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On neutrosophic soft continuous mappings

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Abstract: In this paper, the concept of neutrosophic soft continuous mapping, neutrosophic soft open mapping, neutrosophic soft closed mapping and neutrosophic soft homeomorphism have been introduced along with the investigation of their several characteristics, and verified by proper examples.

Key words: Neutrosophic soft set, neutrosophic soft continuous mapping, neutrosophic soft open mapping, neutrosophic soft closed mapping, neutrosophic soft homeomorphism

1. Introduction

The fuzzy set theory [20] introduced by Zadeh in 1965 is strong mathematical tool for solving uncertainty problems. Researchers in sociology, economics, medical science and other fields are interested in the knowledge that modeling uncertain data is vague and sometimes inadequate. For different field, there are different recommendations for nonclassical and high-grade fuzzy sets since the introduction of fuzzy set theory. The intuitionistic fuzzy set theory [4] introduced by Atanassov, is very useful and applicable among higher order fuzzy sets. However each of these theories has different difficulties, as Molodtsov [13] pointed out. The main reason for these difficulties is the inadequacy of the parameterization tool of theories.

The soft set theory was introduced by Molodtsov as a new mathematical tool for dealing with imprecision and uncertainties. Soft set theory and its application are progressing rapidly in different fields. A lot of studies have been made using soft set and other set theories such as fuzzy soft set, intuitionistic fuzzy soft set etc. [2, 3, 8, 9, 11, 14, 16, 19].

Neutrosophic set theory was first initiated by Smarandache [18] which is a generalization of classical set, fuzzy set, intuitionistic fuzzy soft set etc. Later a combination of neutrosophic set and soft set, neutrosophic soft set introduced by Maji [12]. Using this concept, many studies have been made by different mathematicians [1, 6, 16]. Later this concept has been modified by Ozturk et al. [17]. The neutrosophic soft mapping concepts defined by Alkhazaleh and Marei [1]. After that the concept of neutrosophic soft mapping has been redefined by Yolcu et al.*

The present study introduces notion of neutrosophic soft continuous mapping. We also define neutrosophic soft open mapping, neutrosophic soft closed mapping and neutrosophic soft homeomorphism. Further, we

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investigate some basic operation and related properties of neutrosophic soft continuous mapping. Some related theorems have been proved and studied with different examples.

2. Preliminaries

Definition 2.1 *[18] A neutrosophic set A on the universe of discourse X is defined as:*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T,\ I,\ F : X \to]^-0, 1^+[$ *and* $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.2 *[13] Let X be an initial universe, E be a set of all parameters and $P(X)$ denotes the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$ elements of the soft set (F, E), or as the set of $e-$ approximate elements of the soft set, i.e.

$$(F, E) = \{(e, F(e)) : e \in E,\ F : E \to P(X)\}.$$

Firstly, neutrosophic soft set defined by Maji [12] and later this concept has been modified by Deli and Bromi [7] as given below:

Definition 2.3 *Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $\left(\widetilde{F}, E\right)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$ where $\widetilde{F}$ is called approximate function of the neutrosophic soft set $\left(\widetilde{F}, E\right)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,*

$$\left(\widetilde{F}, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$$

where $T_{\widetilde{F}(e)}(x),\ I_{\widetilde{F}(e)}(x),\ F_{\widetilde{F}(e)}(x) \in [0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{F}(e)$. Since supremum of each $T,\ I,\ F$ is 1 so the inequality $0 \le T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \le 3$ is obvious.

Definition 2.4 *[5] Let $\left(\widetilde{F}, E\right)$ be neutrosophic soft set over the universe set X. The complement of $\left(\widetilde{F}, E\right)$ is denoted by $\left(\widetilde{F}, E\right)^c$ and is defined by:*

$$\left(\widetilde{F}, E\right)^c = \left\{\left(e, \left\langle x, F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x), T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$$

Obvious that, $\left(\left(\widetilde{F}, E\right)^c\right)^c = \left(\widetilde{F}, E\right)$.

Definition 2.5 *[12] Let $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ be two neutrosophic soft sets over the universe set X. $\left(\widetilde{F},E\right)$ is said to be neutrosophic soft subset of $\left(\widetilde{G},E\right)$ if $T_{\widetilde{F}(e)}(x) \le T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \le I_{\widetilde{G}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \ge F_{\widetilde{G}(e)}(x)$, $\forall e \in E$, $\forall x \in X$. It is denoted by $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)$.*

$\left(\widetilde{F},E\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{G},E\right)$ if $\left(\widetilde{F},E\right)$ is neutrosophic soft subset of $\left(\widetilde{G},E\right)$ and $\left(\widetilde{G},E\right)$ is neutrosophic soft subset of $\left(\widetilde{F},E\right)$. It is denoted by $\left(\widetilde{F},E\right) = \left(\widetilde{G},E\right)$.

Definition 2.6 *[17] Let $\left(\widetilde{F}_1,E\right)$ and $\left(\widetilde{F}_2,E\right)$ be two neutrosophic soft sets over the universe set X. Then their union is denoted by $\left(\widetilde{F}_1,E\right) \Cup \left(\widetilde{F}_2,E\right) = \left(\widetilde{F}_3,E\right)$ and is defined by:*

$$\left(\widetilde{F}_3,E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$$

where

$$\begin{aligned} T_{\widetilde{F}_3(e)}(x) &= \max\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\}, \\ I_{\widetilde{F}_3(e)}(x) &= \max\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\}, \\ F_{\widetilde{F}_3(e)}(x) &= \min\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}. \end{aligned}$$

Definition 2.7 *[17] Let $\left(\widetilde{F}_1,E\right)$ and $\left(\widetilde{F}_2,E\right)$ be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $\left(\widetilde{F}_1,E\right) \Cap \left(\widetilde{F}_2,E\right) = \left(\widetilde{F}_3,E\right)$ and is defined by:*

$$\left(\widetilde{F}_3,E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$$

where

$$\begin{aligned} T_{\widetilde{F}_3(e)}(x) &= \min\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\}, \\ I_{\widetilde{F}_3(e)}(x) &= \min\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\}, \\ F_{\widetilde{F}_3(e)}(x) &= \max\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}. \end{aligned}$$

Definition 2.8 *[17] Let $\left\{\left(\widetilde{F}_i,E\right) \middle| i \in I\right\}$ be a family of neutrosophic soft sets over the universe set X. Then*

$$\begin{aligned} \bigcup_{i\in I}\left(\widetilde{F}_i,E\right) &= \left\{\left(e, \left\langle x, \sup\left[T_{\widetilde{F}_i(e)}(x)\right]_{i\in I}, \sup\left[I_{\widetilde{F}_i(e)}(x)\right]_{i\in I}, \inf\left[F_{\widetilde{F}_i(e)}(x)\right]_{i\in I}\right\rangle : x \in X\right) : e \in E\right\}, \\ \bigcap_{i\in I}\left(\widetilde{F}_i,E\right) &= \left\{\left(e, \left\langle x, \inf\left[T_{\widetilde{F}_i(e)}(x)\right]_{i\in I}, \inf\left[I_{\widetilde{F}_i(e)}(x)\right]_{i\in I}, \sup\left[F_{\widetilde{F}_i(e)}(x)\right]_{i\in I}\right\rangle : x \in X\right) : e \in E\right\}. \end{aligned}$$

Definition 2.9 *[17]*

1. *A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over the universe set* X *is said to be null neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 0,\ I_{\widetilde{F}(e)}(x) = 0,\ F_{\widetilde{F}(e)}(x) = 1;\ \forall e \in E, \forall x \in X$. *It is denoted by* $0_{(X,E)}$.

2. *A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over the universe set* X *is said to be absolute neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 1,\ I_{\widetilde{F}(e)}(x) = 1,\ F_{\widetilde{F}(e)}(x) = 0;\ \forall e \in E, \forall x \in X$. *It is denoted by* $1_{(X,E)}$.

   Clearly, $0^c_{(X,E)} = 1_{(X,E)}$ *and* $1^c_{(X,E)} = 0_{(X,E)}$.

Definition 2.10 *[10] Let* $NSS(X,E)$ *be the family of all neutrosophic soft sets over the universe set* X. *Then neutrosophic soft set* $x^e_{(\alpha,\beta,\gamma)}$ *is called a neutrosophic soft point, for every* $x \in X, 0 < \alpha, \beta, \gamma \leq 1,\ e \in E$ *, and defined as follows:*

$$x^e_{(\alpha,\beta,\gamma)}\left(e'\right)(y) = \begin{cases} (\alpha,\beta,\gamma) & \text{if } e' = e \text{ and } y = x, \\ (0,0,1) & \text{if } e' \neq e \text{ or } y \neq x. \end{cases}$$

Definition 2.11 *[10] Let* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over the universe set* X. *We say that* $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right)$ *read as belongs to the neutrosophic soft set* $\left(\widetilde{F}, E\right)$, *whenever* $\alpha \leq T_{\widetilde{F}(e)}(x), \beta \leq I_{\widetilde{F}(e)}(x)$ *and* $\gamma \geq F_{\widetilde{F}(e)}(x)$.

Definition 2.12 *[10] Let* $x^e_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *be two neutrosophic soft points. For the neutrosophic soft points* $x^e_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *over a common universe* X, *we say that the neutrosophic soft points are distinct points if* $x^e_{(\alpha,\beta,\gamma)} \cap\ y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$. *It is clear that* $x^e_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha_1,\beta_1,\gamma_1)}$ *are distinct neutrosophic soft points if and only if* $x \neq y$ *or* $e' \neq e$.

Definition 2.13 *Yolcu et al.*[†] *Let* $NSS(X,E)$ *and* $NSS(Y,E')$ *be two neutrosophic soft classes and* $u : X \to Y$ *and* $v : E \to E'$ *be mappings. Then a mapping* $f = (u,v) : (X,E) \to (Y,E')$ *is defined as follows:*

For a neutrosophic soft set $\left(\widetilde{F}, A\right) \in NSS(X,E)$, $f\left(\left(\widetilde{F}, A\right)\right)$ *is a neutrosophic soft set in* $NSS(Y,E)$ *obtained as follows:*

$$T_{u(F)(e')}(y) = \begin{cases} \sup\limits_{e \in v^{-1}(e') \cap A,\ x \in u^{-1}(y)} T_{F(e)}(x) & , \quad \text{if} \quad u^{-1}(y) \neq \emptyset \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$I_{u(F)(e')}(y) = \begin{cases} \sup\limits_{e \in v^{-1}(e') \cap A,\ x \in u^{-1}(y)} I_{F(e)}(x) & , \quad \text{if} \quad u^{-1}(y) \neq \emptyset \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$F_{u(F)(e')}(y) = \begin{cases} \inf\limits_{e \in v^{-1}(e') \cap A,\ x \in u^{-1}(y)} F_{F(e)}(x) & , \quad \text{if} \quad u^{-1}(y) \neq \emptyset \\ 1 & , \quad \text{otherwise} \end{cases}$$

[†] Yolcu A, Karatas E, Ozturk TY. A new approach to neutrosophic soft mappings and application in decision making (manuscript not published yet).

for $e' \in v(A) \subseteq E', \ y \in Y$.

Definition 2.14 *Yolcu et al.*[‡] *Let* $NSS(X,E)$ *and* $NSS(Y,E')$ *be neutrosophic soft classes and* $u : X \to Y$ *and* $v : E \to E'$ *be mappings. Then a mapping* $f^{-1} : NSS(Y,E') \to NSS(X,E)$ *is defined as follows:*

For a neutrosophic soft set $(\widetilde{G},B)$ *in* $NSS(Y,E')$, $f^{-1}((\widetilde{G},B))$ *is a neutrosophic soft set in* $NSS(X,E)$ *obtained as follows:*

$$T_{u^{-1}(G)(e)}(y) = \begin{cases} T_{G(v(e))}(u(x)) \ , & \text{if} \quad v^{-1}(e) \in B \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$I_{u^{-1}(G)(e)}(y) = \begin{cases} I_{G(v(e))}(u(x)), & \text{if} \quad v^{-1}(e) \in B \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$F_{u^{-1}(G)(e)}(y) = \begin{cases} F_{G(v(e))}(u(x)) \ , & \text{if} \quad v^{-1}(e) \in B \\ 1 & , \quad \text{otherwise} \end{cases}$$

For $e \in v^{-1}(B) \subseteq E$ *and* $x \in X$, $f^{-1}((\widetilde{G},B))$ *is called a neutrosophic soft inverse image of the neutrosophic soft set* $(\widetilde{G},B)$.

Definition 2.15 *Yolcu et al.*[§] *Let* $NSS(X,E)$, $NSS(Y,E)$ *be two neutrosophic soft classes,* $(\widetilde{F},A) \in NSS(X,E)$, $(\widetilde{G},B) \in NSS(Y,E')$. *Then* $f = (u,v) : NSS(X,E) \to NSS(Y,E')$ *be a neutrosophic soft mapping such that* $u : X \to Y, \ v : E \to E'$.

1. *The neutrosophic soft mapping* $f = (u,v)$ *is called a neutrosophic soft injective mapping if for every* $(x_1)^{e_1}_{(\alpha_1,\beta_1,\gamma_1)}, (x_2)^{e_2}_{(\alpha_2,\beta_2,\gamma_2)} \in (\widetilde{F},A)$, $(x_1)^{e_1}_{(\alpha_1,\beta_1,\gamma_1)} \neq (x_2)^{e_2}_{(\alpha_2,\beta_2,\gamma_2)}$ *implies* $f\left((x_1)^{e_1}_{(\alpha_1,\beta_1,\gamma_1)}\right) = (u(x_1),v(e_1)) \neq f\left((x_2)^{e_2}_{(\alpha_2,\beta_2,\gamma_2)}\right) = (u(x_2),v(e_2))$.

2. *The neutrosophic soft mapping* $f = (u,v)$ *is called a neutrosophic soft surjective mapping if there exists a neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F},A)$, *such that* $f(x^e_{(\alpha,\beta,\gamma)}) = y^{e'}_{(\alpha',\beta',\gamma')}$ *for every* $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G},B)$.

3. *The neutrosophic soft mapping* $f = (u,v)$ *is called a neutrosophic soft bijective mapping if* $f = (u,v)$ *is both injective and surjective.*

4. *The neutrosophic soft mapping* $f = (u,v)$ *is called a neutrosophic soft constant mapping if* $f(x^e_{(\alpha,\beta,\gamma)}) = y^{e'}_{(\alpha',\beta',\gamma')}$ *is provided for* $\forall x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F},A)$, $\exists y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G},B)$.

Definition 2.16 *[17] Let* $NSS(X,E)$ *be the family of all neutrosophic soft sets over the universe set* X *and* $\overset{NSS}{\tau} \subset NSS(X,E)$. *Then* $\overset{NSS}{\tau}$ *is said to be a neutrosophic soft topology on* X *if*

1. $0_{(X,E)}$ *and* $1_{(X,E)}$ *belongs to* $\overset{NSS}{\tau}$

‡Yolcu A, Karatas E, Ozturk TY. A new approach to neutrosophic soft mappings and application in decision making (manuscript not published yet).

§Yolcu A, Karatas E, Ozturk TY. A new approach to neutrosophic soft mappings and application in decision making (manuscript not published yet).

2. *The union of any number of neutrosophic soft sets in* $\overset{NSS}{\tau}$ *belongs to* $\overset{NSS}{\tau}$

3. *The intersection of finite number of neutrosophic soft sets in* $\overset{NSS}{\tau}$ *belongs to* $\overset{NSS}{\tau}$.

Then $\left(X, \overset{NSS}{\tau}, E\right)$ *is said to be a neutrosophic soft topological space over* X. *Each members of* $\overset{NSS}{\tau}$ *is said to be neutrosophic soft open set.*

Definition 2.17 *[17] Let* $NSS(X,E)$ *be the family of all neutrosophic soft sets over the universe set* X.

1. *If* $\overset{NSS}{\tau} = \left\{0_{(X,E)}, 1_{(X,E)}\right\}$, *then* $\overset{NSS}{\tau}$ *is said to be the neutrosophic soft indiscrete topology and* $\left(X, \overset{NSS}{\tau}, E\right)$ *is said to be a neutrosophic soft indiscrete topological space over* X.

2. *If* $\overset{NSS}{\tau} = NSS(X,E)$, *then* $\overset{NSS}{\tau}$ *is said to be the neutrosophic soft discrete topology and* $\left(X, \overset{NSS}{\tau}, E\right)$ *is said to be a neutrosophic soft discrete topological space over* X.

Definition 2.18 *[17] Let* $\left(X, \overset{NSS}{\tau}, E\right)$ *be a neutrosophic soft topological space over* X *and* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over* X. *Then* $\left(\widetilde{F}, E\right)$ *is said to be neutrosophic soft closed set iff its complement is a neutrosophic soft open set.*

Definition 2.19 *[10] Let* $(X, \overset{NSS}{\tau}, E)$ *be a neutrosophic soft topological space over* X. *A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *in* $\left(X, \overset{NSS}{\tau}, E\right)$ *is called a neutrosophic soft neighborhood of the neutrosophic soft point* $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right)$, *if there exists a neutrosophic soft open set* $(\widetilde{G}, E)$ *such that* $x^{e}_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E) \subset \left(\widetilde{F}, E\right)$.

Definition 2.20 *[17] Let* $\left(X, \overset{NSS}{\tau}, E\right)$ *be a neutrosophic soft topological space over* X *and* $\left(\widetilde{F}, E\right) \in NSS(X,E)$ *be a neutrosophic soft set.*

1. *The neutrosophic soft interior of* $\left(\widetilde{F}, E\right)$, *denoted* $\left(\widetilde{F}, E\right)^{\circ}$, *is defined as the neutrosophic soft union of all neutrosophic soft open subsets of* $\left(\widetilde{F}, E\right)$. *Clearly,* $\left(\widetilde{F}, E\right)^{\circ}$ *is the biggest neutrosophic soft open set that is contained by* $\left(\widetilde{F}, E\right)$.

2. *The neutrosophic soft closure of* $\left(\widetilde{F}, E\right)$, *denoted* $\overline{\left(\widetilde{F}, E\right)}$, *is defined as the neutrosophic soft intersection of all neutrosophic soft closed supersets of* $\left(\widetilde{F}, E\right)$. *Clearly,* $\overline{\left(\widetilde{F}, E\right)}$ *is the smallest neutrosophic soft closed set that containing* $\left(\widetilde{F}, E\right)$.

3. Neutrosophic soft continuous mappings

Definition 3.1 *Let* $\left(X, \overset{NSS}{\tau_1}, E\right)$ *and* $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ *be two neutrosophic soft topological spaces and* $f = (u,v) : NSS(X,E) \to NSS(Y,E')$ *be a neutrosophic soft mapping. For each neutrosophic soft neighbourhood* $\left(\widetilde{G}, E'\right)$ *of* $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$, *if there exist a neutrosophic soft neighbourhood* $\left(\widetilde{F}, E\right)$ *of neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)} \in NSS(X,E)$ *such that* $f\left(\left(\widetilde{F}, E\right)\right) \subseteq \left(\widetilde{G}, E'\right)$, *then* f *is said to be neutrosophic soft continuous mapping at* $x^e_{(\alpha,\beta,\gamma)}$.

If f *is a neutrosophic soft continuous mapping for all* $x^e_{(\alpha,\beta,\gamma)} \in NSS(X,E)$, *then* f *is called a neutrosophic soft continuous mapping on a neutrosophic soft topological space* $\left(X, \overset{NSS}{\tau_1}, E\right)$.

Theorem 3.2 *Let* $\left(X, \overset{NSS}{\tau_1}, E\right)$ *and* $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ *be two neutrosophic soft topological space and* $f : NSS(X,E) \to NSS(Y,E')$ *be a neutrosophic soft mapping. Then* f *is a neutrosophic soft continuous mapping on a neutrosophic soft topological space* $\left(X, \overset{NSS}{\tau_1}, E\right)$ *iff* $f^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ *is a neutrosophic soft open set, for every* $\left(\widetilde{G}, E'\right) \in \overset{NSS}{\tau_2}$.

Proof Suppose that f is a neutrosophic soft continuous mapping on a neutrosophic soft topological space $\left(X, \overset{NSS}{\tau_1}, E\right)$ and $\left(\widetilde{G}, E'\right) \in \overset{NSS}{\tau_2}$. Let us show that $f^{-1}\left(\left(\widetilde{G}, E'\right)\right) \in \overset{NSS}{\tau_1}$. For every soft point $x^e_{(\alpha,\beta,\gamma)} \in f^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ since $f\left(x^e_{(\alpha,\beta,\gamma)}\right) \in \left(\widetilde{G}, E'\right)$ and f is a neutrosophic soft continuous mapping, then there exists a neutrosophic soft neighbourhood $\left(\widetilde{F}, E\right)$ of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ such that $f\left(\left(\widetilde{F}, E\right)\right) \subseteq \left(\widetilde{G}, E'\right)$. Therefore, $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right) \subseteq f^{-1}\left(\left(\widetilde{G}, E'\right)\right)$. That is $f^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ is a neutrosophic soft open set.

Conversely, let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point of $NSS(X,E)$ and $f\left(x^e_{(\alpha,\beta,\gamma)}\right) \in \left(\widetilde{G}, E'\right)$ be a neutrosophic soft open set in $NSS\left(Y, E'\right)$. Then $x^e_{(\alpha,\beta,\gamma)} \in f^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ is a neutrosophic soft open set in $NSS(X,E)$ and $f\left(f^{-1}\left((\widetilde{G}, E')\right)\right) \subseteq \left(\widetilde{G}, E'\right)$. That is, f is a neutrosophic soft continuous mapping on neutrosophic soft topological space $\left(X, \overset{NSS}{\tau_1}, E\right)$. □

Theorem 3.3 *Let* $\left(X, \overset{NSS}{\tau_1}, E\right)$ *and* $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ *be two neutrosophic soft topological spaces and* $f : NSS(X,E) \to NSS(Y,E')$ *be a neutrosophic soft mapping. Then* f *is a neutrosophic soft continuous mapping on a neutrosophic soft topological spaces* $\left(X, \overset{NSS}{\tau_1}, E\right)$ *iff* $f^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ *is a neutrosophic soft closed set in* X *for every neutrosophic soft closed set* (G, E') *in* Y.

Proof Let $f : NSS(X,E) \to NSS(Y,E')$ be a neutrosophic soft continuous mapping on neutrosophic soft topological spaces $\left(X, \overset{NSS}{\tau_1}, E\right)$ and $\left(\widetilde{G}, E'\right)$ be any neutrosophic soft closed set in Y. Then, since

$f^{-1}\left(\left(\widetilde{G},E'\right)^c\right)=\left(f^{-1}\left(\left(\widetilde{G},E'\right)\right)\right)^c$ and $\left(\widetilde{G},E'\right)^c$ is neutrosophic soft open set, obtained. $\left(f^{-1}\left(\left(\widetilde{G},E'\right)\right)\right)^c$ is neutrosophic soft open set in X. This means that $f^{-1}\left(\left(\widetilde{G},E'\right)\right)$ is a neutrosophic soft closed set in X.

Conversely, suppose that $f^{-1}\left(\left(\widetilde{G},E'\right)\right)$ is a neutrosophic soft closed set in X whenever $\left(\widetilde{G},E'\right)$ is a neutrosophic soft closed set in Y. For any neutrosophic soft open set $\left(\widetilde{H},E'\right)$ in Y, $f^{-1}\left(\left(\widetilde{H},E'\right)^c\right)=\left(f^{-1}\left(\left(\widetilde{H},E'\right)\right)\right)^c$. From the hypothesis, $f^{-1}\left(\left(\widetilde{H},E'\right)^c\right)$ is a neutrosophic soft closed set in X. Therefore $f^{-1}\left(\left(\widetilde{H},E'\right)\right)$ is a neutrosophic soft open set in X. Hence, f is a neutrosophic soft continuous mapping on a neutrosophic soft topological space $\left(X,\overset{NSS}{\tau_1},E\right)$. □

Example 3.4 *Let* $\left(X,\overset{NSS}{\tau_1},E\right)$ *and* $\left(Y,\overset{NSS}{\tau_2},E'\right)$ *be two neutrosophic soft topological spaces and* $f:NSS(X,E)\to NSS(Y,E')$ *be a neutrosophic soft mapping.*

1. If $\overset{NSS}{\tau_1}$ is the neutrosophic soft discrete topology on X, then f is a neutrosophic soft continuous mapping.
2. If $\overset{NSS}{\tau_2}$ is the neutrosophic soft indiscrete topology on Y, then f is a neutrosophic soft continuous mapping.
3. Let $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3\}$, $E=\{e_1,e_2\}$, $E'=\{e_1',e_2'\}$ and $\overset{NSS}{\tau_1}, \overset{NSS}{\tau_2}$ be two neutrosophic soft topology. $\overset{NSS}{\tau_1}, \overset{NSS}{\tau_2}$ defined as follows:

$$\begin{aligned}\overset{NSS}{\tau_1} &= \{0_{(X,E)},1_{(X,E)},(\widetilde{F}_1,E),(\widetilde{F}_2,E),(\widetilde{F}_3,E)\}\\ \overset{NSS}{\tau_2} &= \{0_{(Y,E')},1_{(Y,E')},(\widetilde{G}_1,E'),(\widetilde{G}_2,E'),(\widetilde{G}_3,E')\}\end{aligned}$$

where

$$\begin{aligned}
\left(\widetilde{F}_1,E\right) &= \left\{\begin{array}{c}(e_1,\{<x_1,0.8,0.8,0.3>,<x_2,0,0,1>,<x_3,0,0,1>\})\\(e_2,\{<x_1,0,0,1>,<x_2,0,0,1>,<x_3,0,0,1>\})\end{array}\right\}\\
\left(\widetilde{F}_2,E\right) &= \left\{\begin{array}{c}(e_1,\{<x_1,0.8,0.8,0.3>,<x_2,0.4,0.4,0.2>,<x_3,0,0,1>\})\\(e_2,\{<x_1,0,0,1>,<x_2,0,0,1>,<x_3,0,0,1>\})\end{array}\right\}\\
\left(\widetilde{F}_3,E\right) &= \left\{\begin{array}{c}(e_1,\{<x_1,0.8,0.8,0.3>,<x_2,0.4,0.4,0.2>,<x_3,0,0,1>\})\\(e_2,\{<x_1,0,0,1>,<x_2,0,0,1>,<x_3,0.4,0.4,0.2>\})\end{array}\right\}\\
\left(\widetilde{G}_1,E'\right) &= \left\{\begin{array}{c}(e_1',\{<y_1,0.8,0.8,0.3>,<y_2,0,0,1>,<y_3,0,0,1>\})\\(e_2',\{<y_1,0,0,1>,<y_2,0,0,1>,<y_3,0,0,1>\})\end{array}\right\}\\
\left(\widetilde{G}_2,E'\right) &= \left\{\begin{array}{c}(e_1',\{<y_1,0,0,1>,<y_2,0.8,0.8,0.3>,<y_3,0,0,1>\})\\(e_2',\{<y_1,0,0,1>,<y_2,0,0,1>,<y_3,0,0,1>\})\end{array}\right\}\\
\left(\widetilde{G}_3,E'\right) &= \left\{\begin{array}{c}(e_1',\{<y_1,0,0,1>,<y_2,0.8,0.8,0.3>,<y_3,0.4,0.4,0.2>\})\\(e_2',\{<y_1,0,0,1>,<y_2,0,0,1>,<y_3,0.4,0.4,0.2>\})\end{array}\right\}
\end{aligned}$$

So, neutrosophic soft points of above neutrosophic soft set is as follows:

$$\begin{aligned}
\left(\widetilde{F}_1, E\right) &= \left\{x^{e_1}_{1(0.8,0.8,0.3)}\right\} \\
\left(\widetilde{F}_2, E\right) &= \left\{x^{e_1}_{1(0.8,0.8,0.3)}, x^{e_1}_{2(0.4,0.4,0.2)}\right\} \\
\left(\widetilde{F}_3, E\right) &= \left\{x^{e_1}_{1(0.8,0.8,0.3)}, x^{e_1}_{2(0.4,0.4,0.2)}, x^{e_2}_{3(0.4,0.4,0.2)}\right\} \\
\left(\widetilde{G}_1, E'\right) &= \left\{y^{e_1'}_{1(0.8,0.8,0.3)}\right\} \\
\left(\widetilde{G}_2, E'\right) &= \left\{y^{e_1'}_{2(0.8,0.8,0.3)}\right\} \\
\left(\widetilde{G}_3, E'\right) &= \left\{y^{e_1'}_{2(0.8,0.8,0.3)}, y^{e_1'}_{3(0.4,0.4,0.2)}, y^{e_2'}_{3(0.4,0.4,0.2)}\right\}
\end{aligned}$$

If the mapping $f = (u, v) : NSS(X, E) \to (Y, E')$ defined as follows:

$$\begin{aligned}
u(x_1) &= y_2 \qquad v(e_1) = e_1' \\
u(x_2) &= y_3 \qquad v(e_2) = e_2' \\
u(x_3) &= y_3
\end{aligned}$$

Then $f = (u, v)$ is neutrosophic soft continuous mapping at the neutrosophic soft point $x^{e_1}_{1(0.8,0.8,0.3).}$

4. Consider the above (3).

   The neutrosophic soft open sets on $\overset{NSS}{\tau_2}$ are $\left(\widetilde{G}_1, E'\right), \left(\widetilde{G}_2, E'\right), \left(\widetilde{G}_3, E'\right), 0_{(Y,E')}$ and $1_{(Y,E')}$. Now, we find inverse image of these neutrosophic soft open sets on $\overset{NSS}{\tau_2}$.

$$\begin{aligned}
f^{-1}\left(\left(\widetilde{G}_1, E'\right)\right) &= 0_{(X,E)} \in \overset{NSS}{\tau_1} \\
f^{-1}\left(\left(\widetilde{G}_2, E'\right)\right) &= \left\{\begin{array}{c} (e_1,\{< x_1, 0.8, 0.8, 0.3 >, < x_2, 0, 0, 1 >, < x_3, 0, 0, 1 >\}) \\ (e_2,\{< x_1, 0, 0, 1 >, < x_2, 0, 0, 1 >, < x_3, 0, 0, 1 >\}) \end{array}\right\} = \left(\widetilde{F}_1, E\right) \in \overset{NSS}{\tau_1} \\
f^{-1}\left(\left(\widetilde{G}_3, E'\right)\right) &= \left\{\begin{array}{c} (e_1,\{< x_1, 0.8, 0.8, 0.3 >, < x_2, 0.4, 0.4, 0.2 >, < x_3, 0.4, 0.4, 0.2 >\}) \\ (e_2,\{< x_1, 0, 0, 1 >, < x_2, 0, 0, 1 >, < x_3, 0.4, 0.4, 0.2 >\}) \end{array}\right\} \notin \overset{NSS}{\tau_1}
\end{aligned}$$

   Therefore, f is not neutrosophic soft continuous mapping on $\left(X, \overset{NSS}{\tau_1}, E\right)$.

Theorem 3.5 *Let $\left(X, \overset{NSS}{\tau_1}, E\right)$ and $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ be two neutrosophic soft topological spaces and $f : NSS(X, E) \to NSS(Y, E')$ be a neutrosophic soft mapping. Then f is a neutrosophic soft continuous mapping on a neutrosophic soft topological spaces $\left(X, \overset{NSS}{\tau_1}, E\right)$ if and only if $f^{-1}\left(\left(\widetilde{G}, E'\right)^{\circ}\right) \subseteq f^{-1}\left(\left(\widetilde{G}, E'\right)\right)^{\circ}$ for each $\left(\widetilde{G}, B\right) \in NSS(Y, E')$.*

Proof (:⇒) Let f be a neutrosophic soft continuous mapping and $\left(\widetilde{G},B\right) \in NSS(Y,E')$. Then $f^{-1}\left(\left(\widetilde{G},B\right)^{\circ}\right) \in \overset{NSS}{\tau_1}$ and from $\left(\widetilde{G},B\right)^{\circ} \subseteq \left(\widetilde{G},B\right)$ we have $f^{-1}\left(\left(\widetilde{G},B\right)^{\circ}\right) \subseteq f^{-1}\left(\left(\widetilde{G},B\right)\right)$.

Because of $\left(f^{-1}\left(\left(\widetilde{G},B\right)\right)\right)^{\circ}$ is largest neutrosophic soft open set contained by $f^{-1}\left(\left(\widetilde{G},B\right)\right)$,

$$f^{-1}\left(\left(\widetilde{G},B\right)^{\circ}\right) \subseteq \left(f^{-1}\left(\left(\widetilde{G},B\right)\right)\right)^{\circ}$$

(⇐:) Conversely, let $f^{-1}\left(\left(\widetilde{G},B\right)^{\circ}\right) \subseteq \left(f^{-1}\left(\left(\widetilde{G},B\right)\right)\right)^{\circ}$, for all $\left(\widetilde{G},B\right) \in NSS(Y,E')$. If $\left(\widetilde{G},B\right) \in \overset{NSS}{\tau_2}$, then we have,

$$f^{-1}\left(\left(\widetilde{G},B\right)\right) = f^{-1}\left(\left(\widetilde{G},B\right)^{\circ}\right) \subseteq \left(f^{-1}\left(\left(\widetilde{G},B\right)\right)\right)^{\circ} \subseteq f^{-1}\left(\left(\widetilde{G},B\right)\right)$$

So, $f^{-1}\left(\left(\widetilde{G},B\right)\right) \in \overset{NSS}{\tau_1}$. It means that, f is neutrosophiz soft continuous mapping. □

Theorem 3.6 *Let $\left(X,\overset{NSS}{\tau_1},E\right)$ and $\left(Y,\overset{NSS}{\tau_2},E'\right)$ be two neutrosophic soft topological spaces and $f : NSS(X,E) \to NSS(Y,E')$ be a neutrosophic soft mapping. Then f is a neutrosophic soft continuous mapping on a neutrosophic soft topological spaces $\left(X,\overset{NSS}{\tau_1},E\right)$ if and only if $f\left(\overline{\left(\widetilde{F},E\right)}\right) \subseteq \overline{f\left(\left(\widetilde{F},E\right)\right)}$ for $\left(\widetilde{F},E\right) \in NSS(X,E)$.*

Proof (:⇒) Let f be neutrosophic soft continuous mapping on neutrosophic soft topological space $\left(X,\overset{NSS}{\tau_1},E\right)$ and $\left(\widetilde{F},E\right) \in NSS(X,E)$. Since $\overline{\left(f\left(\left(\widetilde{F},E\right)\right)\right)}$ is a neutrosophic soft closed set in Y, $f^{-1}\left(\overline{f\left(\left(\widetilde{F},E\right)\right)}\right)$ is a neutrosophic soft closed set in X. Then

$$\overline{\left(f^{-1}\left(\overline{f\left(\left(\widetilde{F},E\right)\right)}\right)\right)} = f^{-1}\left(\overline{f\left(\left(\widetilde{F},E\right)\right)}\right) \tag{3.1}$$

and

$$f\left(\left(\widetilde{F},E\right)\right) \subseteq \overline{f\left(\left(\widetilde{F},E\right)\right)}$$

Thus

$$\left(\widetilde{F},E\right) \subseteq f^{-1}\left(f\left(\left(\widetilde{F},E\right)\right)\right) \subseteq f^{-1}\left(\overline{f\left(\left(\widetilde{F},E\right)\right)}\right)$$

from the equality (3.1)

$$\left(\widetilde{F},E\right) \subseteq \overline{\left(f^{-1}\left(\overline{f\left(\left(\widetilde{F},E\right)\right)}\right)\right)} = f^{-1}\left(\overline{f\left(\left(\widetilde{F},E\right)\right)}\right)$$

Hence

$$f\left(\overline{\left(\widetilde{F},E\right)}\right)\subseteq\overline{f\left(\left(\widetilde{F},E\right)\right)}$$

is obtained.

($\Leftarrow$:) Suppose that $f\left(\overline{\left(\widetilde{F},E\right)}\right)\subseteq\overline{f\left(\left(\widetilde{F},E\right)\right)}$ for every $\left(\widetilde{F},E\right)\in NSS(X,E)$.

Let $\left(\widetilde{G},E'\right)$ be any neutrosophic soft closed set in Y, so $\overline{\left(\widetilde{G},E'\right)}=\left(\widetilde{G},E'\right)$

From the hypothesis,

$$f\left(\overline{f^{-1}\left(\left(\widetilde{G},E'\right)\right)}\right)\subseteq\overline{f\left(f^{-1}\left(\left(\widetilde{G},E'\right)\right)\right)}\subseteq\overline{\left(\widetilde{G},E'\right)}=\left(\widetilde{G},E'\right)$$

is obtained. Hence

$$\overline{f^{-1}\left(\left(\widetilde{G},E'\right)\right)}\subseteq f^{-1}\left(\left(\widetilde{G},E'\right)\right)$$

and

$$f^{-1}\left(\left(\widetilde{G},E'\right)\right)\subseteq\overline{f^{-1}\left(\left(\widetilde{G},E'\right)\right)}$$

That is,

$$\overline{f^{-1}\left(\left(\widetilde{G},E'\right)\right)}=f^{-1}\left(\left(\widetilde{G},E'\right)\right)$$

and, so $f^{-1}\left(\left(\widetilde{G},E'\right)\right)$ is a neutrosophic soft closed set in X. Thus f is a neutrosophic soft continuous mapping on neutrosophic soft topological space $\left(X,\overset{NSS}{\tau_1},E\right)$. □

Proposition 3.7 *Let* $\left(X,\overset{NSS}{\tau_1},E\right)$ *and* $\left(Y,\overset{NSS}{\tau_2},E'\right)$ *be two neutrosophic soft topological spaces and* $f:NSS(X,E)\to NSS(Y,E')$ *be a neutrosophic soft continuous mapping. Then for each* $e\in E$, $f:(X,\tau_1^e)\to(Y,\tau_2^e)$ *is a neutrosophic continuous mapping.*

Proof Straightforward. □

Theorem 3.8 *Let* $\left(X,\overset{NSS}{\tau_1},E\right),\left(Y,\overset{NSS}{\tau_2},E'\right)$ *and* $\left(Z,\overset{NSS}{\tau_3},E''\right)$ *be neutrosophic soft topological spaces. If* $f:NSS(X,E)\to NSS(Y,E')$ *and* $g:NSS(Y,E')\to NSS(Z,E'')$ *are neutrosophic soft continuous mapping, the* $g\circ f:NSS(X,E)\to NSS(Z,E'')$ *is a neutrosophic soft continuous mapping.*

Proof Let . $f:NSS(X,E)\to NSS(Y,E')$ and $g:NSS(Y,E')\to NSS(Z,E'')$ are neutrosophic soft continuous mapping and $(H,E'')\in NSS(Z,E'')$. We need to show that $(g\circ f)^{-1}((H,E''))$ is the neutrosophic soft open set in X. We have $(g\circ f)^{-1}((H,E''))=f^{-1}\left(g^{-1}((H,E''))\right)$. Since g is neutrosophic soft continuous mapping, $g^{-1}((H,E''))$ is neutrosophic soft open set in Y. So f is neutrosophic soft continuous mapping. Then

$f^{-1}\left(g^{-1}\left((H,E'')\right)\right)$ is neutrosophic soft open set in X. Hence, $(g\circ f)^{-1}\left((H,E'')\right)$ is neutrosophic soft open set and $g\circ f$ neutrosophic soft continuous mapping. □

Definition 3.9 *Let* $\left(X,\overset{NSS}{\tau_1},E\right)$ *and* $\left(Y,\overset{NSS}{\tau_2},E'\right)$ *be two neutrosophic soft topological spaces and* $f:NSS(X,E)\rightarrow NSS(Y,E')$ *be a neutrosophic soft mapping. Then,*

1. A neutrosophic soft mapping f is called a neutrosophic soft open mapping if $f\left(\left(\widetilde{F},E\right)\right)$ is a neutrosophic soft open set in $NSS(Y,E')$ for each $\left(\widetilde{F},E\right)$ neutrosophic soft open set of $NSS(X,E)$.
2. A neutrosophic soft mapping f is called a neutrosophic soft closed mapping if $f\left(\left(\widetilde{G},E'\right)\right)$ is a neutrosophic soft closed set in $NSS(Y,E')$ for each $\left(\widetilde{F},E\right)$ neutrosophic soft closed set of $NSS(X,E)$.

Corollary 3.10 *Notice that the concept of neutrosophic soft continuous mappingg, neutrosophic soft openness and neutrosophic soft closedness are all independent of each other.*

Example 3.11 *Let* $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3\}$, $E=\{e_1,e_2\}$, $E'=\{e_1^{'},e_2^{'}\}$ *and* $\overset{NSS}{\tau_1},\overset{NSS}{\tau_2}$ *be two neutrosophic soft topology as follows:*

$$\begin{aligned}\overset{NSS}{\tau_1} &= \left\{\{0_{(X,E)},1_{(X,E)},(\widetilde{F}_1,E),(\widetilde{F}_2,E),(\widetilde{F}_3,E)\}\right\}\\ \overset{NSS}{\tau_2} &= \{0_{(Y,E')},1_{(Y,E')},(\widetilde{G}_1,E'),(\widetilde{G}_2,E'),(\widetilde{G}_3,E'),(\widetilde{G}_4,E')\}\end{aligned}$$

where

$$\begin{aligned}\left(\widetilde{F}_1,E\right) &= \left\{\begin{array}{c}(e_1,\{<x_1,0.2,0.5,0.3>,<x_2,0.3,0.7,0.1>,<x_3,0,0,0.1>\})\\ (e_2,\{<x_1,1,0,0>,<x_2,0,0.2,0.5>,<x_3,0.1,0.2,0>\})\end{array}\right\}\\ \left(\widetilde{F}_2,E\right) &= \left\{\begin{array}{c}(e_1,\{<x_1,0.4,0.3,0.1>,<x_2,0.5,0.8,0.1>,<x_3,0.2,0.3,0.5>\})\\ (e_2,\{<x_1,1,0,0>,<x_2,0,0.3,0.4>,<x_3,0.1,0.2,0>\})\end{array}\right\}\\ \left(\widetilde{F}_3,E\right) &= \left\{\begin{array}{c}(e_1,\{<x_1,0.4,0.5,0.1>,<x_2,0.5,0.8,0.1>,<x_3,0.2,0.3,0.5>\})\\ (e_2,\{<x_1,1,0,0>,<x_2,0,0.3,0.4>,<x_3,0.1,0.2,0>\})\end{array}\right\}\end{aligned}$$

$$\begin{aligned}\left(\widetilde{G}_1,E'\right) &= \left\{\begin{array}{c}(e_1^{'},\{<y_1,0.2,0.5,0.1>,<y_2,0.3,0.7,0.1>\})\\ (e_2^{'},\{<y_1,1,0.2,0>,<y_2,0,0.2,0.5>\})\end{array}\right\}\\ \left(\widetilde{G}_2,E'\right) &= \left\{\begin{array}{c}(e_1^{'},\{<y_1,0.4,0.3,0.1>,<y_2,0.5,0.8,0.1>\})\\ (e_2^{'},\{<y_1,1,0.2,0>,<y_2,0,0.3,0.4>\})\end{array}\right\}\\ \left(\widetilde{G}_3,E'\right) &= \left\{\begin{array}{c}(e_1^{'},\{<y_1,0.4,0.5,0.1>,<y_2,0.5,0.8,0.1>\})\\ (e_2^{'},\{<y_1,1,1,0>,<y_2,0,0.3,0.4>\})\end{array}\right\}\\ \left(\widetilde{G}_4,E'\right) &= \left\{\begin{array}{c}(e_1^{'},\{<y_1,0.2,0.3,0.1>,<y_2,0.3,0.7,0.1>\})\\ (e_2^{'},\{<y_1,1,0.2,0>,<y_2,0,0.2,0.5>\})\end{array}\right\}\end{aligned}$$

If the mapping $f = (u, v) : NSS(X, E) \to NSS(Y, E')$ is defined as follows:

$$u(x_1) = y_1 \qquad v(e_1) = e_1^{'}$$
$$u(x_2) = y_2 \qquad v(e_2) = e_2^{'}$$
$$u(x_3) = y_1$$

Then f is a neutrosophic soft open mapping. However f is not a neutrosophic soft continuous and not a neutrosophic soft closed mapping on neutrosophic soft topological space $\left(X, \overset{NSS}{\tau_1}, E\right)$.

Theorem 3.12 *Let* $\left(X, \overset{NSS}{\tau_1}, E\right)$ *and* $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ *be two neutrosophic soft topological spaces and* $f :$ $NSS(X, E) \to NSS(Y, E')$ *be a neutrosophic soft mapping. Then,*

1. f is a neutrosophic soft open mapping iff $f\left(\left(\widetilde{F}, E\right)^{\circ}\right) \subseteq \left(f\left(\left(\widetilde{F}, E\right)\right)\right)^{\circ}$ for each neutrosophic soft set $\left(\widetilde{F}, E\right)$ of $NSS(X, E)$.

2. f is a neutrosophic soft closed mapping iff $\overline{f\left(\left(\widetilde{F}, E\right)\right)} \subseteq \left(f\left(\overline{\left(\widetilde{F}, E\right)}\right)\right)$ for each neutrosophic soft set $\left(\widetilde{F}, E\right)$ of $NSS(X, E)$.

Proof (1) Let f be a neutrosophic soft open mapping and $(\widetilde{F}, E)^{\circ} \in NSS(X, E)$. Then $(\widetilde{F}, E)^{\circ}$ is a neutrosophic open mapping $f\left(\left(\widetilde{F}, E\right)^{\circ}\right) \subseteq \left(\widetilde{F}, E\right)$. Since f is a neutrosophic open mapping $f\left(\left(\widetilde{F}, E\right)^{\circ}\right)$ is a neutrosophic open set in $NSS(Y, E')$ and $f\left(\left(\widetilde{F}, E\right)^{\circ}\right) \subseteq f\left(\left(\widetilde{F}, E\right)\right)$. Thus $f\left(\left(\widetilde{F}, E\right)^{\circ}\right) \subseteq \left(f\left(\left(\widetilde{F}, E\right)\right)\right)^{\circ}$ is obtained.

Conversely, suppose that $\left(\widetilde{F}, E\right)$ is any neutrosophic soft open set in $NSS(X, E)$. Then $\left(\widetilde{F}, E\right) = \left(\widetilde{F}, E\right)^{\circ}$. From the condition of theorem, we have $f\left(\left(\widetilde{F}, E\right)^{\circ}\right) \subseteq \left(f\left(\left(\widetilde{F}, E\right)\right)\right)^{\circ}$. Then $f\left(\left(\widetilde{F}, E\right)\right) = f\left(\left(\widetilde{F}, E\right)^{\circ}\right) \subseteq \left(f\left(\left(\widetilde{F}, E\right)\right)\right)^{\circ} \subseteq f\left(\left(\widetilde{F}, E\right)\right)$. This implics that $f\left(\left(\widetilde{F}, E\right)\right) = \left(f\left(\left(\widetilde{F}, E\right)\right)\right)^{\circ}$. That is, f is a neutrosophic soft open mapping.

(2) Let f be a neutrosophic soft open mapping and $\left(\widetilde{F}, E\right)^{\circ} \in NSS(X, E)$. Since f is a neutrosophic soft closed mapping, $f\left(\overline{\left(\widetilde{F}, E\right)}\right)$ is a neutrosophic soft closed set in $NSS(Y, E')$ and $f\left(\left(\widetilde{F}, E\right)\right) \subseteq f\left(\overline{\left(\widetilde{F}, E\right)}\right)$. Thus $\overline{\left(f\left(\left(\widetilde{F}, E\right)\right)\right)} \subseteq f\left(\overline{\left(\widetilde{F}, E\right)}\right)$ is obtained.

Conversely, suppose that $\left(\widetilde{F}, E\right)$ is any neutrosophic soft closed set in $NSS(X, E)$. Then $\left(\widetilde{F}, E\right) = \overline{\left(\widetilde{F}, E\right)}$. From the condition of theorem,

$\overline{\left(f\left((\widetilde{F},E)\right)\right)} \subseteq f\left(\overline{\left(\widetilde{F},E\right)}\right) = f\left(\left(\widetilde{F},E\right)\right) \subseteq \overline{\left(f\left(\left(\widetilde{F},E\right)\right)\right)}$. Thus means that $\overline{\left(f\left(\left(\widetilde{F},E\right)\right)\right)} = f\left(\left(\widetilde{F},E\right)\right)$. That is, f is a neutrosophic soft closed mapping. □

Corollary 3.13 *A neutrosophic soft mapping f is neutrosophic soft continuous and neutrosophic soft closed iff* $\overline{f\left(\left(\widetilde{F},E\right)\right)} = \left(f\left(\overline{\left(\widetilde{F},E\right)}\right)\right)$

Definition 3.14 *Let $\left(X, \overset{NSS}{\tau_1}, E\right)$ and $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ be two neutrosophic soft topological spaces and $f : NSS(X,E) \rightarrow NSS(Y,E')$ be a neutrosophic soft mapping. Then f is called a neutrosophic soft homeomorphism if,*

1. f is a neutrosophic soft bijection,
2. f is a neutrosophic soft continuous mapping,
3. f^{-1} is a neutrosophic soft continuous mapping.

Theorem 3.15 *Let $\left(X, \overset{NSS}{\tau_1}, E\right)$ and $\left(Y, \overset{NSS}{\tau_2}, E'\right)$ be two neutrosophic soft topological spaces and $f : NSS(X,E) \rightarrow NSS(Y,E')$ be a neutrosophic soft mapping. Then the following conditions are equivalent;*

1. f is a neutrosophic soft homeomorphism,
2. f is a neutrosophic soft continuous and neutrosophic soft closed mapping,
3. f is a neutrosophic soft continuous and neutrosophic soft open mapping.

Proof Straightforward. □

4. Conclusion

Topology, which is an important branch of mathematics, can give many relationships between other scientific fields and matematical models. The motivation of the present paper is to extend the field of the topological structure on the neutrosophic soft sets. We firstly defined neutrosophic soft continuous mapping and investigated several related properties and structural characteristic. Finally we introduce neutrosophic open mapping, neutrosophic closed mapping and neutrosophic soft homeomorphism and supported different examples. We hope that these notions will be useful for the researchers to further promote and advance in neutrosophic soft set theory.

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