Neutrosophic vague Regular Weakly closed sets in Neutrosophic vague topological spaces

Ms. D. Savithiri[1], Dr. C. Janaki[2]

1. Research Scholar, L.R.G. Government Arts College(W), Tirupur, (TN), INDIA.

2.Assistant Professor, Department of Mathematics, L.R.G. Government Arts College(W), Tirupur, (TN), INDIA. Email:

ABSTRACT: The aim of this paper is to introduce a new class of sets called neutrosophic vague rw closed sets and neutrosophic vague rw open sets, neutrosophic vague $rwT_{1/2}$ space and neutrosophic vague rwT_w in neutrosophic vague topological spaces. Further we analyse some of its properties.

KEYWORDS: Neutrosophic vague regular weakly closed set, neutrosophic vague regular weakly open set, neutrosophic vague $rwT_{1/2}$ space, neutrosophic vague rwT_w space, neutrosophic vague rw connected space.

AMS SUBJECT CLASSIFICATION: 54A05, 54A99

I. INTRODUCTION

Topology is the modern version of Geometry. It is commonly defined as the study of shapes and topological spaces. The term topology was introduced by Johann Beredict Listing in the 19th century. Atanassov introduced the degree of membership/falsehood (F) in 1986 and defined the intuitionistic fuzzy set as a generalization of fuzzy sets.

The theory of vague sets was first proposed by Gau and Buehre as an extension of fuzzy set theory in 1993. Smarandache introduced the degree of indeterminacy/neutrality and defined the neutrosophic set which is a generalization of fuzzy set theory and intuitionistic fuzzy sets.

In 2012, Salama and Alblowi used this neutrosophic set and introduced neutrosophic topological spaces. Shawkat Alkhazaleh in 2015 introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set.

II. PRELIMINARIES

Definition 2.1:[11] A neutrosophic vague set A_{NV} (NVS in short) on the universe X written as A_{NV} = $\left\{\left\langle x;\hat{T}_{A_{NV}}(x);\hat{I}_{A_{NV}}(x);\hat{F}_{A_{NV}}(x)\right\rangle;x\in X\right\}$,whose truth membership, indeterminacy membership and false

$$\hat{T}_{A_{NV}}(x)=[T^-,T^+],\hat{I}_{A_{NV}}(x)=[I^-,I^+],$$

membership functions is defined as: $\hat{F}_{A_{NV}}(x)=[F^-,F^+]where$

$$(1)T^+=1-F^-$$
$$(2)F^+=1-T^-$$
$$(3)0^-\le T^-+I^-+F^-\le 2^+$$

Definition 2.2 [11]: Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall$ $u_i \in$ U, $\hat{T}_{A_{NV}}(u_i) \le \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i)\ge \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i)\ge \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where 1≤ i ≤n.

Definition 2.3[11]: The complement of NVS A_{NV} is denoted by A_{NV}^C and is defined by

$\hat{T}^{C}_{A_{NV}}(x) = [1-T^{+}, 1-T^{-}], \hat{I}^{C}_{A_{NV}}(x) = [1-I^{+}, 1-I^{-}],$
$\hat{F}^{C}_{A_{NV}}(x) = [1-F^{+}, 1-F^{-}].$

Definition 2.4[11]: Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x) = [1,1]$; $\hat{I}_{A_{NV}}(x) = [0,0]; \hat{F}_{A_{NV}}(x) = [0,0]$. Then A_{NV} is called unit NVS **(1_{NV} in short)**, where $1 \le i \le n$.

Definition 2.5[11]: Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x) = [0,0]$; $\hat{I}_{A_{NV}}(x) = [1,1]; \hat{F}_{A_{NV}}(x) = [1,1]$. Then A_{NV} is called zero NVS **(0_{NV} in short)**, where $1 \le i \le n$.

Definition 2.6[11]: The union of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy and false membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x) = [\max(T^{-}_{A_{NV_x}}, T^{-}_{B_{NV_x}}), \max(T^{+}_{A_{NV_x}}, T^{+}_{B_{NV_x}})]$$
$$\hat{I}_{C_{NV_x}}(x) = [\min(I^{-}_{A_{NV_x}}, I^{-}_{B_{NV_x}}), \min(I^{+}_{A_{NV_x}}, I^{+}_{B_{NV_x}})]$$
$$\hat{F}_{C_{NV_x}}(x) = [\min(F^{-}_{A_{NV_x}}, F^{-}_{B_{NV_x}}), \min(F^{+}_{A_{NV_x}}, F^{+}_{B_{NV_x}})]$$

Definition 2.7[11]: The intersection of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy and false membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x) = [\min(T^{-}_{A_{NV_x}}, T^{-}_{B_{NV_x}}), \min(T^{+}_{A_{NV_x}}, T^{+}_{B_{NV_x}})]$$
$$\hat{I}_{C_{NV_x}}(x) = [\max(I^{-}_{A_{NV_x}}, I^{-}_{B_{NV_x}}), \max(I^{+}_{A_{NV_x}}, I^{+}_{B_{NV_x}})]$$
$$\hat{F}_{C_{NV_x}}(x) = [\max(F^{-}_{A_{NV_x}}, F^{-}_{B_{NV_x}}), \max(F^{+}_{A_{NV_x}}, F^{+}_{B_{NV_x}})]$$

Definition 2.8:[11] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall$ $u_i \in U$, $\hat{T}_{A_{NV}}(u_i) = \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) = \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) = \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV}, are equal, where $1 \le i \le n$.

Corollary 2.9 [11]: Let A_{NV}, B_{NV} , C_{NV} and D_{NV} be NVSs.

(a) $A_{NV} \subseteq B_{NV}$ and $C_{NV} \subseteq D_{NV} \Rightarrow A_{NV} \cup B_{NV} \subseteq C_{NV} \cup D_{NV}$ and $A_{NV} \cap B_{NV} \subseteq C_{NV} \cap D_{NV}$

(b) $A_{NV} \subseteq B_{NV}$ and $A_{NV} \subseteq C_{NV} \Rightarrow A_{NV} \subseteq B_{NV} \cap C_{NV}$

(c) $A_{NV} \subseteq C_{NV}$ and $B_{NV} \subseteq C_{NV} \Rightarrow A_{NV} \cup B_{NV} \subseteq C_{NV}$

(d) $A_{NV} \subseteq B_{NV}$ and $B_{NV} \subseteq C_{NV} \Rightarrow A_{NV} \subseteq C_{NV}$

(e) $\overline{1}_{NV} = 0_{NV}$ (f) $\overline{0}_{NV} = 1_{NV}$

III. NEUTROSOPHIC VAGUE TOPOLOGICAL SPACES

In this section, we recall some definitions and properties which already exist in neutrosophic vague topological spaces.

Definition 3.1[8]: A neutrosophic vague topology (NVT in short) on X is a family τ of neutrosophic vague sets (NVS in short) in X satisfying the following axioms:

(1) $0_{NV}, 1_{NV} \in \tau$

(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(3) $\cup G_i \in \tau, \forall \{G_i : i \in j\} \subseteq \tau$.

In this case the pair (X,τ) is called neutrosophic vague topological space **(NVTS in short)** and any NVS in τ is known as neutrosophic vague open set **(NVOS in short)** in X. The complement A^c of NVOS in NVTS (X,τ) is called neutrosophic vague closed set **(NVCS in short)** in X.

Definition 3.2[8]: Let (X,τ) be a NVTS and A = $\left\{\left\langle x,[\hat{T}_A,\hat{I}_A,\hat{F}_A\right\rangle\right\}$ be NVS in X. Then the neutrosophic vague interior and neutrosophic vague closure are defined by

(1) NV int(A) = $\cup$ {G/ G is a NVOS in X and G $\subseteq$ A},

(2) NV cl(A) = $\cap$ {K/K is a NVCS in X and A $\subseteq$ K}.

Note that for any NVS A in (X,τ), we have NV cl(A^c) = (NV int(A))^c and NV int (A^c) = (NV cl(A))^c. It can also be shown that NV cl(A) is NVCS and NV int(A) is NVOS in X.

(1) A is NVCS in X if and only if NV cl(A) = A.

(2) A is NVOS in X if and only if NV int(A) = A.

Proposition 3.3[8]: Let A be any NVS in X. Then

(1) NVint (1-A) = 1-(NVcl(A)) and (2) NVcl(1-A) = 1- (NVint(A)).

Proposition 3.4[8]: Let (X,τ) be a NVTS and A, B be NVSs in X. Then the following properties hold:

(a) NVint(A) $\subseteq$ A, (a') A $\subseteq$ NVcl(A)

(b)A$\subseteq$B$\Rightarrow$NVint(A)$\subseteq$NVint(B) (b') A $\subseteq$ B $\Rightarrow$ NVcl(A) $\subseteq$ NVcl(B)

(c)NVint(NVint(A))=NVint(A) (c') NVcl(NVcl(A)) = NVcl(A)

(d)NVint(A$\cap$B)=NVint(A)$\cap$NVint(B) (d') NVcl(A)$\cup$NVcl(B) = NVcl(A$\cup$B).

(e)NVint(1_{NV})=1_{NV}. (e')NVcl(0_{NV}) = 0_{NV}

Proof: (a), (b) and (e) are obvious, (c) follows from (a).

(d) From NVint(A$\cap$B) $\subseteq$ NVint(A) and NVint(A$\cap$B) $\subseteq$ NVint(B) we obtain NVint(A$\cap$B) $\subseteq$ NVint(A) $\cap$ NVint(B) . On the other hand, from the facts NVint(A) $\subseteq$A and NVint(B) $\subseteq$B $\Rightarrow$NVint(A)$\cap$NVint(B) $\subseteq$ A$\cap$B and NVint(A)$\cap$ NVint(B) $\in \tau$ which implies that NVint(A)$\cap$ NVint(B) $\subseteq$ NVint(A$\cap$B) thus we obtain the required result.

(a') – (e') can be easily deduced from (a) – (e).

Definition 3.5[8]: A NVS A = $\left\{\left\langle x,[\hat{T}_A,\hat{I}_A,\hat{F}_A\right\rangle\right\}$ in NVTS (X,τ) is said to be a

(1) Neutrosophic Vague Regular open set **(NVROS in short)** if NVint(NVcl(A)) = A.

(2)Neutrosophic Vague Regular closed set **(NVRCS in short)** if NVcl(NVint(A)) = A.

(3)Neutrosophic Vague Regular semi open set **(NVRSOS in short)** if there exists a NVROS U such that U $\subseteq$ A $\subseteq$ cl(U).

(4) Neutrosophic Vague Semi open set**(NVSOS in short)** if A $\subseteq$ NVcl(NVint (A)).

(5) Neutrosophic Vague Pre-closed set**(NVPCS in short)** NVcl(NVint (A)) $\subseteq$ A.

(6) Neutrosophic Vague α-closed set**(NVαCS in short)** NVcl(NVint(NVcl((A))) $\subseteq$ A.

Definition 3.6[9]:

Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague alpha interior of A (NVαint(A) in short) and neutrosophic vague alpha closure of A (NVαcl(A) in short) are defined by

(1) NV αint(A) = $\cup$\{G/ G is a NVαOS in X and G $\subseteq$ A\}

(2) NV αcl(A) = $\cap$\{K/ K is a NVαCS in X and A $\subseteq$ K\}

Result 3.7[9]: Let A be NVS of a NVTS (X,τ).

(1) NVαcl(A) = A $\cup$ NVcl(NVint(NVcl(A))).

(2) NVαint(A) = A $\cap$ NVint(NVcl(NVint(A))).

Definition 3.8[9]:

Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague pre interior of A **(NVpint(A) in short)** and neutrosophic vague pre closure of A **(NVpcl(A)** in short) are defined by

(1) NV pint(A) = $\cup$\{G/ G is a NVPSOS in X and G $\subseteq$ A\}

(2) NV pcl(A) = $\cap$\{K/ K is a NVPCS in X and A $\subseteq$ K\}

Result 3.9[9]: Let A be NVS of a NVTS (X,τ), then

(1) NV pcl(A) = A $\cup$ NVcl (NVint(A))

(2) NVpint(A) = A $\cap$ NV int (NVcl(A))

Definition 3.10[9]:

A NVS A = $\left\{\left\langle x,[\hat{T}_A,\hat{I}_A,\hat{F}_A\right\rangle\right\}$ in NVTS (X,τ) is said to be a

(1) Neutrosophic Vague w-closed set **(NVWCS in short)** if NV cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is NVSOS in X.

(2) Neutrosophic Vague rg closed set **(NVRGCS in short)** if NV cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is NVROS in X.

(3) Neutrosophic Vague gp closed set **(NVGPCS in short)** if NV pcl(A) $\subseteq$ A whenever A $\subseteq$ U and U is NVOS in X.

(4) Neutrosophic Vague wg closed set **(NVWGCS in short)** if NV cl(NVint(A)) $\subseteq$U whenever A $\subseteq$ U and U is NVOS in X.

(5)Neutrosophic Vague gpr closed set **(NVGPRCS in short)** if NVpcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is NVROS in X.

(6)Neutrosophic Vague rwg closed set **(NVRWGCS in short)** if NVcl(NVint(A)) $\subseteq$ U whenever A $\subseteq$ U and U is NVROS in X.

Definition 3.11[9]: A NVT space (X,τ) is said to be **NV $T_{1/2}$ space** if every NVGC sets are NVC sets in (X,τ).

IV. NEUTROSOPHIC VAGUE RW CLOSED SETS.

In this section we introduce Neutrosophic vague rw closed sets and examine some of its characteristics.

Definition 4.1:

A NVS A is said to be a neutrosophic vague regular weakly closed sets **(NVRWCS in short)** in (X,τ) if NV cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is NVRSOS in X.

The family of all NVRWCSs of a NVTS (X,τ) is denoted by NVRWC(X).

Proposition 4.2:

Every (1) Neutrosophic Vague Closed set (2) Neutrosophic Vague RC closed set (3) Neutrosophic Vague W-closed set is NVRWC set.

Proof: (1) Let A be a NV closed set, i.e., NV cl(A) = A . Let A $\subseteq$ U, where U is NVRSO set. Then Nvcl(A) = A $\subseteq$ U implies that A is NVRW closed set.

(2) Let A be a NVRC set i.e., A = NVcl(NVint(A)). Let U be a NVRSO set such that A $\subseteq$ U. NVcl(A) $\subseteq$ NVcl(NVcl(NVint(A)) $\subseteq$ (NVint(A)) $\subseteq$ NVint(U) $\subseteq$ U implies A is a NVRWC set.

(3) Let A be a NVWC set. Let A $\subseteq$ U and U be a NVRSO set. Since A is a NVWC set and every NVRO set is NVO set, NVcl(A) $\subseteq$ U. Thus A is a NVRWC set.

Remark 4.3: The converse of the above Proposition need not be true as shown in the following example.

Example 4.4:

(1) Let X = {a,b,c} and let τ = {0,G,1} is a NVT on X, where the NVO set

$$G=\left\{x,\frac{a}{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle},\frac{b}{\langle[0.1,0.4];[0.8,0.9];[0.6,0.9]\rangle},\frac{c}{\langle[0.1,0.2];[0.6,0.7];[0.8,0.9]\rangle}\right\}$$

. Then the neutrosophic set

$$A=\left\{x,\frac{a}{\langle[0.3,0.5];[0.6,0.9];[0.5,0.7]\rangle},\frac{b}{\langle[0.2,0.3];[0.5,1];[0.7,0.8]\rangle},\frac{c}{\langle[0.1,0.6];[0.7,0.8];[0.4,0.9]\rangle}\right\}$$

is a NVRWC set in X but it is not a NVCS and NVRCS in (X,τ).

(2) Let X = {a,b,c} and let τ = {0,G,1} is a NVT on X, where the neutrosophic vague open set

$$G=\left\{x,\frac{a}{\langle[0.3,0.6];[0.8,0.9];[0.4,0.7]\rangle},\frac{b}{\langle[0.2,0.4];[0.6,0.9];[0.6,0.8]\rangle},\frac{c}{\langle[0.1,0.5];[0.7,0.8];[0.5,0.9]\rangle}\right\}$$

. Then NV set

$$B=\left\{x,\frac{a}{\langle[0.2,0.6];[0.4,0.7];[0.4,0.8]\rangle},\frac{b}{\langle[0.1,0.4];[0.3,0.8];[0.6,0.9]\rangle},\frac{c}{\langle[0.3,0.3];[0.4,0.5];[0.3,0.7]\rangle}\right\}$$

is a NVRWC set but it is not a NVWC set in X.

Proposition 4.5:

The finite union of two NVRWC subsets of (X,τ) is also a NVRWC subset of X.

Proof: Suppose that A and B are two NVRWC sets in X such that $A \cup B \subseteq U$ where U is a NVRSO set in X. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are NVRWC sets, NVcl(A) $\subseteq$ U and NVcl(B) $\subseteq$ U. By Proposition 3.4, NV cl($A \cup B$) = NV cl(A) $\cup$ NV cl(B) $\subseteq$ U which implies $A \cup B$ is also a NVRWC set in X.

Remark 4.6:

The intersection of two NVRWC sets need not be a NVRWC set in (X,τ) which is proved by following example.

Example 4.7:

Let X = {a,b,c}, $\tau = \{0, F, 1\}$ where F is NVO set in X and

$$F = \left\{ x, \frac{a}{\langle [0.3,0.6];[0.8,0.9];[0.4,0.7] \rangle}, \frac{b}{\langle [0.2,0.4];[0.6,0.9];[0.6,0.8] \rangle}, \frac{c}{\langle [0.1,0.5];[0.7,0.8];[0.5,0.9] \rangle} \right\}$$

. The NV sets

$$A = \left\{ x, \frac{a}{\langle [0.2,0.6];[0.4,0.7];[0.4,0.8] \rangle}, \frac{b}{\langle [0.1,0.4];[0.3,0.8];[0.6,0.9] \rangle}, \frac{c}{\langle [0.3,0.4];[0.4,0.5];[0.6,0.7] \rangle} \right\}$$

and

$$B = \left\{ x, \frac{a}{\langle [0.2,0.4];[0.5,0.7];[0.7,0.8] \rangle}, \frac{b}{\langle [0.1,0.4];[0.8,0.9];[0.6,0.9] \rangle}, \frac{c}{\langle [0.1,0.2];[0.6,0.7];[0.8,0.9] \rangle} \right\}$$

are NVRWC sets but $A \cap B$ is not NVRWC set in (X,τ).

Proposition 4.8:

Every NVRWC set is (1) NVRWGC (2) NVGPRC (3) NVRGC set.

Proof: (i) Let A be a NVRWC set. Let U be a NVRO set such that $A \subseteq U$. Since every NVRO set is NVRSO set and by hypothesis, NVcl(NVint(A)) $\subseteq$ NVcl (A) $\subseteq$ U. Hence A be a NVRWGC set.

(ii) Let A be a NVRWC set. Let $A \subseteq U$, U is NVRO set. Since every NVC set is NVPC set and every NVRO set is NVRSO set, NVpcl(A) $\subseteq$ NVcl(A) $\subseteq$ U. Hence A is NVGPRC set.

(iii) Let A be a NVRWC set. Let $A \subseteq U$ where U is NVRO set. Every NVRO set is NVRSO set. By hypothesis, NVcl(A) $\subseteq$ U. Thus A is NVRGC set.

Remark 4.9: The converse of the above Proposition need not be true as shown in the following example.

Example 4.10:

(1) Let X = {a,b}, $\tau = \{0, G_1, G_2, 1\}$ where the NVO sets G_1 and G_2 are

$$G_1 = \left\{ \frac{a}{\langle [0.6,0.8];[0.1,0.2];[0.2,0.4] \rangle}, \frac{b}{\langle [0.7,0.9];[0.2,0.5];[0.1,0.3] \rangle} \right\},$$

$$G_2 = \left\{ \frac{a}{\langle [0.2,0.4];[0.5,0.6];[0.6,0.8] \rangle}, \frac{b}{\langle [0.3,0.6];[0.7,0.8];[0.4,0.7] \rangle} \right\}.$$ Let the NV set

$$B = \left\{ \frac{a}{\langle [0.1,0.3];[0.6,0.7];[0.7,0.9] \rangle}, \frac{b}{\langle [0.2,0.5];[0.7,0.9];[0.5,0.8] \rangle} \right\}.$$ Then B is both NVGPRC set and NVRWGC set but it is not NVRWC set in (X,τ).

(2) Let X = {a,b}, $\tau = \{0, G_1, G_2, 1\}$ where the NVO sets G_1 and G_2 are

$$G_1=\left\{\frac{a}{\langle[0.2,0.4];[0.7,0.9];[0.6,0.8]\rangle},\frac{b}{\langle[0.3,0.5];[0.6,0.8];[0.5,0.7]\rangle}\right\},$$

$$G_2=\left\{\frac{a}{\langle[0.4,0.9];[0.1,0.3];[0.1,0.6]\rangle},\frac{b}{\langle[0.5,0.7];[0.2,0.6];[0.3,0.5]\rangle}\right\}.$$ Let the NV set

$$A=\left\{\frac{a}{\langle[0.2,0.3];[0.6,0.7];[0.7,0.8]\rangle},\frac{b}{\langle[0.3,0.4];[0.4,0.5];[0.6,0.7]\rangle}\right\}$$ is NVRGC set but it is not a NVRWC set in (X,τ).

Remark 4.11: The concept of NVRWC set is independent with the concept of NVWGC sets and NVGPC sets as shown in the following example.

Example 4.12:

In example 3.10 (2), in the neutrosophic vague topological space (X,τ), the NV set,

$$A=\left\{\frac{a}{\langle[0.3,0.5];[0.3,0.5];[0.5,0.7]\rangle},\frac{b}{\langle[0.2,0.5];[0.3,0.6];[0.5,0.8]\rangle}\right\}$$ is NVRWC set but it is not NVWGC & NVGPC set and the NV set

$$B=\left\{\frac{a}{\langle[0.2,0.3];[0.8,0.9];[0.4,0.8]\rangle},\frac{b}{\langle[0.2,0.5];[0.6,0.9];[0.5,0.8]\rangle}\right\}$$ is both NVWGC set and NVGPC set but it is not NVRWC set.

Proposition 4.13:

If A is NVRWC subset of X such that $A \subseteq B \subseteq$ NVcl(A), then B is a NVRWC set in X.

Proof: Let A be NVRWC set of X such that $A \subset B \subseteq$ NVcl(A). Let U be a NVRSO set of X such that $B \subseteq U$. Then by hypothesis, $A \subseteq U$. Since A is NVRWC, NVcl(A) $\subseteq$ U. Now NV cl(B) $\subseteq$ NV cl(NV cl(A)) = NV cl(A) $\subseteq$ U. Hence B is a NVRWC set in X.

Remark 4.14:

The converse of the above theorem need not be true in general as seen in the following example.

Example 4.15:

Let X = {a,b}, τ = {0, G_1,G_2,1}. Clearly (X,τ) is a NVT on X where the NVO sets G_1 and G_2 are

$$G_1=\left\{\frac{a}{\langle[0.6,0.8];[0.1,0.2];[0.2,0.4]\rangle},\frac{b}{\langle[0.7,0.9];[0.2,0.5];[0.1,0.3]\rangle}\right\},$$

$$G_2=\left\{\frac{a}{\langle[0.2,0.4];[0.5,0.6];[0.6,0.8]\rangle},\frac{b}{\langle[0.3,0.6];[0.7,0.8];[0.4,0.7]\rangle}\right\}.$$

Let the NV sets

$$A=\left\{\frac{a}{\langle[0.2,0.3];[0.6,0.7];[0.7,0.8]\rangle},\frac{b}{\langle[0.4,0.5];[0.4,0.6];[0.5,0.6]\rangle}\right\},$$

$$B=\left\{\frac{a}{\langle[0.2,0.3];[0.3,0.4];[0.7,0.8]\rangle},\frac{b}{\langle[0.4,0.5];[0.3,0.5];[0.5,0.6]\rangle}\right\}$$. Then

$A \subseteq B$, A and B are NVRWC sets, but B is not a subset of NV cl(A).

Proposition 4.16:

If A is NVRO and NVRWC set then A is NVRC set and hence NV clopen.

Proof: Suppose A is NVRO and NVRWC set. As every NVRO set is NVRSO and $A \subseteq A$, then NV cl(A) $\subseteq$ A and also $A \subseteq$ NV cl(A) implies NVcl(A) = A. Thus A is NVC set. Since A is NVRO then A is NVO set. NV cl(NV int(A)) = NV cl(A) = A. Therefore A is NVRC and hence NV clopen.

Definition 4.17:

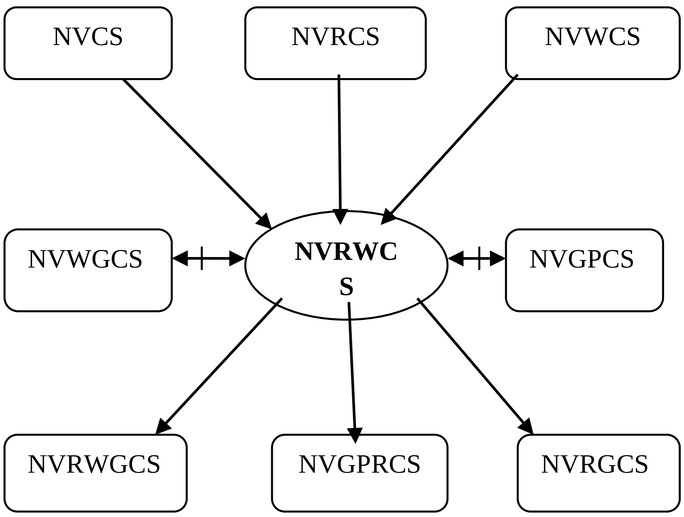
A NVS A is said to be a neutrosophic vague rw open **(NVRWO in short)** set, if A^c is NVRWC set in (X,τ).

Proposition 4.18:

Let A be a NVRWO set of a NVT space (X,τ) and NV int(A) $\subseteq B \subseteq A$. Then $B \in$ NVRWO(X).

Proof: Let A be a NVRWO in X such that NVint(A) $\subseteq B \subseteq A$, which implies $A^c \subseteq B^c \subseteq (\text{NV int } A)^c \Rightarrow A^c \subseteq B^c \subseteq$ NV cl(A^c). By proposition 4.13, B^c is NVRWC set implies that B is NVRWO set.

The above discussions are implemented in the following diagram.



where A ⟶ B means that A implies B but not conversely and A ⟷ means that A and B are independent to each other.

V. APPLICATIONS OF NEUTROSOPHIC VAGUE RW CLOSED SETS.

Definition 5.1: A NVTS (X,τ) is said to be neutrosophic vague rw$T_{1/2}$ space **(NVrw$T_{1/2}$ space in short)** if every NVRWC set is NVC set in X.

Definition 5.2: A NVTS (X,τ) is said to be a neutrosophic vague rwT_w space **(NVrwT_w space in short)** if every NVRWC set is NVWC set.

Proposition 5.3: Every NVrw$T_{1/2}$ space is NVrwT_w space.

Proof: Let X be a NVrw$T_{1/2}$ space and let A be NVRWC set in X. Since every NVWRC set is NVWC set, A is NVWC set. Hence X is NVrwT_w space in (X,τ).

Proposition 5.4: NVrw$T_{1/2}$ space and NV$T_{1/2}$ space are independent to each other.

Proof: Straight forward from the fact that NVGC sets and NVRWC sets are independent to each other.

Definition 5.5: A neutrosophic vague topological space is called **NVRW connected** if there is no proper NV set of X which is both NVRWC set and NVRWO set.

Proposition 5.6: Every NVRW connected space is NV connected.

Proof: Straight forward.

Proposition 5.7:A NVT space is NVRW connected if and only if there exists no non-zero NVRWO sets A and B in X such that A = B^C.

Proof: Necessity: Suppose thst A and B are NVRWO set such that $A \neq \overline{0} \neq B$ and A = B^C. Since A = B^C, B is a NVRWO set, B^C = A is NVRWC set and $B \neq \overline{0}$ implies that $B^C \neq \overline{1} i.e., A \neq \overline{1}.$ Hence there exists a proper NV set A ($A \neq \overline{0}, A \neq \overline{1}$) such that A is both NVRWC and NVRWO set contradicts the hypothesis that X is NVRW connected.

Sufficiency: Let (X,τ) be NVT and A is both NVRWC and NVRWO set in X such that $\overline{0} \neq A \neq \overline{1}.$ Let B=A^C. Then B is NVRWO set and $A \neq \overline{1}$ implies that B = A^C $\neq \overline{0}$, a contradiction. Therefore there is no proper NV set of X which is both NVRWC and NVRWO. Thus the NVT (X,τ) is NVRW connected.

Conclusion: In this paper, we have introduced neutrosophic vague rw closed sets in neutrosophic vague topological spaces and analysed some of its characteristics. In further neutrosophic rw connectedness has been discussed. In future, neutrosophic vague rw closed sets can be extended to some new or existing concepts like continuity.

REFERENCES

[1] Atanassov, Intuitionistic fuzzy sets, in V.Sgurev,ed, *Vii ITKRS Session, Sofia (June 1983 central Sci. and Techn. Library, Bulg Academy of Sciences* (1984).

[2] [2] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy sets and systems* 20 (1986), 87-96.

[3] Chang, C.L. Fuzzy topological spaces, *J Math. Anal.Appl. 1968;24, pp. 182-190*.

[4] Gau, W.L. Buehrer, D.J. Vague sets. IEEE Trans. Systems Man and Cybernet, 1993; 23(2), pp. 610-614.

[5] S. S. Benchelli and R.S. Wali, On RW- closed sets in Topological Spaces, *Bull. Malays. Math. Sci. Soc. (2)* 30(2) (2007), 99-110.

[6] [5] Floretin Smaradhache, Neutrosophic Set - A Generalization of Intuitionistic Fuzzy sets, *Journal of defence Sources Management 1* (2010), 107-116.

[7] F. Smarandache, Neutrosophy and Neutrosophic Logic, *First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico,* Gallup, NM 87301, USA (2002).

[8] Mary Margaret A, Trinita Pricilla M and Shawkat Alkhazaleh, Neutrosophic Vague Topological Spaces, *Neutrosophic Sets and Systems*, Vol. 28, 2019.

[9] Mary Margaret A, Trinita Pricilla M, Neutrosophic Vague Generalized Pre-Closed Sets in Neutrosophic Vague Topological Spaces, *International Journal of Mathematics and its Applications*, Vol5, Issue 4-E(2017), 747-759.

[10] Salama, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. *IOSR Journal of Mathematics*, 2012; 3(4), pp. 31-35.

[11] Shawkat Alkhazaleh. Neutrosophic vague set theory. *Critical Review. 2015*; 10, pp. 29-39.

[12] D.Savithiri, C.Janaki, Neutrosophic rw closed sets in neutrosophic topological spaces, *International Journal of Research and Analytical Reviews (IJRAR),* June 2019, Vol. 6, Issue2, 242-249.

[13] L. A. Zadeh, Fuzzy sets, *Inform. and Control* 8 (1965)