fuzzy forecasting based on twofactors second-order fuzzy-trend logical relationship groups, similarity measures and PSO techniques," *Inf. Sci*, 2017, 391–392, 65–79.
- [26] Rubio A, Bermudez J D, Vercher E. "Improving stock index forecasts by using a new weighted fuzzy-trend time series method," *Expert Syst. Appl.* 2017, 76:12–20.
- [27] Chen S M, Hwang J R. "Temperature prediction using fuzzy time series," *IEEE Transactions on Systems, Man* and Cybernetics. 2000, Part B 30: 263–275.
- [28] Smarandache F. "A unifying field in logics: Neutrosophic logic," *Multiple-Valued Logic*, 1999, 8(3):489-503.
- [29] Yang H L, Zhang C L, Guo Z L. "A hybrid model of single valued neutrosophic sets and rough sets: single valued neutrosophic rough set model," *Soft Computing*, 2016:1-15.
- [30] Wang H, Smarandache F, Zhang Y Q, et al. "Interval Neutrosophic Sets and Logic: Theory and Applications in Computing," *Computer Science*, 2005, 65(4): 87.
- [31] Abdel-Basset M. Mai M. "The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems," *Measurement* 2018, 124:47–55.
- [32] Rıdvan Şahin, Liu P. "Correlation coefficient of singlevalued neutrosophic hesitant fuzzy sets and its applications in decision making," *Neural Computing* and Applications, 2017, 28(6):1387-1395.
- [33] Van L H, Yu V F, Dat L Q, Dung C C, Chou S, Loc N V. "New Integrated Quality Function Deployment Approach Based on Interval Neutrosophic Set for Green Supplier Evaluation and Selection," Sustainability, 2018, 10:838.
- [34] Wu H, Yuan Y, Wei L, et al. "On entropy, similarity measure and cross-entropy of single-valued neutrosophic sets and their application in multi-attribute decision making," *Soft Computing*, 2018:1-10.

- [35] Ulucay V, Deli I, Şahin, Mehmet. "Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making," *Neural Computing* and Applications, 2018, 29:739–748.
- [36] Ye J, Du S. "Some distances, similarity and entropy measures for interval-valued neutrosophic sets and their relationship," *International Journal of Machine Learning & Cybernetics*, 2017(3):1-9.
- [37] Molodtsov D. "Soft set theory—First results," Computers & Mathematics with Applications, 1999, 37(4-5):19-31.
- [38] Peng X, Chong L. "Algorithms for Neutrosophic Soft Decision Making Based on Edas and New Similarity Measure," *Journal of Intelligent & Fuzzy Systems*, 2017, 32(1):955-968.
- [39] Yu H K. "A refined fuzzy time-series model for forecasting," *Physica A Statistical Mechanics & Its Applications*, 2005, 346(3):657-681.
- [40] Wan Y, Si Y W. "Adaptive neuro fuzzy inference system for chart pattern matching in financial time series," *Applied Soft Computing*, 2017, 57:1-18.
- [41] Cheng C H, Yang J H. "Fuzzy Time-Series Model Based on Rough Set Rule Induction For Forecasting Stock Price," *Neurocomputing*, 2018. 302:33–45
- [42] Sumathi I R, Arockiarani I. "Cosine similarity measures of neutrosophic soft set," *Annals of Fuzzy Mathematics and Informatics*,2016.
- [43] Yu H K, Huarng K H, Hsu Y W. "A multivariate heuristic model for fuzzy time-series forecasting," *IEEE Trans, Cybern.* 2007, 37(4):836.
- [44] Chen S M, Chang Y C. "Multi-variable fuzzy forecasting based on fuzzy clustering and fuzzy rule interpolation techniques," *Inf. Sci.*, 2010, 180(24):4772-4783.
- [45] Chen S M, Chen C D. "TAIEX Forecasting Based on Fuzzy Time Series and Fuzzy Variation Groups," *IEEE Transactions on Fuzzy Systems*, 2011, 19(1):1-12.
- [46] Yu H K, Huarng K H. "A neural network-based fuzzy time series model to improve forecasting," *Expert Systems with Applications*, 2010, 37(4):3366-3372.

APPENDIX

.Appendix A

Table A1 Historical training data and fuzzified fluctuation data of TAIEX 1999

Date (YYYY/MM/DD)	TAIEX	Fluctuation	Fuzzified	Date (YYYY/MM/DD)	TAIEX	Fluctuation	Fuzzified	Date (YYYY/MM/DD)	TAIEX	Fluctuation	Fuzzified
1999/1/5	6152.43			1999/2/1	5862.79	-135.53	1	1999/3/8	6431.96	10.23	3
1999/1/6	6199.91	47.48	4	1999/2/2	5749.64	-113.15	2	1999/3/9	6493.43	61.47	4
1999/1/7	6404.31	204.4	5	1999/2/3	5743.86	-5.78	3	1999/3/10	6486.61	-6.82	3
1999/1/8	6421.75	17.44	3	1999/2/4	5514.89	-228.97	1	1999/3/11	6436.8	-49.81	2
1999/1/11	6406.99	-14.76	3	1999/2/5	5474.79	-40.1	3	1999/3/12	6462.73	25.93	3
1999/1/12	6363.89	-43.1	2	1999/2/6	5710.18	235.39	5	1999/3/15	6598.32	135.59	5
1999/1/13	6319.34	-44.55	2	1999/2/8	5822.98	112.8	4	1999/3/16	6672.23	73.91	4
1999/1/14	6241.32	-78.02	2	1999/2/9	5723.73	-99.25	2	1999/3/17	6757.07	84.84	4
1999/1/15	6454.6	213.28	5	1999/2/10	5798	74.27	4	1999/3/18	6895.01	137.94	5
1999/1/16	6483.3	28.7	3	1999/2/20	6072.33	274.33	5	1999/3/19	6997.29	102.28	4
1999/1/18	6377.25	-106.05	2	1999/2/22	6313.63	241.3	5	1999/3/20	6993.38	-3.91	3
1999/1/19	6343.36	-33.89	3	1999/2/23	6180.94	-132.69	1	1999/3/22	7043.23	49.85	4
1999/1/20	6310.71	-32.65	3	1999/2/24	6238.87	57.93	4	1999/3/23	6945.48	-97.75	2
1999/1/21	6332.2	21.49	3	1999/2/25	6275.53	36.66	3	1999/3/24	6889.42	-56.06	2
1999/1/22	6228.95	-103.25	2	1999/2/26	6318.52	42.99	4	1999/3/25	6941.38	51.96	4
1999/1/25	6033.21	-195.74	1	1999/3/1	6312.25	-6.27	3	1999/3/26	7033.25	91.87	4
1999/1/26	6115.64	82.43	4	1999/3/2	6263.54	-48.71	2	1999/3/29	6901.68	-131.57	1
1999/1/27	6138.87	23.23	3	1999/3/3	6403.14	139.6	5	1999/3/30	6898.66	-3.02	3
1999/1/28	6063.41	-75.46	2	1999/3/4	6393.74	-9.4	3	1999/3/31	6881.72	-16.94	3
1999/1/29	5984	-79.41	2	1999/3/5	6383.09	-10.65	3	1999/4/1	7018.68	136.96	5
1999/1/30	5998.32	14.32	3	1999/3/6	6421.73	38.64	3	1999/4/2	7232.51	213.83	5
1999/4/3	7182.2	-50.31	2	1999/5/6	7560.05	-12.11	3	1999/6/5	7639.3	48.86	4
1999/4/6	7163.99	-18.21	3	1999/5/7	7469.33	-90.72	2	1999/6/7	7802.69	163.39	5

VOLUME XX, 2017

2169-3536 (c) 2018 IEEE. Translations and c	ontent mining are permitted for academ	ic research only. Personal use	is also permitted, but	t republication/redistribution	n requires IEEE permission.	See http://www.ieee.org/publication	s standards/publications/rights/index.html f	or more information.
		· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	1		

1999/4/7	7135.89	-28.1	3	1999/5/10	7484.37	15.04	3	1999/6/8	7892.13	89.44	4
1999/4/8	7273.41	137.52	5	1999/5/11	7474.45	-9.92	3	1999/6/9	7957.71	65.58	4
1999/4/9	7265.7	-7.71	3	1999/5/12	7448.41	-26.04	3	1999/6/10	7996.76	39.05	3
1999/4/12	7242.4	-23.3	3	1999/5/13	7416.2	-32.21	3	1999/6/11	7979.4	-17.36	3
1999/4/13	7337.85	95.45	4	1999/5/14	7592.53	176.33	5	1999/6/14	7973.58	-5.82	3
1999/4/14	7398.65	60.8	4	1999/5/15	7576.64	-15.89	3	1999/6/15	7960	-13.58	3
1999/4/15	7498.17	99.52	4	1999/5/17	7599.76	23.12	3	1999/6/16	8059.02	99.02	4
1999/4/16	7466.82	-31.35	3	1999/5/18	7585.51	-14.25	3	1999/6/17	8274.36	215.34	5
1999/4/17	7581.5	114.68	4	1999/5/19	7614.6	29.09	3	1999/6/21	8413.48	139.12	5
1999/4/19	7623.18	41.68	3	1999/5/20	7608.88	-5.72	3	1999/6/22	8608.91	195.43	5
1999/4/20	7627.74	4.56	3	1999/5/21	7606.69	-2.19	3	1999/6/23	8492.32	-116.59	2
1999/4/21	7474.16	-153.58	1	1999/5/24	7588.23	-18.46	3	1999/6/24	8589.31	96.99	4
1999/4/22	7494.6	20.44	3	1999/5/25	7417.03	-171.2	1	1999/6/25	8265.96	-323.35	1
1999/4/23	7612.8	118.2	4	1999/5/26	7426.63	9.6	3	1999/6/28	8281.45	15.49	3
1999/4/26	7629.09	16.29	3	1999/5/27	7469.01	42.38	3	1999/6/29	8514.27	232.82	5
1999/4/27	7550.13	-78.96	2	1999/5/28	7387.37	-81.64	2	1999/6/30	8467.37	-46.9	2
1999/4/28	7496.61	-53.52	2	1999/5/29	7419.7	32.33	3	1999/7/2	8572.09	104.72	4
1999/4/29	7289.62	-206.99	1	1999/5/31	7316.57	-103.13	2	1999/7/3	8563.55	-8.54	3
1999/4/30	7371.17	81.55	4	1999/6/1	7397.62	81.05	4	1999/7/5	8593.35	29.8	3
1999/5/3	7383.26	12.09	3	1999/6/2	7488.03	90.41	4	1999/7/6	8454.49	-138.86	1
1999/5/4	7588.04	204.78	5	1999/6/3	7572.91	84.88	4	1999/7/7	8470.07	15.58	3
1999/5/5	7572.16	-15.88	3	1999/6/4	7590.44	17.53	3	1999/7/8	8592.43	122.36	4
1999/7/9	8550.27	-42.16	3	1999/8/9	7028.01	-21.73	3	1999/9/8	7973.3	27.54	3
1999/7/12	8463.9	-86.37	2	1999/8/10	7269.6	241.59	5	1999/9/9	8025.02	51.72	4
1999/7/13	8204.5	-259.4	1	1999/8/11	7228.68	-40.92	3	1999/9/10	8161.46	136.44	5
1999/7/14	7888.66	-315.84	1	1999/8/12	7330.24	101.56	4	1999/9/13	8178.69	17.23	3
1999/7/15	7918.04	29.38	3	1999/8/13	7626.05	295.81	5	1999/9/14	8092.02	-86.67	2
1999/7/16	7411.58	-506.46	1	1999/8/16	8018.47	392.42	5	1999/9/15	7971.04	-120.98	2
1999/7/17	7366.23	-45.35	2	1999/8/17	8083.43	64.96	4	1999/9/16	7968.9	-2.14	3
1999/7/19	7386.89	20.66	3	1999/8/18	7993.71	-89.72	2	1999/9/17	7916.92	-51.98	2

1999/7/20	7806.85	419.96	5	1999/8/19	7964.67	-29.04	3	1999/9/18	8016.93	100.01	4
1999/7/21	7786.65	-20.2	3	1999/8/20	8117.42	152.75	5	1999/9/20	7972.14	-44.79	2
1999/7/22	7678.67	-107.98	2	1999/8/21	8153.57	36.15	3	1999/9/27	7759.93	-212.21	1
1999/7/23	7724.52	45.85	4	1999/8/23	8119.98	-33.59	3	1999/9/28	7577.85	-182.08	1
1999/7/26	7595.71	-128.81	1	1999/8/24	7984.39	-135.59	1	1999/9/29	7615.45	37.6	3
1999/7/27	7367.97	-227.74	1	1999/8/25	8127.09	142.7	5	1999/9/30	7598.79	-16.66	3
1999/7/28	7484.5	116.53	4	1999/8/26	8097.57	-29.52	3	1999/10/1	7694.99	96.2	4
1999/7/29	7359.37	-125.13	2	1999/8/27	8053.97	-43.6	2	1999/10/2	7659.55	-35.44	3
1999/7/30	7413.11	53.74	4	1999/8/30	8071.36	17.39	3	1999/10/4	7685.48	25.93	3
1999/7/31	7326.75	-86.36	2	1999/8/31	8157.73	86.37	4	1999/10/5	7557.01	-128.47	1
1999/8/2	7195.94	-130.81	1	1999/9/1	8273.33	115.6	4	1999/10/6	7501.63	-55.38	2
1999/8/3	7175.19	-20.75	3	1999/9/2	8226.15	-47.18	2	1999/10/7	7612	110.37	4
1999/8/4	7110.8	-64.39	2	1999/9/3	8073.97	-152.18	1	1999/10/8	7552.98	-59.02	2
1999/8/5	6959.73	-151.07	1	1999/9/4	8065.11	-8.86	3	1999/10/11	7607.11	54.13	4
1999/8/6	6823.52	-136.21	1	1999/9/6	8130.28	65.17	4	1999/10/12	7835.37	228.26	5
1999/8/7	7049.74	226.22	5	1999/9/7	7945.76	-184.52	1	1999/10/13	7836.94	1.57	3
1999/10/14	7879.91	42.97	4	1999/10/20	7666.64	-26.32	3	1999/10/27	7701.22	0.93	3
1999/10/15	7819.09	-60.82	2	1999/10/21	7654.9	-11.74	3	1999/10/28	7681.85	-19.37	3
1999/10/16	7829.39	10.3	3	1999/10/22	7559.63	-95.27	2	1999/10/29	7706.67	24.82	3
1999/10/18	7745.26	-84.13	2	1999/10/25	7680.87	121.24	4	1999/10/30	7854.85	148.18	5
1999/10/19	7692.96	-52.3	2	1999/10/26	7700.29	19.42	3				

Date (YYYY/MM/DD)	Stock Amplitude	Fluctuation	Fuzzified	Date (YYYY/MM/DD)	Stock Amplitude	Fluctuation	Fuzzified	Date (YYYY/MM/DD)	Stock Amplitude	Fluctuation	Fuzzified
1999/1/5	3.10			1999/4/19	0.91	-0.50	2	1999/7/27	2.56	0.84	4
1999/1/6	4.76	1.66	5	1999/4/20	1.74	0.82	4	1999/7/28	1.21	-1.36	1
1999/1/7	3.68	-1.08	1	1999/4/21	3.25	1.51	5	1999/7/29	2.09	0.88	4
1999/1/8	1.90	-1.78	1	1999/4/22	1.35	-1.90	1	1999/7/30	2.81	0.72	4
1999/1/11	1.56	-0.33	2	1999/4/23	1.23	-0.12	3	1999/7/31	2.10	-0.72	2
1999/1/12	1.32	-0.24	3	1999/4/26	1.23	0.01	3	1999/8/2	2.27	0.18	3
1999/1/13	1.15	-0.17	3	1999/4/27	1.80	0.57	4	1999/8/3	2.89	0.61	4
1999/1/14	1.72	0.57	4	1999/4/28	1.26	-0.54	2	1999/8/4	2.24	-0.65	2
1999/1/15	4.46	2.74	5	1999/4/29	1.98	0.72	4	1999/8/5	1.92	-0.31	3
1999/1/16	1.56	-2.91	1	1999/4/30	2.02	0.04	3	1999/8/6	2.95	1.02	5
1999/1/18	1.55	-0.01	3	1999/5/3	1.02	-0.99	1	1999/8/7	4.24	1.30	5
1999/1/19	1.53	-0.01	3	1999/5/4	1.69	0.67	4	1999/8/9	1.23	-3.02	1
1999/1/20	1.08	-0.45	2	1999/5/5	1.11	-0.58	2	1999/8/10	3.27	2.04	5
1999/1/21	1.49	0.41	4	1999/5/6	1.45	0.33	4	1999/8/11	1.58	-1.69	1
1999/1/22	1.69	0.20	3	1999/5/7	1.50	0.05	3	1999/8/12	1.40	-0.17	3
1999/1/25	2.29	0.60	4	1999/5/10	1.08	-0.42	2	1999/8/13	1.97	0.57	4
1999/1/26	2.13	-0.17	3	1999/5/11	1.58	0.50	4	1999/8/16	2.74	0.76	4
1999/1/27	1.23	-0.90	2	1999/5/12	1.33	-0.25	3	1999/8/17	2.05	-0.69	2
1999/1/28	1.42	0.19	3	1999/5/13	1.07	-0.26	3	1999/8/18	1.94	-0.11	3
1999/1/29	1.90	0.48	4	1999/5/14	1.07	0.00	3	1999/8/19	0.92	-1.02	1
1999/1/30	2.12	0.23	3	1999/5/15	1.02	-0.05	3	1999/8/20	1.83	0.91	4
1999/2/1	1.49	-0.63	2	1999/5/17	1.49	0.47	4	1999/8/21	1.50	-0.33	3
1999/2/2	2.84	1.35	5	1999/5/18	2.23	0.74	4	1999/8/23	0.93	-0.58	2
1999/2/3	3.07	0.22	3	1999/5/19	1.12	-1.11	1	1999/8/24	3.00	2.08	5
1999/2/4	4.26	1.20	5	1999/5/20	1.28	0.15	3	1999/8/25	2.22	-0.79	2

Table A2. Historical training data and fuzzified fluctuation data of TAIEX1999

1999/2/5	3.68	-0.58	2	1999/5/21	1.28	0.00	3	1999/8/26	1.86	-0.36	2
1999/2/6	3.55	-0.13	3	1999/5/24	1.33	0.05	3	1999/8/27	1.18	-0.68	2
1999/2/8	1.95	-1.60	1	1999/5/25	1.88	0.54	4	1999/8/30	1.43	0.25	3
1999/2/9	1.33	-0.62	2	1999/5/26	1.48	-0.40	2	1999/8/31	1.30	-0.13	3
1999/2/10	2.48	1.15	5	1999/5/27	0.79	-0.68	2	1999/9/1	1.20	-0.10	3
1999/2/20	1.89	-0.59	2	1999/5/28	1.16	0.37	4	1999/9/2	2.38	1.17	5
1999/2/22	2.88	0.99	4	1999/5/29	1.02	-0.14	3	1999/9/3	1.78	-0.60	2
1999/2/23	3.24	0.36	4	1999/5/31	1.66	0.64	4	1999/9/4	2.12	0.34	4
1999/2/24	1.76	-1.48	1	1999/6/1	1.15	-0.51	2	1999/9/6	1.57	-0.55	2
1999/2/25	1.95	0.19	3	1999/6/2	0.75	-0.41	2	1999/9/7	3.08	1.51	5
1999/2/26	1.48	-0.47	2	1999/6/3	0.56	-0.19	3	1999/9/8	1.73	-1.35	1
1999/3/1	1.10	-0.37	2	1999/6/4	0.46	-0.10	3	1999/9/9	1.08	-0.64	2
1999/3/2	1.82	0.72	4	1999/6/5	0.86	0.39	4	1999/9/10	1.07	-0.02	3
1999/3/3	2.53	0.71	4	1999/6/7	1.55	0.69	4	1999/9/13	1.07	0.00	3
1999/3/4	1.28	-1.25	1	1999/6/8	0.80	-0.75	2	1999/9/14	1.65	0.58	4
1999/3/5	1.84	0.57	4	1999/6/9	0.95	0.16	3	1999/9/15	1.82	0.17	3
1999/3/6	1.28	-0.57	2	1999/6/10	0.91	-0.04	3	1999/9/16	1.11	-0.71	2
1999/3/8	0.75	-0.52	2	1999/6/11	1.05	0.13	3	1999/9/17	1.64	0.54	4
1999/3/9	1.30	0.54	4	1999/6/14	1.36	0.32	3	1999/9/18	0.90	-0.75	2
1999/3/10	0.82	-0.48	2	1999/6/15	1.88	0.52	4	1999/9/20	0.87	-0.03	3
1999/3/11	1.65	0.83	4	1999/6/16	1.64	-0.24	3	1999/9/27	0.07	-0.80	2
1999/3/12	0.89	-0.75	2	1999/6/17	1.07	-0.57	2	1999/9/28	0.51	0.43	4
1999/3/15	2.33	1.43	5	1999/6/21	1.87	0.80	4	1999/9/29	2.64	2.13	5
1999/3/16	1.55	-0.78	2	1999/6/22	2.06	0.19	3	1999/9/30	1.45	-1.19	1
1999/3/17	1.46	-0.09	3	1999/6/23	2.00	-0.06	3	1999/10/1	1.18	-0.27	3
1999/3/18	1.53	0.07	3	1999/6/24	2.19	0.19	3	1999/10/2	1.92	0.74	4
1999/3/19	1.55	0.02	3	1999/6/25	2.80	0.61	4	1999/10/4	1.58	-0.34	2

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2897719, IEEE Access

1999/3/20	1.32	-0.23	3	1999/6/28	1.09	-1.71	1	1999/10/5	2.73	1.15	5
1999/3/22	1.08	-0.24	3	1999/6/29	1.24	0.15	3	1999/10/6	1.00	-1.74	1
1999/3/23	2.77	1.69	5	1999/6/30	1.30	0.06	3	1999/10/7	1.25	0.25	3
1999/3/24	1.24	-1.53	1	1999/7/2	1.31	0.01	3	1999/10/8	1.07	-0.17	3
1999/3/25	1.32	0.08	3	1999/7/3	1.88	0.57	4	1999/10/11	0.96	-0.11	3
1999/3/26	1.13	-0.19	3	1999/7/5	1.45	-0.43	2	1999/10/12	2.10	1.14	5
1999/3/29	2.12	0.98	4	1999/7/6	2.48	1.03	5	1999/10/13	1.13	-0.97	2
1999/3/30	1.92	-0.20	3	1999/7/7	1.28	-1.20	1	1999/10/14	1.20	0.07	3
1999/3/31	1.40	-0.52	2	1999/7/8	0.98	-0.30	3	1999/10/15	1.64	0.44	4
1999/4/1	2.06	0.66	4	1999/7/9	1.32	0.34	4	1999/10/16	1.51	-0.13	3
1999/4/2	1.91	-0.15	3	1999/7/12	1.45	0.13	3	1999/10/18	1.09	-0.41	2
1999/4/3	1.42	-0.50	2	1999/7/13	2.52	1.07	5	1999/10/19	1.41	0.32	3
1999/4/6	0.98	-0.44	2	1999/7/14	5.33	2.80	5	1999/10/20	0.85	-0.56	2
1999/4/7	1.12	0.14	3	1999/7/15	2.80	-2.52	1	1999/10/21	1.68	0.82	4
1999/4/8	1.65	0.53	4	1999/7/16	7.75	4.94	5	1999/10/22	2.23	0.55	4
1999/4/9	1.65	0.00	3	1999/7/17	3.95	-3.79	1	1999/10/25	1.38	-0.85	2
1999/4/12	1.58	-0.07	3	1999/7/19	3.48	-0.47	2	1999/10/26	1.45	0.07	3
1999/4/13	0.85	-0.72	2	1999/7/20	3.06	-0.42	2	1999/10/27	1.39	-0.06	3
1999/4/14	0.84	-0.01	3	1999/7/21	2.43	-0.63	2	1999/10/28	0.85	-0.53	2
1999/4/15	2.07	1.24	5	1999/7/22	1.59	-0.84	2	1999/10/29	1.12	0.26	3
1999/4/16	2.35	0.28	3	1999/7/23	1.45	-0.14	3	1999/10/30	1.14	0.03	3
1999/4/17	1.42	-0.93	2	1999/7/26	1.73	0.28	3				

Date (YYYY/MM/DD)	Volume	Fluctuation	Fuzzified	Date (YYYY/MM/DD)	Volume	Fluctuation	Fuzzified	Date (YYYY/MM/DD)	Volume	Fluctuation	Fuzzified
1999/1/5	43900			1999/4/19	151900	14700	4	1999/7/27	79000	10300	4
1999/1/6	63500	19600	4	1999/4/20	133100	-18800	2	1999/7/28	82800	3800	3
1999/1/7	85100	21600	4	1999/4/21	149100	16000	4	1999/7/29	69100	-13700	2
1999/1/8	98300	13200	4	1999/4/22	105800	-43300	1	1999/7/30	61100	-8000	3
1999/1/11	87100	-11200	2	1999/4/23	149900	44100	5	1999/7/31	60000	-1100	3
1999/1/12	85500	-1600	3	1999/4/26	134800	-15100	2	1999/8/2	53800	-6200	3
1999/1/13	70600	-14900	2	1999/4/27	127300	-7500	3	1999/8/3	71300	17500	4
1999/1/14	73900	3300	3	1999/4/28	114600	-12700	2	1999/8/4	63400	-7900	3
1999/1/15	87200	13300	4	1999/4/29	111300	-3300	3	1999/8/5	58400	-5000	3
1999/1/16	126400	39200	5	1999/4/30	102800	-8500	3	1999/8/6	67200	8800	3
1999/1/18	57800	-68600	1	1999/5/3	106100	3300	3	1999/8/7	91800	24600	4
1999/1/19	56900	-900	3	1999/5/4	138400	32300	5	1999/8/9	73900	-17900	2
1999/1/20	50900	-6000	3	1999/5/5	140400	2000	3	1999/8/10	113700	39800	5
1999/1/21	68300	17400	4	1999/5/6	146500	6100	3	1999/8/11	117300	3600	3
1999/1/22	52700	-15600	2	1999/5/7	113800	-32700	1	1999/8/12	128200	10900	4
1999/1/25	51100	-1600	3	1999/5/10	82700	-31100	1	1999/8/13	172500	44300	5
1999/1/26	48700	-2400	3	1999/5/11	103300	20600	4	1999/8/16	168200	-4300	3
1999/1/27	62300	13600	4	1999/5/12	69100	-34200	1	1999/8/17	164100	-4100	3
1999/1/28	40700	-21600	2	1999/5/13	69500	400	3	1999/8/18	148300	-15800	2
1999/1/29	51700	11000	4	1999/5/14	107500	38000	5	1999/8/19	101900	-46400	1
1999/1/30	51700	0	3	1999/5/15	103800	-3700	3	1999/8/20	139300	37400	5
1999/2/1	39700	-12000	2	1999/5/17	131000	27200	4	1999/8/21	164300	25000	4
1999/2/2	44700	5000	3	1999/5/18	129300	-1700	3	1999/8/23	97300	-67000	1
1999/2/3	57200	12500	4	1999/5/19	151500	22200	4	1999/8/24	104700	7400	3

Table A3. Historical training data and fuzzified fluctuation data of TAIEX1999

1999/2/4	56600	-600	3	1999/5/20	138300	-13200	2	1999/8/25	125300	20600	4
1999/2/5	60700	4100	3	1999/5/21	102300	-36000	1	1999/8/26	149800	24500	4
1999/2/6	67500	6800	3	1999/5/24	110900	8600	3	1999/8/27	101400	-48400	1
1999/2/8	74100	6600	3	1999/5/25	146300	35400	5	1999/8/30	130900	29500	4
1999/2/9	47600	-26500	2	1999/5/26	108600	-37700	1	1999/8/31	151600	20700	4
1999/2/10	55000	7400	3	1999/5/27	126700	18100	4	1999/9/1	192800	41200	5
1999/2/20	57800	2800	3	1999/5/28	108900	-17800	2	1999/9/2	162400	-30400	1
1999/2/22	87000	29200	4	1999/5/29	121500	12600	4	1999/9/3	116200	-46200	1
1999/2/23	108600	21600	4	1999/5/31	111400	-10100	2	1999/9/4	144500	28300	4
1999/2/24	78300	-30300	1	1999/6/1	83400	-28000	2	1999/9/6	127400	-17100	2
1999/2/25	94700	16400	4	1999/6/2	118900	35500	5	1999/9/7	127700	300	3
1999/2/26	61900	-32800	1	1999/6/3	155500	36600	5	1999/9/8	115400	-12300	2
1999/3/1	69700	7800	3	1999/6/4	132800	-22700	2	1999/9/9	106300	-9100	3
1999/3/2	62200	-7500	3	1999/6/5	151300	18500	4	1999/9/10	124700	18400	4
1999/3/3	87100	24900	4	1999/6/7	175200	23900	4	1999/9/13	118600	-6100	3
1999/3/4	99800	12700	4	1999/6/8	176000	800	3	1999/9/14	106200	-12400	2
1999/3/5	72800	-27000	2	1999/6/9	186200	10200	4	1999/9/15	90300	-15900	2
1999/3/6	79800	7000	3	1999/6/10	198000	11800	4	1999/9/16	68400	-21900	2
1999/3/8	50300	-29500	2	1999/6/11	162400	-35600	1	1999/9/17	72500	4100	3
1999/3/9	96100	45800	5	1999/6/14	114700	-47700	1	1999/9/18	70600	-1900	3
1999/3/10	65700	-30400	1	1999/6/15	139400	24700	4	1999/9/20	50000	-20600	2
1999/3/11	75500	9800	3	1999/6/16	156500	17100	4	1999/9/27	10500	-39500	1
1999/3/12	59200	-16300	2	1999/6/17	226700	70200	5	1999/9/28	65300	54800	5
1999/3/15	102500	43300	5	1999/6/21	223600	-3100	3	1999/9/29	128500	63200	5
1999/3/16	141800	39300	5	1999/6/22	216100	-7500	3	1999/9/30	123700	-4800	3
1999/3/17	137100	-4700	3	1999/6/23	229800	13700	4	1999/10/1	107200	-16500	2
1999/3/18	145300	8200	3	1999/6/24	217000	-12800	2	1999/10/2	118200	11000	4

1999/3/19	158800	13500	4	1999/6/25	195300	-21700	2	1999/10/4	106100	-12100	2
1999/3/20	135300	-23500	2	1999/6/28	123800	-71500	1	1999/10/5	93600	-12500	2
1999/3/22	99700	-35600	1	1999/6/29	182500	58700	5	1999/10/6	69100	-24500	2
1999/3/23	126400	26700	4	1999/6/30	194900	12400	4	1999/10/7	85800	16700	4
1999/3/24	97000	-29400	2	1999/7/2	187300	-7600	3	1999/10/8	65800	-20000	2
1999/3/25	93100	-3900	3	1999/7/3	196400	9100	3	1999/10/11	61000	-4800	3
1999/3/26	116000	22900	4	1999/7/5	150100	-46300	1	1999/10/12	116800	55800	5
1999/3/29	90500	-25500	2	1999/7/6	153400	3300	3	1999/10/13	125100	8300	3
1999/3/30	76100	-14400	2	1999/7/7	142500	-10900	2	1999/10/14	103200	-21900	2
1999/3/31	66900	-9200	3	1999/7/8	146700	4200	3	1999/10/15	106500	3300	3
1999/4/1	103200	36300	5	1999/7/9	154100	7400	3	1999/10/16	74700	-31800	1
1999/4/2	151300	48100	5	1999/7/12	121200	-32900	1	1999/10/18	75200	500	3
1999/4/3	136900	-14400	2	1999/7/13	170900	49700	5	1999/10/19	65400	-9800	3
1999/4/6	79500	-57400	1	1999/7/14	137500	-33400	1	1999/10/20	71800	6400	3
1999/4/7	70700	-8800	3	1999/7/15	129100	-8400	3	1999/10/21	69200	-2600	3
1999/4/8	131500	60800	5	1999/7/16	135500	6400	3	1999/10/22	72300	3100	3
1999/4/9	142000	10500	4	1999/7/17	137700	2200	3	1999/10/25	54300	-18000	2
1999/4/12	127800	-14200	2	1999/7/19	112800	-24900	2	1999/10/26	68700	14400	4
1999/4/13	122000	-5800	3	1999/7/20	115900	3100	3	1999/10/27	61900	-6800	3
1999/4/14	143500	21500	4	1999/7/21	148400	32500	5	1999/10/28	60900	-1000	3
1999/4/15	139800	-3700	3	1999/7/22	106900	-41500	1	1999/10/29	92300	31400	5
1999/4/16	165000	25200	4	1999/7/23	82400	-24500	2	1999/10/30	135200	42900	5
1999/4/17	137200	-27800	2	1999/7/26	68700	-13700	2				

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2897719, IEEE Access

2169-3536 (c) 2018 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.