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New type of neutrosophic supra connected space *

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Abstract Neutrosophic α supra-connected space is defined and its properties are studied in this paper. The purpose of this theory is to investigate the common relationship between two objects after dropping an axiom in neutrosophic topological spaces. Also, defined herein is a new compactness in neutrosophic supra topological spaces and some of its properties are investigated.

Key words Neutrosophic supra topology, neutrosophic α supra open set, neutrosophic α supra closed set, neutrosophic α supra connected space, neutrosophic α supra compact sapce.

2020 Mathematics Subject Classification **54D05, 54D30, 54D99, 54E45, 54E99.**

1 Introduction

F. Smarandache [16, 17] developed neutrosophic theory as a generalization of Zadeh's [18] fuzzy set (FS) theory and Atanassov's [2] (IFS) theory. Neutrosophic sets gained attention in many fields such as topology [8, 10–12], image processing, algebra, graph theory, medicine, etc. Fuzzy topological space introduced by D. Coker [3] then Salama and Alblowi [15] defined neutrosophic topology. Later on researchers developed his theory and investigated the various types of open and closed sets, continuous function, homeomorphism in neutrosophic topological spaces.
Kuratowski [4] first used the notion of connected space in general topology. Parimala et al. [7, 9, 13] developed various open and closed sets in nano topological space. Parimala et al. [6, 14] also introduced neutrosophic $\alpha\psi$-closed sets, neutrosophic $\alpha\psi$-connected space. Karthika et al. [5] redefined the notion of neutrosophic topology in the extended range, i.e., the range of neutrosophic components from the unit interval to the complex plane and then studied the relationship between neutrosophic complex $\alpha\psi$

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connectedness and neutrosophic complex connected space and the properties of neutrosophic complex $\alpha\psi$ connected space.

1.1 Motivation and objective

The notion of neutrosophic sets and connected space motivates us to generate this novel neutrosophic α supra connected space. Our objective in this paper is to define the neutrosophic α supra connected space and the notion of the neutrosophic α supra compactness and to study their properties. The purpose of these connected spaces in real life situations is to investigate the common relationship between two objects, such as, two different branded cars, the common symptoms between two diseases, etc. The paper is constructed as follows: in section 2, the basic definitions such as neutrosophic set, neutrosophic supra topological space, neutrosophic α open set, arithmetic operations are discussed. The definition of q-coincident, the interior and the closure of a neutrosophic alpha open set, the neutrosophic α supra connected space and its properties are presented in section 3. The neutrosophic α supra compactness is defined and its properties are investigated in section 4. The conclusions and future directions of work of these novel concepts are presented in section 5.

2 Preliminaries

The definitions which are relevant to our work are presented in this section.

Definition 2.1. [16, 17] A neutrosophic set (NS) D on $\mathfrak{W} \neq \emptyset$ is defined by

$$D = \{\langle \xi, \mu_D(\xi), \sigma_D(\xi), \nu_D(\xi)\rangle : \xi \in \mathfrak{W}\}$$

where MF μ_D, INDF σ_D, NMF ν_D maps from $\mathfrak{W}$ to [0,1] for each $\xi \in \mathfrak{W}$ to D and $0 \leq \mu_D(\xi) + \sigma_D(\xi) + \nu_D(\xi) \leq 3$ for each $\xi \in \mathfrak{W}$.

Definition 2.2. [15] Let $D_1 = \{\langle \xi, \mu_{D_1}(\xi), \sigma_{D_1}(\xi), \nu_{D_1}(\xi)\rangle : \xi \in \mathfrak{W}\}$ and $D_2 = \{\langle \xi, \mu_{D_2}(\xi), \sigma_{D_2}(\xi), \nu_{D_2}(\xi)\rangle : \xi \in \mathfrak{W}\}$ be NSs. Then

(i) $D_1 \subseteq D_2$ if and only if $\mu_{D_1}(\xi) \leq \mu_{D_2}((\xi))$, $\sigma_{D_1}(\xi) \geq \sigma_{D_2}(\xi)$ and $\nu_{D_1}(\xi) \geq \nu_{D_2}(\xi)$;

(ii) ${D_1}^C = \{\langle \xi, \nu_{D_1}(\xi), 1 - \sigma_{D_1}(\xi), \mu_{D_1}(\xi)\rangle : \xi \in \mathfrak{W}\}$;

(iii) $D_1 \cap D_2 = \{\langle \xi, \mu_{D_1}(\xi) \wedge \mu_{D_2}(\xi), \sigma_{D_1}(\xi) \vee \sigma_{D_2}(\xi), \nu_{D_1}(\xi) \vee \nu_{D_2}(\xi)\rangle : \xi \in \mathfrak{W}\}$;

(iv) $D_1 \cup D_2 = \{\langle \xi, \mu_{D_1}(\xi) \vee \mu_{D_2}(\xi), \sigma_{D_1}(\xi) \wedge \sigma_{D_2}(\xi), \nu_{D_1}(\xi) \wedge \nu_{D_2}(\xi)\rangle : \xi \in \mathfrak{W}\}$.

The symbols $\vee, \wedge$ denotes the maximum and minimum operator. The NS $(D_1)^C$ denotes the complement of NS D_1.

Definition 2.3. [15] Let $\mathfrak{D}$ be a family of NSs on $\mathfrak{W}$. The pair $(\mathfrak{W}, \mathfrak{D})$ is called a neutrosophic supra topology, if the following conditions are satisfied:

(T1) $0_{\mathfrak{D}}, 1_{\mathfrak{D}} \in \mathfrak{W}$,

(T2) An arbitrary union of NSs D_i is in $\mathfrak{D}$.

Definition 2.4. [1] A subset D of a NTS $(\mathfrak{W}, \mathfrak{D})$ is called

1. a NPOS, if $D \subseteq (\overline{D})^o$ and a NPCS if $\overline{(D^o)} \subseteq D$,
2. a NSOS, if $D \subseteq \overline{(D^o)}$ and a NSCS if $(\overline{D})^o \subseteq D$,
3. a NαSOS, if $D \subseteq (\overline{(D^o)})^o$ and NαSCS if $\overline{(\overline{D})^o} \subseteq D$.

3 On the neutrosophic α supra connected space

Definition 3.1. The interior and closure of NS D in NTS $\mathfrak{W}$ are denoted by D^o, $\overline{D}$ and defined by $D^o = \cup\{C : C$ is an NαSOS in $\mathfrak{W}$ and C $\subseteq D\}$, $\overline{D} = \cap\{C : C$ is an NαSCS in $\mathfrak{W}$ and C $\supseteq D\}$.

Definition 3.2. Two NαSOSs C and D of $\mathfrak{W}$ are said to be q-coincident if and only if there exists an element $\zeta \in \xi$ such that $C(\zeta) + D(\zeta) > 1$ or, $\mu_C(\xi) > \nu_D(\xi), \sigma_C(\xi) < 1 - \sigma_D(\xi), \nu_C(\xi) < \mu_D(\xi)$.

Lemma 3.3. *For any two NSs D and F of $\mathfrak{W}$, $\neg(DqF)$ if and only if $D \subset F^c$.*

Example 3.4. Let $\mathfrak{W} = \{x, y\}$ and $C = \{\frac{x}{(0.5,0.2,0.2)}, \frac{y}{(0.4,0.4,0.6)}\}$, $D = \{\frac{x}{(0.5,0.3,0.3)}, \frac{y}{(0.2,0.5,0.6)}\}$ be two neutrosophic open sets. Let $\mathfrak{D} = \{0_{\mathfrak{D}}, 1_{\mathfrak{D}}, C, D\}$. We know that each neutrosophic open set is a neutrosophic α open set. C and D are q-coincident since the intersection of these two sets is a non-empty set and also C is not a subset of the complement of D.

Definition 3.5. An NTS is said to be neutrosophic α supra connected space, if the intersection of two NαSOSs C and D is non-empty or, if there does not exist an NαSOS NαSCS F in $\mathfrak{W}$ such that $C \subset F \subset D^c$. An NTS is said to be separation of $\mathfrak{W}$, if it is not a neutrosophic α supra connected space.

Theorem 3.6. *If C, D are NαSOSs in $\mathfrak{W}$ which forms a separation of $\mathfrak{W}$ and U is a neutrosophic α-connected subspace of $\mathfrak{W}$, then U is either in C or in D.*

Proof. Let C, D be an NαSOSs in $\mathfrak{W}$. The intersection of an NαSOS C and a neutrosophic α-connected subspace U is an NαSOS in U and the intersection of an NαSOS D and neutrosophic α-connected subspace is an NαSOS in U. These two NαSOSs are disjoint and their union is a neutrosophic α-connected subspace U. These two NαSOSs constitute a separation of U if these two NαSOSs are nonempty. Thus one of the NαSOSs is empty. Hence the neutrosophic α-connected subspace U is definitely either in C or in D. □

Theorem 3.7. *If an NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space. Then it is a neutrosophic connected space.*

Proof. Let C and D be two neutrosophic open sets in $\mathfrak{W}$. If $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic connected space, then there exists a neutrosophic closed open set F in $\mathfrak{W}$ such that $D \subset F$. Then we know that every neutrosophic open(resp. closed) set is an NαSOS(resp. NαSCS), therefore, F is an NαSOS NαSCS in $\mathfrak{W}$ such that $D \subset F$ and $\neg(FqB)$. Hence $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. This is in contradiction to our hypothesis. Therefore, NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic connected space. □

Theorem 3.8. *An NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space if and only if there is no NαSOS NαSCS E in $\mathfrak{W}$ such that C is a subset of E and E is a subset of the complement of D.*

Proof. Let NTS $(\mathfrak{W}, \mathfrak{D})$ be a neutrosophic α supra connected space. Suppose E is NαSOS NαSCS in $\mathfrak{W}$ such that C is a subset of E and E is a subset of complement of D. This implies that $\neg(EqD)$. Therefore, C is NαSOS NαSCS in $\mathfrak{W}$ such that $C \subset D^c$ and $\neg(EqD)$. Hence $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space, which is a contradiction. Hence there is no NαSOS NαSCS E in $\mathfrak{W}$ such that C is a subset of E and E is a subset of complement of D.
Suppose NTS $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. Then there is an NαSOS NαSCS E in $\mathfrak{W}$ such that C is a subset of E and E is a subset of complement of D. Now, $\neg(EqD)$ which implies that E is a subset of D^c.
$\therefore$ E is an NαSOS NαSCS in $\mathfrak{W}$ such that C is a subset of E and E is a subset of the complement of D, which contradicts our assumption. □

Theorem 3.9. *If an NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space, then NαSOSs $C \neq \emptyset$ and $D \neq \emptyset$.*

Proof. If an NαSOS C is empty, then C is a neutrosophic α supra open neutrosophic α supra closed set in $\mathfrak{W}$. Now to prove $\neg(CqD)$. If NαSOSs C and D are q-coincident, then there is a $\xi \in \mathfrak{W}$ such that $\mu_C(\xi) > \nu_D(\xi)$ or, $\nu_C(\xi) < \mu_D(\xi)$. But $\mu_C(\xi) = 0_{\mathfrak{D}}$ and $\nu_C(\xi) = 1_{\mathfrak{D}}$ for all $\xi \in \mathfrak{W}$. Therefore, there exists no point $\xi \in \mathfrak{W}$ for which $\mu_C(\xi) > \nu_D(\xi)$ or, $\sigma_C(\xi) < 1 - \sigma_D(\xi)$ or, $\nu_C(\xi) < \mu_D(\xi)$. Hence $\neg(CqD)$ and an NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space. This contradicts the hypothesis.
$\therefore$ both NαSOSs $C \neq \emptyset$ and $D \neq \emptyset$. □

Theorem 3.10. *If an NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space and $C \subset C_1$ and $D \subset D_1$, then NαSOSs $C_1 \neq \emptyset$ and $D_1 \neq \emptyset$ are not disjoint sets.*

Proof. Suppose an NTS $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. Then there is a NαSOS NαSCS E in $\mathfrak{W}$ such that $C_1 \subset E$ and $\neg(EqD_1)$. Clearly, $C \subset E$. Now we claim that $\neg(EqB)$. If FqB, then there exists a point $\xi \in \mathfrak{W}$ such that $\mu_E(\xi) > \nu_D(\xi)$ or, $\nu_E(\xi) < \mu_D(\xi)$. Suppose $\xi \in \mathfrak{W}$ such that $\mu_E(\xi) > \nu_B(\xi)$. Now $D \subset D_1$, $\mu_D(\xi) \leq \mu_{D_1}(\xi)$, $\sigma_D(\xi) \geq \sigma_{D_1}(\xi)$ $\nu_D(\xi) \geq \nu_{D_1}(\xi)$. So $\mu_E(\xi) > \nu_{D_1}(\xi)$, $\nu_E(\xi) < \mu_{D_1}(\xi)$ and EqD_1, a contradiction. Consequently, NTS $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. □

Example 3.11. Let $\mathfrak{W} = \{x, y\}$ and $C = \{\frac{x}{(0.5,0.2,0.2)}, \frac{y}{(0.4,0.4,0.6)}\}$, $D = \{\frac{x}{(0.5,0.3,0.3)}, \frac{y}{(0.2,0.5,0.6)}\}$, $D_1 = \{\frac{x}{(0.6,0.1,0.2)}, \frac{y}{(0.7,0.2,0.3)}\}$, $C_1 = \{\frac{x}{(0.5,0.2,0.2)}, \frac{y}{(0.4,0.4,0.4)}\}$ be neutrosophic α open sets on $\mathfrak{W}$. Let $\mathfrak{D} = \{0_{\mathfrak{D}}, 1_{\mathfrak{D}}, C, D, C_1, D_1\}$ be neutrosophic topology on $\mathfrak{W}$ and $C \subseteq C_1, D \subseteq D_1$. Then $(\mathfrak{W}, \mathfrak{D})$ is neutrosophic α connected between C_1 and D_1.

Theorem 3.12. *An NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space iff NαSOSs $\overline{C} \neq \emptyset$ and $\overline{D} \neq \emptyset$ are not disjoint sets.*

Proof. The proof of the necessary part follows from Theorem 3.5, since we know that $C \subset \overline{C}$ and $D \subset \overline{D}$. For the sufficient part we assume that NTS $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. Then NαSOS NαSCS E of $\mathfrak{W}$ is such that $C \subset E$ and $\neg(EqD)$. Since E is an NαSCS and $C \subset E$, $\overline{C} \subset \overline{E}$. Now, $\neg(EqD)$, which implies that E is a subset of the complement of D. Therefore, the interior of E is a subset of the interior of the complement of D. Hence $\neg(Eq\overline{D})$ and $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space between $\overline{C}$ and $\overline{D}$. □

Theorem 3.13. *Let C and D be two NαSOSs in $(\mathfrak{W}, \mathfrak{D})$. If C and D are q-coincident, then $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space.*

Theorem 3.14. *An NTS $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space iff every pair of NαSOSs forms a neutrosophic α supra connected space.*

Proof. Necessity: Let C, D be any pair of neutrosophic α open subsets of $\mathfrak{W}$. Suppose $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. Then there is an NαSOS NαSCS E of $\mathfrak{W}$ such that A is a subset of E and $\neg(AqD)$. E is a nonempty NαSOS NαSCS in $(\mathfrak{W}, \mathfrak{D})$ since, NαSOS C and D are non-empty. Hence $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space.
Sufficiency: Suppose NTS $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. Then there is a proper NαSOS NαSCS $E \neq \emptyset$ of $\mathfrak{W}$. Consequently, $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space between E and E^c, which is a contradiction. □

Theorem 3.15. *Let $(\mathfrak{W}_1, \mathfrak{D}_1)$ be a neutrosophic α subspace of an NTS $(\mathfrak{W}, \mathfrak{D})$ and C, D be neutrosophic α open subsets of $\mathfrak{W}_1$. If $(\mathfrak{W}_1, \mathfrak{D}_1)$ be a neutrosophic α supra connected space then $(\mathfrak{W}, \mathfrak{D})$ is also a neutrosophic α supra connected space.*

Proof. Suppose $(\mathfrak{W}, \mathfrak{D})$ is not a neutrosophic α supra connected space. Then there is an NαSOS NαSCS E of $\mathfrak{W}$ such that C is a subset of E and $\neg(EqD)$. But $U = E \cap \mathfrak{D}_1$. Then U is an NαSOS NαSCS in $\mathfrak{D}_1$ such that C is a subset of U and $\neg(UqD)$. Hence $(\mathfrak{W}_1, \mathfrak{D}_1)$ is not a neutrosophic α supra connected space, a contradiction, thus, $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space. □

Theorem 3.16. *If $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic semi-connected space then $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space.*

Proof. Let C, D be two neutrosophic semi-open sets in $\mathfrak{D}$. Suppose $(\mathfrak{W}, \mathfrak{D})$ be not a neutrosophic α supra connected space. Then there is an NαSOS NαSCS E in $\mathfrak{D}$ such that $C \subset E$ and $\neg(EqD)$. We know that every NαSOS is a neutrosophic semi-open set. Therefore E is a neutrosophic semi-open neutrosophic semi-closed set such that $C \subset E$, which is a contradiction. Hence $(\mathfrak{W}, \mathfrak{D})$ is a neutrosophic α supra connected space. □

4 Neutrosophic α supra compactness

The concepts of a neutrosophic α supra open cover and the neutrosophic α supra compactness in a neutrosophic supra topological space are defined and their properties are studied in this section.

Definition 4.1. 1. Let $\mathcal{N} = \{\langle a, \mu_{D_i}, \sigma_{D_i}, \nu_{D_i}\rangle : i \in J\}$ be a collection of neutrosophic α supra open sets. The collection $\mathcal{N}$ of subsets of neutrosophic supra topological space $(\mathfrak{W}, \mathfrak{D})$ is said to be a neutrosophic α supra cover of $\mathfrak{W}$ if the union of $\{\langle a, \mu_{D_i}, \sigma_{D_i}\nu_{D_i}\rangle : i \in J\}$ is $1_{\mathfrak{D}}$. It is called a neutrosophic α supra open covering of $\mathfrak{W}$ if its NSs are neutrosophic α open subsets of $\mathfrak{W}$.

2. Let $\mathcal{N}_i$ be a subfamily of neutrosophic α supra open covers on $\mathfrak{W}$. A neutrosophic supra topological space is said to be neutrosophic α supra compact if every neutrosophic α supra open covering $\mathcal{N}$ of $\mathfrak{W}$ contains a finite sub collection that also covers $\mathfrak{W}$.

Proposition 4.2. *A neutrosophic supra topological space $(\mathfrak{W}, \mathfrak{D})$ is called neutrosophic α supra compact if and only if every neutrosophic α supra open cover of $\mathfrak{W}$ has a finite neutrosophic α supra subcover.*

Lemma 4.3. *The subspace $\mathfrak{V}$ of $\mathfrak{W}$ is neutrosophic α supra compact if and only if every covering of $\mathfrak{V}$ by sets which are neutrosophic α supra open in $\mathfrak{W}$ contains a finite subcollection covering $\mathfrak{V}$.*

Proof. Suppose $\mathfrak{V}$ is compact, so the collection of neutrosophic α supra open sets $\mathcal{N} = \{\mathcal{N}_i : i \in J\}$ covers $\mathfrak{V}$. Then $\{\mathcal{N}_i \bigcap \mathfrak{V} : i \in J\}$ covers $\mathfrak{V}$. These sets are neutrosophic α supra open sets in $\mathfrak{V}$. The finite subcollection $\{\mathcal{N}_{i_1} \bigcap \mathfrak{V}, \ldots, \mathcal{N}_{i_n} \bigcap \mathfrak{V}\}$ also covers $\mathfrak{V}$. This implies that the finite subcollection $\{\mathcal{N}_{i_1}, \ldots, \mathcal{N}_{i_n}\}$ of $\mathfrak{N}$ covers $\mathfrak{V}$.
Conversely, assume that every neutrosophic α supra open set covering of $\mathfrak{V}$ contains a finite subcollection which also covers $\mathfrak{V}$. Let $\mathcal{B}$ be the family of neutrosophic α supra open sets which covers $\mathfrak{V}$. These neutrosophic α supra open sets are in $\mathfrak{V}$. For each i, choose a neutrosophic α supra open set $\mathcal{N}_i$ in $\mathfrak{W}$ such that $\mathcal{B} = \mathcal{N}_i \bigcap \mathfrak{V}$. The collection of neutrosophic α supra open set $\mathcal{N} = \{\mathcal{N}_i\}$ covers $\mathfrak{V}$, by our assumption, some finite subcollection $\{\mathcal{N}_{i_1}, \ldots, \mathcal{N}_{i_n}\}$ covers $\mathfrak{V}$. Then the finite subcollection $\mathcal{B}_1, \ldots, \mathcal{B}_n$ covers $\mathfrak{V}$. □

Theorem 4.4. *Every neutrosophic α closed subspace of a neutrosophic α supra compact space is neutrosophic α supra compact.*

Theorem 4.5. *If the map $f : \mathfrak{W} \to \mathfrak{V}$ is neutrosophic α supra continuous and $\mathfrak{W}$ is a neutrosophic α supra compact space, then $\mathfrak{V}$ is a neutrosophic α supra compact space.*

Proof. Given that the map $f : \mathcal{W} \longrightarrow \mathfrak{V}$ is a neutrosophic α supra continuous map and $\mathfrak{W}$ is a neutrosophic α supra compact space. Let the collection $\mathcal{C}$ be a covering of neutrosophic α supra open sets $f(\mathfrak{W})$ in $\mathfrak{V}$. Since the map f is continuous, the collection $\{f^{-1}(C) : C \in \mathcal{C}\}$ is a neutrosophic α supra open set covering of X. Hence $f^{-1}(C_1), \ldots, f^{-1}(C_n)$ cover $\mathfrak{W}$. Then the neutrosophic α supra open sets $C_1, \ldots, C_n$ cover $f(\mathfrak{W})$. □

Definition 4.6. A collection of neutrosophic α supra open sets $\mathcal{N}$ of subsets of $\mathfrak{W}$ is said to have finite intersection property if for every finite subcollection $\{\mathcal{N}_1, \ldots, \mathcal{N}_n\}$ of $\mathcal{N}$, the intersection of $\mathcal{N}_1, \ldots, \mathcal{N}_n$ is nonempty.

Theorem 4.7. *Let $\mathfrak{W}$ be a neutrosophic supra topological space. An NSTS $\mathfrak{W}$ is neutrosophic α supra compact if and only if for every collection $\mathcal{N}$ of neutrosophic α supra closed sets in $\mathfrak{W}$ having finite intersection property, $\cap_{A \in \mathcal{N}} A$ of all elements of $\mathcal{N}$ is non-empty.*

Proof. Let $\mathcal{C}$ be a collection of neutrosophic α supra open sets and $\mathcal{D}$ be a neutrosophic α supra closed set, i.e., $\mathcal{D} = \{\mathfrak{W} - C : C \in \mathcal{C}\}$. We know that C covers $\mathfrak{W}$ if and only if $\cap_{D \in \mathcal{D}} D$ of all elements of $\mathcal{D}$ is empty.
The finite subcollection $\{C_1, \ldots, C_n\}$ of $\mathcal{C}$ covers $\mathfrak{W}$ if and only if the intersection of the corresponding subcollection $\{\mathfrak{W} - C_1, \ldots, \mathfrak{W} - C_n\}$ is empty.
Contrarily, if there is no finite subcollection of given neutrosophic α supra open sets $\mathcal{C}$ covering $\mathfrak{W}$, then the given collection $\mathcal{C}$ does not cover $\mathfrak{W}$. This implies that $\cap_{D \in \mathcal{D}}$ is non-empty. This is $\Longrightarrow \Longleftarrow$. □

Theorem 4.8. *A finite union of neutrosophic α supra compact subspaces of $\mathfrak{W}$ is neutrosophic α supra compact.*

5 Conclusions

The neutrosophic α supra connected space is defined in this paper and we studied the relationship between two neutrosophic sets and the properties of neutrosophic α supra connected space. Further the neutrosophic α supra compact space is introduced and its properties were also discussed. The local α connectedness and local α compactness on neutrosophic supra topological space will be our future work.

Abbreviations:

FS	- Fuzzy set
IFS	- Intuitionistic Fuzzy set
MF	- membership function
INDF	- indeterminacy
NMF	- non-membership function
NTS	- Neutrosophic Topological space
NS	- Neutrosophic set
NαSOS	- Neutrosophic alpha supra open set
NαSCS	- Neutrosophic alpha supra closed set

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References

[1] Arokiarani, I., Dhavaseelan, R., Jafari, S. and Parimala, M. (2017). On some new notions and functions in neutrosophic topological spaces, *Neutrosophic Sets Syst.*, 16, 16–19.

[2] Atanassov, K.T. (1986). Intuitionstic fuzzy sets, *Fuzzy Sets and Systems*, 20, 87–96.

[3] Coker, D. (1997). An introduction to fuzzy topological spaces, *Fuzzy Sets and Systems*, 88, 81–89.

[4] Kuratowski, K. (1966). Topology Vol. II (transl.), Academic Press, New York.

[5] Karthika, M., Parimala, M., Jafari, S., Smarandache, F., Alshumrani, Mohammad, Ozel, Cenap and Udhayakumar, R. (2019). Neutrosophic complex $\alpha\psi$ connectedness in neutrosophic complex topological spaces, *Neutrosophic Sets and Systems*, 29,(2019), 158–164.

[6] Parimala, M., Karthika, M., Jafari, Saeid, Smarandache, Florentin and El-Atik, A.A. (2020). Neutrosophic $\alpha\psi$-connectedness,*Journal of Intelligent and Fuzzy Systems*, 38, 853- 857.

[7] Parimala, M., Jafari, S. and Murali, S. (2017). Nano ideal generalized closed sets in nano ideal topological spaces, *Annales Univ. Sci. Budapest.*, 60, 3–11.

[8] Parimala, M., Jeevitha, R., Jafari, S., Smarandache, F. and Udhayakumar, (2018). Neutrosophic $\alpha\psi$-homeomorphism in neutrosophic topological spaces, *Information*, 9, 187, 1-10. ; doi:10.3390/info9080187.

[9] Parimala, M., Jeevitha, R. and Selvakumar, A. (2017). A new type of weakly closed set in ideal topological spaces, *International Journal of Mathematics and its Applications*, 5(4-C), 301–312.

[10] Parimala, M., Karthika, M., Dhavaseelan, R. and Jafari, S. (2018). On neutrosophic supra pre-continuous functions in neutrosophic topological spaces, *New Trends in Neutrosophic Theory and Applications* , 2, 371–383.

[11] Parimala, M., Karthika, M., Jafari, S., Smarandache, F. and Udhayakumar, R. (2019). Neutrosophic nano ideal topological structures, *Neutrosophic Sets and Systems*, 24, 70–77.

[12] Parimala, M. , Karthika, M., Jafari, S., Smarandache, F. and Udhayakumar, R. (2018). Decision-making via neutrosophic support soft topological spaces, *Symmetry*, 10, 1–10. doi:10.3390/sym10060217

[13] Parimala, M. and Perumal, R. (2016). Weaker form of open sets in nano ideal topological spaces, *Global Journal of Pure and Applied Mathematics*, 12(1), 302–305.

[14] Parimala, M., Smarandache, F., Jafari, S. and Udhayakumar, R. (2018). On neutrosophic $\alpha\psi$ -closed sets, *Information*, 9, 103, 1–7.

[15] Salama, A.A. and Alblowi, S.A. (2012). Neutrosophic set and neutrosophic topological spaces, *IOSR J. Math.*, 3, 31–35.

[16] Smarandache, F. (2002) Neutrosophy and neutrosophic logic, *First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics*, University of New Mexico, Gallup, NM, USA, (2002).

[17] Smarandache, F. (1999). A unifying field in logics: neutrosophic logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, USA.

[18] Zadeh, L.A. (1965). Fuzzy sets, *Information and Control*, 18, 338–353.