Novel multiple criteria decision making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs

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Abstract

In this research study, we introduce the concept of bipolar neutrosophic graphs. We present the dominating and independent sets of bipolar neutrosophic graphs. We describe novel multiple criteria decision making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs. We also develop an algorithm for computing domination in bipolar neutrosophic graphs.

Key-words: Bipolar neutrosophic sets, Bipolar neutrosophic graphs, Domination number, decision making, Algorithm.

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1 Introduction

A fuzzy set [30] is an important mathematical structure to represent a collection of objects whose boundary is vague. Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. In 1994, Zhang [32] introduced the notion of bipolar fuzzy sets and relations. Bipolar fuzzy sets are extension of fuzzy sets whose membership degree ranges $[-1, 1]$. The membership degree $(0, 1]$ indicates that the object satisfies a certain property whereas the membership degree $[-1, 0)$ indicates that the element satisfies the implicit counter property. Positive information represent what is considered to be possible and negative information represent what is granted to be impossible. Actually, a variety of decision making problems are based on two-sided bipolar judgements on a positive side and a negative side. Nowadays bipolar fuzzy sets are playing a substantial role in chemistry, economics, computer science, engineering, medicine and decision making problems. Smarandache [23] introduced the idea of neutrosophic probability, sets and logic. Peng *et al.* [20], in 2014, described some operational properties and studied a new approach for multi-criteria decision making problems using neutrosophic sets. Ye [28, 29] discussed trapezoidal neutrosophic sets and simplified neutrosophic sets with applications in multi-criteria decision making problems. The other terminologies and applications of neutrosophic sets can be seen in [24, 28, 29, 9, 8, 11, 25]. In a neutrosophic set, the membership value is associated with truth, false and indeterminacy degrees but there is no restriction on their sum. Deli et al. [10] extended the ideas of bipolar fuzzy sets and neutrosophic sets to bipolar neutrosophic sets and studied its operations and applications in decision making problems.

Graph theory has numerous applications in science and engineering. However, in some cases, some aspects of graph theoretic concepts may be uncertain. In such cases, it is important to deal with uncertainty using the methods of fuzzy sets and logics. Based on Zadeh's fuzzy relations [31] Kaufmann [12] defined a fuzzy graph. The fuzzy relations between fuzzy sets were also considered by Rosenfeld [21] and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya [5] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [17]. The complement of a fuzzy graph was defined by Mordeson [16]. Bhutani and Rosenfeld introduced the concept of M-strong fuzzy graphs in [6] and studied some of their properties. The concept of strong arcs in fuzzy graphs was discussed in [7]. The theory of fuzzy graphs has extended widely by many researchers as it can be seen in [15, 18]. The idea of domination was first arose in chess-board problem in 1862. Somasundaram and Somasundaram [26] introduced domination and independent domination in fuzzy graphs. Gani and Chandrasekaran [19] studied the notion of fuzzy domination and independent domination using strong arcs. Akram [1, 2] introduced bipolar fuzzy graphs and discuss its various properties. Akram and Dudek [3] studied regular bipolar fuzzy graphs. In this research article, we introduce the concept of bipolar neutrosophic graphs. We present the dominating and independent sets of bipolar neutrosophic graphs. We describe an outranking approach for risk analysis and construction of minimum number of radio channels using bipolar neutrosophic sets and bipolar neutrosophic graphs. We also develop an algorithm for computing domination in bipolar neutrosophic graphs.

2 Preliminaries

Let Y be a non-empty universe and $\widetilde{Y^2}$ is the collection of all $2-$element subsets of Y. A pair $G^* = (Y, E)$, where $E \subseteq \widetilde{Y^2}$ is a *graph*. The cardinality of any subset $D \subseteq Y$ is the number of vertices in D, it is denoted by $|D|$.

Definition 2.1. [30, 31] A *fuzzy set* μ in a universe Y is a mapping $\mu : Y \to [0,1]$. A *fuzzy relation* on Y is a fuzzy set ν in $Y \times Y$.

Definition 2.2. [31] If μ is a fuzzy set on Y and ν a fuzzy relation in Y. We can say ν is a fuzzy relation on μ if $\nu(y,z) \leq \min\{\mu(y), \mu(z)\}$ for all $x, y \in Y$.

Definition 2.3. [12] A *fuzzy graph* on a non-empty universe Y is a pair $G = (\mu, \lambda)$, where μ is a fuzzy set on Y and λ is a fuzzy relation in Y such that $\lambda(yz) \leq \min\{\mu(y), \mu(z)\}$ for all y, $z \in Y$. Note that λ is a fuzzy relation on μ, and $\lambda(yz) = 0$ for all $yz \in \widetilde{Y}^2 - E$.

Definition 2.4. [32] A bipolar fuzzy set on a non-empty set Y has the form $C = \{(y, \mu^p(y), \mu^n(y)) : y \in Y\}$ where, $\mu^p : Y \to [0,1]$ and $\mu^n : Y \to [-1,0]$ are mappings.

The positive membership value $\mu^p(y)$ represents the strength of truth or satisfaction of an element y to a certain property corresponding to bipolar fuzzy set C and $\mu^n(x)$ denotes the strength of satisfaction of an element y to some counter property of bipolar fuzzy set C. If $\mu^p(y) \neq 0$ and $\mu^n(y) = 0$, it is the situation when y has only truth satisfaction degree for property C. If $\mu^n(y) \neq 0$ and $\mu^p(y) = 0$, it is the case that y is not satisfying the property of C but satisfying the counter property to C. It is possible for y that $\mu^p(x) \neq 0$ and $\mu^n(x) \neq 0$ when y satisfies the property of C as well as its counter property in some part of Y.

Definition 2.5. [1] Let Y be a nonempty set. A mapping $D = (\mu^p, \mu^n) : Y \times Y \to [0,1] \times [-1,0]$ is a bipolar fuzzy relation on Y such that $\mu^p(xy) \in [0,1]$ and $\mu^n(xy) \in [-1,0]$ for $y,z \in Y$.

Definition 2.6. [1] A bipolar fuzzy graph on Y is a pair $G = (C,D)$ where $C = (\mu_C^p, \mu_C^n)$ is a bipolar fuzzy set on Y and $D = (\mu_D^p, \mu_D^n)$ is a bipolar fuzzy relation in Y such that

$$\mu_D^p(yz) \leq \mu_C^p(y) \wedge \mu_C^p(z) \text{ and } \mu_D^n(yz) \geq \mu_C^n(y) \vee \mu_C^n(z) \text{ for all } y,z \in X.$$

Note that D is a bipolar fuzzy relation on C, and $\mu_D^p(yz) > 0$, $\mu_D^n(yz) < 0$ for $yz \in \widetilde{Y}^2$, $\mu_D^p(yz) = \mu_D^n(yz) = 0$ for $yz \in \widetilde{Y}^2 - E$.

Definition 2.7. [24] A neutrosophic set C on a non-empty set Y is characterized by a truth membership function $t_C : Y \to [0,1]$, indeterminacy membership function $I_C : Y \to [0,1]$ and a falsity membership function $f_C : Y \to [0,1]$. There is no restriction on the sum of $t_C(x)$, $I_C(x)$ and $f_C(x)$ for all $x \in X$.

Definition 2.8. [10] A bipolar neutrosophic set on a empty set Y is an object of the form

$$C = \{(y, t_C^p(y), I_C^p(y), f_C^p(y), t_C^n(y), I_C^n(y), f_C^n(y)) : y \in Y\}$$

where, $t_C^p, I_C^p, f_C^p : Y \to [0,1]$ and $t_C^n, I_C^n, f_C^n : Y \to [-1,0]$. The positive values $t_C^p(y), I_C^p(y), f_C^p(y)$ denote respectively the truth, indeterminacy and false membership degrees of an element $y \in Y$ whereas $t_C^n(y), I_C^n(y), f_C^n(y)$ denote the implicit counter property of the truth, indeterminacy and false membership degrees of the element $y \in Y$ corresponding to the bipolar neutrosophic set C.

3 Bipolar neutrosophic graphs

Definition 3.1. A bipolar neutrosophic relation on a non-empty set Y is a bipolar neutrosophic subset of $Y \times Y$ of the form $D = \{(yz, t_D^p(yz), I_D^p(yz), f_D^p(yz), t_D^n(yz), I_D^n(yz), f_D^n(yz)) : yz \in Y \times Y\}$ where, $t_D^p, I_D^p, f_D^p, t_D^n, I_D^n, f_D^n$ are defined by the the mappings $t_D^p, I_D^p, f_D^p : Y \times Y \to [0,1]$ and $t_D^n, I_D^n, f_D^n : Y \times Y \to [-1,0]$ such that for all $yz \in supp(D)$,

$$0 \leq \sup t_D^p(yz) + \sup I_D^p(yz) + \sup f_D^p(yz) \leq 3 \text{ and } -3 \leq \inf t_D^n(yz) + \inf I_D^n(yz) + \inf f_D^n(yz) \leq 0.$$

Definition 3.2. A bipolar neutrosophic graph on a non-empty set X is a pair $G = (C,D)$, where C is a bipolar neutrosophic set on X and D is a bipolar neutrosophic relation in X such that

$$\begin{aligned}
&t_D^p(yz) \leq t_C^p(y) \wedge t_C^p(z), \quad I_D^p(yz) \leq I_C^p(y) \vee I_C^p(z), \quad f_D^p(yz) \leq f_C^p(y) \vee f_C^p(z),\\
&t_D^n(yz) \geq t_C^n(y) \vee t_C^n(z), \quad I_D^n(yz) \geq I_C^n(y) \wedge I_C^n(z), \quad f_D^n(yz) \geq f_C^n(y) \wedge f_C^n(z) \quad \text{for all } y,z \in Y,\\
&0 \leq \sup t_D^p(y) + \sup I_D^p(y) + \sup f_D^p(y) \leq 3 \text{ and } -3 \leq \inf t_D^n(y) + \inf I_D^n(y) + \inf t_D^n(y) \leq 0 \quad \text{for all } y \in Y,\\
&0 \leq \sup t_D^p(yz) + \sup I_D^p(yz) + \sup f_D^p(yz) \leq 3 \text{ and } -3 \leq \inf t_D^n(yz) + \inf I_D^n(yz) + \inf f_D^n(yz) \leq 0 \quad \text{for all } y,z \in Y.
\end{aligned}$$

Note that $D(yz) = (0,0,0,0,0,0)$ for all $yz \in Y \times Y \setminus E$.

Example 3.1. Here we discuss an example of a bipolar neutrosophic graph such that $Y = \{x,y,z\}$. Let C be a bipolar neutrosophic set on X given in Table.1 and D be a bipolar neutrosophic relation in X given in Table.2. Routine

Table 1	x	y	z
t_C^p	0.3	0.5	0.4
I_C^p	0.4	0.4	0.3
f_C^p	0.5	0.2	0.2
t_C^n	-0.6	-0.1	-0.5
I_C^n	-0.5	-0.8	-0.5
f_C^n	-0.2	-0.2	-0.5

Table 2	xy	yz	xz
t_D^p	0.3	0.3	0.3
I_D^p	0.4	0.4	0.4
f_D^p	0.5	0.2	0.5
t_D^n	-0.1	-0.1	-0.5
I_D^n	-0.8	-0.8	-0.5
f_D^n	-0.2	-0.5	-0.5

calculations show that $G = (C,D)$ is a bipolar neutrosophic graph. The bipolar neutrosophic graph G is shown in Fig. 1.

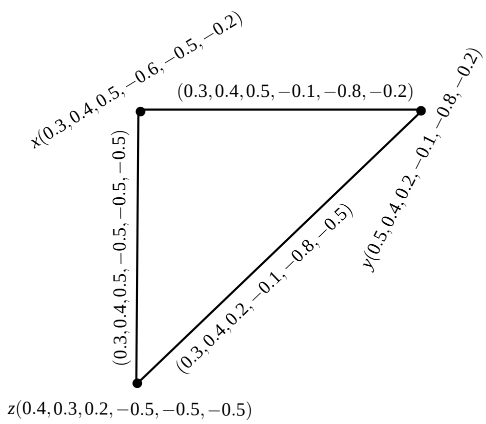

Figure 1: Bipolar neutrosophic graph G

Definition 3.3. Let $G_1 = (C_1,D_1)$ and $G_2 = (C_2,D_2)$ be two bipolar neutrosophic graphs where, C_1 and C_2 are bipolar neutrosophic sets on Y_1 and Y_2, D_1 and D_2 are bipolar neutrosophic relations in Y_1 and Y_2, respectively and $supp(D_1) = E_1$ and $supp(D_2) = E_2$. The *union* of G_1 and G_2 is a pair $G_1 \cup G_2 = (C_1 \cup C_2, D_1 \cup D_2)$ such that

$$t^p_{C_1\cup C_2}(y) = \begin{cases} t^p_{C_1}(y), & y \in Y_1,\ y \notin Y_2 \\ t^p_{C_2}(y), & y \notin Y_1,\ y \in Y_2 \\ t^p_{C_1}(y) \vee t^p_{C_2}(z), & y \in Y_1 \cap Y_2 \end{cases} \quad I^p_{C_1\cup C_2}(y) = \begin{cases} I^p_{C_1}(y), & y \in Y_1,\ y \notin Y_2 \\ I^p_{C_2}(y), & y \notin Y_1,\ y \in Y_2 \\ I^p_{C_1}(y) \wedge I^p_{C_2}(z), & y \in Y_1 \cap Y_2 \end{cases}$$

$$f^p_{C_1\cup C_2}(y) = \begin{cases} f^p_{C_1}(y), & y \in Y_1,\ y \notin Y_2 \\ f^p_{C_2}(y), & y \notin Y_1,\ y \in Y_2 \\ f^p_{C_1}(y) \wedge f^p_{C_2}(z), & y \in Y_1 \cap Y_2 \end{cases} \quad t^n_{C_1\cup C_2}(y) = \begin{cases} t^n_{C_1}(y), & y \in Y_1,\ y \notin Y_2 \\ t^n_{C_2}(y), & y \notin Y_1,\ y \in Y_2 \\ t^n_{C_1}(y) \wedge t^n_{C_2}(z), & y \in Y_1 \cap Y_2 \end{cases}$$

$$I^n_{C_1\cup C_2}(y) = \begin{cases} I^n_{C_1}(y), & y \in Y_1,\ y \notin Y_2 \\ I^n_{C_2}(y), & y \notin Y_1,\ y \in Y_2 \\ I^n_{C_1}(y) \vee I^n_{C_2}(z), & y \in Y_1 \cap Y_2 \end{cases} \quad f^n_{C_1\cup C_2}(y) = \begin{cases} f^n_{C_1}(y), & y \in Y_1,\ y \notin Y_2 \\ f^n_{C_2}(y), & y \notin Y_1,\ y \in Y_2 \\ f^n_{C_1}(y) \vee f^n_{C_2}(z), & y \in Y_1 \cap Y_2 \end{cases}$$

and membership values of edges are

$$t^p_{D_1\cup D_2}(yz)=\begin{cases} t^p_{D_1}(yz), & yz\in E_1,\ yz\notin E_2\\ t^p_{D_2}(yz), & yz\notin E_1,\ yz\in E_2\\ t^p_{D_1}(yz)\vee t^p_{D_2}(yz), & yz\in E_1\cap E_2\end{cases}$$

$$I^p_{D_1\cup D_2}(yz)=\begin{cases} I^p_{D_1}(yz), & yz\in E_1,\ yz\notin E_2\\ I^p_{D_2}(yz), & yz\notin E_1,\ yz\in E_2\\ I^p_{D_1}(yz)\wedge I^p_{D_2}(yz), & yz\in E_1\cap E_2\end{cases}$$

$$f^p_{D_1\cup D_2}(yz)=\begin{cases} f^p_{D_1}(yz), & yz\in E_1,\ yz\notin E_2\\ f^p_{D_2}(yz), & yz\notin E_1,\ yz\in E_2\\ f^p_{D_1}(yz)\wedge f^p_{D_2}(yz), & yz\in E_1\cap E_2\end{cases}$$

$$t^n_{D_1\cup D_2}(yz)=\begin{cases} t^n_{D_1}(yz), & yz\in E_1,\ yz\notin E_2\\ t^n_{D_2}(yz), & yz\notin E_1,\ yz\in E_2\\ t^n_{D_1}(yz)\wedge t^n_{D_2}(yz), & yz\in E_1\cap E_2\end{cases}$$

$$I^n_{D_1\cup D_2}(yz)=\begin{cases} I^n_{D_1}(yz), & yz\in E_1,\ yz\notin E_2\\ I^n_{D_2}(yz), & yz\notin E_1,\ yz\in E_2\\ I^n_{D_1}(yz)\vee I^n_{D_2}(yz), & yz\in E_1\cap E_2\end{cases}$$

$$f^n_{D_1\cup D_2}(yz)=\begin{cases} f^n_{D_1}(yz), & yz\in E_1,\ yz\notin E_2\\ f^n_{D_2}(yz), & yz\notin E_1,\ yz\in E_2\\ f^n_{D_1}(yz)\vee f^n_{D_2}(yz), & yz\in E_1\cap E_2\end{cases}$$

Definition 3.4. The *intersection* of two bipolar neutrosophic graphs $G_1=(C_1,D_1)$ and $G_2=(C_2,D_2)$ is a pair $G_1\cap G_2=(C_1\cap C_2,D_1\cap D_2)$ where, C_1, C_2, D_1 and D_2 are given in Definition 3.3. The membership values of vertices and edges in $G_1\cap G_2$ can be defined as,

$$t^p_{C_1\cap C_2}(y)=t^p_{C_1}(y)\wedge t^p_{C_2}(y),\quad I^p_{C_1\cap C_2}(y)=I^p_{C_1}(y)\vee I^p_{C_2}(y),\quad f^p_{C_1\cap C_2}(y)=f^p_{C_1}(y)\vee f^p_{C_2}(y)$$
$$t^n_{C_1\cap C_2}(y)=t^n_{C_1}(y)\vee t^n_{C_2}(y),\quad I^n_{C_1\cap C_2}(y)=I^n_{C_1}(y)\wedge I^n_{C_2}(y),\quad f^n_{C_1\cap C_2}(y)=f^n_{C_1}(y)\wedge f^n_{C_2}(y),\quad \text{for all}\quad y\in Y_1\cap Y_2.$$

$$t^p_{D_1\cap D_2}(yz)=t^p_{D_1}(yz)\wedge t^p_{D_2}(yz),\quad I^p_{D_1\cap D_2}(yz)=I^p_{D_1}(yz)\vee I^p_{D_2}(yz),\quad f^p_{D_1\cap D_2}(yz)=f^p_{D_1}(yz)\vee f^p_{D_2}(yz)$$
$$t^n_{D_1\cap D_2}(yz)=t^n_{D_1}(yz)\vee t^n_{D_2}(yz),\quad I^n_{D_1\cap D_2}(yz)=I^n_{D_1}(yz)\wedge I^n_{D_2}(yz),\quad f^n_{D_1\cap D_2}(yz)=f^n_{D_1}(yz)\wedge f^n_{D_2}(yz),$$
$$\text{for all}\quad yz\in E_1\cap E_2.$$

Definition 3.5. The *join* of two bipolar neutrosophic graphs $G_1=(C_1,D_1)$ and $G_2=(C_2,D_2)$ is defined by the pair $G_1+G_2=(C_1+C_2,D_1+D_2)$ such that, $C_1+C_2=C_1\cup C_2$ for all $x\in Y_1\cup Y_2$ and

1. $D_1+D_2=D_1\cup D_2$ for all $yz\in E_1\cap E_2$,

2. Let $E^{'}$ be the set of all edges joining the vertices of G_1 and G_2 then for all $yz \in E^{'}$, where $y \in Y_1$ and $z \in Y_2$,

$$t^p_{D_1+D_2}(yz) = t^p_{C_1}(y) \vee t^p_{C_2}(z), \qquad I^p_{D_1+D_2}(yz) = I^p_{C_1}(y) \wedge I^p_{C_2}(z), \qquad f^p_{D_1+D_2}(yz) = f^p_{C_1}(y) \wedge f^p_{C_2}(z),$$
$$t^n_{D_1+D_2}(yz) = t^n_{C_1}(y) \wedge t^n_{C_2}(z), \qquad I^n_{D_1+D_2}(yz) = I^n_{C_1}(y) \vee I^n_{C_2}(z), \qquad f^n_{D_1+D_2}(yz) = f^n_{C_1}(y) \vee f^n_{C_2}(z).$$

Definition 3.6. The *Cartesian product* of two bipolar neutrosophic graphs G_1 and G_2 is denoted by the pair $G_1 \Box G_2 = (C_1 \Box C_2, D_1 \Box D_2)$ and defined as,

$$t^p_{C_1\Box C_2}(y) = t^p_{C_1}(y) \wedge t^p_{D_2}(y), \qquad I^p_{C_1\Box C_2}(y) = I^p_{C_1}(y) \vee I^p_{C_2}(y), \qquad f^p_{C_1\Box C_2}(y) = f^p_{C_1}(y) \vee f^p_{C_2}(y),$$
$$t^n_{C_1\Box D_2}(y) = t^n_{C_1}(y) \vee t^n_{C_2}(y), \qquad I^n_{C_1\Box C_2}(y) = I^n_{C_1}(y) \wedge I^n_{C_2}(y), \qquad f^n_{C_1\Box C_2}(y) = f^n_{C_1}(y) \wedge f^n_{C_2}(y).$$

for all $\quad y \in Y_1 \times Y_2$.

1. $t^p_{D_1\Box D_2}((y_1,y_2)(y_1,z_2)) = t^p_{C_1}(y_1) \wedge t^p_{D_2}(y_2z_2), \quad t^n_{D_1\Box D_2}((y_1,y_2)(y_1,z_2)) = t^n_{C_1}(y_1) \vee t^n_{D_2}(y_2z_2),$
   for all $y_1 \in Y_1, y_2z_2 \in E_2$,
2. $t^p_{D_1\Box D_2}((y_1,y_2)(z_1,y_2)) = t^p_{D_1}(y_1z_1) \wedge t^p_{C_2}(y_2), \quad t^n_{D_1\Box D_2}((y_1,y_2)(z_1,y_2)) = t^n_{D_1}(y_1z_1) \vee t^n_{C_2}(y_2),$
   for all $y_1z_1 \in E_1, y_2 \in Y_2$,
3. $I^p_{D_1\Box D_2}((y_1,y_2)(y_1,z_2)) = I^p_{C_1}(y_1) \vee I^p_{D_2}(y_2z_2), \quad I^n_{D_1\Box D_2}((y_1,y_2)(y_1,z_2)) = I^n_{C_1}(y_1) \wedge I^n_{D_2}(y_2z_2),$
   for all $y_1 \in Y_1, y_2z_2 \in E_2$,
4. $I^p_{D_1\Box D_2}((y_1,y_2)(z_1,y_2)) = I^p_{D_1}(y_1z_1) \vee I^p_{C_2}(y_2), \quad I^n_{D_1\Box D_2}((y_1,y_2)(z_1,y_2)) = I^n_{D_1}(y_1z_1) \wedge I^n_{C_2}(y_2),$
   for all $y_1z_1 \in E_1, y_2 \in Y_2$,
5. $f^p_{D_1\Box D_2}((y_1,y_2)(y_1,z_2)) = f^p_{C_1}(y_1) \vee f^p_{D_2}(y_2z_2), \quad f^n_{D_1\Box D_2}((y_1,y_2)(y_1,z_2)) = f^n_{C_1}(y_1) \wedge f^n_{D_2}(y_2z_2),$
   for all $y_1 \in Y_1, y_2z_2 \in E_2$,
6. $f^p_{D_1\Box D_2}((y_1,y_2)(z_1,y_2)) = f^p_{D_1}(y_1z_1) \vee f^p_{C_2}(y_2), \quad f^n_{D_1\Box D_2}((y_1,y_2)(z_1,y_2)) = f^n_{D_1}(y_1z_1) \wedge f^n_{C_2}(y_2),$
   for all $y_1z_1 \in E_1, y_2 \in Y_2$.

Definition 3.7. The *direct product* of two bipolar neutrosophic graphs $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ is denoted by the pair $G_1 \times G_2 = (C_1 \times C_2, D_1 \times D_2)$ such that,

$$t^p_{C_1\times C_2}(y) = t^p_{C_1}(y) \wedge t^p_{D_2}(y), \qquad I^p_{C_1\times C_2}(y) = I^p_{C_1}(y) \vee I^p_{C_2}(y), \qquad f^p_{C_1\times C_2}(y) = f^p_{C_1}(y) \vee f^p_{C_2}(y),$$
$$t^n_{C_1\times C_2}(y) = t^n_{C_1}(y) \vee t^n_{C_2}(y), \qquad I^n_{C_1\times C_2}(y) = I^n_{C_1}(y) \wedge I^n_{C_2}(y), \qquad f^n_{C_1\times C_2}(y) = f^n_{C_1}(y) \wedge f^n_{C_2}(y),$$

for all $\quad y \in Y_1 \times Y_2$.

1. $t^p_{D_1\times D_2}((y_1,y_2)(z_1,z_2)) = t^p_{D_1}(y_1z_1) \wedge t^p_{D_2}(y_2z_2), \quad t^n_{D_1\times D_2}((y_1,y_2)(z_1,z_2)) = t^n_{D_1}(y_1z_1) \vee t^n_{D_2}(y_2z_2),$
   for all $y_1z_1 \in E_1, y_2z_2 \in E_2$,
2. $I^p_{D_1\times D_2}((y_1,y_2)(z_1,z_2)) = I^p_{D_1}(y_1z_1) \vee I^p_{D_2}(y_2z_2), \quad I^n_{D_1\times D_2}((y_1,y_2)(z_1,z_2)) = I^n_{D_1}(y_1z_1) \wedge I^n_{D_2}(y_2z_2),$
   for all $y_1z_1 \in E_1, y_2z_2 \in E_2$,
3. $f^p_{D_1\times D_2}((y_1,y_2)(z_1,z_2)) = f^p_{D_1}(y_1z_1) \vee f^p_{D_2}(y_2z_2), \quad f^n_{D_1\times D_2}((y_1,y_2)(z_1,z_2)) = f^n_{D_1}(y_1z_1) \wedge f^n_{D_2}(y_2z_2),$
   for all $y_1z_1 \in E_1, y_2z_2 \in E_2$.

Proposition 3.1. *Let G_1 and G_2 be any two bipolar neutrosophic graphs then $G_1 \cup G_2$, $G_1 \cap G_2$, $G_1 + G_2$, $G_1 \square G_2$ and $G_1 \times G_2$ are bipolar neutrosophic graphs.*

Definition 3.8. A bipolar neutrosophic graph $G = (C, D)$ where, $E = supp(D)$, is called *strong bipolar neutrosophic graph* if

$$\begin{aligned}
&t_D^p(yz) = t_C^p(y) \wedge t_C^p(z), \quad I_D^p(yz) = I_C^p(y) \vee I_C^p(z), \quad f_D^p(yz) = f_C^p(y) \vee f_C^p(z),\\
&t_D^n(yz) = t_C^n(y) \vee t_C^n(z), \quad I_D^n(yz) = I_C^n(y) \wedge I_C^n(z), \quad f_D^n(yz) = f_C^n(y) \wedge f_C^n(z) \quad \text{for all} \quad yz \in E.
\end{aligned}$$

Definition 3.9. A bipolar neutrosophic graph $G = (C, D)$ is called *complete bipolar neutrosophic graph* if

$$\begin{aligned}
&t_D^p(yz) = t_C^p(y) \wedge t_C^p(z), \quad I_D^p(yz) = I_C^p(y) \vee I_C^p(z), \quad f_D^p(yz) = f_C^p(y) \vee f_C^p(z),\\
&t_D^n(yz) = t_C^n(y) \vee t_C^n(z), \quad I_D^n(yz) = I_C^n(y) \wedge I_C^n(z), \quad f_D^n(yz) = f_C^n(y) \wedge f_C^n(z) \quad \text{for all} \quad y, z \in Y.
\end{aligned}$$

Definition 3.10. The *complement* of a bipolar neutrosophic graph $G = (C, D)$ is defined as a pair $G^c = (C^c, D^c)$ such that, for all $y \in Y$ and $yz \in \widetilde{Y}^2$,

$$t_{C^c}^p(y) = t_C^p(y), \quad I_{C^c}^p(y) = I_C^p(y), \quad f_{C^c}^p(y) = f_C^p(y), \quad t_{C^c}^n(y) = t_C^n(y), \quad I_{C^c}^n(y) = I_C^n(y), \quad f_{C^c}^p(y) = f_C^p(y).$$

$$\begin{aligned}
&t_{D^c}^p(yz) = t_C^p(y) \wedge t_C^p(z) - t_D^p(yz), \quad I_{D^c}^p(yz) = I_C^p(y) \vee I_C^p(z) - I_D^p(yz), \quad f_{D^c}^p(yz) = f_C^p(y) \vee f_C^p(z) - f_D^p(yz),\\
&t_{D^c}^n(yz) = t_C^n(y) \vee t_C^n(z) - t_D^n(yz), \quad I_{D^c}^n(yz) = I_C^n(y) \wedge I_C^n(z) - I_D^n(yz), \quad f_{D^c}^n(yz) = f_C^n(y) \wedge f_C^n(z) - f_D^n(yz).
\end{aligned}$$

Remark 3.1. A bipolar neutrosophic graph G is said to be *self complementary* if $G = G^c$.

Theorem 3.1. *Let G be a self complementary bipolar neutrosophic graph then,*

$$\begin{aligned}
&\sum_{y \neq z} t_D^p(yz) = \frac{1}{2} \sum_{y \neq z} t_C^p(y) \wedge t_C^p(z), \quad \sum_{y \neq z} I_D^p(yz) = \frac{1}{2} \sum_{y \neq z} I_C^p(y) \vee I_C^p(z), \quad \sum_{y \neq z} f_D^p(yz) = \frac{1}{2} \sum_{y \neq z} f_C^p(y) \vee f_C^p(z),\\
&\sum_{y \neq z} t_D^n(yz) = \frac{1}{2} \sum_{y \neq z} t_C^n(y) \wedge t_C^n(z), \quad \sum_{y \neq z} I_D^n(yz) = \frac{1}{2} \sum_{y \neq z} I_C^n(y) \vee I_C^n(z), \quad \sum_{y \neq z} f_D^n(yz) = \frac{1}{2} \sum_{y \neq z} f_C^n(y) \vee f_C^n(z).
\end{aligned}$$

Theorem 3.2. *Let $G = (C, D)$ be a bipolar neutrosophic graph such that for all $y, z \in Y$,*

$$\begin{aligned}
&t_{D^c}^p(yz) = \frac{1}{2}(t_C^p(y) \wedge t_C^p(z)), \quad I_{D^c}^p(yz) = \frac{1}{2}(I_C^p(y) \vee I_C^p(z)), \quad f_{D^c}^p(yz) = \frac{1}{2}(f_C^p(y) \vee f_C^p(z)),\\
&t_{D^c}^n(yz) = \frac{1}{2}(t_C^n(y) \vee t_C^n(z)), \quad I_{D^c}^n(yz) = \frac{1}{2}(I_C^n(y) \wedge I_C^n(z)), \quad f_{D^c}^n(yz) = \frac{1}{2}(f_C^n(y) \wedge f_C^n(z)).
\end{aligned}$$

Then G is self complementary bipolar neutrosophic graph.

Proof. Let $G^c = (C^c, D^c)$ be the complement of bipolar neutrosophic graph $G = (C, D)$, then by definition. 3.10,

$$\begin{aligned}
t_{D^c}^p(yz) &= t_C^p(y) \wedge t_C^p(z) - t_D^p(yz)\\
t_{D^c}^p(yz) &= t_C^p(y) \wedge t_C^p(z) - \frac{1}{2}(t_C^p(y) \wedge t_C^p(z))\\
t_{D^c}^p(yz) &= \frac{1}{2}(t_C^p(y) \wedge t_C^p(z))\\
t_{D^c}^p(yz) &= t_D^p(yz)
\end{aligned}$$

$$t^n_{D^c}(yz) = t^n_C(y) \vee t^n_C(z) - t^n_D(yz)$$
$$t^n_{D^c}(yz) = t^n_C(y) \vee t^n_C(z) - \frac{1}{2}(t^n_C(y) \vee t^n_C(z))$$
$$t^n_{D^c}(yz) = \frac{1}{2}(t^n_C(y) \vee t^n_C(z))$$
$$t^n_{D^c}(yz) = t^n_D(yz)$$

Similarly, it can be proved that $I^p_{D^c}(yz) = I^p_D(yz)$, $I^n_{D^c}(yz) = I^n_D(yz)$, $f^p_{D^c}(yz) = f^p_D(yz)$ and $f^n_{D^c}(yz) = f^n_D(yz)$. Hence, G is self complementary. □

Definition 3.11. The *degree* of a vertex y in a bipolar neutrosophic graph $G = (C,D)$ is denoted by $\deg(y)$ and defined by the $6-$tuple as,

$$\deg(y) = (\deg^p_t(y),\ \deg^p_I(y),\ \deg^p_f(y),\ \deg^n_t(y),\ \deg^n_I(y),\ \deg^n_f(y)),$$
$$= (\sum_{yz\in E} t^p_D(yz),\ \sum_{yz\in E} I^p_D(yz),\ \sum_{yz\in E} f^p_D(yz),\ \sum_{yz\in E} t^n_D(yz),\ \sum_{yz\in E} I^n_D(yz),\ \sum_{yz\in E} f^n_D(yz)).$$

The term degree is also referred as *neighborhood degree*.

Definition 3.12. The closed neighborhood degree of a vertex y in a bipolar neutrosophic graph is denoted by $\deg[y]$ and defined as,

$$\deg[y] = (\deg^p_t[y],\ \deg^p_I[y],\ \deg^p_f[y],\ \deg^n_t[y],\ \deg^n_I[y],\ \deg^n_f[y],$$
$$= (\deg^p_t(y) + t^p_C(y),\ \deg^p_I(y) + I^p_C(y),\ \deg^p_f(y) + f^p_C(y),\ \deg^n_t(y) + t^n_C(y),\ \deg^n_I(y) + t^n_C(y),$$
$$\deg^n_f(y) + f^p_C(y)).$$

Definition 3.13. A bipolar neutrosophic graph G is known as a *regular* bipolar neutrosophic graph if all vertices of G have same degree. A bipolar neutrosophic graph G is known as a *totally regular* bipolar neutrosophic graph if all vertices of G have same closed neighborhood degree.

Theorem 3.3. *A complete bipolar neutrosophic graph is totally regular.*

Theorem 3.4. *Let $G = (C,D)$ be a bipolar neutrosohic graph then $C = (t^p, I^p, f^p, t^n, I^n, f^n)$ is a constant function if and only if the following statements are equivalent:*

(1) *G is a regular bipolar neutrosophic graph,*

(2) *G is totally regular bipolar neutrosophic graph.*

Proof. Assume that C is a constant function and for all $y \in Y$,

$$t^p_C(y) = k_t,\ I^p_C(y) = k_I,\ f^p_C(y) = k_f,\ t^n_C(y) = k^{'}_t,\ I^n_C(y) = k^{'}_I,\ f^n_C(y) = k^{'}_f$$

where, $k_t,\ k_I,\ k_f,\ k^{'}_t,\ k^{'}_I,\ k^{'}_f$ are constants.

$(1) \Rightarrow (2)$ Suppose that G is a regular bipolar neutrosophic graph and $\deg(y) = (p_t,\ p_I,\ p_f,\ n_t,\ n_I,\ n_f)$ for all $y \in Y$. Now consider,

$\deg[y] = (\deg^p_t(y) + t^p_C(y), \deg^p_I(y) + I^p_C(y), \deg^p_f(y) + f^p_C(y), \deg^n_t(y) + t^n_C(y), \deg^n_I(y) + t^n_C(y), \deg^n_f(y) + f^p_C(y)) = (p_t +$

$k_t, p_I+k_I, p_f+k_f, n_t+k_t^{'}, n_I+k_I^{'}, n_f+k_f^{'})$ for all $y \in Y$.
It is proved that G is totally regular bipolar neutrosophic graph.
$(2) \Rightarrow (1)$ Suppose that G is totally regular bipolar neutrosophic graph and for all $y \in Y$ $\deg[y] = (p_t^{'},\ p_I^{'},\ p_f^{'},\ n_t^{'},\ n_I^{'},\ n_f^{'})$.

$$(\deg_t^p(y)+k_t, \deg_I^p(y)+k_I, \deg_f^p(y)+k_f, \deg_t^n(y)+k_t^{'}, \deg_I^n(y)+k_I^{'}, \deg_f^n(y)+k_f^{'}) = (p_t^{'},\ p_I^{'},\ p_f^{'},\ n_t^{'},\ n_I^{'},\ n_f^{'}),$$
$$\deg_t^p(y), \deg_I^p(y), \deg_f^p(y), \deg_t^n(y), \deg_I^n(y), \deg_f^n(y)) + (k_t, k_I, k_f, k_t^{'}, k_I^{'}, k_f^{'}) = (p_t^{'},\ p_I^{'},\ p_f^{'},\ n_t^{'},\ n_I^{'},\ n_f^{'}),$$
$$(\deg_t^p(y), \deg_I^p(y), \deg_f^p(y), \deg_t^n(y), \deg_I^n(y), \deg_f^n(y)) = (p_t^{'}-k_t,\ p_I^{'}-k_I,\ p_f^{'}-k_f,\ n_t^{'}-k_t^{'},\ n_I^{'}-k_I^{'},\ n_f^{'}-k_f^{'}),$$

for all $y \in Y$. Thus G is a regular bipolar neutrosophic graph.
Conversely, assume that the conditions are equivalent. Let $\deg(y) = (c_t,\ c_I,\ c_f,\ d_t,\ d_I,\ d_f)$ and $\deg[y] = (c_t^{'},\ c_I^{'},\ c_f^{'},\ d_t^{'},\ d_I^{'},\ d_f^{'})$.
Since by definition of closed neighborhood degree for all $y \in Y$,
$\deg[y] = \deg(y) + (t_C^p(y), I_C^p(y), f_C^p(y), t_C^n(y), I_C^n(y), f_C^p(y))$,
$\Rightarrow\ (t_C^p(y), I_C^p(y), f_C^p(y), t_C^n(y), I_C^n(y), f_C^p(y)) = \deg[y] - \deg(y)$,
$\Rightarrow\ (t_C^p(y), I_C^p(y), f_C^p(y), t_C^n(y), I_C^n(y), f_C^p(y)) = (c_t^{'}-c_t, c_I^{'}-c_I, c_f^{'}-c_f, d_t^{'}-d_t, d_I^{'}-d_I, d_f^{'}-d_f)$,
for all $y \in Y$. Hence $C = (c_t^{'}-c_t, c_I^{'}-c_I, c_f^{'}-c_f, d_t^{'}-d_t, d_I^{'}-d_I, d_f^{'}-d_f)$, a constant function which completes the proof. □

Definition 3.14. A bipolar neutrosophic graph G is said to be *irregular* if at least two vertices have distinct degrees. If all vertices do not have same closed neighborhood degrees then G is known as *totally irregular* bipolar neutrosophic graph.

Theorem 3.5. *Let $G = (C, D)$ be a bipolar neutrosophic graph and $C = (t_C^p, I_C^p, f_C^p, t_C^n, I_C^n, f_C^n)$ be a constant function then G is an irregular bipolar neutrosophic graph if and only if G is a totally irregular bipolar neutrosophic graph.*

Proof. Assume that G is an irregular bipolar neutrosophic graph then at least two vertices of G have distinct degrees. Let y and z be two vertices such that $\deg(y) = (r_1, r_2, r_3, s_1, s_2, s_3)$ and $\deg(z) = (r_1^{'}, r_2^{'}, r_3^{'}, s_1^{'}, s_2^{'}, s_3^{'})$ where, $r_i \neq r_i^{'}$, for some $i = 1, 2, 3$.
Since, C is a constant function let $C = (k_1, k_2, k_3, l_1, l_2, l_3)$. Therefore,

$$\deg[y] = \deg(y) + (k_1,\ k_2,\ k_3,\ l_1,\ l_2,\ l_3)$$
$$\deg[y] = (r_1+k_1, r_2+k_2, r_3+k_3, s_1+l_1, s_2+l_2, s_3+l_3)$$
$$\text{and} \quad \deg[z] = (r_1^{'}+k_1, r_2^{'}+k_2, r_3^{'}+k_3, s_1^{'}+l_1, s_2^{'}+l_2, s_3^{'}+l_3).$$

Clearly $r_i + k_i \neq r_i^{'} + k_i$, for some $i = 1, 2, 3$ therefore y and z have distinct closed neighborhood degrees. Hence G is a totally irregular bipolar neutrosophic graph.
The converse part is similar. □

Definition 3.15. If $G = (C, D)$ be a bipolar neutrosophic graph and y, z are two vertices in G then we say that y *dominates* z if

$$t_D^p(yz) = t_C^p(y) \wedge t_C^p(z), \quad I_D^p(yz) = I_C^p(y) \vee I_C^p(z), \quad f_D^p(yz) = f_C^p(y) \vee f_C^p(z),$$
$$t_D^n(yz) = t_C^n(y) \vee t_C^n(z), \quad I_D^n(yz) = I_C^n(y) \wedge I_C^n(z), \quad f_D^n(yz) = f_C^n(y) \wedge f_C^n(z).$$

A subset $D^{'} \subseteq Y$ is a *dominating set* if for each $z \in Y \setminus D^{'}$ there exists $y \in D^{'}$ such that y dominates z. A dominating set $D^{'}$ is minimal if for every $y \in D^{'}$, $D^{'} \setminus \{y\}$ is not a dominating set. The *domination number* of G is the minimum cardinality among all minimal dominating sets of G, denoted by $\lambda(G)$.

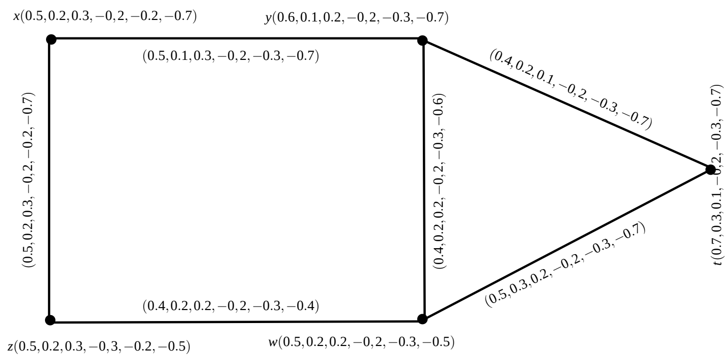

Figure 2: Bipolar neutrosophic graph G.

Example 3.2. Consider a bipolar neutrosophic graph as shown in Fig.2. The set $\{x,w\}$ is a minimal dominating set and $\lambda(G) = 2$

Theorem 3.6. *If G_1 and G_2 are two bipolar neutrosophic graphs with $D_1^{'}$ and $D_2^{'}$ as dominating sets then $\lambda(G_1 \cup G_2) = \lambda(G_1) + \lambda(G_2) - |D_1^{'} \cap D_2^{'}|$.*

Proof. Since $D_1^{'}$ and $D_2^{'}$ are dominating sets of G_1 and G_2, $D_1^{'} \cup D_2^{'}$ is a dominating set of $G_1 \cup G_2$. Therefore, $\lambda(G_1 \cup G_2) \leq |D_1^{'} \cup D_2^{'}|$. It only remains to show that $D_1^{'} \cup D_2^{'}$ is the minimal dominating set. On contrary, assume that $D^{'} = D_1^{'} \cup D_2^{'} \setminus \{y\}$ is a minimal dominating set of $G_1 \cup G_2$. There are two cases,
Case 1. If $y \in D_1^{'}$ and $x \notin D_2^{'}$, then $D_1^{'} \setminus \{y\}$ is not a dominating set of G_1 which implies that $D_1^{'} \cup D_2^{'} \setminus \{y\} = D^{'}$ is not a dominating set of $G_1 \cup G_2$. A contradiction, hence $D_1^{'} \cup D_2^{'}$ is a minimal dominating set and

$$\lambda(G_1 \cup G_2) = |D_1^{'} \cup D_2^{'}|,$$
$$\Rightarrow \qquad \lambda(G_1 \cup G_2) = \lambda(G_1) + \lambda(G_2) - |D_1^{'} \cap D_2^{'}|.$$

Case 2. If $y \in D_2^{'}$ and $y \notin D_1^{'}$, same contradiction can be obtained. □

Theorem 3.7. *If G_1 and G_2 are two bipolar neutrosophic graphs with $Y_1 \cap Y_2 \neq \emptyset$ then,*

$$\lambda(G_1 + G_2) = \min\{\lambda(G_1), \lambda(G_2), 2\}.$$

Proof. Let $y_1 \in Y_1$ and $y_2 \in Y_2$, since $G_1 + G_2$ is a bipolar neutrosophic graph, we have

$$t^p_{D_1+D_2}(y_1y_2) = t^p_{C_1+C_2}(y_1) \wedge t^p_{C_1+C_2}(y_2), \qquad t^n_{D_1+D_2}(y_1y_2) = t^n_{C_1+C_2}(y_1) \vee t^n_{C_1+C_2}(y_2)$$
$$I^p_{D_1+D_2}(y_1y_2) = I^p_{C_1+C_2}(y_1) \vee I^p_{C_1+C_2}(y_2), \qquad I^n_{D_1+D_2}(y_1y_2) = I^n_{C_1+C_2}(y_1) \wedge I^n_{C_1+C_2}(y_2)$$
$$f^p_{D_1+D_2}(y_1y_2) = f^p_{C_1+C_2}(y_1) \vee f^p_{C_1+C_2}(y_2), \qquad f^n_{D_1+D_2}(y_1y_2) = f^n_{C_1+C_2}(y_1) \wedge f^n_{C_1+C_2}(y_2).$$

Hence any vertex of G_1 dominates all vertices of G_2 and similarly any vertex of G_2 dominates all vertices of G_1. So, $\{y_1, y_2\}$ is a dominating set of $G_1 + G_2$. If D is a minimum dominating set of $G_1 + G_2$, then D is one of the following forms,

1. $D = D_1$ where, $\lambda(G_1) = |D_1|$,
2. $D = D_2$ where, $\lambda(G_2) = |D_2|$,
3. $D = \{y_1, y_2\}$ where, $y_1 \in Y_1$ and $y_2 \in Y_2$. $\{y_1\}$ and $\{y_2\}$ are not dominating sets of G_1 or G_2, respectively.

Hence,

$$\lambda(G_1 + G_2) = \min\{\lambda(G_1), \lambda(G_2), 2\}.$$

□

Theorem 3.8. *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two bipolar neutrosophic graphs. If for $y_1 \in X_1$, $C_1(y_1) > \mathbf{0}$ where, $\mathbf{0} = (0,0,0,0,0,0)$, and y_2 dominates z_2 in G_2 then (y_1, y_2) dominates (y_1, z_2) in $G_1 \square G_2$.*

Proof. Since y_2 dominates z_2 therefore,

$$\begin{aligned}
t^p_{D_2}(y_2z_2) &= t^p_{C_2}(y_2) \wedge t^p_{C_2}(z_2), & I^p_{D_2}(y_2z_2) &= I^p_{C_2}(y_2) \vee I^p_{C_2}(z_2), & f^p_{D_2}(y_2z_2) &= f^p_{C_2}(y_2) \vee f^p_{C_2}(z_2),\\
t^n_{D_2}(y_2z_2) &= t^n_{C_2}(y_2) \vee t^n_{C_2}(z_2), & I^n_{D_2}(y_2z_2) &= I^n_{C_2}(y_2) \wedge I^n_{C_2}(z_2), & f^n_{D_2}(y_2z_2) &= f^n_{C_2}(y_2) \wedge f^n_{C_2}(z_2).
\end{aligned}$$

For $y_1 \in Y_1$, take $(y_1, z_2) \in Y_1 \times Y_2$. By definition 3.6,

$$\begin{aligned}
t^p_{D_1 \square D_2}((y_1,y_2)(y_1,z_2)) &= t^p_{C_1}(y_1) \wedge t^p_{D_2}(y_2z_2),\\
&= t^p_{C_1}(y_1) \wedge \{t^p_{C_2}(y_2) \wedge t^p_{C_2}(z_2)\},\\
&= \{t^p_{C_1}(y_1) \wedge t^p_{C_2}(y_2)\} \wedge \{t^p_{C_1}(y_1) \wedge t^p_{C_2}(z_2)\},\\
&= t^p_{C_1 \square C_2}(y_1,y_2) \wedge t^p_{C_1 \square C_2}(y_1,z_2).
\end{aligned}$$

$$\begin{aligned}
t^n_{D_1 \square D_2}((y_1,y_2)(y_1,z_2)) &= t^n_{C_1}(y_1) \vee t^n_{D_2}(y_2z_2),\\
&= t^n_{C_1}(y_1) \vee \{t^n_{C_2}(y_2) \vee t^n_{C_2}(z_2)\},\\
&= \{t^n_{C_1}(y_1) \vee t^n_{C_2}(y_2)\} \vee \{t^n_{C_1}(y_1) \vee t^n_{C_2}(z_2)\},\\
&= t^n_{C_1 \square C_2}(y_1,y_2) \vee t^n_{C_1 \square C_2}(y_1,z_2).
\end{aligned}$$

Similarly, it can be proved that

$$\begin{aligned}
I^p_{D_1 \square D_2}((y_1,y_2)(y_1,z_2)) &= I^p_{C_1 \square C_2}(y_1,y_2) \vee I^p_{C_1 \square C_2}(y_1,z_2),\\
I^n_{D_1 \square D_2}((y_1,y_2)(y_1,z_2)) &= I^n_{C_1 \square C_2}(y_1,y_2) \wedge I^n_{C_1 \square C_2}(y_1,z_2),\\
f^p_{D_1 \square D_2}((y_1,y_2)(y_1,z_2)) &= f^p_{C_1 \square C_2}(y_1,y_2) \vee f^p_{C_1 \square C_2}(y_1,z_2),\\
f^n_{D_1 \square D_2}((y_1,y_2)(y_1,z_2)) &= f^n_{C_1 \square C_2}(y_1,y_2) \wedge f^n_{C_1 \square C_2}(y_1,z_2).
\end{aligned}$$

Hence (y_1,y_2) dominates (y_1,z_2) and the proof is complete. □

Proposition 3.2. *If G_1 and G_2 are bipolar neutrosophic graphs and for $z_2 \in Y_2$, $C_2(z_2) > \mathbf{0}$ where, $\mathbf{0} = (0,0,0,0,0,0)$, y_1 dominates z_1 in G_1 then (y_1,z_2) dominates (z_1,z_2) in $G_1 \Box G_2$.*

Theorem 3.9. *If $D_1^{'}$ and $D_2^{'}$ are minimal dominating sets of $G_1 = (C_1,D_1)$ and $G_2 = (C_2,D_2)$, respectively. Then $D_1^{'} \times X_2$ and $X_1 \times D_2^{'}$ are dominating sets of $G_1 \Box G_2$ and*

$$\lambda(G_1 \Box G_2) \leq |D_1^{'} \times Y_2| \wedge |Y_1 \times D_2^{'}|. \tag{3.1}$$

Proof. To prove inequality 3.1, we need to show that $D_1^{'} \times Y_2$ and $Y_1 \times D_2^{'}$ are dominating sets of $G_1 \Box G_2$. Let $(z_1,z_2) \notin D_1^{'} \times Y_2$ then, $z_1 \notin D_1^{'}$. Since $D_1^{'}$ is a dominating set of G_1, there exists $y_1 \in D_1^{'}$ that dominates z_1. By theorem 3.2, (y_1,z_2) dominates (z_1,z_2) in $G_1 \Box G_2$. Since (z_1,z_2) was taken to be arbitrary therefore, $D_1^{'} \times Y_2$ is a dominating set of $G_1 \Box G_2$. Similarly, $Y_1 \times D_2^{'}$ is a dominating set if $G_1 \Box G_2$. Hence the proof. □

Theorem 3.10. *Let $D_1^{'}$ and $D_2^{'}$ be the dominating sets of $G_1 = (C_1,D_1)$ and $G_2 = (C_2,D_2)$, respectively. Then $D_1^{'} \times D_2^{'}$ is a dominating set of the direct product $G_1 \times G_2$ and*

$$\lambda(G_1 \times G_2) = |D_1^{'} \times D_2^{'}|. \tag{3.2}$$

Proof. Let $(z_1,z_2) \in Y_1 \times Y_2 \setminus D_1^{'} \times D_2^{'}$ then $z_1 \in Y_1 \setminus D_1^{'}$ and $z_2 \in Y_2 \setminus D_2^{'}$. Since, $D_1^{'}$ and $D_2^{'}$ are dominating sets there exist $y_1 \in D_1^{'}$ and $y_2 \in D_2^{'}$ such that y_1 dominates z_1 and y_2 dominates z_2. Consider,

$$\begin{aligned}
t^p_{D_1 \times D_2}((y_1,y_2)(z_1,z_2)) &= t^p_{D_1}(y_1z_1) \wedge t^p_{D_2}(y_2z_2),\\
&= \{t^p_{C_1}(y_1) \wedge t^p_{C_1}(z_1)\} \wedge \{t^p_{C_2}(y_2) \wedge t^p_{C_2}(z_2)\},\\
&= \{t^p_{C_1}(y_1) \wedge t^p_{C_2}(y_2)\} \wedge \{t^p_{C_1}(z_1) \wedge t^p_{C_2}(z_2)\},\\
&= t^p_{C_1 \times C_2}(y_1,y_2) \wedge t^p_{C_1 \times C_2}(z_1,z_2).
\end{aligned}$$

It shows that (y_1,y_2) dominates (z_1,z_2). Since (y_1,y_2) was taken to be arbitrary therefore, every element of $Y_1 \times Y_2 \setminus D_1^{'} \times D_2^{'}$ is dominated by some element of $D_1^{'} \times D_2^{'}$. It only remains to show that $D_1^{'} \times D_2^{'}$ is a minimal dominating set. On contrary, assume that $D^{'}$ is a minimal dominating set of $G_1 \times G_2$ such that $|D^{'}| < |D_1^{'} \times D_2^{'}|$. Let $(t_1,t_2) \in D_1^{'} \times D_2^{'}$ such that $(t_1,t_2) \notin D^{'}$ i.e., $t_1 \in D_1^{'}$ and $t_2 \in D_2^{'}$ then there exist $t_1^{'} \in Y_1 \setminus D_1^{'}$ and $t_2^{'} \in Y_2 \setminus D_2^{'}$ which are only dominated by t_1 and t_2, respectively. Hence no element other than (t_1,t_2) dominates $(t_1^{'},t_2^{'})$ so $(t_1,t_2) \in D^{'}$. A contradiction, thus $\lambda(G_1 \times G_2) = |D_1^{'} \times D_2^{'}|$. □

Corollary 3.1. *If G_1 and G_2 are two bipolar neutrosophic graphs, y_1 dominates z_1 in G_1 and y_2 dominates z_2 in G_2 then (y_1,z_1) dominates (y_2,z_2) in $G_1 \times G_2$.*

Definition 3.16. In a bipolar neutrosophic graph two vertices y and z are *independent* if

$$\begin{aligned}
&t^p_D(yz) < t^p_C(y) \wedge t^p_C(z), \quad I^p_D(yz) < I^p_C(y) \vee I^p_C(z), \quad f^p_D(yz) < f^p_C(y) \vee f^p_C(z),\\
&t^n_D(yz) > t^n_C(y) \vee t^n_C(z), \quad I^n_D(yz) > I^n_C(y) \wedge I^n_C(z), \quad f^n_D(yz) > f^n_C(y) \wedge f^n_C(z).
\end{aligned} \tag{3.3}$$

An *independent set* N of a bipolar neutrosophic graph is a subset N of Y such that for all $y,z \in N$ equations 3.3 are satisfied. An independent set is *maximal* if for every $t \in Y \setminus N$, $N \cup \{t\}$ is not an independent set. An *independent* *number* is the maximal cardinality among all maximal independent sets of a bipolar neutrosophic graph. It is denoted by $\alpha(G)$.

Theorem 3.11. *If G_1 and G_2 are bipolar neutrosophic graphs on Y_1 and Y_2, respectively such that $Y_1 \cap Y_2 = \emptyset$ then $\alpha(G_1 \cup G_2) = \alpha(G_1) + \alpha(G_2)$.*

Proof. Let N_1 and N_2 be maximal independent sets of G_1 and G_2. Since $N_1 \cap N_2 = \emptyset$ therefore, $N_1 \cup N_2$ is a maximal independent set of $G_1 \cup G_2$. Hence $\alpha(G_1 \cup G_2) = \alpha(G_1) + \alpha(G_2)$. □

Theorem 3.12. *Let G_1 and G_2 be two bipolar neutrosophic graphs then $\alpha(G_1 + G_2) = \alpha(G_1) \vee \alpha(G_2)$.*

Proof. Let N_1 and N_2 be maximal independent sets. Since every vertex of G_1 dominates every vertex of G_2 in $G_1 + G_1$. Hence, maximal independent set of $G_1 + G_2$ is either N_1 or N_2. Thus, $\alpha(G_1 + G_2) = \alpha(G_1) \vee \alpha(G_2)$. □

Theorem 3.13. *If N_1 and N_2 are maximal independent sets of G_1 and G_2, respectively and $Y_1 \cap Y_2 = \emptyset$. Then $\alpha(G_1 \square G_2) = |N_1 \times N_2| + |N|$ where, $N = \{(y_i, z_i) : y_i \in Y_1 \setminus N_1, z_i \in Y_2 \setminus N_2, y_i y_{i+1} \in E_1, z_i z_{i+1} \in E_2,\ i = 1,2,3,\cdots\}$.*

Proof. N_1 and N_2 are maximal independent sets of G_1 and G_2, respectively. Clearly, $N_1 \times N_2$ is an independent set of $G_1 \square G_2$ as no vertex of $N_1 \times N_2$ dominates any other vertex of $N_1 \times N_2$.
Consider the set of vertices $N = \{(y_i, z_i) : y_i \in Y_1 \setminus N_1, z_i \in Y_2 \setminus N_2, y_i y_{i+1} \in E_1, z_i z_{i+1} \in E_2\}$. It can be seen that no vertex $(y_i, z_i) \in N$ for each $i = 1,2,3,\cdots$ dominates $(y_{i+1}, z_{i+1}) \in N$ for each $i = 1,2,3,\cdots$. Hence $N' = (N_1 \times N_2) \cup N$ is an independent set of $G_1 \square G_2$.
Assume that $S = N' \cup \{(y_i, z_j)\}$, for some $i \neq j$, $y_i \in Y_1 \setminus N_1$ and $z_j \in Y_2 \setminus N_2$, is a maximal independent set. Without loss of generality, assume that $j = i+1$ then, (y_i, z_j) is dominated by (y_i, z_i). A contradiction, hence N' is a maximal independent set and $\alpha(G_1 \square G_2) = |N'| = |N_1 \times N_2| + |N|$ □

Theorem 3.14. *If D_1' and D_2' are minimal dominating sets of G_1 and G_2 then, $Y_1 \times Y_2 \setminus D_1' \times D_2'$ is a maximal independent set of $G_1 \times G_2$ and $\alpha(G_1 \times G_2) = n_1 n_2 - \lambda(G_1 \times G_2)$ where, n_1 and n_2 are the number of vertices in G_1 and G_2.*

The proof is obvious.

Theorem 3.15. *An independent set of a bipolar neutrosophic graph $G = (C, D)$ is maximal if and only if it is independent and dominating.*

Proof. If N is a maximal independent set of G, then for every $y \in Y \setminus N$, $N \cup \{y\}$ is not an independent set. For every vertex $y \in Y \setminus N$, there exists some $z \in N$ such that

$$t_D^p(yz) = t_C^p(y) \wedge t_C^p(z), \quad I_D^p(yz) = I_C^p(y) \vee I_C^p(z), \quad f_D^p(yz) = f_C^p(y) \vee f_C^p(z),$$
$$t_D^n(yz) = t_C^n(y) \vee t_C^n(z), \quad I_D^n(yz) = I_C^n(y) \wedge I_C^n(z), \quad f_D^n(yz) = f_C^n(y) \wedge f_C^n(z).$$

Thus y dominates x and hence N is both independent and dominating set.
Conversely, assume that D is both independent and dominating set but not maximal independent set. So there exists

a vertex $x \in X \setminus N$ such that $N \cup \{x\}$ is an independent set i.e., no vertex in N dominates x, a contradiction to the fact that N is a dominating set. Hence N is maximal. □

Theorem 3.16. *Any maximal independent set of a bipolar neutrosophic graph is a minimal dominating set.*

Proof. If N is a maximal independent set of a bipolar neutrosophic graph then by Theorem 3.15, N is a dominating set. Assume that N is not a minimal dominating set then, there always exist at least one $z \in N$ for which $N \setminus \{z\}$ is a dominating set. On the other hand if $N \setminus \{z\}$ dominates $X \setminus \{N \setminus \{z\}\}$, at least one vertex in $N \setminus \{z\}$ dominates z. A contradiction to the fact that N is an independent set of bipolar neutrosophic graph G. Hence N is a minimal dominating set. □

4 Multiple criteria decision making methods

Multiple criteria decision making refers to making decisions in the presence of multiple, usually conflicting, criteria. Multiple criteria decision making problems are common in everyday life. In this section, we present multiple criteria decision making methods for the identification of risk in decision support systems. The method is explained by an example for prevention of accidental hazards in chemical industry. The application of domination in bipolar neutrosophic graphs is given for the construction of transmission stations.

(1) An outranking approach for safety analysis using bipolar neutrosophic sets

The proposed methodology can be implemented in various fields in different ways e.g., multi-criteria decision making problems with bipolar neutrosophic information. However, our main focus is the identification of risk assessments in industry which is described in the following steps.

The bipolar neutrosophic information consists of a group of risks\alternatives $R = \{r_1, r_2, \cdots, r_n\}$ evaluated on the basis of criteria $C = \{c_1, c_2, \cdots, c_m\}$. Here $r_i,\ i = 1, 2, \cdots, n$ is the possibility for the criteria $c_k,\ k = 1, 2, \cdots, m$ and r_{ik} are in the form of bipolar neutrosophic values. This method is suitable if we have a small set of data and experts are able to evaluate the data in the form of bipolar neutrosophic information. Take the values of r_{ik} as $r_{ik} = (t^p_{ik}, I^p_{ik}, f^p_{ik}, t^n_{ik}, I^n_{ik}, f^n_{ik})$.

Step 1. Construct the table of the given data.

Step 2. Determine the average values using the following bipolar neutrosophic average operator,

$$A_i = \frac{1}{n}\Big(\sum_{j=1}^{m} t^p_{ij} - \prod_{j=1}^{m} t^p_{ij},\ \prod_{j=1}^{m} I^p_{ij},\ \prod_{j=1}^{m} f^p_{ij},\ \prod_{j=1}^{m} t^n_{ij},\ \sum_{j=1}^{m} I^n_{ij} - \prod_{j=1}^{m} I^n_{ij},\ \sum_{j=1}^{m} f^n_{ij} - \prod_{j=1}^{m} f^n_{ij}\Big), \tag{4.1}$$

for each $i = 1, 2, \cdots, n$.

Step 3. Construct the weighted average matrix.

Choose the weight vector $\mathbf{w} = (w_1, w_2, \cdots, w_n)$. According to the weights for each alternative, the weighted average table can be calculated by multiplying each average value with the corresponding weight as:

$$\beta_i = A_i w_i, \qquad i = 1, 2, \cdots, n.$$

Step 4. Calculate the normalized value for each alternative\risk β_i using the formula,

$$\alpha_i = \sqrt{(t_i^p)^2 + (I_i^p)^2 + (f_i^p)^2 + (1-t_i^n)^2 + (-1+I_i^n)^2 + (-1+f_i^n)^2}, \tag{4.2}$$

for each $i = 1, 2, \cdots, n$. The resulting table indicate the preference ordering of the alternatives\risks. The alternative\risk with maximum α_i value is most dangerous or more preferable.

Example 4.1. Chemical industry is a very important part of human society. These industries contain large amount of organic and inorganic chemicals and materials. Many chemical products have a high risk of fire due to flammable materials, large explosions and oxygen deficiency etc. These accidents can cause the death of employs, damages to building, destruction of machines and transports, economical losses etc. Therefore, it is very important to prevent these accidental losses by identifying the major risks of fire, explosions and oxygen deficiency.
A manager of a chemical industry Y wants to prevent such types of accidents that caused the major loss to company in the past. He collected data from witness reports, investigation teams and near by chemical industries and found that the major causes could be the chemical reactions, oxidizing materials, formation of toxic substances, electric hazards, oil spill, hydrocarbon gas leakage and energy systems. The witness reports, investigation teams and industries have different opinions. There is a bipolarity in people's thinking and judgement. The data can be considered as bipolar neutrosophic information. The bipolar neutrosophic information about company Y old accidents is given in Table 1

Table 1: Bipolar neutrosophic Data

	Fire	Oxygen Deficiency	Large Explosion
Chemical Exposures	(0.5,0.7,0.2,-0.6,-0.3,-0.7)	(0.1,0.5,0.7,-0.5,-0.2,-0.8)	(0.6,0.2,0.3,-0.4,0.0,-0.1)
Oxidizing materials	(0.9,0.7,0.2,-0.8,-0.6,-0.1)	(0.3,0.5,0.2,-0.5,-0.5,-0.2)	(0.9,0.5,0.5,-0.6,-0.5,-0.2)
Toxic vapour cloud	(0.7,0.3,0.1,-0.4,-0.1,-0.3)	(0.6,0.3,0.2,-0.5,-0.3,-0.3)	(0.5,0.1,0.2,-0.6,-0.2,-0.2)
Electric Hazard	(0.3,0.4,0.2,-0.6,-0.3,-0.7)	(0.9,0.4,0.6,-0.1,-0.7,-0.5)	(0.7,0.6,0.8,-0.7,-0.5,-0.1)
Oil Spill	(0.7,0.5,0.3,-0.4,-0.2,-0.2)	(0.2,0.2,0.2,-0.7,-0.4,-0.4)	(0.9,0.2,0.7,-0.1,-0.6,-0.8)
Hydrocarbon gas leakage	(0.5,0.3,0.2,-0.5,-0.2,-0.2)	(0.3,0.2,0.3,-0.7,-0.4,-0.3)	(0.8,0.2,0.1,-0.1,-0.9,-0.2)
Ammonium Nitrate	(0.3,0.2,0.3,-0.5,-0.6,-0.5)	(0.9,0.2,0.1,0.0,-0.6,-0.5)	(0.6,0.2,0.1,-0.2,-0.3,-0.5)

By applying the bipolar neutrosophic average operator 4.1 on Table 1, the average values are given in Table.2.

Table 2: Bipolar neutrosophic average values

	Average Value
Chemical Exposures	(0.39,0.023,0.014,-0.04,-0.167,-0.515)
Oxidizing materials	(0.619,0.032,0.001,-0.08,-0.483,-0.165)
Toxic vapour cloud	(0.53,0.003,0.001,-0.04,-0.198,-0.261)
Electric Hazard	(0.570,0.032,0.032,-0.014,-0.465,-0.422)
Oil Spill	(0.558,0.007,0.014,-0.009,-0.384,-0.445)
Hydrocarbon gas leakage	(0.493,0.004,0.002,-0.011,-0.543,-0.229)
Ammonium Nitrate	(0.546,0.003,0.001,0.0,-0.464,-0.417)

With regard to the weight vector $(0.35, 0.80, 0.30, 0.275, 0.65, 0.75, 0.50)$ associated to each cause of accident, the weighted average values are obtained by multiplying each average value with corresponding weight and are given in Table 3.

Table 3: Bipolar neutrosophic weighted average table

	Average Value
Chemical Exposures	(0.1365,0.0081,0.0049,-0.0140,-0.0585,-0.1803)
Oxidizing materials	(0.4952,0.0256,0.0008,-0.0640,-0.3864,-0.1320)
Toxic vapour cloud	(0.1590,0.0009,0.0003,-0.012,-0.0594,-0.0783)
Electric Hazard	(0.2850,0.0160,0.0160,-0.0070,-0.2325,-0.2110)
Oil Spill	(0.1535,0.0019,0.0039,-0.0025,-0.1056,-0.1224)
Hydrocarbon gas leakage	(0.3205,0.0026,0.0013,-0.0072,-0.3530,-0.1489)
Ammonium Nitrate	(0.4095,0.0023,0.0008,0.0,-0.3480,-0.2110)

Using formula 4.2, the resulting normalized values are shown in Table 4.

Table 4: Normalized values

	Normalized value
Chemical Exposures	1.5966
Oxidizing materials	1.5006
Toxic vapour cloud	1.6540
Electric Hazard	1.6090
Oil Spill	1.4938
Hydrocarbon gas leakage	1.6036
Ammonium Nitrate	1.5089

The accident possibilities can be placed in the following order: Toxic vapour cloud $\succ$ Electric Hazard $\succ$ Hydrocarbon gas leakage $\succ$ Chemical Exposures $\succ$ Ammonium Nitrate $\succ$ Oxidizing materials $\succ$ Oil Spill where, the symbol $\succ$ represents partial ordering of objects. It can be easily seen that the formation of toxic vapour clouds, electrical and energy systems and hydrocarbon gas leakage are the major dangers to the chemical industry. There is a very little danger due to oil spill. Chemical Exposures, oxidizing materials and ammonium nitrate has an average accidental danger. Therefore, industry needs special precautions to prevent the major hazards that could happen due the formation of toxic vapour clouds.

(2) Domination in bipolar neutrosophic graphs

Domination has a wide variety of applications in communication networks, coding theory, fixing surveillance cameras, detecting biological proteins and social networks etc. Consider the example of a TV channel that wants to set up transmission stations in a number of cities such that every city in the country get access to the channel signals from at least one of the stations. To reduce the cost for building large stations it is required to set up minimum number of stations. This problem can be represented by a bipolar neutrosophic graph in which vertices represent the cities and there is an edge between two cities if they can communicate directly with each other. Consider the network of ten cities $\{C_1, C_2, \cdots, C_{10}\}$. In the bipolar neutrosophic graph, the degree of each vertex represents the level of signals it can transmit to other cities and the bipolar neutrosophic value of each edge represents the degree of communication between the cities. The graph is shown in Figure.3. $D = \{C_8, C_{10}\}$ is the minimum dominating set. It is concluded that

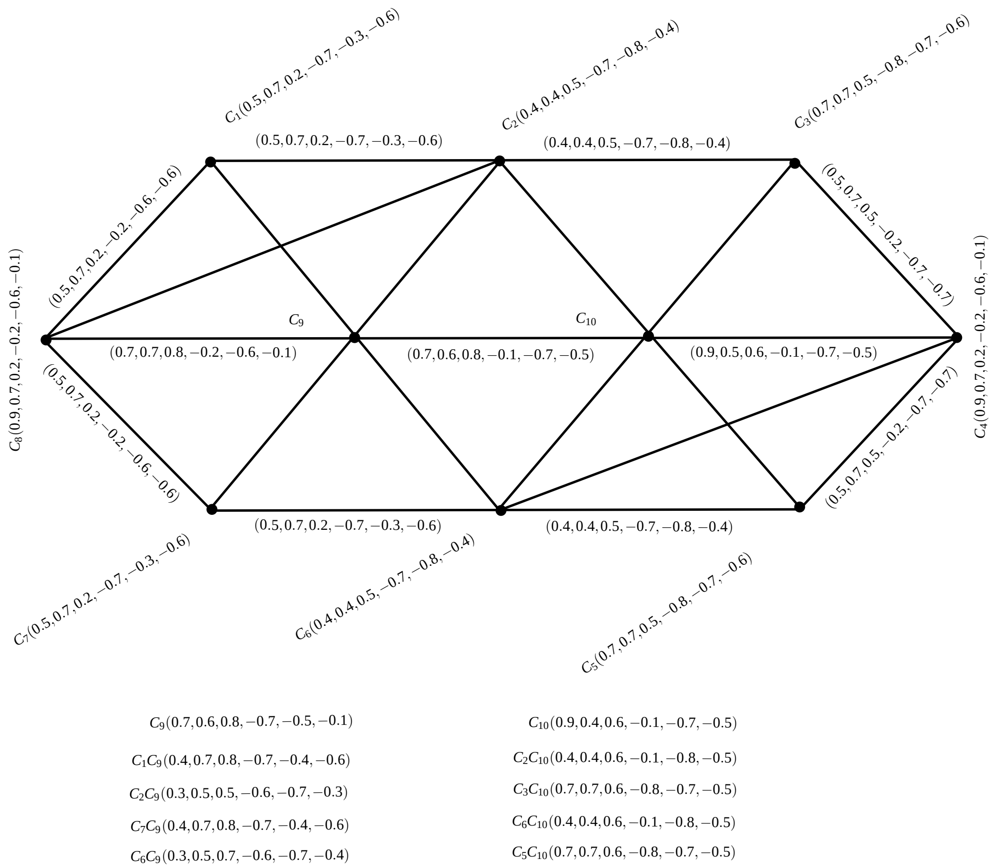

Figure 3: Domination in bipolar neutrosophic graph

building only two large transmitting stations in C_8 and C_{10}, a high economical benefit can be achieved.

The method of calculating the minimum number of stations in described in the following algorithm 1.
Algorithm 1

1. Enter the total number of possible locations n.
2. Input the adjacency matrix $[C_{ij}]_{n\times n}$ of transmission stations $C_1, C_2, \ldots, C_n$.
3. $k = 0, D = \emptyset$
4. do i from $1 \to n$
   do j from $i+1 \to n$
   if $(t^p, I^p, f^p, t^n, I^n, f^n)(C_iC_j) = (t^p, I^p, f^p, t^n, I^n, f^n)(C_i) \wedge (t^p, I^p, f^p, t^n, I^n, f^n)(C_j)$ then
   $C_i \in D$, $k = k+1$, $x_k = C_i$
   end if
   end do
   end do
5. Arrange $X \setminus D = \{x_{k+1}, x_{k+2}, \ldots, x_n\} = J,\ \ p = 0,\ \ q = 1$
6. do i from $1 \to k$
   $D^{'} = D \setminus x_{k-i+1},\ \ x_{k-i+1} = x_{n+1}$
   do j from $k \to n+1$
   do m from $1 \to k-1$
   if $(t^p, I^p, f^p, t^n, I^n, f^n)(x_mx_j) = (t^p, I^p, f^p, t^n, I^n, f^n)(x_m) \wedge (t^p, I^p, f^p, t^n, I^n, f^n)(x_j)$ then
   $D = D^{'},\ \ p = p+1,\ \ \ k = k-1,\ \ d_q = x_i,\ \ q = q+1,$ stop the loop
   else if $(m = k-1)$ then
   $D = D, D^{'} = \emptyset$
   end if
   end do
   end do
   end do
7. if $(D \cup (\cup_{i=1}^{q} d_i) \cup J = X)$ then
   D is a minimal dominating set.
   else
   There is no dominating set.
   end if

5 Conclusions

Bipolar fuzzy graph theory has many applications in science and technology, especially in the fields of neural networks, operations research, artificial intelligence and decision making. A bipolar neutrosophic graph is a generalization of the notion bipolar fuzzy graph. We have introduced the idea of bipolar neutrosophic graph and operations on bipolar neutrosophic graphs. Some properties of regular, totally regular, irregular and totally irregular bipolar neutrosophic graphs are discussed in detail. We have investigated the dominating and independent sets of certain graph products. Two applications of bipolar neutrosophic sets and bipolar neutrosophic graphs are studied in chemical industry and construction of radio channels. We are extend our research of fuzzification to (1) Bipolar fuzzy rough graphs; (2) Bipolar fuzzy rough hypergraphs, (3) Bipolar fuzzy rough neutrosophic graphs, and (4) Decision support systems based on bipolar neutrosophic graphs.

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