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NEW TYPE OF NEUTROSOPHIC OFF GRAPHS

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ABSTRACT. In this paper, the concept of neutrosophic off graph, total neutrosophic off graph and middle neutrosophic off graph are introduced. Several interesting properties along with the examples are established.

1. INTRODUCTION

In 1965, Zadeh [11] introduced the notion of a fuzzy subset of a set as a method for representing uncertainity. A. Rosenfeld [5] was introduced the idea of fuzzy graph. The properties of fuzzy graphs are very much useful in obtaining solutions to many problems like traffic congestion problem, networking, etc,. M.Akram [1] introduced the concept of Strong intuitionistic fuzzy graphs. NagoorGani [3] introduced busy Nodes and free Nodes, order, degree, size in intuitionistic fuzzy graph. The concepts of total and middle intuitionistic fuzzy graph was introduced by NagoorGani and Rahman [4]. R.Narmada Devi [6,7] were introduced the concepts of neutrosophic complex $\mathcal{N}$-continuity and neutrosophic complex graphs. Moreover, F. Smarandache [8,9] introduced the idea of neutrosophic set theory and their logic. Also he introduced the concept of

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neutrosophic off set [10]. S. Broumi [2] was studied the properties of single valued neutrosophic graphs. In this paper the concepts of neutrosophic off graph and their interesting properties are discussed.

2. Preliminaries

Definition 1. ***[10]*** *Let $\mathcal{U}$ be a universe of discourse and the neutrosophic set A_3 in $\mathcal{U}$. Let $T(x), I(x), F(x)$ be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in \mathcal{U}$, with respect to the set A_3 : $T(x), I(x), F(x) : \mathcal{U} \to [\Psi, \Omega]$ where $\Psi < 0 < 1 < \Omega$ and Ψ is called underlimit, while Ω is called overlimit and $T(x), I(x), F(x) \in [\Psi, \Omega]$. A Single-Valued Neutrosophic Offset(A_3 is defined as: $A_3 = (x, < T(x), I(x), F(x) >), x \in \mathcal{U}$ such that there exist some elements in A_3 that have at least one neutrosophic component that is > 1, and at least another neutrosophic component that is < 0.*

Definition 2. ***[4]*** *The busy value of a node v of an IFG $G =< V, E >$ is $(D_\mu(v), D_\gamma(v))$ where $D_\mu(v) = \sum \mu_1(v) \wedge \mu_1(v_i)$ and $D_\gamma(v) = \sum \gamma_1(v) \wedge \gamma_1(v_i)$ where v_i are neighbours of v. The busy value of an $IFGG$ is defined to be the sum of the busy values of an IFG G is defined to be the sum of the busy values of all nodes of G. (i.e.)$D(G) = (\sum_i D_\mu(v_i), \sum_i D_\gamma(v_i))$ where v_i are nodes of G.*

3. New View on Neutrosophic Off Graphs

In this section, the concept of neutrosophic off graph is introduced. Some types of neutrosophic off graph are studied. Several interesting properties along with the examples are established.

Definition 3. *A pair $G = (A, B)$ is called a neutrosophic off graph (in short., $\mathcal{N}$offG,)on a crisp graph $G^* = (V, E)$, where $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ is a neutrosophic off set on V, for every $x \in V$ and $B = \langle xy, T_B(xy), I_B(xy), F_B(xy)\rangle$ is a $Noff$ set on E such that*

(i) $T_B(xy) \leq \min[T_A(x), T_A(y)]$,
(ii) $I_B(xy) \leq \min[I_A(x), I_A(y)]$ *and*
(iii) $F_B(xy) \geq \max[F_A(x), F_A(y)]$, *for every $xy \in E \subseteq V \times V$.*

Then A and B are neutrosophic vertex off set on V and neutrosophic edge off set on E respectively.

Definition 4. *Let G be any a $\mathcal{N}offG$ of a crisp graph G^*. Then $H = (A_1, B_1)$ is called a neutrosophic off subgraph($\mathcal{N}offsubG$) if*

(i) $T_{A_1}(x) = T_A(x), I_{A_1}(x) = I_A(x), F_{A_1}(x) = F_A(x)$*, for all* $x \in V_1 \subseteq V$*,*
(ii) $T_{B_1}(xy) = T_B(xy), I_{B_1}(xy) = I_B(xy), F_{B_1}(xy) = F_B(xy)$*, for all* $xy \in E_1 \subseteq E$.

Definition 5. *A $\mathcal{N}offG$ $C(G) = (C(A), C(B))$ is called a complement of a $\mathcal{N}offG$ G if*

$$C(A) = A, T_{C(B)}(xy) = [T_A(x) \wedge T_A(y)] - T_B(xy),$$
$$I_{C(B)}(xy) = [I_A(x) \wedge I_A(y)] - I_B(xy) \text{ and}$$
$$F_{C(B)}(xy) = [F_A(x) \vee F_A(y)] - F_B(xy),$$

for every $xy \in E$.

Definition 6. *A $\mathcal{N}offG$ G is called a complete $\mathcal{N}offG$(in short.,$\mathfrak{comp}\mathcal{N}offG$) if $T_B(xy) = \min[T_A(x), T_A(y)]$, $I_B(xy) = \min[I_A(x), I_A(y)]$ and $F_B(xy) = \max[F_A(x), F_A(y)]$, for every $x, y \in V$.*

Definition 7. *A $\mathcal{N}offG$ G is called a $\mathfrak{Strong}\mathcal{N}offG$ if $T_B(xy) = \min[T_A(x), T_A(y)]$, $I_B(xy) = \min[I_A(x), I_A(y)]$ and $F_B(xy) = \max[F_A(x), F_A(y)]$, for every $xy \in E$.*

Definition 8. *Let G be any $\mathcal{N}offG$ of a crisp graph G^*. Then the* ***degree of a vertex*** *$x \in V$ is $deg_G(x) = \langle T_{deg_G}(x), I_{deg_G}(x), F_{deg_G}(x)\rangle$ where $T_{deg_G}(x) = \sum_{x \neq y} T_B(xy), I_{deg_G}(x) = \sum_{x \neq y} I_B(xy),$ and $F_{deg_G}(x) = \sum_{x \neq y} F_B(xy)$.*

Proposition 3.1. *Let G be any $\mathcal{N}offG$ of a crisp graph G^*. Then $\sum_j deg_G(x_j) = \langle 2(\sum_{x \neq y} T_B(xy)), 2(\sum_{x \neq y} I_B(xy)), 2(\sum_{x \neq y} F_B(xy))\rangle$.*

Definition 9. *Let G be any $\mathcal{N}offG$ of a crisp graph G^*. Then the*

(i) ***size*** *of a $\mathcal{N}offG$ G is defined by $S(G) = \langle T_S(G), I_S(G), F_S(G)\rangle$ where $T_S(G) = \sum_{x \neq y} T_B(xy)$, $I_S(G) = \sum_{x \neq y} I_B(xy)$, and $F_S(G) = \sum_{x \neq y} F_B(xy)$.*
(ii) ***order*** *of a $\mathcal{N}offG$ G is defined by $Ord(G) = \langle T_{Ord}(G), I_{Ord}(G), F_{Ord}(G)\rangle$ where $T_{Ord}(G) = \sum_{x_i \in V} T_A(x_i)$, $I_{Ord}(G) = \sum_{x_i \in V} I_A(x_i)$, and $F_{Ord}(G) = \sum_{x_i \in V} F_B(x_i)$.*

(iii) ***busy value of a vertex*** $v \in V$ *of* G *is* $Busy_G(v) = \langle T_{busy_G}(v), I_{busy_G}(v), F_{busy_G}(v)\rangle$ *where* $T_{busy_G}(v) = \sum_i T_A(v) \wedge T_A(v_i), I_{busy_G}(v) = \sum_i I_A(v) \wedge I_A(v_i)$ *and* $F_{busy_G}(v) = \sum_i F_A(v) \vee F_A(v_i)$ *where* v_i *are neighbours of* v*.*

(iv) ***busy value of a*** $\mathcal{N}$***off****G* G *is* $Busy(G) = \langle T_{busy}(G), I_{busy}(G), F_{busy}(G)\rangle$ *where* $T_{busy}(G) = \sum_{v\in V} T_{busy_G}(v), I_{busy}(G) = \sum_{v\in V} I_{busy_G}(v)$ *and* $F_{busy}(G) = \sum_{v\in V} F_{busy_G}(v)$*.*

Proposition 3.2. *Let* G *be any* $\mathcal{N}$*offG of a crisp graph* G^**. Let* $v \in V$*. Then* $deg_{C(G)}(v) = Busy_G(v) - deg_G(v)$*.*

Remark 3.1. *From the definition of a complete* $\mathcal{N}$*offG G, it follows that for each vertex* $v \in V$*,* $Busy(v) = deg_G(v)$*.*

Definition 10. *Let* G *be any* $\mathcal{N}$*offG of a crisp graph* G^**. Then A node or vertex* $v \in V$ *of a* $\mathcal{N}$*offG* G *is said to be a* **busy node (or) busy vertex** *if* $T_A(v) \leq T_{deg_G}(v), I_A(v) \leq I_{deg_G}(v)$*and* $F_A(v) \geq F_{deg_G}(v)$*; otherwise it is called a* ***free node****. An edge* $uv \in E$ *of a* $\mathcal{N}$*offG* G *is said to be an* ***effective edge*** *if* $T_B(uv) = T_A(u) \wedge T_A(v), I_B(uv) = I_A(u) \wedge I_A(v)$ *and* $F_B(uv) = F_A(u) \vee F_A(v)$*.*

Proposition 3.3. *Let* G *be any* $\mathcal{N}$*offG of a crisp graph* G^* *If* G *has effective edges then it has atleast one busy vertex.*

Definition 11. *Let* G *be any* $\mathcal{N}$*offG of a crisp graph* G^**. Let* $T(G) = (A_T, B_T)$ *be a total* $\mathcal{N}$*offG of* G *which is defined as follows: Let the vertex set of* $T(G)$ *be* $V \cup E$*. Then the* $Noff$ *set* A_T *is defined on* $V \cup E$ *as follows: for* $w \in V \cup E$*,*

(i) $T_{A_T}(w) = T_A(u), I_{A_T}(w) = I_A(u)$ *and* $F_{A_T}(w) = F_A(u), w = u \in V$*.*
(ii) $T_{A_T}(w) = T_B(e), I_{A_T}(w) = I_B(e)$ *and* $F_{A_T}(w) = F_B(e), w = e \in E$*.*

The $Noff$ *set* B_T *is defined on* $V \cup E \times V \cup E$ *as follows:*
case(a) $T_{B_T}((u,v)) = T_B(u,v)$*,* $I_{B_T}((u,v)) = I_B(u,v)$ *and* $F_{B_T}((u,v)) = F_B(u,v), u, v \in V$*.*
case(b) $T_{B_T}((u,e)) = T_A(u) \wedge T_B(e)$ *if* $u \in V$*,* $e \in E$ *and the vertex* u *lies on the edge* e*, otherwise* $T_{B_T}((u,e)) = 0$*.* $I_{B_T}((u,e)) = I_A(u) \wedge I_B(e)$ *if* $u \in V$*,* $e \in E$ *and the vertex* u *lies on the edge* e*, otherwise* $I_{B_T}((u,e)) = 0$*.* $F_{B_T}((u,e)) = F_A(u) \vee F_B(e)$ *if* $u \in V$*,* $e \in E$ *and the vertex* u *lies on the edge* e*, otherwise* $F_{B_T}((u,e)) = 0$*.*
case(c) $T_{B_T}((e_j,e_k)) = T_B(e_j) \wedge T_B(e_k)$ *if* $e_j, e_k \in E$ *and the edges* e_j *and* e_k *have a common vertex, otherwise* $T_{B_T}((e_j,e_k)) = 0$*.* $I_{B_T}((e_j,e_k)) = I_B(e_j) \wedge I_B(e_k)$ *if*

$e_j, e_k \in E$ and the edges e_j and e_k have a common vertex, otherwise $I_{B_T}((e_j, e_k)) = 0$. $F_{B_T}((e_j, e_k)) = F_B(e_j) \vee F_B(e_k)$ if $e_j, e_k \in E$ and the edges e_j and e_k have a common vertex, otherwise $F_{B_T}((e_j, e_k)) = 0$.

By the Definition, $T_{B_T}((u,v)) \leq T_{A_T}(u) \wedge T_{A_T}(v), I_{B_T}((u,v)) \leq I_{A_T}(u) \wedge I_{A_T}(v)$ and $F_{B_T}((u,v)) \geq F_{A_T}(u) \vee F_{A_T}(v)$, for every $u, v \in V \cup E$. Then $T(G) = (A_T, B_T)$ is a total $\mathcal{N}$off$G$ of G.

Example 3.1. *The following graph is a example of total $\mathcal{N}$offG.*

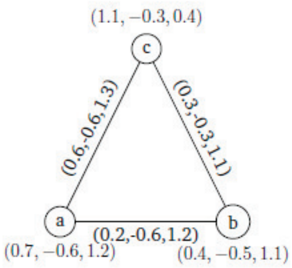

FIGURE 1. G

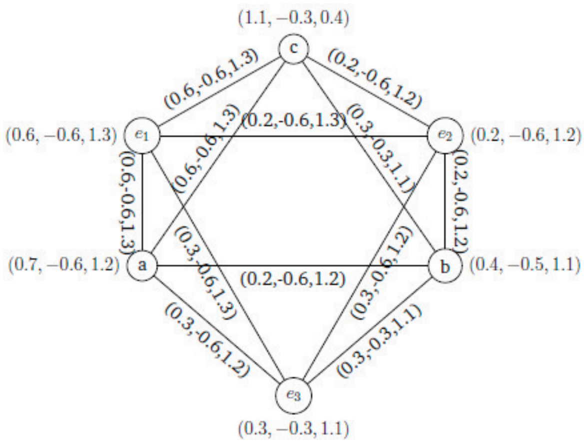

FIGURE 1. T(G)

Proposition 3.4. *Let G be any $\mathcal{N}$offG of a crisp graph G^*. If G is a $\mathfrak{Strong}\mathcal{N}$offG, then $S(T(G)) = 3S(G) + \langle \sum_{e_j,e_k \in E} T_B(e_j) \wedge T_B(e_k),\ \sum_{e_j,e_k \in E} I_B(e_j) \wedge I_B(e_k),\ \sum_{e_j,e_k \in E} F_B(e_j) \vee F_B(e_k) \rangle$.*

Definition 12. *Let G be any $\mathcal{N}$offG of a crisp graph G^*. Let $M(G) = (A_M, B_M)$ be a middle $\mathcal{N}$offG of G which is defined as follows: Let the vertex set of $M(G)$ be $V \cup E$. Then the $Noff$ set A_M is defined on VUE as follows: for $w \in V \cup E$,*

(i) $T_{A_M}(w) = T_A(u), I_{A_M}(w) = I_A(u)$ *and* $F_{A_M}(w) = F_A(u), w = u \in V$.
(ii) $T_{A_M}(w) = T_B(e), I_{A_M}(w) = I_B(e)$ *and* $F_{A_M}(w) = F_B(e), w = e \in E$.

The $Noff$ set B_M is defined on $V \cup E \times V \cup E$ as follows:
case(a) $T_{B_M}((u,v)) = 0, I_{B_M}((u,v)) = 0$ *and* $F_{B_M}((u,v)) = 0, u, v \in V$.
case(b) *$T_{B_M}((u,e)) = T_B(e)$ if $u \in V$, $e \in E$ and the vertex u lies on the edge e, otherwise $T_{B_M}((u,e)) = 0$. $I_{B_M}((u,e)) = I_B(e)$ if $u \in V$, $e \in E$ and the vertex u*

lies on the edge e*,otherwise* $I_{B_M}((u,e)) = 0$. $F_{B_M}((u,e)) = F_B(e)$ *if* $u \in V$, $e \in E$ *and the vertex* u *lies on the edge* e*,otherwise* $F_{B_M}((u,e)) = 0$.
case(c) $T_{B_M}((e_j,e_k)) = T_B(e_j) \wedge T_B(e_k)$ *if* $e_j, e_k \in E$ *and the edges* e_j *and* e_k *have a common vertex, otherwise* $T_{B_M}((e_j,e_k)) = 0$. $I_{B_M}((e_j,e_k)) = I_B(e_j) \wedge I_B(e_k)$ *if* $e_j, e_k \in E$ *and the edges* e_j *and* e_k *have a common vertex, otherwise* $I_{B_M}((e_j,e_k)) = 0$. $F_{B_M}((e_j,e_k)) = F_B(e_j) \vee F_B(e_k)$ *if* $e_j, e_k \in E$ *and the edges* e_j *and* e_k *have a common vertex,otherwise* $F_{B_M}((e_j,e_k)) = 0$.

By the Definition, $T_{B_M}((u,v)) \leq T_{A_M}(u) \wedge T_{A_M}(v), I_{B_M}((u,v)) \leq I_{A_M}(u) \wedge I_{A_M}(v)$ and $F_{B_M}((u,v)) \geq F_{A_M}(u) \vee F_{A_M}(v)$, for every $u, v \in V \cup E$. Then $M(G) = (A_M, B_M)$ is a middle $\mathcal{N}$off$G$ of G.

Example 3.2. *The following graph is a example of middle* $\mathcal{N}$*offG of* G.

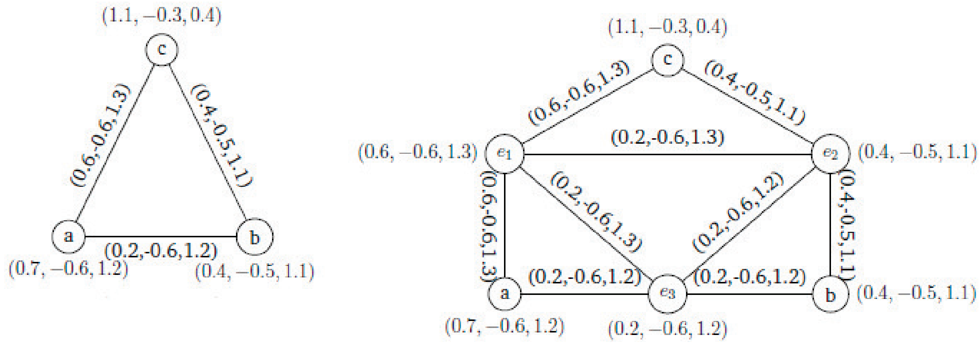

FIGURE 2. G and T(G)

Proposition 3.5. *Let* $T(G) = (A_T, B_T)$ *be a total* $\mathcal{N}$*offG of a* $\mathfrak{Strong}\mathcal{N}$*offG* G. *Then*

(i) $deg_{T(G)}(u) = 2deg_G(u)$ *for all* $u \in V$.
(ii) $deg_{T(G)}(e_i) = Busy_{T(G)}(e_i)$, *if* $e_i \in E$.

Proposition 3.6. *Let* G *be any* $\mathcal{N}$*offG of a crisp graph* G^*. *If* G *is a* $\mathfrak{Strong}\mathcal{N}$*offG, then* $S(M(G)) = 2S(G) + \langle \sum_{e_j,e_k \in E} T_B(e_j) \wedge T_B(e_k), \sum_{e_j,e_k \in E} I_B(e_j) \wedge I_B(e_k), \sum_{e_j,e_k \in E} F_B(e_j) \vee F_B(e_k) \rangle$.

Proposition 3.7. *Let* $M(G) = (A_M, B_M)$ *be a middle* $\mathcal{N}$*offG of a* $\mathfrak{Strong}\mathcal{N}$*offG* G. *Then*

(i) $deg_{M(G)}(u) = deg_G(u)$ *for all* $u \in V$.

(ii) *If* $u = e_j$ *and* $e_j, e_k \in E$ *are adjacent in* G^**, then*

$$deg_{M(G)}(e_j) = 2\langle \sum_{e_j \in E} T_B(e_j), \sum_{e_j \in E} I_B(e_j), \sum_{e_j \in E} F_B(e_j)\rangle + \langle \sum_{e_k \in E} T_B(e_j) \wedge T_B(e_k), \sum_{e_k \in E} I_B(e_j) \wedge I_B(e_k), \sum_{e_k \in E} F_B(e_j) \vee F_B(e_k)\rangle.$$

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