Neutrosophic Weakly $\pi$ Generalized Continuous Mapping

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Abstract- Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic continuity is called Neutrosophic weakly $\pi$ generalized continuous in Neutrosophic topological spaces and also discussed about properties and characterization Neutrosophic weakly $\pi$ generalized continuous.

Keywords – NS W$\pi$G open set, NS W$\pi$G closed set Neutrosophic weakly $\pi$ generalized continuous, Neutrosophic topological spaces

I. INTRODUCTION

A.A. Salama introduced Neutrosophic topological spaces by using Smarandache’s Neutrosophic sets. I. Arokiarani [2] et al, introduced Neutrosophic $\alpha$-closed sets. P. Ishwarya, [8] et.al, introduced and studied about on Neutrosophic semi-open sets in Neutrosophic topological spaces. Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic continuity is called Neutrosophic weakly $\pi$ generalized continuous in Neutrosophic topological spaces and also discussed about properties and characterization Neutrosophic weakly $\pi$ generalized continuous.

II. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Definition 2.1 [7]

Let X be a non-empty fixed set. A Neutrosophic set $A$ is an object having the form

$A = \{ <x, \eta_A(x), \sigma_A(x), \gamma_A(x) > : x \in X \}$

where $\eta_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent Neutrosophic topological spaces the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set $A$.

Remark 2.2 [7]

A Neutrosophic set $A = \{ <x, \eta_A(x), \sigma_A(x), \gamma_A(x) > : x \in X \}$ can be identified to an ordered triple $<\eta_A, \sigma_A, \gamma_A >$ in $\mathbb{R}^{3}$. On $X$.

Remark 2.3 [7]

We shall use the symbol $A = <x, \eta_A, \sigma_A, \gamma_A >$ for the Neutrosophic set $A = \{ <x, \eta_A(x), \sigma_A(x), \gamma_A(x) > : x \in X \}$.

Example 2.4 [7]

Every Neutrosophic set $A$ is a non-empty set in $X$ is obviously on Neutrosophic set having the form $A = \{ <x, \eta_A(x), 1-(\eta_A(x) + \gamma_A(x)), \gamma_A(x) > : x \in X \}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set $0_N$ and $1_N$ in $X$ as follows:

$0_N$ may be defined as:

$0_1 = \{ <x, 0, 0, 1 > : x \in X \}$

$0_2 = \{ <x, 0, 1, 1 > : x \in X \}$

$0_3 = \{ <x, 0, 1, 0 > : x \in X \}$

$0_4 = \{ <x, 0, 0, 0 > : x \in X \}$

$1_N$ may be defined as:

$1_1 = \{ <x, 1, 0, 0 > : x \in X \}$

Volume IX Issue I JANUARY 2020
Let \( A = \{ x, \eta_A(x), \sigma_A(x), \gamma_A(x) \} \) be a Neutrosophic set on \( X \), then the complement of the set \( A \) is defined as:
\[ A^C = \{ x, 1 - \sigma_A(x), 1 - \gamma_A(x) \} : x \in X \]

**Definition 2.6** [7]
Let \( X \) be a non-empty set, and Neutrosophic sets \( A \) and \( B \) in the form:
\[ A = \{ x, \eta_A(x), \sigma_A(x), \gamma_A(x) : x \in X \} \]
\[ B = \{ x, \eta_B(x), \sigma_B(x), \gamma_B(x) : x \in X \} \]
Then the family \( \tau \) of Neutrosophic subsets in \( X \) satisfying the following axioms:
(i) \( 0_N \subseteq A, 0_N \subseteq B \)
(ii) \( A \subseteq 1_N, 1_N \subseteq A \)

**Definition 2.8** [7]
Let \( X \) be a non-empty set, and \( A = \{ x, \eta_A(x), \sigma_A(x), \gamma_A(x) \} \), \( B = \{ x, \eta_B(x), \sigma_B(x), \gamma_B(x) \} \) be two Neutrosophic sets. Then:
(i) \( A \cap B \) defined as:
\[ A \cap B = \{ x, \eta_A(x) \wedge \eta_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \} \]
(ii) \( A \cup B \) defined as:
\[ A \cup B = \{ x, \eta_A(x) \vee \eta_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \} \]

**Proposition 2.9** [7]
For any Neutrosophic set \( A \), then the following conditions hold:
(i) \( 0_N \subseteq A \subseteq 0_N \)
(ii) \( A \subseteq 1_N, 1_N \subseteq A \)

**Definition 2.10** [11]
A Neutrosophic topology is a non-empty set \( X \) is a family \( \tau_N \) of Neutrosophic subsets in \( X \) satisfying the following axioms:
(i) \( 0_N, 1_N \subseteq \tau_N \)
(ii) \( G_1 \cap G_2 \in \tau_N \) for any \( G_1, G_2 \in \tau_N \)
(iii) \( U \subseteq \tau_N \) for any family \( \{ G_i \} \subseteq \tau_N \)

The pair \( (X, \tau_N) \) is called a Neutrosophic topological space.

**Example 2.11** [11]
Let \( X = \{ x \} \) and
\[ A_1 = \{ x, 0.6, 0.6, 0.5 : x \in X \} \]
\[ A_2 = \{ x, 0.5, 0.7, 0.9 : x \in X \} \]
\[ A_3 = \{ x, 0.6, 0.7, 0.5 : x \in X \} \]
\[ A_4 = \{ x, 0.5, 0.6, 0.9 : x \in X \} \]
Then the family \( \tau_N = \{ 0_N, 1_N, A_1, A_2, A_3, A_4 \} \) is called a Neutrosophic topological space on \( X \).

**Definition 2.12** [11]
Let \( (X, \tau_N) \) be Neutrosophic topological spaces and \( A = \{ x, \eta_A(x), \sigma_A(x), \gamma_A(x) : x \in X \} \) be a Neutrosophic set in \( X \). Then the Neutrosophic closure and Neutrosophic interior of \( A \) are defined by:
\[ \text{Neu-cl}(A) = i \{ K : K \text{ is a Neutrosophic closed set in } X \} \]
\[ \text{Neu-int}(A) = U \{ G : G \text{ is a Neutrosophic open set in } X \} \]

**Definition 2.13**
Let \( (X, \tau_N) \) be a Neutrosophic topological space. Then \( A \) is called Neutrosophic regular closed set if \( A = \text{Neu-cl}(\text{Neu-int}(A)) \).

**Definition 2.14**
Let \( (X, \tau_N) \) be a Neutrosophic topological space. Then \( A \) is called
(i). Neutrosophic regular open set \([2](\text{Neu-ROS in short})\) if \(A=\text{Neu-Int}(\text{Neu-Cl}(A))\),
(ii). Neutrosophic \(\alpha\)-open set \([2](\text{Neu-\(\alpha\)OS in short})\) if \(A \subseteq \text{Neu-Int}(\text{Neu-Cl}(\text{Neu-Int}(A)))\),
(iii). Neutrosophic semi open set \([9](\text{Neu-SOS in short})\) if \(A \subseteq \text{Neu-Cl}(\text{Neu-Int}(A))\),
(iv).Neutrosophic pre open set \([13]\) (Neu-POS in short) if \(A \subseteq \text{Neu-Int}(\text{Neu-Cl}(A))\).

**Definition 2.15**

Let \((X, \tau_X)\) be a Neutrosophic topological space. Then \(A\) is called
(i).Neutrosophic generalized closed set\([4]\)(Neu-GCS in short) if \(\text{Neu-cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a Neu-OS in \(X\),
(ii).Neutrosophic generalized semi closed set\([12]\)(Neu-GSCS in short) if \(\text{Neu-scl}(A) \subseteq U\) Whenever \(A \subseteq U\) and \(U\) is a Neu-OS in \(X\),
(iii).Neutrosophic \(\alpha\) generalized closed set \([8]\)(Neu-\(\alpha\)CS in short) if \(\text{Neu-\(\alpha\)cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a Neu-OS in \(X\),
(iv).Neutrosophic generalized alpha closed set \([8]\) (Neu-\(\alpha\)GCS in short) if \(\text{Neu-\(\alpha\)cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a Neu-\(\alpha\)OS in \(X\).

The complements of the above mentioned Neutrosophic closed sets are called their respective Neutrosophic open sets.

**Definition 2.18:**\([5]\)

Let \(f\) be a mapping from an NSTS \((X, NS_x)\) into NSTS \((Y, \sigma)\). Then \(f\) is said to be Neutrosophic continuous (NS cts) if, \(f^{-1}(B)\in NSGCS(X)\) for every NSCS, \(B\) in \(Y\).

**Definition 2.18:**\([5]\)

Let \(f\) be a mapping from an NSTS \((X, NS_x)\) into NSTS \((Y, NS_y)\). Then \(f\) is said to be Neutrosophic continuous (NS cts) if, \(f^{-1}(B)\in NSOS(X)\) for every \(B\in \sigma\).

**Definition 2.19:**\([12]\)

A mapping \(f:\ (X, NS_x)\rightarrow (Y, NS_y)\) is called Neutrosophic generalized semi continuous (NSG cts) if, \(f^{-1}(B)\in NSGSCS(X, NS_x)\) for every NSCS, \(B\) of \((Y, NS_y)\).

**Definition 2.20:**

A mapping \(f:\ (X, NS_x)\rightarrow (Y, NS_y)\) is called Neutrosophic \(\alpha\) continuous (NS\(\alpha\) cts) if, \(f^{-1}(B)\in NS\alphaGCS\) in \((X, NS_x)\) for every NSCS, \(B\) of \((Y, NS_y)\).

**Definition 2.17:**

Let \(f\) be a mapping from an NSTS \((X, NS_x)\) into NSTS \((Y, NS_y)\). Then \(f\) is said to be
i) Neutrosophic semi continuous \([15]\) (NS(S) cts) if, \(f^{-1}(B)\in NS(S)O(X)\) for every \(B\in \sigma\),
ii) Neutrosophic \(\alpha\) continuous \([15]\) (NS(\(\alpha\)) cts) if, \(f^{-1}(B)\in NS(\alpha)O(X)\) for every \(B\in \sigma\),
iii)Neutrosophic pre continuous \([15]\) (NS(P) cts) if, \(f^{-1}(B)\in NS(P)O(X)\) for every \(B\in \sigma\),
iv)Neutrosophic regular continuous \([15]\) (NS(R) cts) if, \(f^{-1}(B)\in NSRO(X)\) for every \(B\in \sigma\).

### 3. NEUTROSOPHIC WEAKLY \(\pi\) GENERALIZED CONTINUOUS MAPPINGS

In this section, Neutrosophic weakly \(\pi\) generalized continuous mappings is defined. Some of its properties are derived.

**Definition 3.1:**

A mapping \(f:\ (X, NS_x)\rightarrow (Y, NS_y)\) is called an Neutrosophic weakly \(\pi\) generalized continuous mapping (NS(\(\pi\)G) cts) if, \(f^{-1}(B)\) is a NS(\(\pi\)G)CS in \((X, NS_x)\) for every NSCS, \(B\) of \((Y, NS_y)\).

**Example 3.2:**

Let \(X=\{a,b\}, Y=\{u,v\}\) and
\[
G_1=\langle x, \begin{pmatrix} 2 \times 10^{-10} & 5 \times 10^{-10} & 6 \times 10^{-10} \\ 2 \times 10^{-10} & 5 \times 10^{-10} & 7 \times 10^{-10} \end{pmatrix} \rangle, \]
\[
G_2=\langle y, \begin{pmatrix} 6 \times 10^{-10} & 4 \times 10^{-10} \\ 7 \times 10^{-10} & 5 \times 10^{-10} & 2 \times 10^{-10} \end{pmatrix} \rangle. \]
Then
\[
\text{NS}_x=\{0_{NS_x}, G_1, 1_{NS_x}\} \text{ and } \text{NS}_y=\{0_{NS_y}, G_2, 1_{NS_y}\} \text{ are NSTs on } X \text{ and } Y \text{ respectively.}
\]
Define a mapping \(f:\ (X, NS_x)\rightarrow (Y, NS_y)\) by \(f(a)=u\) and \(f(b)=v\).
Then \(f\) is a NS(\(\pi\)G)CTS mapping.

**Proposition 3.3:**

Every NSCTS mapping is a NS(\(\pi\)G)CTS mapping but not conversely.

**Proof:**

Let \(f:\ (X, NS_x)\rightarrow (Y, NS_y)\) be a NSCTS mapping. Let \(B\) be a NSCS in \(Y\). Since \(f\) is NSCTS mapping, \(f^{-1}(B)\) is a NSCS in \(X\). Since every NSCS is a NS(\(\pi\)G)CS, \(f^{-1}(B)\) is a NS(\(\pi\)G)CS in \(X\). Therefore \(f\) is a NS(\(\pi\)G)CTS mapping.
Example 3.4:
Let X = {a,b}, Y = {u,v} and
\[ G_1 = \langle \begin{pmatrix} 0.1 & 0.5 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.6 & 0.7 \\ 0.1 & 0.1 \end{pmatrix} \rangle, \]
\[ G_2 = \langle \begin{pmatrix} 0.1 & 0.5 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.3 & 0.1 \\ 0.1 & 0.1 \end{pmatrix} \rangle. \]
Then NS_{G_1} = \{ 0_{NS}, G_1, 1_{NS} \} and NS_{G_2} = \{ 0_{NS}, G_2, 1_{NS} \} are NSTs on X and Y respectively.

Define a mapping \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) by \( f(a) = u \) and \( f(b) = v \).

The NSS, \( B = \langle \begin{pmatrix} 0.1 & 0.5 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.6 & 0.7 \\ 0.1 & 0.1 \end{pmatrix} \rangle \) is NSCS in Y. Therefore \( f^{-1}(B) \) is NS(WπG)CS in X, but not NSCS in X. Therefore \( f \) is a NS(WπG)CTS mapping but not a NSTS mapping.

Proposition 3.5:
Every NS(α) continuous mapping is a NS(WπG)CTS mapping but not conversely.

Proof:
Let \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) be a NS(α) continuous mapping. Let B be a NSCS in Y. Then by definition \( f^{-1}(B) \) is a NS(α)CS in X. Since every NS(α)CS is a NS(WπG)CS, \( f^{-1}(B) \) is a NS(WπG)CS in X. Thus \( f \) is a NS(WπG)CTS mapping.

Example 3.6:
Let X = {a,b}, Y = {u,v} and
\[ G_1 = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{pmatrix} \rangle, \]
\[ G_2 = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.3 \\ 0.1 & 0.1 \end{pmatrix} \rangle. \]
Then NS_{G_1} = \{ 0_{NS}, G_1, 1_{NS} \} and NS_{G_2} = \{ 0_{NS}, G_2, 1_{NS} \} are NSTs on X and Y respectively.

Define a mapping \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) by \( f(a) = u \) and \( f(b) = v \).

The NSS, \( B = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{pmatrix} \rangle \) is NSCS in Y. Then \( f^{-1}(B) \) is NS(WπG)CS in X, but not NS(α)CS in X. Therefore \( f \) is a NS(WπG)CTS mapping but not a NS(α) continuous mapping.

Proposition 3.7:
Every NS(R) CTS mapping is a NS(WπG)CTS mapping but not conversely.

Proof:
Let \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) be a NS(R) CTS mapping. Let B be a NSCS in Y. Then by definition \( f^{-1}(B) \) is a NS(R)CS in X. Since every NS(R)CS is a NS(WπG)CS, \( f^{-1}(B) \) is a NS(WπG)CS in X. Thus \( f \) is a NS(WπG)CTS mapping.

Example 3.8:
Let X = {a,b}, Y = {u,v} and
\[ G_1 = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{pmatrix} \rangle, \]
\[ G_2 = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.3 \\ 0.1 & 0.1 \end{pmatrix} \rangle. \]
Then NS_{G_1} = \{ 0_{NS}, G_1, 1_{NS} \} and NS_{G_2} = \{ 0_{NS}, G_2, 1_{NS} \} are NSTs on X and Y respectively.

Define a mapping \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) by \( f(a) = u \) and \( f(b) = v \).

The NSS, \( B = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{pmatrix} \rangle \) is NSCS in Y. Then \( f^{-1}(B) \) is NS(WπG)CS in X, but not NS(R)CS in X. Therefore \( f \) is a NS(WπG)CTS mapping but not a NS(R) CTS mapping.

Proposition 3.9:
Every NS(P) CTS mapping is a NS(WπG)CTS mapping but not conversely.

Proof:
Let \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) be a NS(P) CTS mapping. Let B be a NSCS in Y. Then by definition \( f^{-1}(B) \) is a NS(P)CS in X. Since every NS(P)CS is a NS(WπG)CS, \( f^{-1}(B) \) is a NS(WπG)CS in X. Thus \( f \) is a NS(WπG)CTS mapping.

Example 3.10:
Let X = {a,b}, Y = {u,v} and
\[ G_1 = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{pmatrix} \rangle, \]
\[ G_2 = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.3 \\ 0.1 & 0.1 \end{pmatrix} \rangle. \]
Then NS_{G_1} = \{ 0_{NS}, G_1, 1_{NS} \} and NS_{G_2} = \{ 0_{NS}, G_2, 1_{NS} \} are NSTs on X and Y respectively.

Define a mapping \( f: (X, NS_{G_1}) \rightarrow (Y, NS_{G_2}) \) by \( f(a) = u \) and \( f(b) = v \).

The NSS, \( B = \langle \begin{pmatrix} 0.5 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{pmatrix} \rangle \) is NSCS in Y. Then \( f^{-1}(B) \) is NS(WπG)CS in X, but not NS(P)CS in X. Therefore \( f \) is a NS(WπG)CTS mapping but not a NS(P) CTS mapping.
Proposition 3.11:
Every NS(G)CTS mapping is a NS(WπG)CTS mapping but not conversely.

Proof:
Let \( f: (X, NS_X) \rightarrow (Y, NS_Y) \) be a NS(G)CTS mapping. Let \( B \) be a NSCS in \( Y \). Since \( f \) is a NS(G)CTS mapping, \( f^{-1}(B) \) is a NS(G)CS in \( X \). Since every NS(G)CS is a NS(WπG)CS, \( f^{-1}(B) \) is a NS(WπG)CS in \( X \). Thus \( f \) is a NS(WπG)CTS mapping.

Example 3.12: Let \( X = \{a,b\}, Y = \{u,v\} \) and
\[
G_1 = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle,
G_2 = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{2}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{7}{10} \\
\frac{5}{10}
\end{array} \right) \rangle.
\]
Then \( NS_{T_1} = \{ 0_{NS} \text{, } G_1 \text{, } 1_{NS} \} \) and \( NS_{T_2} = \{ 0_{NS} \text{, } G_2 \text{, } 1_{NS} \} \) are NSTs on \( X \) and \( Y \) respectively.

Define a mapping \( f: (X, NS_X) \rightarrow (Y, NS_Y) \) by \( f(a) = u \) and \( f(b) = v \).
The NSS, \( B = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle \) is NSCS in \( Y \), \( f^{-1}(B) \) is NS(WπG)CS in \( X \) but not NS(G)CS in \( X \).
Therefore \( f \) is NS(WπG)CTS mapping but not a NS(G)CTS mapping.

Proposition 3.13:
Every NS(αG) continuous mapping is a NS(WπG)CTS mapping but not conversely.

Proof:
Let \( f: (X, NS_X) \rightarrow (Y, NS_Y) \) be a NS(αG) continuous mapping. Let \( B \) be a NSCS in \( Y \). Then by definition, \( f^{-1}(B) \) is a NS(αG)CS in \( X \). Since every NS(αG)CS is a NS(WπG)CS, \( f^{-1}(B) \) is a NS(WπG)CS in \( X \). So, \( f \) is a NS(WπG)CTS mapping.

Example 3.14:
Let \( X = \{a,b\}, Y = \{u,v\} \) and
\[
G_1 = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle,
G_2 = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{2}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{7}{10} \\
\frac{5}{10}
\end{array} \right) \rangle.
\]
Then \( NS_{T_1} = \{ 0_{NS} \text{, } G_1 \text{, } 1_{NS} \} \) and \( NS_{T_2} = \{ 0_{NS} \text{, } G_2 \text{, } 1_{NS} \} \) are NSTs on \( X \) and \( Y \) respectively.

Define a mapping \( f: (X, NS_X) \rightarrow (Y, NS_Y) \) by \( f(a) = u \) and \( f(b) = v \).
The NSS, \( B = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle \) is NSCS in \( Y \), \( f^{-1}(B) \) is NS(WπG)CS in \( X \), but not NS(αG)CS in \( X \).
Therefore \( f \) is NS(WπG)CTS mapping but not a NS(αG) continuous mapping.

Remark 3.15:
NSS continuous mapping and NS(WπG)CTS mapping are independent to each other.

Example 3.16:
Let \( X = \{a,b\}, Y = \{u,v\} \) and
\[
G_1 = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle,
G_2 = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{2}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{7}{10} \\
\frac{5}{10}
\end{array} \right) \rangle.
\]
Then \( NS_{T_1} = \{ 0_{NS} \text{, } G_1 \text{, } 1_{NS} \} \) and \( NS_{T_2} = \{ 0_{NS} \text{, } G_2 \text{, } 1_{NS} \} \) are NSTs on \( X \) and \( Y \) respectively.

Define a mapping \( f: (X, NS_X) \rightarrow (Y, NS_Y) \) by \( f(a) = u \) and \( f(b) = v \).
Then \( f \) is NSS continuous mapping but not a NS(WπG)CTS mapping, since \( B = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle \) is a NSSCS in \( Y \), but \( f^{-1}(B) = \langle \left( \begin{array}{c}
\frac{5}{10} \\
\frac{1}{10}
\end{array} \right), \left( \begin{array}{c}
\frac{6}{10} \\
\frac{2}{10}
\end{array} \right) \rangle \) is not a NS(WπG)CS in \( X \).

Example 3.18:
Let \( X = \{a,b\}, Y = \{u,v\} \) and
\[ G_1 = \langle x, \left( \frac{2}{10}, \frac{5}{10} \right) \rangle, \]
\[ G_2 = \langle y, \left( \frac{5}{10}, \frac{5}{10} \right) \rangle. \]

Then
\[ N_{x_1} = \{ 0_{NS}, G_1, 1_{NS} \} \] and \[ N_{x_2} = \{ 0_{NS}, G_2, 1_{NS} \} \] are NSTs on X and Y respectively.

Define a mapping \( f: (X, N_{x_1}) \rightarrow (Y, N_{x_2}) \) by \( f(a) = u \) and \( f(b) = v \).

Then \( f \) is NS(GS) CTS mapping, but not a NS(WπG)CTS mapping,

since \[ B = \langle y, \left( \frac{2}{10}, \frac{5}{10} \right) \rangle \] is a NS(GS)CS in Y,

but \[ f^{-1}(B) = \langle x, \left( \frac{2}{10}, \frac{5}{10} \right) \rangle \] is not a NS(WπG)CS in X.

Example 3.19:

Let \( X = \{ a, b \}, Y = \{ u, v \} \) and
\[ G_1 = \langle x, \left( \frac{5}{10}, \frac{3}{10} \right) \rangle, \]
\[ G_2 = \langle y, \left( \frac{3}{10}, \frac{5}{10} \right) \rangle. \]

Then \( N_{x_1} = \{ 0_{NS}, G_1, 1_{NS} \} \) and \[ N_{x_2} = \{ 0_{NS}, G_2, 1_{NS} \} \] are NSTs on X and Y respectively.

Define a mapping \( f: (X, N_{x_1}) \rightarrow (Y, N_{x_2}) \) by \( f(a) = u \) and \( f(b) = v \).

Then \( f \) is NS(WπG)CTS mapping, but not a NS(GS) CTS mapping,

since \[ B = \langle y, \left( \frac{2}{10}, \frac{5}{10} \right) \rangle \] is a NS(WπG)CS in Y,

but \[ f^{-1}(B) = \langle x, \left( \frac{2}{10}, \frac{5}{10} \right) \rangle \] is not a NS(GS)CS in X.

### 4. APPLICATIONS OF NEUTROSOPHIC WEAKLY π GENERALIZED CLOSED MAPPING

**Definition 4.1:**

An NSTS \((X, N_{x_2})\) is called a Neutrosophic \(wπT1/2\) space (NSwπT1/2) if every NS\(WπGCS\) in X is an NSCS in X.

**Proposition 4.2:**

A mapping \( f: (X, N_{x_1}) \rightarrow (Y, N_{x_2}) \) is NS(WπG) CTS, then the inverse image of each NSOS in Y is a NS(WπG)OS in X.

**Proof:**

Let \( B \) be a NSOS in Y. This implies \( B^C \) is NSCS in Y. Since \( f \) is NS(WπG) CTS, \( f^{-1}(B^C) \) is NS(WπG)CS in X. Since \( f^{-1}(B^C) = (f^{-1}(B))^C \), \( f^{-1}(B) \) is a NS(WπG)OS in X.

**Proposition 4.3:**

Let \( f: X \rightarrow Y \) be a NS(WπG)CTS mapping and X be a NSwπT1/2 space. Then \( f \) is a NSCTS mapping.
Proof: Let X be a NSwπT1/2 space and B be a NSCS in Y. Then by definition (4.1), \( f^{-1}(B) \) is a NS(WπG)CS in X. So, \( f^{-1}(B) \) is a NSCS in X. Therefore \( f \) is a NSCTS mapping.

**Proposition 4.4:**
A mapping \( f: (X, N_S) \rightarrow (Y, N_S) \) be a NS(WπG)CTS mapping and \( g: (Y, N_S) \rightarrow (Z, N_S) \) is NS continuous, then \( g \circ f: (X, N_S) \rightarrow (Z, N_S) \) is a NS(WπG) CTS.

Proof: Let D be a NSCS in Z. Then by definition \( g^{-1}(D) \) is a NSCS in Y. Since \( f \) is a NS(WπG)CTS mapping, inverse image of a NSCS in Y is a NS(WπG)CS in X. i.e., \( f^{-1} (g^{-1}(D)) = (g 
\circ f)^{-1}(D) \) is a NS(WπG)CS in X. Therefore \( g \circ f \) is a NS(WπG)CTS mapping.

V. CONCLUSION

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In this paper, we introduced the concept of NS(WπG)CTS in Neutrosophic Topological Spaces. This shall be extended in the future Research with some applications

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