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NEUTROSOPHIC STRONGLY α-GENERALIZED SEMI CLOSED SETS

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ABSTRACT. The purpose of this paper is to introduce and study the concepts of Neutrosophic strongly α-generalized semi-closed sets and Neutrosophic strongly α-generalized semi-open sets. Some of their properties are explored.

1. INTRODUCTION AND PRELIMINARIES

A.A. Salama [9] introduced Neutrosophic topological spaces by using Smarandache's [4,5] Neutrosophic sets. I.Arokiarani et al. [1] introduced Neutrosophic α-closed sets. P. Ishwarya et al. [6] introduced and studied about Neutrosophic semi-open sets in Neutrosophic topological spaces. Neutrosophic Generalized semi-closed sets are introduced by V.K. Shanthi et al. [10] and then D. Jayanthi [7] initiated Neutrosophic αg closed sets. V. Banu Priya et al. [2] introduced Neutrosophic αgs-closed sets. Aim of this present paper is, to introduce and investigate about new kind of Neutrosophic closed sets called Neutrosophic strongly α-generalized semi-closed sets and Neutrosophic strongly α generalized semi open sets and its properties are discussed in detail.

Definition 1.1. *[4,5] Let X be a non empty set and Neutrosophic sets A and B in the form $A = \{\langle x, \eta_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$, $B = \{\langle x, \eta_B(x), \sigma_B(x), \nu_B(x)\rangle \mid x \in X\}$ then*

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(1) *the complement of the set A, A^c defined as* $A^c = \{\langle x, \nu_A(x), 1-\sigma_A(x), \eta_A(x)\rangle \mid x \in X\}$;
(2) $A \subseteq B$ *defined as* $A \subseteq B \Leftrightarrow \eta_A(x) \leqslant \eta_B(x), \sigma_A(x) \leqslant \sigma_B(x)$ *and* $\nu_A(x) \geqslant \nu_B(x)$ *for all* $x \in X$;
(3) $A \cap B$ *defined as* $A \cap B = \langle x, \eta_A(x) \wedge \eta_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle$;
(4) $A \cup B$ *defined as* $A \cup B = \langle x, \eta_A(x) \vee \eta_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle$.

Definition 1.2. *[9] A Neutrosophic topology on a non empty set X is a family τ_N of Neutrosophic subsets in X satisfying the following axioms:*

(1) $0_N, 1_N \in \tau_N$;
(2) $G_1 \cap G_2 \in \tau_N$ *for any* $G_1, G_2 \in \tau_N$;
(3) $\cup G_i \in \tau_N$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau_N$;

the pair (X, τ_N) is called a Neutrosophic topological space. The elements in τ_N are called as Neutrosophic open sets. The Neutrosophic set A is closed if and only if A^c is Neutrosophic open.

Definition 1.3. *Let (X, τ_N) be Neutrosophic topological spaces. The Neutrosophic closure and Neutrosophic interior of A are defined by*

(1) $N\text{-}cl(A) = \cap\{K \mid K$ *is a Neutrosophic closed set in X and* $A \subseteq K\}$;
(2) $N\text{-}int(A) = \cup\{G \mid G$ *is a Neutrosophic open set in X and* $G \subseteq A\}$.

Definition 1.4. *Let (X, τ_N) be a Neutrosophic topological space. The subset A is:*

(1) *Neutrosophic regular closed set [1] (N-RCS in short) if* $A = N\text{-}cl(N\text{-}int(A))$.
(2) *Neutrosophic α closed set [1] (N-αCS in short) if* $N\text{-}cl(N\text{-}int(N\text{-}cl((A))) \subseteq (A)$.
(3) *Neutrosophic semi closed set [6] (N-SCS in short) if* $N\text{-}int(N\text{-}cl(A)) \subseteq A$.
(4) *Neutrosophic pre closed set [11] (N-PCS in short) if* $N\text{-}cl(N\text{-}int(A)) \subseteq A$.
(5) *Neutrosophic semipreclosed set [8](N-$SPCS$ in short) if* $N\text{-}int(N\text{-}cl(N\text{-}int(A) \subseteq A$.
(6) *Neutrosophic generalised closed set [3] (N-GCS in short) if* $N\text{-}cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is a N-OS in X.*
(7) *Neutrosophic generalised semi closed set [10] (N-$GSCS$ in short) if* $N\text{-}scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is a N-OS in X.*
(8) *Neutrosophic α generalised closed set [7] (N-αGCS in short) if* $N\text{-}\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is a N-OS in X.*

(9) *Neutrosophic α generalised semi closed set [2] (N-$\alpha GSCS$ in short) if N-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a N-SOS in X.*

2. Neutrosophic strongly α-generalized semi closed sets

Definition 2.1. *A NS A in (X, τ) is said to be a Neutrosophic strongly α- generalized semi-closed set (briefly $Ns\ \alpha$ GSCS) $N\alpha cl(A) \subseteq U^*$ whenever $A \subseteq U^*$ and U^* is a NGSOS in (X, τ) and the family of all $Ns\alpha GSCS$ of a NTS (X, τ) is denoted by $Ns\alpha GSC(X)$.*

Example 1. *Let X = $\{a, b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \langle x, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}) \rangle$. Then the NS $A = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}) \rangle$ is a $Ns\alpha GSCS$ in (X, τ).*

Theorem 2.1. *Every NCS in (X, τ) is a $Ns\alpha GSCS$ but not conversely.*

Proof. Assume that A is a NCS in (X, τ). Let us consider a NS $A \subseteq U^*$where U^* is a $NGSOS$ in X. Since $N\alpha cl(A) \subseteq Ncl(A)$ and A is a NCS in X, $N\alpha cl(A) \subseteq Ncl(A) = A \subseteq U^*$and $U^*$is $NGSOS$. That is $N\alpha cl(A) \subseteq U$. Therefore, A is $Ns\alpha GSCS$ in X. ☐

Example 2. *Let X = $\{a, b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \langle x, (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}) \rangle$. Then the NS $A = \langle x, (\frac{1}{10}, \frac{1}{2}, \frac{9}{10}), (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}) \rangle$ is $Ns\alpha GSCS$ but not a NCS in X.*

Theorem 2.2. *Every $N\alpha CS$ in (X, τ) is a $Ns\alpha GSCS$ in (X, τ) but not conversely.*

Proof. Let A be a $N\alpha CS$ in X. Let us consider a NS $A \subseteq U^*$is a $NGSOS$ in (X, τ). Since A is a $N\alpha CS$, $N\alpha cl(A) = A$. Hence $N\alpha cl(A) \subseteq U^*$ whenever $A \subseteq U^*$ and U^* is $NGSOS$. Therefore, A is a $Ns\alpha GSCS$ in X. ☐

Example 3. *Let X = $\{a, b\}$. Let $\tau = \{0_N, V_1, V_2, , 1_N\}$ be a NT on X, where $V_1 = \langle x, (\frac{2}{5}, \frac{1}{2}, \frac{1}{2}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}) \rangle$ and $V_2 = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{1}{5}, \frac{1}{2}, \frac{7}{10}) \rangle$. Consider a NS $A = \langle x, (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}), (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}) \rangle$ which is $Ns\alpha GSCS$ but not $N\alpha CS$, since $Ncl(Nin(NclA))) = 1_N \nsubseteq A$.*

Theorem 2.3. *Every $NRCS$ in (X, τ) is a $Ns\alpha GSCS$ in (X, τ) but not conversely.*

Proof. Let A be a $NRCS$ in (X, τ). Since every $NRCS$ is a NCS, A is a NCS in X. Hence by Theorem 2.1, A is a $Ns\alpha GSCS$ in X. ☐

Example 4. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \left\langle x, (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}) \right\rangle$. Consider ANS $A = \left\langle x, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}) \right\rangle$ which is a $Ns\alpha GSCS$ but not $NRCS$ in X as $Ncl(Nint(A)) = 0_N \subset A$.*

Theorem 2.4. *Every $Ns\alpha GSCS$ in (X,τ) is a $N\alpha GSCS$ in (X,τ) but not conversely.*

Proof. Assume that A is a $Ns\alpha GSCS$ in (X,τ). Let us consider NS $A \subseteq U^*$ where $U^*$is a $NSOS$ in X. Since every $NSOS$ is a $NGSOS$ and by hypothesis $N\alpha cl(A) \subseteq U^*$, whenever $A \subseteq U^*$and $U^*$is a $NGSOS$ in X. We have $N\alpha cl(A) \subseteq U^*$, whenever $A \subseteq U^*$and $U^*$is a $NSOS$ in X. Hence A is a $N\alpha GSCS$ in X. ☐

Example 5. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \left\langle x, (\frac{3}{10}, \frac{1}{2}, \frac{2}{5}), (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}) \right\rangle$. Then the NS $A = \left\langle x, (\frac{1}{2}, \frac{1}{2}, \frac{3}{10}), (\frac{1}{5}, \frac{1}{2}, \frac{3}{10}) \right\rangle$ is a $N\alpha GSCS$ but not a $Ns\alpha GSCS$ in X.*

Remark 2.1. *A NP closedness is independent of $Ns\alpha GS$ closedness.*

Example 6. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X,where $V = \left\langle x, (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}), (\frac{1}{5}, \frac{1}{2}, \frac{3}{10}) \right\rangle$. Then the NS $A = \left\langle x, (\frac{2}{5}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}\frac{2}{5}) \right\rangle$ is $NPCS$ but not $Ns\alpha GSCS$.*

Example 7. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \left\langle x, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}) \right\rangle$. Then the NS $A = \left\langle x, (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}) \right\rangle$ is $Ns\alpha GSCS$ but not a $NPCS$.*

Remark 2.2. *A NSP closedness is independent of $Ns\alpha GS$ closedness.*

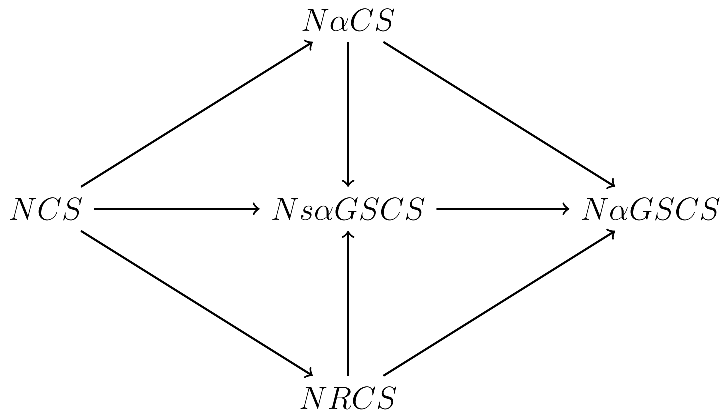
Example 8. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \left\langle x, (\frac{3}{5}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{1}{5}) \right\rangle$. Then the NS $A = \left\langle x, (\frac{7}{10}, \frac{1}{2}, \frac{1}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{10}) \right\rangle$ is $NSPCS$ but not $Ns\alpha GSCS$.*

Example 9. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \left\langle x, (\frac{3}{5}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{1}{5}) \right\rangle$. Then the NS $A = \left\langle x, (\frac{1}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{1}{10}, \frac{1}{2}, \frac{1}{2}) \right\rangle$ is $Ns\alpha GSCS$ but not $NSPCS$.*

Remark 2.3. *A $N\gamma CS$ in (X,τ) need not be a $Ns\alpha GSCS$.*

Example 10. *Let $X = \{a,b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \left\langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2}) \right\rangle$ .Then the NS $A = \left\langle x, (\frac{2}{5}, \frac{1}{2}, \frac{1}{2}), (\frac{2}{5}, \frac{1}{2}, \frac{2}{5}) \right\rangle$ is $N\gamma CS$ but not $Ns\alpha GSCS$.*

The relations between various types of Neutrosophic closed sets are given in the following diagram.



The reverse implications are not true in general.

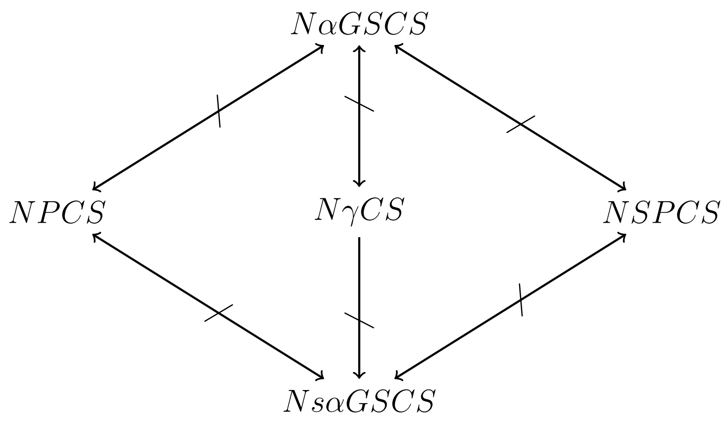
Remark 2.4. *The intersection of any two $Ns\alpha GSCS$ is not a $Ns\alpha GSCS$ in general as can be seen in the following example.*

Example 11. *Let $X = \{a, b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$. Then the NS $A = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{7}{10}, \frac{1}{2}, \frac{3}{10})\rangle$ and $B = \langle x, (\frac{9}{10}, \frac{1}{2}, \frac{1}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ are $Ns\alpha GSCS$ in X. Now $A \cap B = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ $\subseteq U^* = \langle x, (\frac{2}{5}, \frac{1}{2}, \frac{3}{10}), (\frac{1}{2}, \frac{1}{2}, \frac{2}{5})\rangle$ and U^* is $NGSOS$ in X. But $N\alpha cl(A \cap B) = 1_N \nsubseteq U^*$. Therefore, $A \cap B$ is not a $Ns\alpha GSCS$ in X.*

Theorem 2.5. *Let (X, τ) be a NTS. Then for every $A \in Ns\alpha GSC(X)$ and for every NS B in X, $A \subseteq B \subseteq N\alpha cl(A)$ implies $B \in Ns\alpha GSC(X)$.*

Proof. Let $B \subseteq U^*$ where $U^*$is a $NGSOS$ in X. Since $A \subseteq B, A \subseteq U^*$. Since A is a $Ns\alpha GSCS$ in X, $N\alpha cl(A) \subseteq U^*$. By hypothesis $B \subseteq N\alpha cl(A)$. This implies $N\alpha cl(B) \subseteq N\alpha cl(A) \subseteq U^*$. Therefore, $N\alpha cl(B) \subseteq U^*$. Hence B is a $Ns\alpha GSCS$ in X. □

The independent relations between various types of Neutrosophic closed sets are given in the following diagram.



In this diagram, $A \not\leftrightarrow B$ denotes A and B are independent and $A \not\rightarrow B$ denotes A need not be B.

Theorem 2.6. *If A is a $NGSOS$ and a $Ns\alpha GSCS$, then A is a $N\alpha CS$ in X.*

Proof. Let A be a $NGSOS$ in X. Since $A \subseteq A$, by hypothesis $N\alpha cl(A) \subseteq A$. But always $A \subseteq N\alpha cl(A)$. Therefore, $N\alpha cl(A) = A$. Hence A is a $N\alpha CS$ in X. □

Theorem 2.7. *Let (X, τ) be a NTS. Then A is a $Ns\alpha GSCS$ if and only if $A\bar{q}F$ implies $N\alpha cl(A)\bar{q}F$ for every $NGSCS$ F of X.*

Proof. Necessary Part: Let F be a $NGSCS$ and $A\bar{q}F$. Then $A \subseteq \acute{F}$ where $\acute{F}$ is a $NGSOS$ in X. By assumption $N\alpha cl(A) \subseteq \acute{F}$. Hence $N\alpha cl(A)\bar{q}F$.

Sufficient Part: Let F be $NGSCS$ in X such that $A \subseteq \acute{F}$. By hypothesis, $A\bar{q}F$ implies $N\alpha cl(A)\bar{q}F$. This implies $N\alpha cl(A) \subseteq \acute{F}$ whenever $A \subseteq \acute{F}$ and $\acute{F}$ is a $NGSOS$ in X. Hence A is a $Ns\alpha GSCS$ in X. □

3. Neutrosophic strongly α -generalized semi open sets

In this section we introduce Neutrosophic strongly α -generalized semi-open sets and study some of its properties.

Definition 3.1. *A NS A is said to be Neutrosophic strongly α-generalized semi-open set (briefly $Ns\alpha GSOS$) in (X, τ) if the complement A^c is a $Ns\alpha GSCS$ in X. The family of all $Ns\alpha GSOS$ of a NTS (X, τ) is denoted by $Ns\alpha GSO(X)$.*

Theorem 3.1. *For any NTS (X, τ), every NOS is a $Ns\alpha GSOS$ but not conversely.*

Proof. Let A be a NOS in X. Then A^c is a NCS in X. By Theorem 2.1, A^c is a $Ns\alpha GSCS$ in X. Hence A is a $Ns\alpha GSOS$ in X. □

Example 12. *Let $X = \{a, b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \langle x, (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}) \rangle$. Consider the NS $A = \langle x, (\frac{9}{10}, \frac{1}{2}, \frac{1}{10}), (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}) \rangle$. Since A^c is a $Ns\alpha GSCS$, A is a $Ns\alpha GSOS$ but not NOS in X.*

Theorem 3.2. *In any NTS (X, τ) every $N\alpha OS$ is a $Ns\alpha GSOS$ but not conversely.*

Proof. Let A be a $N\alpha OS$ in X. Then A^c is a $N\alpha CS$ in X. By Theorem 2.2, A^c is a $Ns\alpha GSCS$ in X. Hence A is a $Ns\alpha GSOS$ in X. □

Example 13. *Let $X = \{a, b\}$. Let $\tau = \{0_N, V_1, V_2, , 1_N\}$ be a NT on X, where $V_1 = \langle x, (\frac{2}{5}, \frac{1}{2}, \frac{1}{2}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}) \rangle$ and $V_2 = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{1}{5}, \frac{1}{2}, \frac{7}{10}) \rangle$. Then the NS $A = \langle x, (\frac{1}{10}, \frac{1}{2}, \frac{4}{5}), (\frac{1}{10}, \frac{1}{2}, \frac{4}{5}) \rangle$ is a $Ns\alpha GSOS$ in X but not a $N\alpha OS$ in X.*

Theorem 3.3. *In any NTS (X, τ), every $NROS$ is a $Ns\alpha GSOS$ but not conversely.*

Proof. Let A be a $NROS$ in X. Then A^c is a $NRCS$ in X. By Theorem 2.3, A^c is a $Ns\alpha GSCS$ in X. Hence A is a $Ns\alpha GSOS$ in X. □

Example 14. *Let $X = \{a, b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \langle x, (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5}) \rangle$. Then the NS $A = \langle x, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}) \rangle$ is a $Ns\alpha GSOS$ in X but not a $NROS$ in X.*

Theorem 3.4. *In any NTS (X, τ), every $Ns\alpha GSOS$ is a $N\alpha GSOS$ but not conversely.*

Proof. Let A be a $Ns\alpha GSOS$ in X. Then A^c is a $Ns\alpha GSCS$ in X. By Theorem 2.4, A^c is a $N\alpha GSCS$ in X. Hence A is a $N\alpha GSOS$ in X. □

Example 15. *Let $X = \{a, b\}$. Let $\tau = \{0_N, V, 1_N\}$ be a NT on X, where $V = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{2}{5}), (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}) \rangle$. Then the NS $A = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{1}{2}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{10}) \rangle$ is a $N\alpha GSOS$ in X but not a $Ns\alpha GSOS$ in X.*

Remark 3.1. *The union of any two $Ns\alpha GSOS$ is not a $Ns\alpha GSOS$ in general.*

Example 16. *Let $X = \{a, b\}$. Let $\tau = \{0_N, V_1, V_2, 1_N\}$ be a NT on X, where $V_1 = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5}) \rangle$. $V_2 = \langle x, (\frac{1}{10}, \frac{1}{2}, \frac{9}{10}), (\frac{1}{2}, \frac{1}{2}, \frac{2}{5}) \rangle$ are $Ns\alpha GSOS$ in X.Now $V_1 \cup V_2 = \langle x, (\frac{3}{5}, \frac{1}{2}, \frac{3}{10}), (\frac{1}{2}, \frac{1}{2}, \frac{2}{5}) \rangle$ is not a $Ns\alpha GSOS$ in X.*

Theorem 3.5. *A NS A of a NTS (X, τ) is a $Ns\alpha GSOS$ if and only if $F \subseteq \alpha int(A)$ whenever F is a $NGSCS$ in X and $F \subseteq A$.*

Proof. Necessary Part: Let A be a $Ns\alpha GSOS$ in X. Let F be a $NGSCS$ in X and $F \subseteq A$. Then $\acute{F}$ is a $NGSOS$ in X such that $A' \subseteq \acute{F}$. Since A' is a $Ns\alpha GSCS$, we have $N\alpha cl(A') \subseteq \acute{F}$. Hence $(N\alpha int(A')) \subseteq \acute{F}$. Therefore, $F \subseteq N\alpha int(A)$. Sufficient Part: Let A be a NS in X and let $F \subseteq N\alpha int(A)$ whenever F is a $NGSCS$ in X and $F \subseteq A$. Then $A' \subseteq \acute{F}$ and $\acute{F}$ is a $NGSOS$. By hypothesis, $(N\alpha int(A')) \subseteq \acute{F}$, which implies $N\alpha cl(A') \subseteq \acute{F}$. Therefore, A is a $Ns\alpha GSCS$ in X. Hence A is a $Ns\alpha GSOS$ in X. □

Theorem 3.6. *If A is a $Ns\alpha GSOS$ in (X, τ), then A is a $NGSOS$ in (X, τ).*

Proof. Let A be a $Ns\alpha GSOS$ in X. This implies A is a $N\alpha GSOS$ in X. Since every $N\alpha GSOS$ is a $NGSOS$, A is a $NGSOS$ in X. □

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