NEW TYPE OF NEUTROSOPHIC OFF GRAPHS

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ABSTRACT. In this paper, the concept of neutrosophic off graph, total neutrosophic off graph and middle neutrosophic off graph are introduced. Several interesting properties along with the examples are established.

1. INTRODUCTION

In 1965, Zadeh [11] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. A. Rosenfeld [5] was introduced the idea of fuzzy graph. The properties of fuzzy graphs are very much useful in obtaining solutions to many problems like traffic congestion problem, networking, etc.. M.Akram [1] introduced the concept of Strong intuitionistic fuzzy graphs. NagoorGani [3] introduced busy Nodes and free Nodes, order, degree, size in intuitionistic fuzzy graph. The concepts of total and middle intuitionistic fuzzy graph was introduced by NagoorGani and Rahman [4]. R.Narmada Devi [6,7] were introduced the concepts of neutrosophic complex \( N \)-continuity and neutrosophic complex graphs. Moreover, F. Smarandache [8,9] introduced the idea of neutrosophic set theory and their logic. Also he introduced the concept of

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neutrosophic off set [10]. S. Broumi [2] was studied the properties of single valued neutrosophic graphs. In this paper the concepts of neutrosophic off graph and their interesting properties are discussed.

2. Preliminaries

**Definition 1.** [10] Let \( \mathcal{U} \) be a universe of discourse and the neutrosophic set \( A_3 \) in \( \mathcal{U} \). Let \( T(x), I(x), F(x) \) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element \( x \in \mathcal{U} \), with respect to the set \( A_3 : T(x), I(x), F(x) : \mathcal{U} \rightarrow [\Psi, \Omega] \) where \( \Psi < 0 < 1 < \Omega \) and \( \Psi \) is called underlimit, while \( \Omega \) is called overlimit and \( T(x), I(x), F(x) \in [\Psi, \Omega] \). A Single-Valued Neutrosophic Offset \( A_3 \) is defined as: \( A_3 = (x, < T(x), I(x), F(x) >), x \in \mathcal{U} \) such that there exist some elements in \( A_3 \) that have at least one neutrosophic component that is \( > 1 \), and at least another neutrosophic component that is \( < 0 \).

**Definition 2.** [4] The busy value of a node \( v \) of an IFG \( G = \langle V, E \rangle \) is \( (D_\mu(v), D_\gamma(v)) \) where \( D_\mu(v) = \sum \mu_1(v) \wedge \mu_1(v_i) \) and \( D_\gamma(v) = \sum \gamma_1(v) \wedge \gamma_1(v_i) \) where \( v_i \) are neighbours of \( v \). The busy value of an IFGG is defined to be the sum of the busy values of an IFG \( G \) is defined to be the sum of the busy values of all nodes of \( G \). (i.e.) \( D(G) = (\sum_i D_\mu(v_i), \sum_i D_\gamma(v_i)) \) where \( v_i \) are nodes of \( G \).

3. New View on Neutrosophic Off Graphs

In this section, the concept of neutrosophic off graph is introduced. Some types of neutrosophic off graph are studied. Several interesting properties along with the examples are established.

**Definition 3.** A pair \( G = (A, B) \) is called a neutrosophic off graph (in short, NoffG, ) on a crisp graph \( G^* = (V, E) \), where \( A = \langle x, T_A(x), I_A(x), F_A(x) \rangle \) is a neutrosophic off set on \( V \), for every \( x \in V \) and \( B = \langle xy, T_B(xy), I_B(xy), F_B(xy) \rangle \) is a Noff set on \( E \) such that

(i) \( T_B(xy) \leq \min[T_A(x), T_A(y)] \),

(ii) \( I_B(xy) \leq \min[I_A(x), I_A(y)] \) and

(iii) \( F_B(xy) \geq \max[F_A(x), F_A(y)] \), for every \( xy \in E \subseteq V \times V \).
Then $A$ and $B$ are neutrosophic vertex off set on $V$ and neutrosophic edge off set on $E$ respectively.

Definition 4. Let $G$ be any a $N_{off}G$ of a crisp graph $G^*$. Then $H = (A_1, B_1)$ is called a neutrosophic off subgraph($N_{off sub}G$) if

(i) $T_{A_1}(x) = T_A(x), I_{A_1}(x) = I_A(x), F_{A_1}(x) = F_A(x)$, for all $x \in V_1 \subseteq V$,
(ii) $T_{B_1}(xy) = T_B(xy), I_{B_1}(xy) = I_B(xy), F_{B_1}(xy) = F_B(xy)$, for all $xy \in E_1 \subseteq E$.

Definition 5. A $N_{off}G C(G) = (C(A), C(B))$ is called a complement of a $N_{off}G$ $G$ if

$$
C(A) = A, T_{C(B)}(xy) = [T_A(x) \land T_A(y)] - T_B(xy),
I_{C(B)}(xy) = [I_A(x) \land I_A(y)] - I_B(xy) \text{ and }
F_{C(B)}(xy) = [F_A(x) \lor F_A(y)] - F_B(xy),
$$

for every $xy \in E$.

Definition 6. A $N_{off}G G$ is called a complete $N_{off}G$(in short., $compN_{off}G$) if $T_B(xy) = \min[T_A(x), T_A(y)], I_B(xy) = \min[I_A(x), I_A(y)]$ and $F_B(xy) = \max[F_A(x), F_A(y)]$, for every $x, y \in V$.

Definition 7. A $N_{off}G G$ is called a Strong $N_{off}G$ if $T_B(xy) = \min[T_A(x), T_A(y)], I_B(xy) = \min[I_A(x), I_A(y)]$ and $F_B(xy) = \max[F_A(x), F_A(y)]$, for every $xy \in E$.

Definition 8. Let $G$ be any $N_{off}G$ of a crisp graph $G^*$. Then the degree of a vertex $x \in V$ is $deg_{G^*}(x) = \langle T_{deg_{G^*}}(x), I_{deg_{G^*}}(x), F_{deg_{G^*}}(x) \rangle$ where $T_{deg_{G^*}}(x) = \sum_{x \neq y} T_B(xy), I_{deg_{G^*}}(x) = \sum_{x \neq y} I_B(xy)$, and $F_{deg_{G^*}}(x) = \sum_{x \neq y} F_B(xy)$.

Proposition 3.1. Let $G$ be any $N_{off}G$ of a crisp graph $G^*$. Then $\sum_j deg_{G^*}(x_j) = <2(\sum_{x \neq y} T_B(xy)), 2(\sum_{x \neq y} I_B(xy)), 2(\sum_{x \neq y} F_B(xy))>.$

Definition 9. Let $G$ be any $N_{off}G$ of a crisp graph $G^*$. Then the

(i) size of a $N_{off}G G$ is defined by $S(G) = \langle T_S(G), I_S(G), F_S(G) \rangle$ where $T_S(G) = \sum_{x \neq y} T_B(xy), I_S(G) = \sum_{x \neq y} I_B(xy)$, and $F_S(G) = \sum_{x \neq y} F_B(xy)$.
(ii) order of a $N_{off}G G$ is defined by $Ord(G) = \langle T_{Ord}(G), I_{Ord}(G), F_{Ord}(G) \rangle$ where $T_{Ord}(G) = \sum_{x_i \in V} T_A(x_i), I_{Ord}(G) = \sum_{x_i \in V} I_A(x_i)$, and $F_{Ord}(G) = \sum_{x_i \in V} F_B(x_i)$.
(iii) **busy value of a vertex** \( v \in V \) of \( G \) is \( \text{Busy}_G(v) = \langle T_{\text{busy}_G}(v), I_{\text{busy}_G}(v), F_{\text{busy}_G}(v) \rangle \) where \( T_{\text{busy}_G}(v) = \sum_i T_A(v) \land T_A(v_i), I_{\text{busy}_G}(v) = \sum_i I_A(v) \land I_A(v_i) \) and \( F_{\text{busy}_G}(v) = \sum_i F_A(v) \lor F_A(v_i) \) where \( v_i \) are neighbours of \( v \).

(iv) **busy value of a \( \text{Noff} \) \( G \)** is \( \text{Busy}(G) = \langle T_{\text{busy}}(G), I_{\text{busy}}(G), F_{\text{busy}}(G) \rangle \) where \( T_{\text{busy}}(G) = \sum_{v \in V} T_{\text{busy}_G}(v), I_{\text{busy}}(G) = \sum_{v \in V} I_{\text{busy}_G}(v) \) and \( F_{\text{busy}}(G) = \sum_{v \in V} F_{\text{busy}_G}(v) \).

**Proposition 3.2.** Let \( G \) be any \( \text{Noff} \) \( G \) of a crisp graph \( G^* \). Let \( v \in V \). Then \( \deg_{G^*}(v) = \text{Busy}_G(v) - \deg_G(v) \).

**Remark 3.1.** From the definition of a complete \( \text{Noff} \) \( G \), it follows that for each vertex \( v \in V \), \( \text{Busy}(v) = \deg_G(v) \).

**Definition 10.** Let \( G \) be any \( \text{Noff} \) \( G \) of a crisp graph \( G^* \). Then a node or vertex \( v \in V \) of a \( \text{Noff} \) \( G \) is said to be a **busy node (or) busy vertex** if \( T_A(v) \leq T_{\deg_G}(v), I_A(v) \leq I_{\deg_G}(v) \) and \( F_A(v) \geq F_{\deg_G}(v) \); otherwise it is called a **free node**.

An edge \( uv \in E \) of a \( \text{Noff} \) \( G \) is said to be an **effective edge** if \( T_B(uv) = T_A(u) \land T_A(v), I_B(uv) = I_A(u) \land I_A(v) \) and \( F_B(uv) = F_A(u) \lor F_A(v) \).

**Proposition 3.3.** Let \( G \) be any \( \text{Noff} \) \( G \) of a crisp graph \( G^* \) If \( G \) has effective edges then it has atleast one busy vertex.

**Definition 11.** Let \( G \) be any \( \text{Noff} \) \( G \) of a crisp graph \( G^* \). Let \( T(G) = (A_T, B_T) \) be a total \( \text{Noff} \) of \( G \) which is defined as follows: Let the vertex set of \( T(G) \) be \( V \cup E \). Then the \( \text{Noff} \) set \( A_T \) is defined on \( V \cup E \) as follows: for \( w \in V \cup E \),

(i) \( T_A(w) = T_A(u), I_A(w) = I_A(u) \) and \( F_A(w) = F_A(u), w = u \in V \).

(ii) \( T_A(w) = T_B(e), I_A(w) = I_B(e) \) and \( F_A(w) = F_B(e), w = e \in E \).

The \( \text{Noff} \) set \( B_T \) is defined on \( V \cup E \times V \cup E \) as follows:

**case(a)** \( T_{B_T}((u, v)) = T_B(u, v), I_{B_T}((u, v)) = I_B(u, v) \) and \( F_{B_T}((u, v)) = F_B(u, v), u, v \in V \).

**case(b)** \( T_{B_T}((u, e)) = T_A(u) \land T_B(e) \) if \( u \in V, e \in E \) and the vertex \( u \) lies on the edge \( e \), otherwise \( T_{B_T}((u, e)) = 0 \). \( I_{B_T}((u, e)) = I_A(u) \land I_B(e) \) if \( u \in V, e \in E \) and the vertex \( u \) lies on the edge \( e \), otherwise \( I_{B_T}((u, e)) = 0 \). \( F_{B_T}((u, e)) = F_A(u) \lor F_B(e) \) if \( u \in V, e \in E \) and the vertex \( u \) lies on the edge \( e \), otherwise \( F_{B_T}((u, e)) = 0 \).

**case(c)** \( T_{B_T}((e_j, e_k)) = T_B(e_j) \land T_B(e_k) \) if \( e_j, e_k \in E \) and the edges \( e_j \) and \( e_k \) have a common vertex, otherwise \( T_{B_T}((e_j, e_k)) = 0 \). \( I_{B_T}((e_j, e_k)) = I_B(e_j) \land I_B(e_k) \) if
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Let the edges \( e_j, e_k \in E \) have a common vertex, otherwise \( I_{BT}((e_j, e_k)) = 0 \). \( F_{BT}((e_j, e_k)) = F_B(e_j) \lor F_B(e_k) \) if \( e_j, e_k \in E \) and the edges \( e_j \) and \( e_k \) have a common vertex, otherwise \( F_{BT}((e_j, e_k)) = 0 \).

By the Definition, \( T_{BT}((u, v)) \leq T_A(u) \land T_A(v), I_{BT}((u, v)) \leq I_A(u) \land I_A(v) \) and \( F_{BT}((u, v)) \geq F_A(u) \lor F_A(v) \), for every \( u, v \in V \cup E \). Then \( T(G) = (A_T, B_T) \) is a total \( N_{off}G \) of \( G \).

Example 3.1. The following graph is an example of total \( N_{off}G \).

![Figure 1. G](image1)

![Figure 1. T(G)](image2)

Proposition 3.4. Let \( G \) be any \( N_{off}G \) of a crisp graph \( G^* \). If \( G \) is a Strong \( N_{off}G \), then \( S(T(G)) = 3S(G) + \sum_{e_j, e_k \in E} T_B(e_j) \land T_B(e_k), \sum_{e_j, e_k \in E} I_B(e_j) \land I_B(e_k), \sum_{e_j, e_k \in E} F_B(e_j) \lor F_B(e_k)) \).

Definition 12. Let \( G \) be any \( N_{off}G \) of a crisp graph \( G^* \). Let \( M(G) = (A_M, B_M) \) be a middle \( N_{off}G \) of \( G \) which is defined as follows: Let the vertex set of \( M(G) \) be \( V \cup E \). Then the \( N_{off} \) set \( A_M \) is defined on \( V \cup E \times V \cup E \) as follows: for \( w \in V \cup E \),

(i) \( T_A(w) = T_A(u), I_A(w) = I_A(u) \) and \( F_A(w) = F_A(u), w = u \in V \).

(ii) \( T_A(w) = T_B(e), I_A(w) = I_B(e) \) and \( F_A(w) = F_B(e), w = e \in E \).

The \( N_{off} \) set \( B_M \) is defined on \( V \cup E \times V \cup E \) as follows:

- **case(a)** \( T_{BM}((u, v)) = 0, I_{BM}((u, v)) = 0 \) and \( F_{BM}((u, v)) = 0, u, v \in V \).
- **case(b)** \( T_{BM}((u, e)) = T_B(e) \) if \( u \in V, e \in E \) and the vertex \( u \) lies on the edge \( e \), otherwise \( T_{BM}((u, e)) = 0, I_{BM}((u, e)) = I_B(e) \) if \( u \in V, e \in E \) and the vertex \( u \)
lies on the edge $e$, otherwise $I_{BM}((u, e)) = 0$. $F_{BM}((u, e)) = F_B(e)$ if $u \in V$, $e \in E$ and the vertex $u$ lies on the edge $e$, otherwise $F_{BM}((u, e)) = 0$.

**case(c)** $T_{BM}((e_j, e_k)) = T_B(e_j) \land T_B(e_k)$ if $e_j, e_k \in E$ and the edges $e_j$ and $e_k$ have a common vertex, otherwise $T_{BM}((e_j, e_k)) = 0$. $I_{BM}((e_j, e_k)) = I_B(e_j) \land I_B(e_k)$ if $e_j, e_k \in E$ and the edges $e_j$ and $e_k$ have a common vertex, otherwise $I_{BM}((e_j, e_k)) = 0$. $F_{BM}((e_j, e_k)) = F_B(e_j) \lor F_B(e_k)$ if $e_j, e_k \in E$ and the edges $e_j$ and $e_k$ have a common vertex, otherwise $F_{BM}((e_j, e_k)) = 0$.

By the Definition, $T_{BM}((u, v)) \leq T_{AM}(u) \land T_{AM}(v)$, $I_{BM}((u, v)) \leq I_{AM}(u) \land I_{AM}(v)$ and $F_{BM}((u, v)) \geq F_{AM}(u) \lor F_{AM}(v)$, for every $u, v \in V \cup E$. Then $M(G) = (A_M, B_M)$ is a middle $N$off $G$ of $G$.

**Example 3.2.** The following graph is an example of middle $N$off $G$ of $G$.

![Figure 2. G and T(G)](image)

**Proposition 3.5.** Let $T(G) = (A_T, B_T)$ be a total $N$off $G$ of a Strong $N$off $G$ $G$. Then

(i) $deg_{T(G)}(u) = 2deg_G(u)$ for all $u \in V$.

(ii) $deg_{T(G)}(e_i) = Busy_{T(G)}(e_i)$, if $e_i \in E$.

**Proposition 3.6.** Let $G$ be any $N$off $G$ of a crisp graph $G^*$. If $G$ is a Strong $N$off $G$, then $S(M(G)) = 2S(G) + \langle \sum_{e_j, e_k \in E} T_B(e_j) \land T_B(e_k), \sum_{e_j, e_k \in E} I_B(e_j) \land I_B(e_k), \sum_{e_j, e_k \in E} F_B(e_j) \lor F_B(e_k) \rangle$.

**Proposition 3.7.** Let $M(G) = (A_M, B_M)$ be a middle $N$off $G$ of a Strong $N$off $G$ $G$. Then
(i) \( \deg_{M(G)}(u) = \deg_G(u) \) for all \( u \in V \).

(ii) If \( u = e_j \) and \( e_j, e_k \in E \) are adjacent in \( G^* \), then

\[
\deg_{M(G)}(e_j) = 2 \left( \sum_{e_j \in E} T_B(e_j), \sum_{e_j \in E} I_B(e_j), \sum_{e_j \in E} F_B(e_j) \right) + \left( \sum_{e_k \in E} T_B(e_j) \wedge T_B(e_k), \sum_{e_k \in E} I_B(e_j) \wedge I_B(e_k), \sum_{e_k \in E} F_B(e_j) \vee F_B(e_k) \right).
\]

References


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