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Neutrosophic Vague Generalized Pre-Closed Sets in Neutrosophic Vague Topological Spaces

Research Article

A.Mary Margaret[1]* and M.Trinita Pricilla[1]

1 Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.

Abstract: The aim of this paper is to introduce and develop a new class of sets namely neutrosophic vague generalized pre-closed sets in neutrosophic vague topological space. Further we have analyse the properties of neutrosophic vague generalized pre-open sets. Also some applications namely neutrosophic vague $T_{1/2}$ space, neutrosophic vague ${}_pT_{1/2}$ space and neutrosophic vague ${}_{gp}T_{1/2}$ space are introduced.

Keywords: Neutrosophic vague topological space, neutrosophic vague generalized pre-closed sets, neutrosophic vague $T_{1/2}$ space, neutrosophic vague ${}_pT_{1/2}$ space and neutrosophic vague ${}_{gp}T_{1/2}$ space.

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1. Introduction

In 1970, Levine [7] initiated the study of generalized closed sets. Zadeh [16] introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set as a mathematical tool to solve problems and vagueness in everyday life. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The theory of fuzzy topology was introduced by C.L.Chang [4] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. Atanassov [3] introduced the degree of nonmembership/falsehood (F) in 1986 and defined the intuitionistic fuzzy set as a generalization of fuzzy sets. The theory of vague sets was first proposed by Gau and Buehre [6] as an extension of fuzzy set theory in 1993. Then, Smarandache [14] introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and defined the neutrosophic set. He has coined the words neutrosophy and neutrosophic. Neutrosophic set is a generalization of fuzzy set theory and intuitionistic fuzzy sets. Shawkat Alkhazaleh [13] in 2015 introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. In this paper we introduce the concept of neutrosophic vague generalized pre-closed sets and neutrosophic vague generalized pre-open sets and their properties are obtained. Also its relationship with other existing sets are compared and discussed with examples.

2. Preliminaries

Definition 2.1 ([13]). *A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \left\{\left\langle x; \hat{T}_{A_{NV}}(x); \hat{I}_{A_{NV}}(x); \hat{F}_{A_{NV}}(x)\right\rangle; x \in X\right\}$, whose truth membership, indeterminacy membership and false membership*

* *E-mail: marymargaret.a11@gmail.com*

functions is defined as:

$$\hat{T}_{A_{NV}}(x) = \left[T^-, T^+\right], \hat{I}_{A_{NV}}(x) = \left[I^-, I^+\right], \hat{F}_{A_{NV}}(x) = \left[F^-, F^+\right]$$

where,

(1). $T^+ = 1 - F^-$

(2). $F^+ = 1 - T^-$ *and*

(3). $^-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2.2 ([13]). *Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall\, u_i \in U, \hat{T}_{A_{NV}}(u_i) \leq \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) \geq \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) \geq \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.*

Definition 2.3 ([13]). *The complement of NVS A_{NV} is denoted by A^c_{NV} and is defined by*

$$\hat{T}^c_{A_{NV}}(x) = \left[1 - T^+, 1 - T^-\right], \hat{I}^c_{A_{NV}}(x) = \left[1 - I^+, 1 - I^-\right], \hat{F}^c_{A_{NV}}(x) = \left[1 - F^+, 1 - F^-\right].$$

Definition 2.4 ([13]). *Let A_{NV} be NVS of the universe U where $\forall\, u_i \in U$, $\hat{T}_{A_{NV}}(x) = [1, 1]$; $\hat{I}_{A_{NV}}(x) = [0, 0]$; $\hat{F}_{A_{NV}}(x) = [0, 0]$. Then A_{NV} is called unit NVS, where $1 \leq i \leq n$.*

Definition 2.5 ([13]). *Let A_{NV} be NVS of the universe U where $\forall\, u_i \in U$, $\hat{T}_{A_{NV}}(x) = [0, 0]$; $\hat{I}_{A_{NV}}(x) = [1, 1]$; $\hat{F}_{A_{NV}}(x) = [1, 1]$. Then A_{NV} is called zero NVS, where $1 \leq i \leq n$.*

Definition 2.6 ([13]). *The union of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,*

$$\begin{aligned}
\hat{T}_{C_{NV}}(x) &= \left[\max\left(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}\right), \max\left(T^+_{A_{NV_x}}, T^+_{B_{NV_x}}\right)\right] \\
\hat{I}_{C_{NV}}(x) &= \left[\min\left(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}\right), \min\left(I^+_{A_{NV_x}}, I^+_{B_{NV_x}}\right)\right] \\
\hat{F}_{C_{NV}}(x) &= \left[\min\left(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}\right), \min\left(F^+_{A_{NV_x}}, F^+_{B_{NV_x}}\right)\right].
\end{aligned}$$

Definition 2.7 ([13]). *The intersection of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,*

$$\begin{aligned}
\hat{T}_{C_{NV}}(x) &= \left[\min\left(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}\right), \min\left(T^+_{A_{NV_x}}, T^+_{B_{NV_x}}\right)\right] \\
\hat{I}_{C_{NV}}(x) &= \left[\max\left(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}\right), \max\left(I^+_{A_{NV_x}}, I^+_{B_{NV_x}}\right)\right] \\
\hat{F}_{C_{NV}}(x) &= \left[\max\left(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}\right), \max\left(F^+_{A_{NV_x}}, F^+_{B_{NV_x}}\right)\right].
\end{aligned}$$

Definition 2.8 ([13]). *Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall\, u_i \in U, \hat{T}_{A_{NV}}(u_i) = \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) = \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) = \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV}, are called equal, where $1 \leq i \leq n$.*

Definition 2.9. *Let (X, τ) be topological space. A subset A of X is called:*

(1). semi closed set (SCS in short) [8] if $int\,(cl\,(A)) \subseteq A$,

(2). pre- closed set (PCS in short) [11] if $cl\left(int\left(A\right)\right) \subseteq A$,

(3). semi-pre closed set (SPCS in short) [1] if $int\left(cl\left(int\left(A\right)\right)\right) \subseteq A$,

(4). α *-closed set (* α *CS in short) [12] if* $cl\left(int\left(cl\left(A\right)\right)\right) \subseteq A$,

(5). regular closed set (RCS in short) [15] if $A = cl\left(int\left(A\right)\right)$.

Definition 2.10. *Let* (X, τ) *be topological space. A subset* A *of* X *is called:*

(1). generalized closed (briefly, g-closed) [7] if $cl\left(A\right) \subseteq U$ *, whenever* $A \subseteq U$ *and* U *is open in* X.

(2). generalized semi closed (briefly, gs-closed) [2] if $scl\left(A\right) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* X.

(3). a-generalized closed (briefly, ag-closed) [9] if $\alpha cl\left(A\right) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* X.

(4). generalized pre-closed (briefly, gp-closed) [10] if $pcl\left(A\right) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* X.

(5). generalized semi-pre closed (briefly, gsp-closed) [5] if $spcl\left(A\right) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* X.

3. Neutrosophic Vague Topological Space

In this section we introduce neutrosophic vague topology.

Definition 3.1. *A neutrosophic vague topology (NVT in short) on* X *is a family* τ *of neutrosophic vague sets (NVS in short) in* X *satisfying the following axioms:*

(1). $0_{NV}, 1_{NV} \in \tau$

(2). $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$

(3). $\cup G_i \in \tau, \forall \ \{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called neutrosophic vague topological space (NVTS in short) and any NVS in* τ *is known as neutrosophic vague open set (NVOS in short) in* X*. The complement* A^c *of NVOS in NVTS* (X, τ) *is called neutrosophic vague closed set (NVCS in short) in* X.

Definition 3.2. *Let* (X, τ) *be NVTS and* $A = \left\{\left\langle x, \left[\hat{T}_A, \hat{I}_A, \hat{F}_A\right]\right\rangle\right\}$ *be NVS in* X*. Then the neutrosophic vague interior and neutrosophic vague closure are defined by*

(1). $NVint\left(A\right) = \cup\left\{G/G \text{isaNVOSin} X \text{and} G \subseteq A\right\}$,

(2). $NVcl\left(A\right) = \cap\left\{K/K \text{isaNVCSin} X \text{and} A \subseteq K\right\}$.

Note that for any NVS A in (X, τ) , we have $NVcl\left(A^c\right) = \left(NVint\left(A\right)\right)^c$ and $NVint\left(A^c\right) = \left(NVcl\left(A\right)\right)^c$. It can be also shown that $NVcl\left(A\right)$ is NVCS and $NVint\left(A\right)$ is NVOS in X.

(1). A is NVCS in X if and only if $NVcl\left(A\right) = A$.

(2). A is NVOS in X if and only if $NVint\left(A\right) = A$.

Proposition 3.3. *Let* A *be any NVS in* X*. Then*

(1). $NVint\left(1 - A\right) = 1 - \left(NVcl\left(A\right)\right)$ *and*

(2). $NVcl\,(1-A) = 1-(NVint\,(A))$.

Proof.

(1). By definition $NVcl\,(A) = \cap\{K/K \text{isaNVCSin} X \text{and} A \subseteq K\}$.

$$\begin{aligned} 1-(NVcl\,(A)) &= 1-\cap\{K/K\text{isaNVCSin}X\text{and}A \subseteq K\} \\ &= \cup\{1-K/K\text{isaNVCSin}X\text{and}A \subseteq K\} \\ &= \cup\{G/G\text{isanNVOSin}X\text{and}G \subseteq 1-A\} \\ &= NVint\,(1-A) \end{aligned}$$

(2). The proof is similar to (1). □

Proposition 3.4. *Let (X,τ) be a NVTS and A , B be NVSs in X. Then the following properties hold:*

(a). $NVint\,(A) \subseteq A$,

(a'). $A \subseteq NVcl\,(A)$

(b). $A \subseteq B \Rightarrow NVint\,(A) \subseteq NVint\,(B)$,

(b'). $A \subseteq B \Rightarrow NVcl\,(A) \subseteq NVcl\,(B)$

(c). $NVint\,(NVint\,(A)) = NVint\,(A)$,

(c'). $NVcl\,(NVcl\,(A)) = NVcl\,(A)$

(d). $NVint\,(A \cap B) = NVint\,(A) \cap NVint\,(B)$,

(d'). $NVcl\,(A \cup B) = NVcl\,(A) \cup NVcl\,(B)$

(e). $NVint\,(1_{NV}) = 1_{NV}$,

(e'). $NVcl\,(0_{NV}) = 0_{NV}$.

Proof. (a), (b) and (e) are obvious, (c) follows from (a).

(d) From $NVint\,(A \cap B) \subseteq NVint\,(A)$ and $NVint\,(A \cap B) \subseteq NVint\,(B)$ we obtain $NVint\,(A \cap B) \subseteq NVint\,(A) \cap NVint\,(B)$.

On the other hand, from the facts $NVint\,(A) \subseteq A$ and $NVint\,(B) \subseteq B \Rightarrow NVint\,(A) \cap NVint\,(B) \subseteq A \cap B$ and $NVint\,(A) \cap NVint\,(B) \in \tau$ we see that $NVint\,(A) \cap NVint\,(B) \subseteq NVint\,(A \cap B)$, for which we obtain the required result.

(a')-(e') They can be easily deduced from (a)-(e). □

Definition 3.5. *A NVS $A = \left\{\left\langle x, \left[\hat{T}_A, \hat{I}_A, \hat{F}_A\right]\right\rangle\right\}$ in NVTS (X,τ) is said to be*

(1). Neutrosophic Vague semi closed set (NVSCS in short) if $NVint\,(NVcl\,(A)) \subseteq A$,

(2). Neutrosophic Vague semi open set (NVSOS in short) if $A \subseteq NVcl\,(NVint\,(A))$,

(3). Neutrosophic Vague pre-closed set (NVPCS in short) if $NVcl\,(NVint\,(A)) \subseteq A$,

(4). Neutrosophic Vague pre-open set (NVPOS in short) if $A \subseteq NVint\,(NVcl\,(A))$,

(5). Neutrosophic Vague α-closed set (NV α CS in short) if $NVcl\,(NVint\,(NVcl\,(A))) \subseteq A$,

(6). Neutrosophic Vague α-open set (NVαOS in short) if $A \subseteq NVint\,(NVcl\,(NVint\,(A)))$,

(7). Neutrosophic Vague semi pre- closed set (NVSPCS in short) if $NVint\,(NVcl\,(NVint\,(A))) \subseteq A$,

(8). Neutrosophic Vague semi pre-open set (NVSPOS in short) if $A \subseteq NVcl\,(NVint\,(NVcl\,(A)))$,

(9). Neutrosophic Vague regular open set (NVROS in short) if $A = NVint\,(NVcl\,(A))$,

(10). Neutrosophic Vague regular closed set (NVRCS in short) if $A = NVcl\,(NVint\,(A))$.

Definition 3.6. *Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague semi interior of A ($NV\mathrm{s}int(A)$ in short) and neutrosophic vague semi closure of A ($NV\mathrm{s}cl(A)$ in short) are defined by*

(1). $NV\mathrm{s}int(A) = \cup\{G/G \mathrm{isaNVSOSin} X \mathrm{and} G \subseteq A\}$,

(2). $NV\mathrm{s}cl(A) = \cap\{K/K \mathrm{isaNVSCSin} X \mathrm{and} A \subseteq K\}$.

Result 3.7. *Let A be NVS of a NVTS (X,τ) , then*

(1). $NV\mathrm{s}cl(A) = A \cup NVint(NVcl(A))$,

(2). $NV\mathrm{s}int(A) = A \cap NVcl(NVint(A))$.

Definition 3.8. *Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague alpha interior of A ($NV\alpha int(A)$ in short) and neutrosophic vague alpha closure of A ($NV\alpha cl(A)$ in short) are defined by*

(1). $NV\alpha int(A) = \cup\{G/G \mathrm{isaNV\alpha OSin} X \mathrm{and} G \subseteq A\}$,

(2). $NV\alpha cl(A) = \cap\{K/K \mathrm{isaNV\alpha CSin} X \mathrm{and} A \subseteq K\}$.

Result 3.9. *Let A be NVS of a NVTS (X,τ) , then*

(1). $NV\alpha cl(A) = A \cup NVcl(NVint(NVcl(A)))$,

(2). $NV\alpha int(A) = A \cap NVint(NVcl(NVint(A)))$.

Definition 3.10. *Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague semi-pre interior of A ($NV\mathrm{sp}int(A)$ in short) and neutrosophic vague semi-pre closure of A ($NV\mathrm{sp}cl(A)$ in short) are defined by*

(1). $NV\mathrm{sp}int(A) = \cup\{G/G \mathrm{isaNVSPOSin} X \mathrm{and} G \subseteq A\}$,

(2). $NV\mathrm{sp}cl(A) = \cap\{K/K \mathrm{isaNVSPCSin} X \mathrm{and} A \subseteq K\}$.

Definition 3.11. *A NVS A of a NVTS (X,τ) is said to be neutrosophic vague generalized closed set (NVGCS in short) if $NVcl(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X.*

Definition 3.12. *A NVS A of a NVTS (X,τ) is said to be neutrosophic vague generalized semi closed set (NVGSCS in short) if $NV\mathrm{s}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X.*

Definition 3.13. *A NV A of a NVTS (X,τ) is said to be neutrosophic vague alpha generalized closed set ($NV\alpha GCS$ in short) if $NV\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X.*

Definition 3.14. *A NVS A of a NVTS (X,τ) is said to be neutrosophic vague generalized semi-pre closed set (NVGSPCS in short) if $NV\mathrm{sp}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X.*

Definition 3.15. *Let (X,τ) be a NVTS and $A = \left\{\left\langle x, \left[\hat{T}_A, \hat{I}_A, \hat{F}_A\right]\right\rangle\right\}$ be a NVS in X. The neutrosophic vague pre interior of A and denoted by $NV\mathrm{pint}(A)$ is defined to be the union of all neutrosophic vague pre-open sets of X which are contained in A. The intersection of all neutrosophic vague pre-closed sets containing A is called the neutrosophic pre-closure of A and is denoted by $NV\mathrm{pcl}(A)$.*

(1). $NV\mathrm{pint}(A) = \cup\{G/G \mathrm{isaNVPOSin} X \mathrm{and} G \subseteq A\}$,

(2). $NV\mathrm{pcl}(A) = \cap\{K/K \mathrm{isaNVPCSin} X \mathrm{and} A \subseteq K\}$.

Result 3.16. *Let A be NVS of a NVTS (X,τ) , then*

(1). $NV\mathrm{pcl}(A) = A \cup NVcl(NVint(A))$,

(2). $NV\mathrm{pint}(A) = A \cap NVint(NVcl(A))$.

4. Neutrosophic Vague Generalized Pre-closed Sets

In this section we introduce neutrosophic vague generalized pre-closed set and their properties are analysed.

Definition 4.1. *A NVS A is said to be neutrosophic vague generalized pre-closed set (NVGPCS in short) in (X,τ) if $NV\mathrm{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X. The family of all NVGPCSs of a NVTS (X,τ) is denoted by NVGPC (X).*

Example 4.2. *Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a NVT on X, where $G = \left\{x, \frac{a}{\langle[0.6,0.8];[0.3,0.5];[0.2,0.4]\rangle}, \frac{b}{\langle[0.4,0.5];[0.2,0.6];[0.5,0.6]\rangle}\right\}$. Then the NVS $A = \left\{x, \frac{a}{\langle[0.6,0.7];[0.5,0.8];[0.3,0.4]\rangle}, \frac{b}{\langle[0.2,0.5];[0.4,0.6];[0.5,0.8]\rangle}\right\}$ is NVGPCS in X.*

Theorem 4.3. *Every NVCS is NVGCS but not conversely.*

Proof. Let A be NVCS in X. Suppose U is NVOS in X, such that $A \subseteq U$. Then $NVcl(A) = A \subseteq U$. Hence A is NVGCS in X. □

Example 4.4. *Let $X = \{a,b,c\}$ and let $\tau = \{0,G,1\}$ be a NVT on X, where $G = \left\{x, \frac{a}{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle}, \frac{b}{\langle[0.1,0.4];[0.8,0.9];[0.6,0.9]\rangle}, \frac{c}{\langle[0.1,0.2];[0.6,0.7];[0.8,0.9]\rangle}\right\}$. Then the NVS $A = \left\{x, \frac{a}{\langle[0.3,0.5];[0.6,0.9];[0.5,0.7]\rangle}, \frac{b}{\langle[0.2,0.3];[0.5,1];[0.7,0.8]\rangle}, \frac{c}{\langle[0.1,0.6];[0.7,0.8];[0.4,0.9]\rangle}\right\}$ is NVGCS in X but not NVCS in X. Since $NVcl(A) = \left\{x, \frac{a}{\langle[0.7,0.8];[0.3,0.5];[0.2,0.3]\rangle}, \frac{b}{\langle[0.6,0.9];[0.1,0.2];[0.1,0.4]\rangle}, \frac{c}{\langle[0.8,0.9];[0.3,0.4];[0.1,0.2]\rangle}\right\} \neq A$.*

Theorem 4.5. *Every NVCS is NV α CS but not conversely.*

Proof. Let A be NVCS in X. Since $NVint(A) \subseteq A$, and $NVcl(A) = A$, which implies $NVint(NVcl(A)) \subseteq NVcl(A)$, so $NVcl(NVint(NVcl(A))) \subseteq A$. Hence A is NV α CS in X. □

Example 4.6. *Let $X = \{a,b,c\}$ and let $\tau = \{0,G_1,G_2,1\}$ be NVT on X , where $G_1 = \left\{x, \frac{a}{\langle[0.2,0.5];[0.6,0.9];[0.5,0.8]\rangle}, \frac{b}{\langle[0.1,0.2];[0.1,0.3];[0.8,0.9]\rangle}, \frac{c}{\langle[0.3,0.4];[0.5,0.7];[0.6,0.7]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.6,0.8];[0.2,0.3];[0.2,0.4]\rangle}, \frac{b}{\langle[0.5,0.7];[0.1,0.2];[0.3,0.5]\rangle}, \frac{c}{\langle[0.7,0.9];[0.4,0.6];[0.1,0.3]\rangle}\right\}$. Then the NVS $A = \left\{x, \frac{a}{\langle[0.1,0.4];[0.3,0.4];[0.6,0.9]\rangle}, \frac{b}{\langle[0.7,0.9];[0.8,0.9];[0.1,0.3]\rangle}, \frac{c}{\langle[0.1,0.2];[0.3,0.6];[0.8,0.9]\rangle}\right\}$ is NV α CS in X but not NVCS in X. Since $NVcl(A) = \left\{x, \frac{a}{\langle[0.5,0.8];[0.1,0.4];[0.2,0.5]\rangle}, \frac{b}{\langle[0.8,0.9];[0.7,0.9];[0.1,0.2]\rangle}, \frac{c}{\langle[0.6,0.7];[0.3,0.5];[0.3,0.4]\rangle}\right\} \neq A$.*

Theorem 4.7. *Every NVCS is NVPCS but not conversely.*

Proof. Suppose A is NVCS in X. Since $NVint(A) \subseteq A$, $NVcl(NVint(A)) \subseteq NVcl(A) = A$, which implies $NVcl(NVint(A)) \subseteq A$. Thus A is NVPCS in X. □

Example 4.8. *Let $X = \{a,b\}$ and let $\tau = \{0,G_1,G_2,1\}$ be NVT on X , where $G_1 = \left\{x, \frac{a}{\langle[0.5,0.8];[0.2,0.3];[0.2,0.5]\rangle}, \frac{b}{\langle[0.6,0.9];[0.3,0.7];[0.1,0.4]\rangle}\right\}$ and $G_2 = \left\{x, \frac{a}{\langle[0.4,0.7];[0.8,0.9];[0.3,0.6]\rangle}, \frac{b}{\langle[0.2,0.5];[0.6,0.8];[0.5,0.8]\rangle}\right\}$. Then the NVS $A = \left\{x, \frac{a}{\langle[0.2,0.6];[0.7,0.9];[0.4,0.8]\rangle}, \frac{b}{\langle[0.3,0.8];[0.5,0.7];[0.2,0.7]\rangle}\right\}$ is NVPCS in X but not NVCS in X.*

Theorem 4.9. *Every NV α CS is NVPCS but not conversely.*

Proof. Assume that A is NV α CS in X. Then $NVcl\,(NVint\,(NVcl\,(A))) \subseteq A$. Since $A \subseteq NVcl\,(A)$, which implies $NVcl\,(NVint\,(A)) \subseteq A$. Hence A is NVPCS in X. □

Example 4.10. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X *, where* $G_1 = \left\{x, \frac{a}{\langle[0.7,0.9];[0.3,0.5];[0.1,0.3]\rangle}, \frac{b}{\langle[0.6,0.8];[0.1,0.3];[0.2,0.4]\rangle}, \frac{c}{\langle[0.8,1];[0.2,0.6];[0,0.2]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.5,0.7];[0.4,0.8];[0.3,0.5]\rangle}, \frac{b}{\langle[0.4,0.6];[0.3,0.7];[0.4,0.6]\rangle}, \frac{c}{\langle[0.2,0.4];[0.8,0.9];[0.6,0.8]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.3,0.6];[0.1,0.2];[0.4,0.7]\rangle}, \frac{b}{\langle[0.2,0.8];[0.7,0.9];[0.2,0.8]\rangle}, \frac{c}{\langle[0.4,0.7];[0.1,0.3];[0.3,0.6]\rangle}\right\}$ *is NVPCS in* X *but not NV* α *CS in* X. *Since* $NVcl\,(NVint\,(NVcl\,(A))) = 1 \not\subset A$.

Theorem 4.11. *Every NVRCS is NVCS but not conversely.*

Proof. Let A be NVRCS in X. Then $A = NVcl\,(NVint\,(A)) \Rightarrow NVcl\,(A) = NVcl\,(NVint\,(A))$. Therefore $NVcl\,(A) = A$. Hence, A is NVCS in X. □

Example 4.12. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *is NVT on* X *, where* $G = \left\{x, \frac{a}{\langle[0.4,0.8];[0.8,0.9];[0.2,0.6]\rangle}, \frac{b}{\langle[0.2,0.4];[0.6,0.7];[0.6,0.8]\rangle}, \frac{c}{\langle[0.1,0.3];[0.6,0.9];[0.7,0.9]\rangle}\right\}$. *The NVS* $A = \left\{x, \frac{a}{\langle[0.2,0.6];[0.1,0.2];[0.4,0.8]\rangle}, \frac{b}{\langle[0.6,0.8];[0.3,0.4];[0.2,0.4]\rangle}, \frac{c}{\langle[0.7,0.9];[0.1,0.4];[0.1,0.3]\rangle}\right\}$ *is a NVCS in* X *but not NVRCS in* X.

Theorem 4.13. *Every NV* α *CS is NVSCS but not conversely.*

Proof. Let A be NV α CS in X. Then $NVcl\,(NVint\,(Nvcl\,(A))) \subseteq A$. Since $A \subseteq NVcl\,(A)$, so $NVint\,(NVcl\,(A)) \subseteq A$. Hence, A is neutrosophic vague semi closed set in X. □

Example 4.14. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X*, where* $G_1 = \left\{x, \frac{a}{\langle[0.3,0.6];[0.8,0.9];[0.4,0.7]\rangle}, \frac{b}{\langle[0.2,0.4];[0.6,0.9];[0.6,0.8]\rangle}, \frac{c}{\langle[0.1,0.5];[0.7,0.8];[0.5,0.9]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.5,0.7];[0.1,0.2];[0.3,0.5]\rangle}, \frac{b}{\langle[0.7,0.9];[0.2,0.5];[0.1,0.3]\rangle}, \frac{c}{\langle[0.6,0.8];[0.3,0.4];[0.2,0.4]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.4,0.7];[0.8,0.9];[0.3,0.6]\rangle}, \frac{b}{\langle[0.5,0.7];[0.1,0.9];[0.3,0.5]\rangle}, \frac{c}{\langle[0.4,0.9];[0.7,0.8];[0.1,0.6]\rangle}\right\}$ *is NVSCS in* X *but not NV* α *CS in* X.

Theorem 4.15. *Every NVPCS is NVSPCS but not conversely.*

Proof. Let A be NVPCS in X. By hypothesis $NVcl\,(NVint\,(A)) \subseteq A$. Therefore $NVint\,(NVcl\,(NVint\,(A))) \subseteq NVint\,(A) \subseteq A$. Therefore $NVint\,(NVcl\,(NVint\,(A))) \subseteq A$. Hence A is NVSPCS in X. □

Example 4.16. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X*, where* $G_1 = \left\{x, \frac{a}{\langle[0.9,1];[0.2,0.3];[0,0.1]\rangle}, \frac{b}{\langle[0.7,0.9];[0.4,0.5];[0.1,0.3]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.1,0.5];[0.7,0.9];[0.5,0.7]\rangle}, \frac{b}{\langle[0.4,0.6];[0.8,0.9];[0.4,0.6]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.7,0.8];[0.5,0.6];[0.2,0.3]\rangle}, \frac{b}{\langle[0.6,0.8];[0.8,0.9];[0.2,0.4]\rangle}\right\}$ *is NVSPCS in* X *but not NVPCS in* X.

Theorem 4.17. *Every NVCS is NVGPCS but not conversely.*

Proof. Let A be NVCS in X and let $A \subseteq U$ and U be NVOS in X. Since $NVpcl\,(A) \subseteq NVcl\,(A)$ and A is NVCS in X, $NVpcl\,(A) \subseteq NVcl\,(A) = A \subseteq U$. Therefore A is NVGPCS in X. □

Example 4.18. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G, 1\}$ *be NVT on* X*, where* $G = \left\{x, \frac{a}{\langle[0.4,0.7];[0.6,0.8];[0.3,0.6]\rangle}, \frac{b}{\langle[0.3,0.5];[0.4,0.7];[0.5,0.7]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.2,0.6];[0.4,0.7];[0.4,0.8]\rangle}, \frac{b}{\langle[0.1,0.4];[0.3,0.8];[0.6,0.9]\rangle}\right\}$ *is NVGPCS in* X *but not NVCS in* X.

Theorem 4.19. *Every NVGCS is NVGPCS but not conversely.*

Proof. Let A be NVGCS in X and let $A \subseteq U$ and U is NVOS in (X, τ). Since $NVpcl\,(A) \subseteq NVcl\,(A)$ and by hypothesis, $NVpcl\,(A) \subseteq U$. Therefore A is NVGPCS in X. □

Example 4.20. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X *, where* $G_1 = \left\{x, \frac{a}{\langle[0.1,0.4];[0.6,0.7];[0.6,0.9]\rangle}, \frac{b}{\langle[0.2,0.5];[0.7,0.9];[0.5,0.8]\rangle}\right\}$ *and* $G_2 = \left\{x, \frac{a}{\langle[0.7,0.9];[0.2,0.6];[0.1,0.3]\rangle}, \frac{b}{\langle[0.8,0.9];[0.4,0.5];[0.1,0.2]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.8,0.9];[0.2,0.3];[0.1,0.2]\rangle}, \frac{b}{\langle[0.6,0.8];[0.1,0.2];[0.2,0.4]\rangle}\right\}$ *is NVGPCS in* X *but not NVGCS in* X *since* $NVcl(A) = 1 \not\subseteq G$.

Theorem 4.21. *Every NV* α *CS is NVGPCS but not conversely.*

Proof. Let A be NV α CS in X and let $A \subseteq U$ and U be NVOS in X. By hypothesis, $NVcl(NVint(NVcl(A))) \subseteq A$. Since $A \subseteq NVcl(A), NVcl(NVint(A)) \subseteq NVcl(NVint(NVcl(A))) \subseteq A$. Hence $NV\mathrm{pcl}(A) \subseteq A \subseteq U$. Therefore A is NVGPCS in X. □

Example 4.22. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X *, where* $G_1 = \left\{x, \frac{a}{\langle[0.6,0.9];[0.3,0.5];[0.1,0.4]\rangle}, \frac{b}{\langle[0.7,0.8];[0.2,0.4];[0.2,0.3]\rangle}\right\}$ *and* $G_2 = \left\{x, \frac{a}{\langle[0.1,0.3];[0.5,0.8];[0.7,0.9]\rangle}, \frac{b}{\langle[0.4,0.5];[0.6,0.8];[0.5,0.6]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.8,0.9];[0.1,0.4];[0.1,0.2]\rangle}, \frac{b}{\langle[0.7,0.9];[0.2,0.3];[0.1,0.3]\rangle}\right\}$ *is NVGPCS in* X *but not NV* α *CS in* X *since* $NVcl(NVint(NVcl(A))) = 1 \not\subseteq A$.

Theorem 4.23. *Every NVRCS is NVGPCS but not conversely.*

Proof. Let A be a NVRCS in X. By Definition 3.5, $A = NVcl(NVint(A))$. This implies $NVcl(A) = NVcl(NVint(A))$. Therefore $NVcl(A) = A$. That is A is NVCS in X. By Theorem 4.17, A is NVGPCS in X. □

Example 4.24. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X *, where* $G_1 = \left\{x, \frac{a}{\langle[0.1,0.4];[0.5,0.7];[0.6,0.9]\rangle}, \frac{b}{\langle[0.2,0.4];[0.6,0.8];[0.6,0.8]\rangle}\right\}$ *and* $G_2 = \left\{x, \frac{a}{\langle[0.6,0.9];[0.4,0.5];[0.1,0.4]\rangle}, \frac{b}{\langle[0.7,0.8];[0.2,0.4];[0.2,0.3]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.7,0.9];[0.3,0.5];[0.1,0.3]\rangle}, \frac{b}{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle}\right\}$ *is NVGPCS in* X *but not NVRCS in* X *since* $NVcl(NVint(A)) = \left\{x, \frac{a}{\langle[0.6,0.9];[0.3,0.5];[0.3,0.4]\rangle}, \frac{b}{\langle[0.6,0.8];[0.2,0.4];[0.2,0.4]\rangle}\right\} \neq A$.

Theorem 4.25. *Every NVPCS is NVGPCS but not conversely.*

Proof. Let A be NVPCS in X and let $A \subseteq U$ and U is NVOS in X. By Definition 3.5, $NVcl(NVint(A)) \subseteq A$. This implies $NV\mathrm{pcl}(A) = A \cup NVcl(NVint(A)) \subseteq A$. Therefore $NV\mathrm{pcl}(A) \subseteq U$. Hence A is NVGPCS in X. □

Example 4.26. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *be NVT on* X *, where* $G = \left\{x, \frac{a}{\langle[0.5,0.7];[0.3,0.6];[0.3,0.5]\rangle}, \frac{b}{\langle[0.4,0.8];[0.2,0.5];[0.2,0.6]\rangle}, \frac{c}{\langle[0.2,0.6];[0.4,0.5];[0.4,0.8]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.6,0.7];[0.2,0.5];[0.3,0.4]\rangle}, \frac{b}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle}, \frac{c}{\langle[0.5,0.8];[0.2,0.4];[0.2,0.5]\rangle}\right\}$ *is NVGPCS in* X *but not NVPCS in* X *since* $NVcl(NVint(A)) = 1 \not\subseteq A$.

Theorem 4.27. *Every NV* α *GCS is NVGPCS but not conversely.*

Proof. Let A be NV α GCS in X and let $A \subseteq U$ and U is NVOS in (X, τ). By Result 3.9, $A \cup NVcl(NVint(NVcl(A))) \subseteq U$. This implies $NVcl(NVint(NVcl(A))) \subseteq U$ and $NVcl(NVint(A)) \subseteq U$. Thus $NVpcl(A) = A \cup NVcl(NVint(A)) \subseteq U$. Hence A is NVGPCS in X. □

Example 4.28. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X, *where* $G_1 = \left\{x, \frac{a}{\langle[0.6,0.8];[0.1,0.2];[0.2,0.4]\rangle}, \frac{b}{\langle[0.7,0.9];[0.2,0.5];[0.1,0.3]\rangle}\right\}$ *and* $G_2 = \left\{x, \frac{a}{\langle[0.2,0.4];[0.5,0.6];[0.6,0.8]\rangle}, \frac{b}{\langle[0.3,0.6];[0.7,0.8];[0.4,0.7]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.8,0.9];[0.2,0.7];[0.1,0.2]\rangle}, \frac{b}{\langle[0.5,0.7];[0.3,0.4];[0.3,0.5]\rangle}\right\}$ *is NVGPCS in* X *but not NV* α *GCS in* X *since* $NV\alpha cl(A) = 1 \not\subseteq G$.

Theorem 4.29. *Every NVGPCS is NVSPCS but not conversely.*

Proof. Let A be NVGPCS in X , this implies $NV\mathrm{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X. By hypothesis $NVcl(NVint(A)) \subseteq A$. Therefore $NVint(NVcl(NVint(A))) \subseteq NVint(A) \subseteq A$. Therefore $NVint(NVcl(NVint(A))) \subseteq A$. Hence A is NVSPCS in X. □

Example 4.30. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *is NVT on* X *, where* $G = \left\{x, \frac{a}{\langle[0.1,0.3];[0.4,0.6];[0.7,0.9]\rangle}, \frac{b}{\langle[0.2,0.4];[0.7,0.9];[0.6,0.8]\rangle}, \frac{c}{\langle[0.3,0.4];[0.8,0.9];[0.6,0.7]\rangle}\right\}$. *Then the NVS* $A = G$ *is NVSPCS in* X *but not NVGPCS in* X.

Theorem 4.31. *Every NVGPCS is NVGSPCS but not conversely.*

Proof. Let A be NVGPCS in X and let $A \subseteq U$ and U is NVOS in X. By hypothesis $NVcl(NVint(A)) \subseteq A \subseteq U$. Therefore $NVint(NVcl(NVint(A))) \subseteq NVint(U) \subseteq U$. This implies $NV\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X. Therefore A is NVGSPCS in X. □

Example 4.32. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *be NVT on* X*, where* $G = \left\{x, \frac{a}{\langle[0.2,0.4];[0.6,0.8];[0.6,0.8]\rangle}, \frac{b}{\langle[0.1,0.3];[0.4,0.6];[0.7,0.9]\rangle}, \frac{c}{\langle[0.4,0.5];[0.7,0.8];[0.5,0.6]\rangle}\right\}$. *Then the NVS* $A = G$ *is NVGSPCS in* X *but not NVGPCS in* X.

Proposition 4.33. *NVSCS and NVGPCS are independent to each other.*

Example 4.34. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *be NVT on* X*, where* $G = \left\{x, \frac{a}{\langle[0.2,0.6];[0.7,0.8];[0.4,0.8]\rangle}, \frac{b}{\langle[0.1,0.4];[0.5,0.6];[0.6,0.9]\rangle}, \frac{c}{\langle[0.3,0.5];[0.7,0.8];[0.5,0.7]\rangle}\right\}$. *Then the NVS* $A = G$ *is NVSCS in* X *but not NVGPCS in* X.

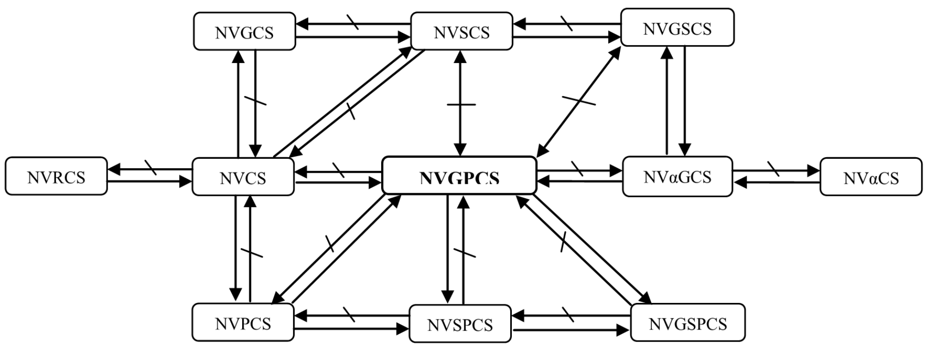
Example 4.35. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *be NVT on* X *, where* $G = \left\{x, \frac{a}{\langle[0.4,0.7];[0.1,0.2];[0.3,0.6]\rangle}, \frac{b}{\langle[0.5,0.6];[0.2,0.6];[0.4,0.5]\rangle}, \frac{c}{\langle[0.6,0.8];[0.4,0.5];[0.2,0.4]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.2,0.5];[0.6,0.7];[0.5,0.8]\rangle}, \frac{b}{\langle[0.3,0.4];[0.4,0.6];[0.6,0.7]\rangle}, \frac{c}{\langle[0.5,0.7];[0.6,0.7];[0.3,0.5]\rangle}\right\}$ *is NVGPCS in* X *but not NVSCS in* X *since* $NVint(NVcl(A)) = 1 \not\subset A$.

Proposition 4.36. *NVGSCS and NVGPCS are independent to each other.*

Example 4.37. *Let* $X = \{a, b, c\}$ *and let* $\tau = \{0, G, 1\}$ *be NVT on* X *, where* $G = \left\{x, \frac{a}{\langle[0.4,0.5];[0.1,0.2];[0.5,0.6]\rangle}, \frac{b}{\langle[0.3,0.6];[0.2,0.7];[0.4,0.7]\rangle}, \frac{c}{\langle[0.2,0.7];[0.1,0.3];[0.3,0.8]\rangle}\right\}$. *Then the NVS* $A = G$ *is NVGSCS in* X *but not NVGPCS in* X *since* $A \subseteq G$ *but* $NV\text{pcl}(A) = 1 \not\subset G$.

Example 4.38. *Let* $X = \{a, b\}$ *and let* $\tau = \{0, G_1, G_2, 1\}$ *be NVT on* X*, where* $G_1 = \left\{x, \frac{a}{\langle[0.3,0.5];[0.7,0.8];[0.5,0.7]\rangle}, \frac{b}{\langle[0.2,0.4];[0.6,0.7];[0.6,0.8]\rangle}\right\}$ *and* $G_2 = \left\{x, \frac{a}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle}, \frac{b}{\langle[0.8,0.9];[0.2,0.5];[0.1,0.2]\rangle}\right\}$. *Then the NVS* $A = \left\{x, \frac{a}{\langle[0.2,0.5];[0.8,0.9];[0.5,0.8]\rangle}, \frac{b}{\langle[0.1,0.3];[0.7,0.8];[0.7,0.9]\rangle}\right\}$ *is NVGPCS in* X *but not NVGSCS in* X.

Remark 4.39. *We have the following implications by summing up the above theorems.*



In this diagram by $A \to B$ *we mean* A *implies* B *but not conversely and* $A \nleftrightarrow B$ *means* A *and* B *are independent of each other. None of them is reversible.*

Remark 4.40. *The union of any two NVGPCSs is not NVGPCS in general as seen in the following example.*

Example 4.41. *Let* $X = \{a, b, c\}$ *and let* $G = \left\{x, \frac{a}{\langle[0.4,0.7];[0.2,0.3];[0.3,0.6]\rangle}, \frac{b}{\langle[0.5,0.6];[0.3,0.4];[0.4,0.5]\rangle}, \frac{c}{\langle[0.6,0.7];[0.3,0.5];[0.3,0.4]\rangle}\right\}$. *Then* $\tau = \{0, G, 1\}$ *is NVT on* X *and the NVSs* $A = \left\{x, \frac{a}{\langle[0.6,0.9];[0.6,0.7];[0.1,0.4]\rangle}, \frac{b}{\langle[0.3,0.4];[0.5,0.6];[0.6,0.7]\rangle}, \frac{c}{\langle[0.4,0.5];[0.6,0.7];[0.5,0.6]\rangle}\right\}$, $B = \left\{x, \frac{a}{\langle[0.7,0.8];[0.1,0.2];[0.2,0.3]\rangle}, \frac{b}{\langle[0.6,0.8];[0.3,0.4];[0.2,0.4]\rangle}, \frac{c}{\langle[0.6,0.9];[0.1,0.3];[0.1,0.4]\rangle}\right\}$ *are NVGPCSs in* X *but* $A \cup B$ *is not NVGPCS in* X.

5. Neutrosophic Vague Generalized Pre-open Set

In this section we introduce neutrosophic vague generalized pre-open set and their properties are deliberated.

Definition 5.1. *A NVS A is said to be neutrosophic vague generalized pre-open set (NVGPOS in short) in (X,τ) if the complement A^c is NVGPCS in (X,τ). The family of all NVGPOSs of NVTS (X,τ) is denoted by NVGPO (X).*

Example 5.2. *Let $X = \{a,b,c\}$ and let $\tau = \{0,G,1\}$ is NVT on X , where $G = \left\{x, \frac{a}{\langle[0.6,0.8];[0.2,0.5];[0.2,0.4]\rangle}, \frac{b}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle}, \frac{c}{\langle[0.6,0.9];[0.1,0.2];[0.1,0.4]\rangle}\right\}$. Then the NVS $A = \left\{x, \frac{a}{\langle[0.3,0.5];[0.1,0.4];[0.5,0.7]\rangle}, \frac{b}{\langle[0.6,0.7];[0.2,0.5];[0.3,0.4]\rangle}, \frac{c}{\langle[0.6,0.8];[0.5,0.6];[0.2,0.4]\rangle}\right\}$ is NVGPOS in X.*

Theorem 5.3. *For any NVTS (X,τ) , we have the following results.*

(1). Every NVOS is NVGPOS but not conversely.

(2). Every NVROS is NVGPOS but not conversely.

(3). Every NV α OS is NVGPOS but not conversely.

(4). Every NVPOS is NVGPOS but not conversely.

The converse of the above theorem need not be true which can be seen from the following examples.

Example 5.4. *Let $X = \{a,b\}$ and $G_1 = \left\{x, \frac{a}{\langle[0.2,0.5];[0.6,0.7];[0.5,0.8]\rangle}, \frac{b}{\langle[0.3,0.6];[0.5,0.7];[0.4,0.7]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.6,0.7];[0.2,0.4];[0.3,0.4]\rangle}, \frac{b}{\langle[0.8,0.9];[0.4,0.5];[0.1,0.2]\rangle}\right\}$. Then $\tau = \{0,G_1,G_2,1\}$ is NVT on X. The NVS $A = \left\{x, \frac{a}{\langle[0.4,0.7];[0.5,0.6];[0.3,0.6]\rangle}, \frac{b}{\langle[0.3,0.6];[0.4,0.5];[0.4,0.7]\rangle}\right\}$ is NVGPOS in X but not NVOS in X.*

Example 5.5. *Let $X = \{a,b\}$ and $G_1 = \left\{x, \frac{a}{\langle[0.7,0.9];[0.1,0.2];[0.1,0.3]\rangle}, \frac{b}{\langle[0.8,0.9];[0.2,0.5];[0.1,0.2]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle}, \frac{b}{\langle[0.4,0.6];[0.2,0.4];[0.4,0.6]\rangle}\right\}$. Then $\tau = \{0,G_1,G_2,1\}$ is NVT on X. The NVS $A = \left\{x, \frac{a}{\langle[0.5,0.6];[0.4,0.7];[0.5,0.8]\rangle}, \frac{b}{\langle[0.2,0.5];[0.5,0.8];[0.5,0.8]\rangle}\right\}$ is NVPSOS in X but not NVROS in X.*

Example 5.6. *Let $X = \{a,b,c\}$ and $G_1 = \left\{x, \frac{a}{\langle[0.2,0.4];[0.7,0.8];[0.6,0.8]\rangle}, \frac{b}{\langle[0.3,0.5];[0.6,0.9];[0.5,0.7]\rangle}, \frac{c}{\langle[0.4,0.5];[0.6,0.7];[0.5,0.6]\rangle}\right\}$, $G_2 = \left\{x, \frac{a}{\langle[0.6,0.8];[0.2,0.5];[0.2,0.4]\rangle}, \frac{b}{\langle[0.7,0.9];[0.1,0.4];[0.1,0.3]\rangle}, \frac{c}{\langle[0.7,0.8];[0.5,0.6];[0.2,0.3]\rangle}\right\}$. Then $\tau = \{0,G_1,G_2,1\}$ is a NVT on X. The NVS $A = \left\{x, \frac{a}{\langle[0.4,0.5];[0.1,0.2];[0.3,0.5]\rangle}, \frac{b}{\langle[0.3,0.6];[0.5,0.6];[0.4,0.7]\rangle}, \frac{c}{\langle[0.4,0.5];[0.2,0.3];[0.5,0.6]\rangle}\right\}$ is NVGPOS in X but not NV α OS in X.*

Example 5.7. *Let $X = \{a,b,c\}$ and $G = \left\{x, \frac{a}{\langle[0.7,0.8];[0.2,0.5];[0.2,0.3]\rangle}, \frac{b}{\langle[0.6,0.8];[0.1,0.4];[0.2,0.4]\rangle}, \frac{c}{\langle[0.7,0.9];[0.2,0.3];[0.1,0.3]\rangle}\right\}$. Then $\tau = \{0,G,1\}$ is NVT on X. The NVS $A = \left\{x, \frac{a}{\langle[0.1,0.2];[0.6,0.8];[0.8,0.9]\rangle}, \frac{b}{\langle[0.2,0.3];[0.8,0.9];[0.7,0.8]\rangle}, \frac{c}{\langle[0.1,0.2];[0.7,0.9];[0.8,0.9]\rangle}\right\}$ is NVGPOS in X but not NVPOS in X.*

Remark 5.8. *The intersection of any two NVGPOSs is not NVGPOS in general and it is shown in the following example.*

Example 5.9. *Let $X = \{a,b,c\}$ and let $G = \left\{x, \frac{a}{\langle[0.3,0.5];[0.6,0.7];[0.5,0.7]\rangle}, \frac{b}{\langle[0.4,0.5];[0.6,0.8];[0.5,0.6]\rangle}, \frac{c}{\langle[0.2,0.4];[0.7,0.9];[0.6,0.8]\rangle}\right\}$. Then $\tau = \{0,G,1\}$ is NVT on X and the NVSs $A = \left\{x, \frac{a}{\langle[0.4,0.6];[0.5,0.7];[0.4,0.6]\rangle}, \frac{b}{\langle[0.4,0.7];[0.1,0.4];[0.3,0.6]\rangle}, \frac{c}{\langle[0.5,0.9];[0.2,0.6];[0.1,0.5]\rangle}\right\}$, $B = \left\{x, \frac{a}{\langle[0.4,0.7];[0.4,0.8];[0.3,0.6]\rangle}, \frac{b}{\langle[0.3,0.5];[0.4,0.5];[0.5,0.7]\rangle}, \frac{c}{\langle[0.2,0.6];[0.3,0.6];[0.4,0.8]\rangle}\right\}$ are NVGPOSs in X but $A \cap B$ is not NVGPOS in X.*

Theorem 5.10. *Let (X,τ) be NVTS. If $A \in \mathrm{NVGPO}(X)$ then $V \subseteq NVint(NVcl(A))$ whenever $V \subseteq A$ and V is NVCS in X.*

Proof. Let $A \in \text{NVGPO}(X)$. Then A^c is NVGPCS in X. Therefore $NV\text{pcl}(A^c) \subseteq U$ whenever $A^c \subseteq U$ and U is NVOS in X. That is $NVcl(NVint(A^c)) \subseteq U$. This implies $U^c \subseteq NVint(NVcl(A))$ whenever $U^c \subseteq A$ and U^c is NVCS in X. Replacing U^c by V, we get $V \subseteq NVint(NVcl(A))$ whenever $V \subseteq A$ and V is NVCS in X. □

Theorem 5.11. *Let (X,τ) be NVTS. Then for every $A \in \text{NVGPO}(X)$ and for every $B \in \text{NVS}(X)$, $NV\text{p}int(A) \subseteq B \subseteq A$ implies $B \in \text{NVGPO}(X)$.*

Proof. By hypothesis $A^c \subseteq B^c \subseteq (NV\text{p}int(A))^c$. Let $B^c \subseteq U$ and U be NVOS. Since $A^c \subseteq B^c$, $A^c \subseteq U$. But A^c is NVGPCS, $NV\text{pcl}(A^c) \subseteq U$. Also $B^c \subseteq (NV\text{p}int(A))^c = NV\text{pcl}(A^c)$. Therefore $NV\text{pcl}(B^c) \subseteq NV\text{pcl}(A^c) \subseteq U$. Hence B^c is NVGPCS. Which implies B is NVGPOS of X. □

Theorem 5.12. *A NVS A of NVTS (X,τ) is NVGPOS if and only if $F \subseteq NV\text{p}int(A)$ whenever F is NVCS and $F \subseteq A$.*

Proof. Necessity: Suppose A is NVGPOS in X. Let F be NVCS and $F \subseteq A$. Then F^c is NVOS in X such that $A^c \subseteq F^c$. Since A^c is NVGPCS, we have $NV\text{pcl}(A^c) \subseteq F^c$. Hence $(NV\text{p}int(A))^c \subseteq F^c$. Therefore $F \subseteq NV\text{p}int(A)$.

Sufficiency: Let A be NVS of X and let $F \subseteq NV\text{p}int(A)$ whenever F is NVCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is NVOS. By hypothesis, $(NV\text{p}int(A))^c \subseteq F^c$. Which implies $NV\text{pcl}(A^c) \subseteq F^c$. Therefore A^c is NVGPCS of X. Hence A is NVGPOS of X. □

Corollary 5.13. *A NVS A of a NVTS (X,τ) is NVGPOS if and only if $F \subseteq NVint(NVcl(A))$ whenever F is NVCS and $F \subseteq A$.*

Proof. Necessity: Suppose A is NVGPOS in X. Let F be NVCS and $F \subseteq A$. Then F^c is NVOS in X such that $A^c \subseteq F^c$. Since A^c is NVGPCS, we have $NV\text{pcl}(A^c) \subseteq F^c$. Therefore $NVcl(NVint(A^c)) \subseteq F^c$. Hence $(NVint(NVcl(A)))^c \subseteq F^c$. This implies $F \subseteq NVint(NVcl(A))$.

Sufficiency: Let A be NVS of X and let $F \subseteq NVint(NVcl(A))$ whenever F is NVCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is NVOS. By hypothesis, $(NVint(NVcl(A)))^c \subseteq F^c$. Hence $NVcl(NVint(A^c)) \subseteq F^c$, which implies $NV\text{pcl}(A^c) \subseteq F^c$. Hence A is NVGPOS of X. □

Theorem 5.14. *For a NVS A, A is NVOS and NVGPCS in X if and only if A is NVROS in X.*

Proof. Necessity: Let A be NVOS and NVGPCS in X. Then $NV\text{pcl}(A) \subseteq A$. This implies $NVcl(NVint(A)) \subseteq A$. Since A is NVOS, it is NVPOS. Hence $A \subseteq NVint(NVcl(A))$. Therefore $A = NVint(NVcl(A))$. Hence A is NVROS in X.

Sufficiency: Let A be NVROS in X. Therefore $A = NVint(NVcl(A))$. Let $A \subseteq U$ and U is NVOS in X. This implies $NV\text{pcl}(A) \subseteq A$. Hence A is NVGPCS in X. □

6. Applications of Neutrosophic Vague Generalized Pre-closed Sets

In this section we provide some applications of neutrosophic vague generalized pre-closed sets.

Definition 6.1. *A NVTS (X,τ) is said to be neutrosophic vague $T_{1/2}$ space ($NVT_{1/2}$ in short) if every NVGCS in X is NVCS in X.*

Definition 6.2. *A NVTS (X,τ) is said to be neutrosophic vague ${}_pT_{1/2}$ space ($NV_pT_{1/2}$ in short) if every NVPCS in X is NVCS in X.*

Definition 6.3. *A NVTS (X,τ) is said to be neutrosophic vague ${}_{gp}T_{1/2}$ space (NV${}_{gp}T_{1/2}$ in short) if every NVGPCS in X is NVCS in X.*

Definition 6.4. *A NVTS (X,τ) is said to be a neutrosophic vague ${}_{gp}T_p$ space (NV ${}_{gp}T_p$ in short) if every NVGPCS in X is NVPCS in X.*

Theorem 6.5. *Every NV $T_{1/2}$ space is NV ${}_{gp}T_p$ space. But the converse is not true in general.*

Proof. Let X be NV $T_{1/2}$ space and let A be NVGCS in X, we know that every NVGCS is NVGPCS, hence A is NVGPCS in X. By hypothesis A is NVCS in X. Since every NVCS is NVPCS, A is NVPCS in X. Hence X is NV ${}_{gp}T_p$ space. □

Example 6.6. *Let $X = \{a,b\}$ and $G = \left\{x, \frac{a}{\langle[0.6,0.7];[0.1,0.3];[0.3,0.4]\rangle}, \frac{b}{\langle[0.8,0.9];[0.2,0.4];[0.1,0.2]\rangle}\right\}$. Then $\tau = \{0,G,1\}$ is NVT on X. Let $A = \left\{x, \frac{a}{\langle[0.7,0.8];[0.2,0.5];[0.2,0.3]\rangle}, \frac{b}{\langle[0.3,0.5];[0.6,0.7];[0.5,0.7]\rangle}\right\}$. Then (X,τ) is NV $_{\mathrm{gp}}\mathrm{T}_{\mathrm{p}}$ space. But it is not NV $\mathrm{T}_{1/2}$ space since A is NVGCS but not NVCS in X.*

Theorem 6.7. *Every NV ${}_{gp}T_{1/2}$ space is NV ${}_{gp}T_p$ space. But the converse is not true in general.*

Proof. Let X be NV ${}_{gp}T_{1/2}$ space and let A be NVGPCS in X. By hypothesis A is NVCS in X. Since every NVCS is NVPCS, A is NVPCS in X. Hence X is NV ${}_{gp}T_p$ space. □

Example 6.8. *Let $X = \{a,b,c\}$ and $G = \left\{x, \frac{a}{\langle[0.5,0.7];[0.2,0.4];[0.3,0.5]\rangle}, \frac{b}{\langle[0.3,0.8];[0.1,0.3];[0.2,0.7]\rangle}, \frac{c}{\langle[0.4,0.7];[0.2,0.6];[0.3,0.6]\rangle}\right\}$. Then $\tau = \{0,G,1\}$ is NVT on X. Let $A = \left\{x, \frac{a}{\langle[0.4,0.8];[0.2,0.8];[0.2,0.6]\rangle}, \frac{b}{\langle[0.2,0.5];[0.1,0.4];[0.5,0.8]\rangle}, \frac{c}{\langle[0.1,0.6];[0.2,0.5];[0.4,0.9]\rangle}\right\}$. Then X is NV $_{\mathrm{gp}}\mathrm{T}_{1/2}$ space. But it is not NV $_{\mathrm{gp}}\mathrm{T}_{\mathrm{p}}$ space since A is NVGPCS but not NVCS in X.*

Theorem 6.9. *Let (X,τ) be NVTS and X is NV ${}_{gp}T_{1/2}$ space then,*

(1). Any union of NVGPCSs is NVGPCS

(2). Any intersection of NVGPOSs is NVGPOS.

Proof.

(1). Let $\{A_i\}_{i\in J}$ is a collection of NVGPCSs in NV ${}_{gp}T_{1/2}$ space (X,τ). Therefore every NVGPCS is NVCS. But the union of NVCS is NVCS. Hence the union of NVGPCS is NVGPCS in X.

(2). It can be proved by taking complement of (1). □

Theorem 6.10. *A NVTS X is NV ${}_{gp}T_{1/2}$ space if and only if* $\mathrm{NVGPO}\,(X) = \mathrm{NVPO}\,(X)$.

Proof. Necessity: Let A be NVGPOS in X, then A^c is NVGPCS in X. By hypothesis A^c is NVGPCS in X. Therefore A is NVPOS in X. Hence $\mathrm{NVGPO}\,(X) = \mathrm{NVPO}\,(X)$.

Sufficiency: Let A be NVGPCS in X. Then A^c is NVGPOS in X. By hypothesis A^c is NVGPOS in X. Therefore A is NVPCS in X. Hence X is NV ${}_{gp}T_p$ space. □

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