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Research Article

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Neutrosophic triplet normed space

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Abstract: In this paper; new properties for neutrosophic triplet groups are introduced. A notion of neutrosophic triplet metric space is given and properties of neutrosophic triplet metric spaces are studied. Neutrosophic triplet vector space and neutrosophic triplet normed space are also studied and some of their properties are given. Furthermore,we also show that these neutrosophic triplet notionsare different from the classical notions.

Keywords: Neutrosophic triplet groups, neutrosophic triplet metric spaces, neutrosophic triplet vector spaces, neutrosophic triplet normed spaces

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1 Introduction

There are many uncertainties in our daily life. Classic methods cannot always explain these uncertainties. The concept of fuzzy set theory is introduced by Zadeh in [1] to overcome uncertainties. Although fuzzy sets are used in many applications, it does not explain the indeterminancy states because it has only membership (truth) function. Then the concept of intuitionistic fuzzy sets theory is introduced by Atanassov in [2] to overcome uncertainties. The theory deals with states of truth, falsity and indeterminancy. However these states have been defined as dependent on each other. Finally the concept of neutrosophic set theory is introduced by Smarandache in [3]. In this theory the states of truth, falsity and indeterminancy are defined as independent on each other. By utilizing the idea of neutrosophic theory, Kandasamy and Smarandache introduced neutrosophic algebraic structures in [4, 5]. Florentin Smarandache and Mumtaz Ali introduced neutrosophic triplet theory in [6] and neutrosophic triplet groups in [7, 8]. The neutrosophic triplet set is completely different from the classical sets, since for each element "a" in neutrosophic triplet set N together with a binary operation *; there exist a neutral of "a" called neut(a) such that a*neut(a) = neut(a)*a = a and an opposite of "a" called anti(a) such that a*anti(a) = anti(a)*a = neut(a). Where; neut(a) is different from the classical algebraic unitary element. A neutrosophic triplet is of the form <a, neut(a), anti(a) >. Also, Florentin Smarandache and Mumtaz Ali studied the neutrosophic triplet field [9, 10] and the neutrosophic triplet ring [10, 11]. Recently some researchers have been dealing with neutrosophic set theory. For example, Broumi, Bakali, Talea and Smarandache studied the single valued neutrosophic graphs [12] and interval valued neutrosophic graphs [13]. Broumi, Bakali, Talea, Smarandache and Vladareanu studied SV-Trapezoidal neutrosophic numbers [14] and neutrosophic shortest path problem [15]. Liu and Shi studied interval neutrosophic hesitant set [16] and neutrosophic uncertain linguistic number [17]. Liu and Tang studied some power generalized aggregation operators based on the interval neutrosophic numbers [18] and Multi-criteria group decision-making based on interval neutrosophic uncertain linguistic variables and Choquet integral [19]. Liu and Wang studied interval neutrosophic prioritized OWA operator [20]. Liu and Teng studied multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator [21]. Liu, Zhang, Liu, and Wang studied multi-valued neutrosophic number bonferroni mean operators [22]. P Liu studied the aggregation operators based on Archimedean t-conorm and t-norm for the single valued neutrosophic numbers [23]. Also, in [24–31] neutrosophic set theory was studied.

In this paper; we give new properties for neurosophic triplet groups and we introduced neutrosophic triplet metric, neutrosophic triplet vector space and neutrosophic triplet normed space. These neutrosophic triplet structures have been studied by Şahin and Kargın in [24] However; we used these structures to show these structures are different from the classical structures. Also, we give new properties and new definitions for these structures. In this paper; in section 2; some preliminary results for neutrosophic triplet groups are given. In section 3; new properties for neutrosophic triplet group are given. In sec-

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tion 4; neutrosophic triplet metric space is defined and some properties of a neutrosophic triplet metric space are given.It is show that neutrosophic triplet metric different from the classical metric. Also, the convergence of a sequence and a Cauchy sequence in a neutrosophic triplet metric space are defined. In section 5; neutrosophic triplet vector space is defined and some properties of neutrosophic triplet vector space are given. Also, it is show that neutrosophic triplet vector spaces are different from the classical vector spaces. In section 6; a neutrosophic triplet normed space is defined and some properties of neutrosophic triplet normed space are given.The convergence of a sequence and a Cauchy sequence in the neutrosophic triplet normed space are defined.Also, it is show that neutrosophic triplet normed spaces are different from the classical normed space. However; it is show that if certain conditions are met; every classical normed space can be a neutrosophic triplet normed space at the same time. In section 7; conclusions are given.

2 Preliminaries

Definition 2.1 [7]: Let N be a set together with a binary operation *. Then, N is called a neutrosophic triplet set if for any a ∈ N, there exists a neutral of "a" called neut(a), different from the classical algebraic unitary element, and an opposite of "a" called anti(a), with neut(a) and anti(a) belonging to N, such that:
a*neut(a) = neut(a)*a = a,
and
a*anti(a) = anti(a)*a = a.

The elements a, neut(a) and anti(a) are collectively called as neutrosophic triplet, and we denote it by (a, neut(a), anti(a)). Here, we mean neutral of aand apparently, "a" is just the first coordinate of a neutrosophic triplet and it is not a neutrosophic triplet. For the same element "a" in N, there may be more neutrals toit neut(a)'s and more opposites of it anti(a)'s.

Definition 2.2[7]: Let (N,*) be a neutrosophic triplet set. Then, N is called a neutrosophic triplet group, if the following conditions are satisfied.

(1) If (N,*) is well-defined, i.e. for any a,b ∈ N, one has a*b ∈ N.
(2) If (N,*) is associative, i.e. (a*b)*c = a*(b*c) for all a,b,c ∈ N.

The neutrosophic triplet group, in general, is not a group in the classical algebraic way.

One can consider that neutrosophic neutrals are replacing the classical unitary element, and the neutrosophic opposites are replacing the classical inverse elements.

Definition 2.3 [7]: Let (N,*) be a neutrosophic triplet group. Then N is called a commutative neutrosophic triplet group if for all a,b ∈ N, we have a*b = b*a.

Proposition 2.4 [7]: Let (N,*) be a neutrosophic triplet group with respect to * and a,b,c ∈ N;

(1) a*b = a*c if and only if neut(a)*b = neut(a)*c
(2) b*a = c*a if and only if b*neut(a) = c*neut(a)
(3) if anti(a)*b = anti(a)*c, then neut(a)*b = neut(a)*c
(4) if b*anti(a) = c*anti(a), then b*neut(a) = c*neut(a)

Theorem 2.5[7]: Let (N,*) be a commutative neutrosophic triplet group with respect to * and a,b ∈ N;

(i) neut(a)*neut(b) = neut(a*b);
(ii) anti(a)*anti(b) = anti(a*b);

Theorem 2.6[7]: Let (N,*) be a commutative neutrosophic triplet group with respect to * and a ∈ N;

(i) neut(a)*neut(a) = neut(a);
(ii) anti(a)*neut(a) = neut(a)* anti(a) = anti(a);

Definition 2.7 [9, 11]: Let (NTF,*,#) be a neutrosophic triplet set together with two binary operations* and #. Then (NTF,*,#) is called neutrosophic triplet field if the following conditions hold.

(1) (NTF,*) is a commutative neutrosophic triplet group with respect to *.
(2) (NTF,#) is a neutrosophic triplet group with respect to #.
(3) a#(b*c) = (a#b)*(a#c) and (b*c)#a = (b#a)*(c#a) for all a,b,c ∈ NTF.

3 New properties for neutrosophic triplet groups

Firstly; we define neutrosophic triplet subset and for neutrosophic triplet groups, we give new properties as theorems.

Definition 3.1: Let(N, *) be a neutrosophic triplet set. For a subset S⊂N; if (S, *) is a neutrosophic triplet set itself, then S is called the neutrosophic triplet subset.

Theorem 3.2: Let (N,*) be a neutrosophic triplet group with no zero divisors and with respect to *. For a ∈ N;

If a = neut(a), then there exists an anti(a) such that neut(a) = anti(a) = a.

Proof: By the definition of neutrosophic triplet set; as a*anti(a) = anti(a)*a = a we have anti(a)*(anti(a)*a) = anti(a)*a. Thus; anti(a)*neut(a) = neut(a).

If a = neut(a); by Theorem 2.6, we have anti(a)*neut(a) = anti(a) and therefore we have a anti(a) such that neut(a) = anti(a) = a

Theorem 3.3: Let (N,*) be a neutrosophic triplet group with no zero divisors and with respect to *. For a ∈ N;

(i) neut(neut(a)) = neut(a)
(ii) anti(neut(a)) = neut(a))
(iii) anti(anti(a)) = a
(iv) neut(anti(a)) = neut(a)

Proof :

(i) By the definition of neutrosophic triplet set; we have neut(a)*neut(neut(a)) = neut(a). By Proposition 2.4, we have a*neut(a)*neut(neut(a)) = a*neut(a). Thus; a*neut(neut(a)) = a.
(ii) By the definiton of neutrosophic triplet set; we have neut(neut(a)) = neut(a).
(iii) Using i), as neut(neut(a)) = neut(a); there exists a element b ∈ N such that neut(a) = b. Since neut(a) = b, we have neut(b) = b. By Theorem 3.2, as neut(b) = b, we have neut(b) = anti(b). Thus; we have neut(neut(a)) = anti(neut(a)) = neut(a).
(iv) By the definition of neutrosophic triplet set, as a*anti(a) = anti(a)*a = a; we have anti(anti(a))*anti(a) = anti(a)*anti(anti(a)) = neut(a). Thus; anti(anti(a)) = a.
(v) By the definition of neutrosophic triplet set, as a*anti(a) = anti(a)*a = a; we have anti(a)*neut(anti(a)) = neut(anti(a))*anti(a) = anti(a). By proposition 2.4; we have neut(a)*neut(anti(a)) = neut(a). Again from the proposition 2.4; we have a*neut(anti(a)) = a. Thus; neut(anti(a)) = neut(a).

4 Neutrosophic triplet metric space

Definition 4.1: Let(N,*) be a neutrosophic triplet set and let x*y ∈ N for all x,y ∈ N. If the function d:NxN→ $\mathbb{R}^+\cup\{0\}$ satisfies the following conditions; d is called a neutrosophic triplet metric. For all x,y,z ∈ N;

(a) d(x,y) ≥ 0;
(b) If x = y; then d(x,y) = 0
(c) d(x,y) = d(y,x)
(d) If there exists any element y ∈ N such that d(x,z) ≤ d(x,z*neut(y)), then d(x,z*neut(y)) ≤ d(x,y)+ d(y,z).

Furthermore; ((N,*), d) space is called neutrosophic triplet metric space.

Corollary 4.2: The neutrosophic triplet metric is generally different from the classical metric, since for there is no "*" binary operation different from "d" and neut(x) element for any element x in classical metric.

Example 4.3: Let X be a set and P(X) be the power set of X, namely the set of all subsets of X and s(A) be number of elements in A ∈ P(X). Then (P(X), ∪) is a neutrosophic triplet set, since for A∪A = A∪A = A and A∪A = A∪A = A. Thus, we can take neut(A) = A, anti(A) = A for all A ∈ P(X). Also, A∪A ∈ P(X), for all A ∈ P(X). Later, we define the function d: P(X)xP(X)→ $\mathbb{R}^+\cup\{0\}$, d(A,B) = |s(A)-s(B)| and we show that "d" is a neutrosophic triplet metric.

a), b) and c) are clear.

d) For ∅ ∈ P(X), since for d(A,B) = d(A,B∪∅); d(A,B∪∅) = d(A,B) = |s(A)-s(B)|. From the triangle inequality; it is clear |s(A)-s(B)| ≤ |s(A)-s(C)|+|s(C)-s(B)|. Thus; d(A,B∪∅) ≤ d(A,C)+d(C,B). Also, ((P(X),∪),d) is a neutrosophic triplet metric space.

Now let's define the convergence of a sequence and Cauchy sequence in the neutrosophic triplet metric space.

Definition 4.4: Let((N,*), d) be a neutrosophic triplet metric space and $\{x_n\}$ be a sequence in this space and x ∈ N . For all ε > 0; such that

d(x, $\{x_n\}$) < ε

for all n ≥ M, if there exist M ∈ ℕ ; then $\{x_n\}$ converges to x ∈ N. It is denoted by

$\lim_{n\to\infty} x_n$ = x or$x_n\to$ x.

Definition 4.5: Let((N,*), d) be a neutrosophic triplet metric space and $\{x_n\}$ be a sequence in this space.

For all ε > 0; such that

d($\{x_m\}$, $\{x_n\}$) < ε

for all n ≥ M , if there exist M ∈ ℕ ; then the sequence $\{x_n\}$ is a Cauchy sequence.

Theorem 4.6: Let((N,*), d) be a neutrosophic triplet metric space and $\{x_n\}$ be a sequence in this space. If $\{x_n\}$ is convergent and, d($\{x_n\}$,$\{x_m\}$) ≤ d($\{x_n\}$,$\{x_m\}$*neut(x)) for any x ∈ N; then $\{x_n\}$ is a Cauchy sequence.

Proof: As $\{x_n\}$ is convergent; d(x,$\{x_n\}$) < ε/2 for all n ≥ M or d(x, $\{x_m\}$) < ε/2 for all m ≥ M. For all n, m ≥ M, as d($\{x_n\}$,$\{x_m\}$) ≤ d($\{x_n\}$,$\{x_m\}$*neut(x)); d($\{x_n\}$,$\{x_m\}$) ≤ d($\{x_n\}$,$\{x_m\}$*neut(x)) ≤ d(x, $\{x_n\}$)+ d(x, $\{x_m\}$) = ε/2 + ε/2 . Therefore; by the definition of Cauchy sequence, $\{x_n\}$ is a Cauchy sequence.

Definition 4.7: Let ((N,⋆), d) be a neutrosophic triplet metric space. If every $\{x_n\}$ cauchy sequence in this space is convergent; then this space is called a complete neutrosophic triplet metric space.

5 Neutrosophic triplet vector spaces

Now let's define the neutrosophic triplet vector space which has much more properties than the neutrosophic triplet sets. Thus, we will obtain more specific structure that is wider than neutrosophic triplet sets.

Definition 5.1: Let(NTF,$\star_1$, $\#_1$) be a neutrosophic triplet field and let (NTV,$\star_2$, $\#_2$) be a neutrosophic triplet set together with binary operations "$\star_2$" and "$\#_2$". Then (NTV,$\star_2$, $\#_2$) is called a neutrosophic triplet vector space if the following conditions hold. For all u, v ∈ NTV and for all k ∈ NTF; such that u$\star_2$v ∈ NTV and u $\#_2$k ∈ NTV;

(1) (u$\star_2$v)$\star_2$t = u$\star_2$ (v$\star_2$t), for every u,v,t ∈ NTV
(2) u$\star_2$v = v$\star_2$u, for every u,v ∈ NTV
(3) (v$\star_2$u)$\#_2$k = (v$\#_2$k)$\star_2$(u$\#_2$k), for all k ∈ NTF and for all u,v ∈ NTV
(4) (k$\star_1$t)$\#_2$u = (k$\#_2$v)$\star_1$(u$\#_2$v), for all k,t ∈ NTF and for all u ∈ NTV
(5) (k$\#_1$t)$\#_2$u = k$\#_1$(t$\#_2$u), for all k,t ∈ NTF and for all u ∈ NTV
(6) For all u ∈ NTV; such that u $\#_2$neut(k) = neut(k)$\#_2$ u = u, there exists any neut(k) ∈ NTF

Here; the condition 1) and 2) indicate that the neutrosophic triplet set (NTV,$\star_2$) is a commutative neutrosophic triplet group.

Corollary 5.2: By the condition 6) of the neutrosophic triplet vector space definition; for every u ∈ NTV; neut(k) ∈ NTF that satisfies u $\#_2$ neut(k) = neut(k)$\#_2$ u = u need not be unique.Therefore; neutrosophic triplet vector space is generally different from the classical vector space.

Example 5.3: Let X = {1,2} be a set and P(X) = {∅, {1}, {2}, {1, 2}} be power set of X and let (P(X), ⋆) be a neutrosophic triplet set. Where ⋆ = ∪, neut(∅) = neut({1}) = neut({2}) = ∅, neut({1, 2}) = {1} and anti(A) = A for A ∈ P(X) and for "⋆". Then (P(X), ∪,∩) is a neutrosophic triplet field, since for neut(A) = A, anti(A) = A for "∪, ∩". Now, we show that (P(X), ⋆, ∩) is a neutrosophic triplet vector space on (P(X), ∪,∩) neutrosophic triplet field.

1), 2) and 3) are clear.
4) (A∪B)∩C = (A∩C)∪(B∩C), for all A, B, C ∈ P(X).
5) (A∩B)∩C = A∩(B∩C)for all A, B, C ∈ P(X).
6) For all A ∈ P(X); such that A∩ neut(X) = neut(X)∩ A = A, there exists neut(X) = X ∈ P(X).

Thus; (P(X), ⋆, ∩) is a neutrosophic triplet vector space on (P(X), ∪, ∩) neutrosophic triplet field.

Theorem 5.4: Let(NTV, ⋆,#) be a neutrosophic triplet vector space on a neutrosophic triplet field. If (NTV, ⋆,#) is satisfies the following condition, (NTV, ⋆,#) is also a neutrosophic triplet field.

(i) a#b ∈ NTV; for all a, b ∈ NTV;
(ii) a#(b#c) = (a#b)#c; for all a,b,c ∈ NTV;
(iii) a#(b⋆c) = (a#b)⋆(a#c) and (b⋆c)#a = (b#a)⋆(c#a); for all a,b,c ∈ NTV;

Proof: As(NTV, ⋆,#) is a neutrosophic triplet vector space; (NTV, ⋆) is a commutative neutrosophic triplet group. From 1), 2), (NTV, #) is a neutrosophic triplet group and from 3) (NTV, ⋆,#) is satisfies the condition ofneutrosophic triplet field. Thus; (NTV, ⋆,#) is a neutrosophic triplet field.

Corollary 5.5: Let(NTV, ⋆,#) be a neutrosophic triplet vector space on a neutrosophic triplet field. If (NTV, ⋆,#) satisfies following conditions, then (NTV, ⋆,#) is a neutrosophic triplet vector space on itself.

(a) a#b ∈ NTV; for all a, b ∈ NTV
(b) a#(b#c) = (a#b)#c; for a,b,c ∈ NTV
(c) a#(b⋆c) = (a#b)⋆(a#c) and (b⋆c)#a = (b#a)⋆(c#a); for all a,b,c ∈ NTV

Proof: From the Theorem 5.4; (NTV, ⋆,#) is a neutrosophic triplet field. Thus; we suppose that

NTV = NTF, $\star_1$ = $\star_2$ = ⋆ and $\#_1$ = $\#_2$ = #.

Now, we show that (NTV, ⋆,#) satisfies the condition in definition 5.1 (definition of neutrosophic triplet vector space). As NTV = NTF and from the condition a); we have a⋆b ∈ NTV and a#b ∈ NTV; for a,b ∈ NTV. From the definition of NTV; NTV satisfies condition 1) and 2) in definition 5.1. As $\star_1$ = $\star_2$ = ⋆ and $\#_1$ = $\#_2$ = # and from the condition c), NTV satisfies condition 3),4) and 5). As NTV = NTF, $\star_1$ = $\star_2$ = ⋆ and $\#_1$ = $\#_2$ = #, 6) is clear. Thus; as a), b) and c) are satisfied; then (NTV, ⋆,#) is a neutrosophic triplet vector space on itself.

Definition 5.6: Let(NTV,$\star_2$, $\#_2$) be a neutrosophic triplet vector space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field and S⊂NTV. If (S,$\star_2$, $\#_2$) is a neutrosophic triplet vector space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field, (S,$\star_2$, $\#_2$) is called the neutrosophic triplet subvector space of (NTV,$\star_2$, $\#_2$).

Example 5.7: From the example 5.3, as (P(X),⋆,∩) is a neutrosophic triplet vector space on (P(X),∪,∩) neutrosophic triplet field, where A⋆B = A∪B, neut(∅) = neut({1})

= neut({2}) = ∅, neut({1, 2}) = {1} and anti(A) = A for A ∈ P(X) and for "*". For any S⊂P(X); (S, *, ∩) is a neutrosophic triplet subvector space of (P(X), *, ∩).

Theorem 5.8: Let(NTV,$*_2$, $\#_2$) be a neutrosophic triplet vector space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field and (S,$*_2$, $\#_2$) be a neutrosophic triplet subset of (NTV,$*_2$, $\#_2$). (S,$*_2$, $\#_2$) is neutrosophic triplet subvector space of (NTV,$*_2$, $\#_2$) if and only if

(a) $a *_2 b \in S$; for all a,b ∈ S,
(b) $a \#_2 c \in S$; for all a ∈ S and c ∈ NTF,

Proof: As (S,$*_2$, $\#_2$) is a neutrosophic triplet subvector space; a) and b) are clear. On the contrary if a) and b) are satisfied; then from the definition neutrosophic triplet vector space and as (S,$*_2$, $\#_2$) is a neutrosophic triplet subset of (NTV,$*_2$, $\#_2$), (S,$*_2$, $\#_2$) satisfies the condition of a neutrosophic triplet vector space.

6 Neutrosophic triplet normed space

Now let's define the neutrosophic triplet normed spaces on the neutrosophic triplet vector space in chapter 5, which have much more properties than neutrosophic triplet metric spaces. Thus, we will obtain a more specific structure that is wider than the neutrosophic triplet metric space in chapter 4.

Definition 6.1: Let (NTV,$*_2$, $\#_2$) be a neutrosophic triplet vector space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field. If $\|.\|$:NTV → $\mathbb{R}^+ \cup \{0\}$ function satisfies following condition; $\|.\|$ is called neutrosophic triplet normed on (NTV,$*_2$, $\#_2$).

Where; f: NTF X NTV → $\mathbb{R}^+ \cup \{0\}$, f(α,x) = f(anti(α), anti(x)) is a function and for every x,y ∈ NTV and α ∈ NTF;

(a) $\|x\| \geq 0$;
(b) If x = neut(x), then $\|x\| = 0$
(c) $\|\alpha \#_2 x\| = f(\alpha,x).\|x\|$
(d) $\|anti(x)\| = \|x\|$
(e) If $\|x *_2 y\| \leq \|x *_2 y *_2 neut(k)\|$; then $\|x *_2 y *_2 neut(k)\| \leq \|x\| + \|y\|$, for any k ∈ NTV.

Furthermore on (NTV,$*_2$,$\#_2$), the neutrosophic triplet vector space defined by $\|.\|$ is called a neutrosophic triplet normed space and is denoted by ((NTV,$*_2$,$\#_2$), $\|.\|$).

Example 6.2: From example 5.3; (P(X), *, ∩) is a neutrosophic triplet vector space on

(P(X), ∪,∩) neutrosophic triplet field. Then taking

f: P(X) X P(X) → $\mathbb{R}^+ \cup \{0\}$, f(A,B) = s(A∩B)/s(B), $\|.\|$:P(X) → $\mathbb{R}^+ \cup \{0\}$. Now we show that;$\|A\|$ = s(A) is a neutrosophic triplet norm and ((P(X),*, ∩), $\|.\|$) is a neutrosophic triplet normed space. Where; s(A) is number of elements in A ∈ P(X)and neut(A) = ∅ , anti(A) = A for "*".

(a) $\|A\|$ = s(A) ≥ 0.
(b) If A = neut(A) = ∅, then $\|x\| = \|\emptyset\| = 0$.
(c) $\|A \cap B\|$ = s(A∩B) = [s(A ∩B)/s(B)]. s(B) = f(A,B).$\|B\|$.
(d) As, A = anti(A), it is clear that $\|anti(A)\| = \|A\|$.
(e) As A*B = A∪B we can take $\|A*B\|$ = s(A*B) = s((A*B)*∅) = $\|A*B*\emptyset\|$ = $\|A*B*neut(C)\|$ for any C ∈ P(X). So; $\|A*B\| = \|A*B*neut(C)\| \leq \|A\|+\|B\|$, for any C ∈ NTV.

It is clear by definition 6.1 that neutrosophic triplet normed spaces are generally different from classical normed spaces, since for there is not any "f" function in classical normed space. However; if certain conditions are met; every classical normed space can be a neutrosophic triplet normed space at the same time.

Theorem 6.3: Let ((V, +, .), $\|.\|$) be a normed space on any F field. If we take f(α,x) = |α|; then

$\|.\|$ Norm provides the neutrosophic triplet norm condition.

Proof: As ((V, +, .),$\|.\|$) is a normed space; anti(α) = -α; for α ∈ F, we have

f(α, x) = f(anti(α), anti(x)).

(a) From the definition of norm, we have $\|x\| \geq 0$, for ∀x ∈ V;
(b) From the definition of a vector space; we have neut(x) = 0, for ∀x ∈ V; thus; if x = neut(x); then $\|x\|$ = 0.
(c) As f(α) = |α|; we have $\|\alpha.x\| = |\alpha|.\|x\| = f(\alpha).\ \|x\|$.
(d) From the definition of vector space; we have anti(x) = -x, for ∀x ∈ V; thus; $\|anti(x)\| = \|-x\| = \|-1.x\| = |1|.\|x\| = \|x\|$.
(e) From the definition of vector space; we have neut(x) = 0, for ∀x ∈ V; thus; we have $\|x + y\| = \|x + y + neut(k)\| = \|x + y\| \leq \|x\| + \|y\|$.

Proposition 6.4: Let ((NTV, $*_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field. Then, the function d: NTVxNTV→ $\mathbb{R}$ defined by

d(x,y) = $\|x *_2 anti(y)\|$ provides neutrosophic triplet metric space conditions.

Proof: Let x,y,z ∈ NTV. From the definition of neutrosophic triplet norm;

(1) d(x,y) = $\|x *_2 anti(y)\|) \geq 0$;
(2) If x = y then; d(x,y) = $\|x *_2 anti(y)\| = \|y *_2 anti(y)\| = \|neut(y)\| = 0$; Thus; we have d(x,y) = 0
(3) As $\|anti(x)\| = \|x\|$; we have d(x,y) = $\|x *_2 anti(y)\|$ = $\|anti(x *_2 anti(y))\|$. From the theorem 2.5 and

theorem 3.3; we have d(x,y) = $\|$anti(x$\star_2$ anti (y))$\|$ = $\|$anti(x)$\star_2$ anti(anti (y))$\|$ = $\|$anti(x)$\star_2$ y$\|$. As NTV is a commutative group with respect to"$\star_2$"; we have d(x,y) = $\|$anti(x)$\star_2$ y$\|$ = $\|y\star_2$ anti(x)$\|$ = d(y,x).

(4) For any k $\in$ NTV ; suppose that d(x,z) = $\|$x$\star_2$ anti(z)$\|$ ≤ $\|$x$\star_2$ anti(z)$\star_2$ $neut(k)\|$; then $\|$x$\star_2$ anti(z)$\|$ ≤ $\|$x$\star_2$ anti(z)$\star_2$ $neut(k)\|$ = $\|$x$\star_2$ anti(z)$\star_2 k\star_2$ $anti(k)\|$. As NTV is a commutative group with respect to "$\star_2$"; we have $\|$x$\star_2$ anti(z)$\star_2 k\star_2$ $anti(k)\|$ = $\|$(x$\star_2$ $anti$ (k))$\star_2$(anti (z) $\star_2$ k)$\|$ ≤ $\|$x$\star_2$ $anti(k)\|$ + $\|$k$\star_2$ $anti$ (z))$\|$. Thus; if d(x,z) ≤ d(x,z$\star_2$ neut(k)); then d(x,z$\star_2$ neut(k)) ≤ d(x,k)+d(k,z)

Corollary 6.5: Every neutrosophic triplet normed space is a neutrosophic triplet metric space. But the opposite is not always true.

Definition 6.6: Let ((NTV, $\star_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field. d: NTVx NTV$\rightarrow$ $\mathbb{R}$ neutrosophic triplet metric define by

d(x,y) = $\|$ x$\star_2$ anti(y)$\|$

is called the neutrosophic triplet normed spacereduced by (NTV, $\star_2$, $\#_2$).

Now let's define the convergence of a sequence and a Cauchy sequence in the neutrosophic triplet normed space with respect to neutrosophic triplet metric which is reduced by neutrosophic triplet normed space.

Definition 6.7: Let ((NTV,$\star_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ be a sequence in this space and d be a neutrosophic triplet metric reduced by ((NTV,$\star_2$, $\#_2$), $\|.\|$). For all ε > 0, x $\in$ NTV such that for all n ≥ M

d(x,$\{x_n\}$) = $\|$x$\star_2$ anti(x_n})$\|$ < ε

if there exists a M $\in$ $\mathbb{N}$; $\{x_n\}$ sequence converges to x. It is denoted by

$\lim_{n\to\infty} x_n$ = x or $x_n\rightarrow$ x

Definition 6.8: Let ((NTV,$\star_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ be a sequence in this space and d be a neutrosophic triplet metric reduced by ((NTV,$\star_2$, $\#_2$), $\|.\|$). For all ε > 0 such that for all n,m ≥ M

d(x_m,$\{x_n\}$) = $\|x_m\star_2$ anti($\{x_n\}$)$\|$ < ε

if there exists a M $\in$ $\mathbb{N}$; $\{x_n\}$ sequence is called Cauchy sequence.

Definition 6.9: Let ((NTV,$\star_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ be a sequence in this space and "d" be a neutrosophic triplet metric reduced by ((NTV,$\star_2$, $\#_2$), $\|.\|$). If each $\{x_n\}$ cauchy sequence in this space is convergent to d reduced neutrosophic triplet metric; ((NTV,$\star_2$, $\#_2$), $\|.\|$) is called neutrosophic triplet Banach space.

Theorem 6.10: Let ((NTV,$\star_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ and $\{y_n\}$ be sequences in ((NTV,$\star_2$, $\#_2$), $\|.\|$) such that $\{x_n\}\rightarrow$ x $\in$ NTV and $\{y_n\}\rightarrow$ y $\in$ NTV.

If $\|$x$\|$≤$\|$x$\star_2$ neut(y)$\|$ and $\|$y$\|$ ≤ $\|$y$\star_2$ neut(x)$\|$; then

(i) $|\|x\| - \|y\||$ ≤ $\|x\star_2$ $anti(y)\|$
(ii) $\lim_{n\to\infty}\|x_n\| = \|x\|$
(iii) $\lim_{n\to\infty}(x_n+ y_n)$ = x+y

Proof:

(i) As $\|$x$\|$ ≤ $\|$x$\star_2$ neut(y)$\|$; we have $\|$x$\|$ ≤ $\|$x$\star_2$ neut(y)$\|$ = $\|$x$\star_2$ anti(y)$\star_2 y\|$. From the definition of neutrosophic triplet norm;
$\|$x$\|$ ≤ $\|$x$\star_2$ neut(y)$\|$ = $\|$x$\star_2$ anti(y)$\star_2 y\|$ = $\|$x$\star_2$ anti(y)$\star_2$ $y\star_2$ neut(y)$\|$ ≤ $\|$x$\star_2$ anti(y)$\|$+$\|$y$\|$. Thus;

$$\|x\| - \|y\| \le \|x\star_2 \text{ anti}(y)\| \quad (1)$$

As $\|$y$\|$ ≤ $\|$y$\star_2$ neut(x)$\|$; we have $\|$y$\|$ ≤ $\|$y$\star_2$ neut(x)$\|$ = $\|$y$\star_2$ anti(x)$\star_2$ $x\|$. From the definition of neutrosophic triplet norm;
$\|$y$\|$ ≤ $\|$y$\star_2$ neut(x)$\|$ = $\|$y$\star_2$ anti(x)$\star_2$ $x\|$ = $\|$y$\star_2$ anti(x)$\star_2$ $x\star_2$ neut(x)$\|$ ≤ $\|$y$\star_2$ anti(x)$\|$+$\|$x$\|$. Thus;

$$\|y\| - \|x\| \le \|x\star_2 \text{ anti}(y)\| \quad (2)$$

Using (1) and (2); we have $|\|x\| - \|y\||$ ≤ $\|x\star_2$ $anti(y)\|$.

(ii) As $\{x_n\}$ → x and from the condition i); we have $|\|x\| - \|x_n\||$ ≤ $\|x\star_2$ $anti(x_n)\|$ < ε. Thus; $\lim_{n\to\infty}\|x_n\| = \|x\|$.

(iii) As $\{x_n\}\rightarrow$ x, $\{y_n\}\rightarrow$ y, $\|$x$\|$ ≤ $\|$x$\star_2$ neut(y)$\|$ and $\|$y$\|$ ≤ $\|$y$\star_2$ neut(x)$\|$; we have $\|(x_n\star_2$ $y_n)\star_2$ anti(x$\star_2$ y)$\|$ = $\|$(x$\star_2$ anti(x_n)$\star_2$ y$\star_2$ anti(y_n)$\|$ ≤ $\|$(x$\star_2$ anti(x_n) $\|$+$\|$ y$\star_2$ anti(y_n)$\|$ < ε. Thus; $\lim_{n\to\infty}(x_n+ y_n)$ = x+y.

Theorem 6.11: Let ((NTV,$\star_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$\star_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ and $\{y_n\}$ be sequences in ((NTV,$\star_2$, $\#_2$), $\|.\|$) such that $\{x_n\}$ → x $\in$ NTV. If $\lim_{n\to\infty}\|x_n\star_2$ $anti(y_n)\|$ = 0 and $\|$x$\star_2$ $anti(y_n)\|$ ≤ $\|$x$\star_2$ $anti(y_n)\star_2$ $neut(x_n)\|$; then $\lim_{n\to\infty} y_n$ = x;

Proof: As $\|$x$\star_2$ $anti(y_n)\|$ ≤ $\|$x$\star_2$ $anti(y_n)\star_2$ $neut(x_n)\|$; we have $\|$x$\star_2$ $anti(y_n)\|$ ≤ $\|$x$\star_2$ $anti(y_n)\star_2$ $neut(x_n)\|$ = $\|$x$\star_2$ $anti(y_n)\star_2$ $x_n\star_2$ $anti(x_n)\|$ = $\|$(x$\star_2$ x_n)$\star_2$($anti(y_n)\star_2$ $anti(x_n)$)$\|$ ≤ $\|x\star_2$ $anti$ $(x_n)\|$ + $\|x_n\star_2$ $anti$ $(y_n)\|$. Thus; as $\lim_{n\to\infty}\|x_n\star_2$ $anti(y_n)\|$ = 0 and $\{x_n\}$ → x; we have $\|$x$\star_2$ $anti(y_n)\|$ ≤ ε and $\lim_{n\to\infty} y_n$ = x.

7 Conclusion

In this paper; we give new properties for neutrosophic triplet groups and we introduced neutrosophic triplet metric space, neutrosophic triplet vector space and neutrosophic triplet normed space. We also show that this neutrosophic triplet notions different from the classical notions. Theseneutrosophic triplet notions have several extraordinary properties compared to the classical notions. We also studied some interesting properties of this newly born structure. We give rise to a new field or research called neutrosophic triplet structures.

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