On $\alpha\omega$-closed sets and its connectedness in terms of neutrosophic topological spaces

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Abstract

The aim of this paper is to introduce the notion of neutrosophic $\alpha\omega$-closed sets and study some of the properties of neutrosophic $\alpha\omega$-closed sets. Further, we investigated neutrosophic $\alpha\omega$-continuity, neutrosophic $\alpha\omega$-irresoluteness, neutrosophic $\alpha\omega$-connectedness and neutrosophic contra $\alpha\omega$-continuity along with examples.

Keywords: neutrosophic topology, neutrosophic $\alpha\omega$-closed set, neutrosophic $\alpha\omega$-continuous function and neutrosophic contra $\alpha\omega$-continuous mappings.

1 Introduction

Zadeh [19] introduced truth (t) or the degree of membership of an object in fuzzy set theory. The falsehood (f) or the degree of non-membership of an object along with membership of an object introduced by Atanassov [4,5,6] in intuitionistic fuzzy set. Neutrosophic (i) or the degree of indeterminacy of an object along with membership and non-membership of an objects for incomplete, imprecise, indeterminate information is introduced by Smarandache [16,17] in 1998. The neutrosophic triplet set consist of three components $(t, f, i) = (\text{truth}, \text{falsehood}, \text{indeterminacy})$. The neutrosophic topological spaces introduced and developed by Salama et al., [15]. This leads to many investigation among researchers in the field of neutrosophic topology and their application in decision making algorithms [8,11,12,13,14]. Arokiarani et al.,[3] introduced and studied $\alpha$-open sets in neutrosophic topological spaces. Devi et al., [7,9,10] introduced $\alpha\omega$-closed sets in general topology, fuzzy topology and intuitionistic fuzzy topology. In this article, we introduce neutrosophic $\alpha\omega$-closed sets in neutrosophic topological spaces. Also, we introduce and investigate neutrosophic $\alpha\omega$-continuous function and neutrosophic contra $\alpha\omega$-continuous mappings.

2 Preliminaries

Let $(X, \tau)$ be the neutrosophic topological space(NTS). Each neutrosophic set(NTS) in $(X, \tau)$ is called a neutrosophic open set(NOS) and its complement is called a neutrosophic closed set (NCS).

We provide some of the basic definitions in neutrosophic sets. These are very useful in the sequel.

Definition 2.1. [17] A neutrosophic set (NS) $A$ is an object of the following form

$U = \{(u, \mu_U(u), \nu_U(u), \omega_U(u)) : u \in X\}$

where the mappings $\mu_U : X \rightarrow I$, $\nu_U : X \rightarrow I$ and $\omega_U : X \rightarrow I$ denote the degree of membership (namely $\mu_U(u)$), the degree of indeterminacy (namely $\nu_U(u)$) and the degree of nonmembership (namely $\omega_U(u)$) for
each element \( u \in X \) to the set \( U \), respectively and \( 0 \leq \mu_U(u) + \nu_U(u) + \omega_U(u) \leq 3 \) for each \( u \in X \).

**Definition 2.2.** [17] Let \( U \) and \( V \) be NSs of the form \( U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u) \rangle : u \in X \} \) and \( V = \{\langle u, \mu_V(u), \nu_V(u), \omega_V(u) \rangle : u \in X \} \). Then

(i) \( U \subseteq V \) if and only if \( \mu_U(u) \leq \mu_V(u) \), \( \nu_U(u) \geq \nu_V(u) \) and \( \omega_U(u) \geq \omega_V(u) \);

(ii) \( \overline{U} = \{\langle u, \nu_U(u), \mu_U(u), \omega_U(u) \rangle : u \in X \} \);

(iii) \( U \cap V = \{\langle u, \mu_U(u) \land \mu_V(u), \nu_U(u) \lor \nu_V(u), \omega_U(u) \lor \omega_V(u) \rangle : u \in X \} \);

(iv) \( U \cup V = \{\langle u, \mu_U(u) \lor \mu_V(u), \nu_U(u) \land \nu_V(u), \omega_U(u) \land \omega_V(u) \rangle : u \in X \} \).

We will use the notation \( U = \langle u, \mu_U, \nu_U, \omega_U \rangle \) instead of \( U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u) \rangle : u \in X \} \). The NSs \( 0_x \) and \( 1_x \) are defined by \( 0_x = \{\langle u, 0, 0, 1 \rangle : u \in X \} \) and \( 1_x = \{\langle u, 1, 0, 0 \rangle : u \in X \} \).

Let \( r, s, t \in [0, 1] \) such that \( 0 \leq r + s + t \leq 3 \). A neutrosophic point \( (NP) \) \( p_{(r,s,t)} \) is a neutrosophic set defined by

\[
p_{(r,s,t)}(u) = \begin{cases} (r,s,t)(x) & \text{if } u = p \\ (0,1,1) & \text{otherwise} \end{cases}
\]

Let \( f \) be a mapping from an ordinary set \( X \) into an ordinary set \( Y \). If \( V = \{\langle y, \mu_V(y), \nu_V(y), \omega_V(y) \rangle : y \in Y \} \) is a NS in \( Y \), then the inverse image of \( V \) under \( f \) is a NS defined by

\[
f^{-1}(V) = \{\langle u, f^{-1}(\mu_V(u)), f^{-1}(\nu_V(v)), f^{-1}(\omega_V(v)) \rangle : u \in X \}
\]

The image of NS \( U = \{\langle v, \mu_U(v), \nu_U(v), \omega_U(v) \rangle : v \in Y \} \) under \( f \) is a NS defined by \( f(U) = \{\langle v, f(\mu_U(v)), f(\nu_U(v)), f(\omega_U(v)) \rangle : v \in Y \} \) where

\[
f(\mu_U)(v) = \begin{cases} \sup_{u \in f^{-1}(v)} \mu_U(u), & \text{if } f^{-1}(v) \neq 0 \\ 0 & \text{otherwise} \end{cases}
\]

\[
f(\nu_U)(v) = \begin{cases} \inf_{u \in f^{-1}(v)} \nu_U(u), & \text{if } f^{-1}(v) \neq 0 \\ 1 & \text{otherwise} \end{cases}
\]

\[
f(\omega_U)(v) = \begin{cases} \inf_{u \in f^{-1}(v)} \omega_U(u), & \text{if } f^{-1}(v) \neq 0 \\ 1 & \text{otherwise} \end{cases}
\]

for each \( v \in Y \).

**Definition 2.3.** [15] A neutrosophic topology (NT) in a nonempty set \( X \) is a family \( \tau \) of NSs in \( X \) satisfying the following axioms:

(NT1) \( 0_x, 1_x \in \tau \);

(NT2) \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \);

(NT3) \( \cup G_i \in \tau \) for any arbitrary family \( \{G_i : i \in J\} \subseteq \tau \).

**Definition 2.4.** [15] Let \( U \) be a NS in NTS \( X \). Then

\( \text{Nint}(U) = \cup \{O : O \text{ is an NOS in } X \text{ and } O \subseteq U\} \)

is called a neutrosophic interior of \( U \);

\( \text{Ncl}(U) = \cap \{O : O \text{ is an NCS in } X \text{ and } O \supseteq U\} \)

is called a neutrosophic closure of \( U \).

**Definition 2.5.** [15] Let \( p_{(r,s,t)} \) be a NP in NTS \( X \). A NS \( U \) in \( X \) is called a neutrosophic neighborhood (NN) of \( p_{(r,s,t)} \) if there exists a NOS \( V \) in \( X \) such that \( p_{(r,s,t)} \in V \subseteq U \).

**Definition 2.6.** [3] A subset \( U \) of a neutrosophic space \( (X, \tau) \) is called

1. a neutrosophic pre-open set if \( U \subseteq \text{Nint} (\text{Ncl}(U)) \) and a neutrosophic pre-closed set if \( \text{Ncl}(\text{Nint}(U)) \subseteq U \);

2. a neutrosophic semi-open set if \( U \subseteq \text{Ncl}(\text{Nint}(U)) \) and a neutrosophic semi-closed set if \( \text{Nint}(\text{Ncl}(U)) \subseteq U \);

3. a neutrosophic \( \alpha \)-open set if \( U \subseteq \text{Nint} (\text{Ncl}(\text{Nint}(U))) \) and a neutrosophic \( \alpha \)-closed set if \( \text{Ncl}(\text{Nint}(\text{Ncl}(U))) \subseteq U \).
The pre-closure (resp. semi-closure, $\alpha$-closure) of a subset $U$ of a neutrosophic space $(X, \tau)$ is the intersection of all pre-closed (resp. semi-closed, $\alpha$-closed) sets that contain $U$ and is denoted by $Npc(U)$ (resp. $Nscl(U)$, $N\alpha cl(U)$).

3 On neutrosophic $\alpha\omega$-closed sets

**Definition 3.1.** A subset $A$ of a neutrosophic topological space $(X, \tau)$ is called

1. a neutrosophic $\alpha\omega$-closed set if $Ncl(U) \subseteq G$ whenever $U \subseteq G$ and $G$ is neutrosophic semi-open in $(X, \tau)$.

2. a neutrosophic $\omega\alpha$-closed ($N\alpha\omega$-closed) set if $N\omega cl(U) \subseteq G$ whenever $U \subseteq G$ and $G$ is an $N\alpha$-open set in $(X, \tau)$. Its complement is called a neutrosophic $\alpha\omega$-open ($N\alpha\omega$-open) set.

**Definition 3.2.** Let $U$ be a NS in NTS $X$. Then

$N\omega int(U) = \bigcup\{O : O$ is an $N\omega OS$ in $X$ and $O \subseteq U\}$ is said to be a neutrosophic $\omega\alpha$-interior of $U$;

$N\omega cl(U) = \bigcap\{O : O$ is an $N\alpha CS$ in $X$ and $O \supseteq U\}$ is said to be a neutrosophic $\omega\alpha$-closure of $U$.

**Theorem 3.3.** Every $N\alpha$-closed set and $N$-closed set are $N\alpha\omega$-closed set.

**Proof.** Let $U$ be an $N\alpha$-closed set, then $U = N\alpha cl(U)$. Let $U \subseteq G$, $G$ is $N\alpha$-open. Since $U$ is $N\alpha$-closed, $N\omega cl(U) \subseteq N\alpha cl(U) \subseteq G$. Thus $U$ is $N\alpha\omega$-closed.

**Theorem 3.4.** Every neutrosophic semi-closure set in a neutrosophic set is an $N\alpha\omega$-closed.

**Proof.** Let $U$ be a $N$semi-closed set in $(X, \tau)$, then $U = Nscl(U)$. Let $U \subseteq G$, $G$ is $N\alpha$-open in $(X, \tau)$. Since $U$ is $N$semi-closed, $N\omega cl(U) \subseteq Nscl(U) \subseteq G$. This shows that $U$ is $N\alpha\omega$-closed set.

The converses of the above theorems are not true as explained in Example 3.5.

**Example 3.5.** Let $X = \{u, v, w\}$ and neutrosophic sets $A, B, C$ be defined by:

$$A = \{(0.1, 0.4, 0.7), (0.9, 0.6, 0.3), (0.9, 0.6, 0.3)\}$$

$$B = \{(0.6, 0.6, 0.4), (0.2, 0.7, 0.8), (1, 0.6, 0.5)\}$$

$$C = \{(0.1, 0.4, 0.8), (0.2, 0.6, 0.4), (0.6, 0.5, 0.9)\}$$

Let $\tau = \{0, A, 1\}$. Then $B$ is $N\omega\alpha$-closed in $(X, \tau)$ but not $N\alpha$-closed and thus it is not $N$-closed and $C$ is $N\omega\alpha$-closed in $(X, \tau)$ but not $N$semi-closed.

**Theorem 3.6.** Let $(X, \tau)$ be a NTS and let $U \in NS(X)$. If $U$ is $N\omega\alpha$-closed set and $U \subseteq V \subseteq N\omega cl(U)$, then $V$ is $N\omega\alpha$-closed set.

**Proof.** Let $G$ be a $N\omega$-open set such that $V \subseteq G$. Since $U \subseteq V$, then $U \subseteq G$. But $U$ is $N\omega\alpha$-closed, so $N\omega cl(U) \subseteq G$. Since $V \subseteq N\omega cl(U)$, $N\omega cl(V) \subseteq N\omega cl(U)$ and hence $N\omega cl(V) \subseteq G$. Therefore $V$ is a $N\alpha\omega$-closed set.

**Theorem 3.7.** Let $U$ be a $N\alpha\omega$-open set in $X$ and $N\omega int(U) \subseteq V \subseteq U$, then $V$ is $N\alpha\omega$-open.

**Proof.** Suppose $U$ is $N\alpha$-open in $X$ and $N\omega int(U) \subseteq V \subseteq U$. Then $\overline{V}$ is $N\alpha$-closed and $\overline{V} \subseteq \overline{\overline{U}} \subseteq N\omega cl(U)$. Then $\overline{U}$ is a $N\alpha\omega$-closed set by theorem 3.5. Hence $V$ is a $N\alpha\omega$-open set in $X$.

**Theorem 3.8.** A NS $U$ in a NTS $(X, \tau)$ is a $N\alpha\omega$-open set if and only if $V \subseteq N\omega int(U)$ whenever $V$ is a $N\alpha$-closed set and $V \subseteq U$.

**Proof.** Let $U$ be a $N\alpha\omega$-open set and let $V$ be a $N\alpha$-closed set such that $V \subseteq U$. Then $\overline{U} \subseteq \overline{V}$ and hence $N\omega cl(U) \subseteq \overline{V}$, since $\overline{U}$ is $N\alpha$-closed. But $N\omega cl(\overline{U}) = N\omega int(U)$, thus $V \subseteq N\omega int(U)$.

Conversely, suppose that the condition is satisfied, then $N\omega cl(U) \subseteq \overline{V}$ whenever $\overline{V}$ is $N\alpha$-open set and $\overline{U} \subseteq \overline{V}$. This implies that $N\omega cl(\overline{U}) \subseteq \overline{V} = G$ where $G$ is $N\alpha$-open set and $\overline{U} \subseteq G$. Therefore $\overline{U}$ is $N\alpha\omega$-closed set and hence $U$ is $N\alpha\omega$-open.

**Theorem 3.9.** Let $U$ be a $N\alpha\omega$-closed subset of $(X, \tau)$. Then $N\omega cl(U) - U$ does not contain any non-empty $N\alpha\omega$-closed set.

**Proof.** Assume that $U$ is a $N\alpha\omega$-closed set. Let $F$ be a non-empty $N\alpha\omega$-closed set, such that $F \subseteq
Corollary 3.10. Let $U$ be a $N\alpha\omega$-closed set of $(X, \tau)$. Then $N\omega cl(U)-U$ does not contain any non-empty $N$-closed set.

Proof. The proof follows from the Theorem 3.9.

Theorem 3.11. If $U$ is both $N\omega$-open and $N\alpha\omega$-closed set, then $U$ is a $N\omega$-closed set.

Proof. Since $U$ is both $N\omega$-open and $N\alpha\omega$-closed set in $X$, then $N\omega cl(U) \subseteq U$. Also we have $U \subseteq N\omega cl(U)$. This gives that $N\omega cl(U) = U$. Therefore $U$ is a $N\omega$-closed set in $X$.

4 On neutrosophic $\alpha\omega$-continuity, connectedness and contra-continuity

Definition 4.1. Let $(X, \tau)$ and $(Y, \sigma)$ be any two neutrosophic topological spaces.

1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a neutrosophic $\alpha\omega$-continuous (briefly, $N\alpha\omega$-continuous) function if the inverse image of every open set in $Y$ is a $N\alpha\omega$-open set in $X$.
   Equivalently, if the inverse image of every open set in $(Y, \sigma)$ is $N\alpha\omega$-open in $(X, \tau)$;

2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a neutrosophic $\alpha\omega$-irresolute (briefly, $N\alpha\omega$-irresolute) function if the inverse image of every $N\alpha\omega$-open set in $Y$ is a $N\alpha\omega$-open set in $X$.
   Equivalently, if the inverse image of every $N\alpha\omega$-open set in $(Y, \sigma)$ is $N\alpha\omega$-open in $(X, \tau)$;

Definition 4.2. A NTS $(X, \tau)$ is said to be neutrosophic-$\alpha\omega T_{1/2}(N\alpha\omega T_{1/2}$ in short) space if every $N\alpha\omega C$ in $X$ is an $NC$ in $X$.

Definition 4.3. Let $(X, \tau)$ be any neutrosophic topological space. $(X, \tau)$ is said to be neutrosophic $\alpha\omega$-disconnected (shortly $N\alpha\omega$-disconnected) if there exists a $N\alpha\omega$-open and $N\alpha\omega$-closed set $\bar{F}$ such that $\bar{F} \neq 0_\omega$ and $\bar{F} \neq 1_\omega$. $(X, \tau)$ is said to be neutrosophic $\alpha\omega$-connected if it is not neutrosophic $\alpha\omega$-disconnected.

Theorem 4.4. Every $N\alpha\omega$-connected space is neutrosophic connected.

Proof. For a $N\alpha\omega$-connected $(X, \tau)$ space and let $(X, \tau)$ not be neutrosophic connected. Hence, there exists a proper neutrosophic set $\bar{F} = <\mu_{\bar{F}}, \sigma_{\bar{F}}, \nu_{\bar{F}}>$, $\bar{F} \neq 0_\omega$ and $\bar{F} \neq 1_\omega$, such that $\bar{F}$ is both neutrosophic open and neutrosophic closed in $(X, \tau)$. Since every neutrosophic open set is $N\alpha\omega$-open and neutrosophic closed set is $N\alpha\omega$-closed, $X$ is not neutrosophic connected. Therefore, $(X, \tau)$ is neutrosophic connected. However, the converse is not true.

Example 4.5. Let $X = \{u, v, w\}$ and neutrosophic sets $A, B$ and $C$ be defined by:

\[
A = \{(0.4, 0.5, 0.5), (0.4, 0.5, 0.5), (0.5, 0.5, 0.5)\}
\]
\[
B = \{(0.7, 0.6, 0.5), (0.7, 0.6, 0.5), (0.3, 0.4, 0.5)\}
\]
\[
C = \{(0.5, 0.6, 0.5), (0.5, 0.6, 0.5), (0.5, 0.6, 0.5)\}
\]

Let $\tau = \{0_\omega, A, B, 1_\omega\}$. It is obvious that $(X, \tau)$ is NTS. Now, $(X, \tau)$ is neutrosophic connected. However, it is not a $N\alpha\omega$-connected.

Theorem 4.6. Let $(X, \tau)$ be a neutrosophic $\alpha\omega T_{1/2}$ space. $(X, \tau)$ is neutrosophic connected iff $(X, \tau)$ is $N\alpha\omega$-connected.

Proof. Let $(X, \tau)$ is neutrosophic connected. Suppose that $(X, \tau)$ is not $N\alpha\omega$-connected, and there exists a neutrosophic set $\bar{F}$ which is both $N\alpha\omega$-open and $N\alpha\omega$-closed. Since $(X, \tau)$ is neutrosophic $\alpha\omega T_{1/2}$, $\bar{F}$ is both neutrosophic open and neutrosophic closed. Therefore, $(X, \tau)$ is not a neutrosophic connected which is contradiction to our hypothesis. Hence, $(X, \tau)$ is $N\alpha\omega$-connected.

Conversely, let $(X, \tau)$ is $N\alpha\omega$-connected. Suppose that $(X, \tau)$ is not neutrosophic connected, and there exists a neutrosophic set $\bar{F}$ such that $\bar{F}$ is both $NC$s and $NO$s in $(X, \tau)$. Since the neutrosophic open set is $N\alpha\omega$-open and the neutrosophic closed set is $N\alpha\omega$-closed, $(X, \tau)$ is not $N\alpha\omega$-connected. Hence, $(X, \tau)$ is neutrosophic connected.
Theorem 4.7. Suppose \((X, \tau)\) and \((Y, \sigma)\) are any two NTSs. If \(g : (X, \tau) \to (Y, \sigma)\) is \(N\omega\)-continuous surjection and \((X, \tau)\) is \(N\omega\)-connected, then \((Y, \sigma)\) is neutrosophic connected.

**Proof.** Suppose that \((Y, \sigma)\) is not neutrosophic connected, such that the neutrosophic set \(F\) is both neutrosophic open and neutrosophic closed in \((Y, \sigma)\). Since \(g\) is \(N\omega\)-continuous, \(g^{-1}(F)\) is \(Nomega\)-open and \(Nomega\)-closed in \((Y, \sigma)\). Thus, \((Y, \sigma)\) is not \(Nomega\)-connected. Hence, \((Y, \sigma)\) is neutrosophic connected.

Theorem 4.8. Let \(g : (X, \tau) \to (Y, \sigma)\) be a function. Then the following conditions are equivalent.

(i) \(g\) is \(Nomega\)-continuous;

(ii) The inverse \(f^{-1}(U)\) of each \(N\)-open set \(U\) in \(X\) is \(Nomega\)-open set in \(X\).

**Proof.** It is clear, since \(g^{-1}(U) = g^{-1}(Y, \sigma)\) for each \(N\)-open set \(U\) of \(Y\).

Theorem 4.9. If \(g : (X, \tau) \to (Y, \sigma)\) be a \(Nomega\)-continuous mapping, then the following statements holds:

(i) \(g(Nomega\text{Ncl}(U)) \subseteq Ncl(g(U))\), for all neutrosophic set \(U\) in \(X\);

(ii) \(Nomega\text{Ncl}(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))\), for all neutrosophic set \(V\) in \(Y\).

**Proof.**

(i) Since \(Ncl(g(U))\) is neutrosophic closed set in \(Y\) and \(g\) is \(Nomega\)-continuous, then \(g^{-1}(Ncl(g(U)))\) is \(Nomega\)-closed in \(X\). Now, since \(U \subseteq g^{-1}(Ncl(g(U)))\). So, \(Nomega\text{cl}(U) \subseteq g^{-1}(Ncl(g(U)))\). Therefore, \(g(Nomega\text{Ncl}(U)) \subseteq Ncl(g(U))\).

(ii) By replacing \(U\) with \(V\) in (i), we obtain \(g(Nomega\text{Ncl}(g^{-1}(V))) \subseteq Ncl(g(g^{-1}(V))) \subseteq Ncl(V)\). Hence \(Nomega\text{cl}(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))\).

Theorem 4.10. Let \(g\) be a function from a NTS \((X, \tau)\) to a NTS \((Y, \sigma)\). Then the following statements are equivalent.

(i) \(g\) is a neutrosophic \(omega\)-continuous function.

(ii) For every \(NP\) \(p_{(r,s,t)} \in X\) and each NN \(U\) of \(g(p_{(r,s,t)})\), there exists a \(Nomega\)-open set \(V\) such that \(p_{(r,s,t)} \in V \subseteq g^{-1}(U)\).

(iii) For every \(NP\) \(p_{(r,s,t)} \in X\) and each NN \(U\) of \(g(p_{(r,s,t)})\), there exists a \(Nomega\)-open set \(V\) such that \(p_{(r,s,t)} \in V \subseteq g^{-1}(U)\).

**Proof.** (i) \(\Rightarrow\) (ii). If \(p_{(r,s,t)}\) is a \(NP\) in \(X\) and also if \(U\) be a NN of \(g(p_{(r,s,t)})\), then there exists a NOS \(W\) in \(Y\) such that \(g(p_{(r,s,t)}) \in W \subseteq U\). we have \(g\) is neutrosophic \(omega\)-continuous, \(V = g^{-1}(W)\) is a \(Nomega\)OS and \(p_{(r,s,t)} \in g^{-1}(g(p_{(r,s,t)})) \subseteq g^{-1}(W) = V \subseteq g^{-1}(U)\).

Thus (ii) is a valid statement.

(ii) \(\Rightarrow\) (iii). Let \(p_{(r,s,t)}\) be a \(NP\) in \(X\) and take \(U\) be a NN of \(g(p_{(r,s,t)})\). Then there exists a \(Nomega\)OS \(U\) such that \(p_{(r,s,t)} \in V \subseteq g^{-1}(U)\) by (ii). Thus, we have \(p_{(r,s,t)} \in V \subseteq g^{-1}(U)\) \(\subseteq U\). Hence (iii) is valid.

(iii) \(\Rightarrow\) (i). Let \(V\) be a NOS in \(Y\) and let \(p_{(r,s,t)} \in g^{-1}(V)\). Then \(g(p_{(r,s,t)}) \in g(g^{-1}(V)) \subseteq V\). Since \(V\) is a NOS, it follows that \(V\) is a NN of \(g(p_{(r,s,t)})\) so from (iii), there exists a \(Nomega\)OS \(U\) such that \(p_{(r,s,t)} \in U\) and \(g(U) \subseteq V\). This implies that \(p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V)\).

Then, we know that \(g^{-1}(V)\) is a \(Nomega\)OS in \(X\). Thus \(g\) is neutrosophic \(omega\)-continuous.

Definition 4.11. A function is said to be a neutrosophic contra \(omega\)-continuous function if the inverse image of each NOS \(V\) in \(Y\) is a \(Nomega\)CS in \(X\).

Theorem 4.12. Let \(g : (X, \tau) \to (Y, \sigma)\) be a function. Then, the following assertions are equivalent:

(i) \(g\) is a neutrosophic contra \(omega\)-continuous function;
(ii) $g^{-1}(V)$ is a $N\omega$ CS in $X$, for each NOS $V$ in $Y$.

**Proof.** (i) $\Rightarrow$ (ii) Let $g$ be any neutrosophic contra $\omega$-continuous function and let $V$ be any NOS in $Y$. Then, $\overline{V}$ is a NCS in $Y$. By the assumption $g^{-1}(\overline{V})$ is a $N\omega$CS in $X$. Hence, we get that $g^{-1}(V)$ is a $N\omega$CS in $X$.

The converse of the theorem can be done in the same sense.

**Theorem 4.13.** Let $g : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an NTS $X$ into an NTS $Y$. The mapping $g$ is neutrosophic contra $\omega$-continuous if $Ncl(g(U)) \subseteq g(N\omega int(U))$, for each NS $U$ in $X$.

**Proof.** Let $V$ be any NCS in $X$. Then, $Ncl(V) = V$, and also $g$ is onto, by assumption, it shows that $g(N\omega int(g^{-1}(V))) \supseteq Ncl(g(g^{-1}(V))) = Ncl(V) = V$. Hence $g^{-1}(g(N\omega int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Since $g$ is an into mapping, we have $N\omega int(g^{-1}(V)) = g^{-1}(g(N\omega int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Therefore $N\omega int(g^{-1}(V)) = g^{-1}(V)$, so $g^{-1}(V)$ is a $N\omega$OS in $X$. Hence $g$ is a neutrosophic contra $\omega$-continuous mapping.

**Theorem 4.14.** Let $g : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:

(i) $g$ is a neutrosophic contra $\omega$-continuous mapping;

(ii) for each NP $p_{(r,s,t)}$ in $X$ and NCS $V$ containing $g(p_{(r,s,t)})$ there exists $N\omega$OS $U$ in $X$ containing $p_{(r,s,t)}$ such that $A \subseteq f^{-1}(B)$;

(iii) for each NP $p_{(r,s,t)}$ in $X$ and NCS $V$ containing $p_{(r,s,t)}$ there exists $N\omega$OS $U$ in $X$ containing $p_{(r,s,t)}$ such that $g(U) \subseteq V$.

**Proof.** (i) $\Rightarrow$ (ii) Let $g$ be an neutrosophic contra $\omega$-continuous mapping, let $V$ be any NCS in $Y$ and let $p_{(r,s,t)}$ be a NP in $X$ and such that $g(p_{(r,s,t)}) \in V$. Then $p_{(r,s,t)} \in g^{-1}(V) = N\omega int(g^{-1}(V))$. Let $U = N\omega int(g^{-1}(V))$. Then $U$ is an $N\omega$OS and $U = N\omega int(g^{-1}(V)) \subseteq g^{-1}(V)$.

(ii) $\Rightarrow$ (iii) The results follows from the evident relations $g(U) \subseteq g(g^{-1}(V)) \subseteq V$.

(iii) $\Rightarrow$ (i) Let $V$ be any NCS in $Y$ and let $p_{(r,s,t)}$ be a NP in $X$ such that $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in V$. According to the assumption, there exists an $N\omega$OS $U$ in $X$ such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. Hence $p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V)$. Therefore $p_{(r,s,t)} \in U = \omega int(U) \subseteq N\omega int(g^{-1}(V))$. Since, $p_{(r,s,t)}$ is an arbitrary NP and $g^{-1}(V)$ is the union of all NPs in $g^{-1}(V)$, we obtain that $g^{-1}(V) \subseteq N\omega int(g^{-1}(V))$. Thus $g$ is a neutrosophic contra $N\omega$OS-continuous mapping.

**Corollary 4.15.** Let $X$, $X_1$ and $X_2$ be NTSs, $p_1 : X \to X_1 \times X_2$ ($i = 1, 2$) and $p_2 : X \to X_1 \times X_2$ are the projections of $X_1 \times X_2$ onto $X_i$, ($i = 1, 2$). If $g : X \to X_1 \times X_2$ is a neutrosophic contra $\omega$-continuous, then $p_1 g$ are also neutrosophic contra $\omega$-continuous mappings.

**Proof.** The proof follows from the fact that the projections are all neutrosophic continuous functions.

**Theorem 4.16.** Let $g : (X_1, \tau) \to (Y_1, \sigma)$ be a function. If the graph $h : X_1 \to X_1 \times Y_1$ of $g$ is neutrosophic contra $\omega$-continuous, then $g$ is neutrosophic contra $\omega$-continuous.

**Proof.** For every NOS $V$ in $Y_1$ holds $g^{-1}(V) = 1 \land g^{-1}(V) = h^{-1}(1 \times V)$. Since $h$ is a neutrosophic contra $\omega$-continuous function and $1 \times V$ is a NOS in $X_1 \times Y_1$, $g^{-1}(V)$ is a $N\omega$CS in $X_1$, so $g$ is a neutrosophic contra $\omega$-continuous mapping.

5 Conclusions

In this paper, we introduced and investigated the neutrosophic $\omega$ closed sets and its properties. Also, we investigated the continuity, irresolute, connectedness and contra-continuity in terms of neutrosophic $\omega$ closed sets.

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