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On Neutrosophic Extended Triplet LA-hypergroups and Strong Pure LA-semihypergroups

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Abstract: We introduce the notions of neutrosophic extended triplet LA-semihypergroup, neutrosophic extended triplet LA-hypergroup, which can reflect some symmetry of hyperoperation and discuss the relationships among them and regular LA-semihypergroups, LA-hypergroups, regular LA-hypergroups. In particular, we introduce the notion of strong pure neutrosophic extended triplet LA-semihypergroup, get some special properties of it and prove the construction theorem about it under the condition of asymmetry. The examples in this paper are all from Python programs.

Keywords: LA-semihypergroup; LA-hypergroup; neutrosophic extended triplet LA-semihypergroup; neutrosophic extended triplet LA-hypergroup

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1. Introduction and Preliminaries

Left almost semigroup (abbreviated as LA-semigroup, some researchers also call it Abel Grassmann's groupoid), a non-associative and noncommutative algebraic structure, was first proposed by Kazim and Naseeruddin in Reference [1]. Hyperstructure theory was first introduced by Marty in Reference [2]. In the following decades and nowadays, various hyperstructures are widely studied and applied [3–6]. In Reference [7], Hila and Dine extended the concept of LA-semigroup to LA-semihypergroup and investigated several properties of LA-semihypergroups. Since then, many researchers have been done a lot of studies in this field [8–13].

In recent years, as an application of idea of neutrosophic set, the new notion of neutrosophic triplet group (NTG) was firstly introduced by F. Smarandache and M. Ali in Reference [14]. Soon after, M. Gulistan, S. Nawaz and N. Hassan applied the idea of NTG to LA-semihypergroup, proposed the concept of NTG-LA-semihypergroup and got some interesting results in Reference [15]. Meanwhile, F. Smarandache extended the concept of NTG to neutrosophic triplet extended group (NETG) in Reference [16]. Later, some research articles in this field are published. F. Smarandache, X.H. Zhang, X.G. An and Q.Q. Hu investigated properties and structures of NETG in Reference [17]; T.G. Jaíyéolá and F. Smarandache obtained some conclusions on neutrosophic triplet groups and discussed their applications in Reference [18]; The new concept of NET-Abel-Gassmann's Groupoid was introduced and the relationships of NETGs and regular semigroups were studied in Reference [19]; X.H. Zhang and X.Y. Wu prove that the construction theorem of NETG in Reference [20]; The concept of generalized neutrosophic extended group were proposed by Y.C. Ma and the relationships of NETGs and generalized groups were studied in References [21–22]. In particular, the notions of NET-semihypergroup and NET-hypergroup were introduced by X.H. Zhang, F. Smarandache and Y.C. Ma and the decomposition theorem of PWC-NET-semihypergroup was proved in Reference [23]. For the study of some related algebraic systems, please refer to Reference [24–26].

In this study, we apply the concept of NETG to LA-semihypergroup and introduce the new notions of NET-LA-semihypergroup, NET-LA-hypergroup, SPNET-LA-semihypergroup; Further, we discuss their properties, relations and so forth.

First of all, recall some conclusions and definitions on LA-semihypergroups.

Definition 1. *[7] We say that a mapping*

$$\circ : H \times H \to P^*(H)$$

is a binary hyperoperation, if H is a nonempty set, P(H) is power set of H and $P^*(H) = P(H)/\phi$.

Definition 2. *[7] (H, ∘) is a binary hypergroupoid, if H is a nonempty set and ∘ is a binary hyperoperation. In addition, we write*

$$X \circ Y = \bigcup_{a \in X,\, b \in Y} (a \circ b)\,,\; X \circ a = X \circ \{a\}\,,\; a \circ Y = \{a\} \circ Y\,,$$

where $a \in H$, $X \subseteq H$, $Y \subseteq H$ *and* $X \neq \phi$, $Y \neq \phi$.

Definition 3. *[7] A binary hypergroupoid (H, ∘) is an LA-semihypergroup, if*

$$(a \circ b) \circ c = (c \circ b) \circ a \quad (1)$$

for all a, b, c ∈ H, that is

$$\bigcup_{s \in (a \circ b)} (s \circ c) = \bigcup_{t \in (c \circ b)} (t \circ a)\cdot \quad (2)$$

By Equation (1), we know that every LA-semihypergroup (H, ∘) satisfies

$$(a \circ b) \circ (c \circ d) = (a \circ c) \circ (b \circ d) \quad (3)$$

for all a, b, c, d ∈ H.

Note that, the Equations (1) and (3) are all set equations. If we replace all the elements in the equations (1) and (3) with nonempty subsets of H, these equations still hold.

Definition 4. *[7] (T, ∘) is a sub LA-semihypergroup of (H, ∘), if the following conditions hold:*

(a) $T \subseteq H$, $T \neq \phi$;
(b) $m \circ n \subseteq T$ *for all m, n ∈ T;*
(c) (H, ∘) is an LA-semihypergroup.

Definition 5. *[8] Suppose (H, ∘) is an LA-semihypergroup. An element a ∈ H is regular if there is an element t ∈ H such that*

$$a \in a \circ t \circ a.$$

Furthermore, (H, ∘) is a regular LA-semihypergroup if each element of H is regular.

Definition 6. *[7] Suppose (H, ∘) is an LA-semihypergroup. (H, ∘) is an LA-hypergroup if it satisfies*

$$t \circ H = H \circ t = H$$

for all t ∈ H.

Definition 7. *[8] Suppose $(H, \circ)$ is an LA-semihypergroup. An element $e \in H$ is*

(a) *a left identity, if $a \in e \circ a$ for each $a \in H$;*
(b) *a right identity, if $a \in a \circ e$ for each $a \in H$;*
(c) *an identity, if $a \in (e \circ a) \cap (a \circ e)$ for each $a \in H$;*
(d) *a pure left identity, if $a = e \circ a$ for each $a \in H$;*
(e) *a pure right identity, if $a = a \circ e$ for each $a \in H$;*
(f) *a pure identity, if $a = (e \circ a) \cap (a \circ e)$ for each $a \in H$;*
(g) *a scalar identity, if $a = e \circ a = a \circ e$ for each $a \in H$.*

In addition, we say that $x \in H$ is an inverse of $a \in H$ if x satisfies

$$e \in (a \circ x) \cap (x \circ a),$$

where e is an identity of $(H, \circ)$.

Definition 8. *$(H, \circ)$ is a regular LA-hypergroup, if it satisfies the following conditions:*

(a) *$(H, \circ)$ is an LA-hypergroup;*
(b) *There exists $e \in H$ such that e is identity of $(H, \circ)$;*
(c) *Every element $a \in H$ has at least one inverse.*

Definition 9. *[16] A nonempty set M is said to be a neutrosophic extended triplet set if to any given $a \in M$, there are $s \in M$ and $t \in M$, in such a way that*

$$a \circ s = s \circ a = a \tag{4}$$

$$a \circ t = t \circ a = s, \tag{5}$$

where $\circ$ is a binary operation on M, s is an extend neutral of 'a', t is an opposite of 'a' about s, (a, s, t) is a neutrosophic extend triplet.

Definition 10. *[14,16] A semihypergroup $(H, \circ)$ is said to be an NET-semihypergroup if to any given $a \in H$, there are $s \in H$ and $t \in H$, in such a way that*

$$a \in (s \circ a) \cap (a \circ s), \tag{6}$$

$$s \in (t \circ a) \cap (a \circ t). \tag{7}$$

In addition, for a certain $a \in H$, we say that (a, s, t) is a hyper-neutrosophic-triplet and use $\{\}_{neut(a)}$ for the set of all s that satisfy Formula (6) and (7). For a certain $s \in \{\}_{neut(a)}$, we use $\{\}_{anti(a)_s}$ for the set of all t that satisfy Formula (7).

2. Neutrosophic Extended Triplet LA-Semihypergroups and Neutrosophic Extended Triplet LA-Hypergroups

Definition 11. *An LA-semihypergroup $(L, *)$ is said to be*

(a) *a left neutrosophic extended triplet LA-semihypergroup (LNET-LA-semihypergroup) if to any given $a \in L$, there are $p \in L$ and $q \in L$, in such a way that*

$$a \in p * a \tag{8}$$

$$p \in q * a. \tag{9}$$

Furthermore, for a certain a ∈ L, p, q and (a, p, q) are called left neutral of a, left opposite of a and left hyper-neutrosophic-triplet respectively. $\{\}_{lneut(a)}$ *is used to represent the set of all p that satisfy Formula (8), (9) and for a certain p ∈* $\{\}_{lneut(a)}$, $\{\}_{lanti(a)_p}$ *is used to represent the set of all q that satisfy Formula (9).*

(b) *a right neutrosophic extended triplet LA-semihypergroup (RNET-LA-semihypergroup), if to any given a ∈ L, there are s ∈ L and t ∈ L, in such a way that*

$$a \in a * s \tag{10}$$

$$s \in a * t. \tag{11}$$

Furthermore, for a certain a ∈ H, (a, s, t) is called right-hyper-neutrosophic-triplet. $\{\}_{rneut(a)}$ *is used to represent the set of all s that satisfy Formula (10), (11) and for a certain s ∈* $\{\}_{rneut(a)}$, $\{\}_{ranti(a)_s}$ *is used to represent the set of all t that satisfy Formula (11).*

(c) *a neutrosophic extended triplet LA-semihypergroup (NET-LA-semihypergroup), if to any given a ∈ L, there are m ∈ L and n ∈ L, in such a way that*

$$a \in (m * a) \cap (a * m) \tag{12}$$

$$m \in (n * a) \cap (a * n). \tag{13}$$

Furthermore, for a certain a ∈ L, (a, m, n) is called a hyper-neutrosophic-triplet, $\{\}_{neut(a)}$ *is used to represent the set of all m that satisfy Formula (12), (13) and for a certain m ∈* $\{\}_{neut(a)}$, $\{\}_{anti(a)_m}$ is used to represent the set of all n that satisfy Formula (13).

Example 1. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 1).*

Table 1. The binary hypergroupoid (*L*, ∗).

∗	0	1	2
0	0	0	0
1	0	1	0
2	0	0	{0, 2}

By Python program 1, (L, ∗) is an LA-semihypergroup (please see Figure 1).

Python program 1 Verification of LA-semihypergroup 1

```
T = [ [[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]] ]
count = 0
for x in range(3):
for y in range(3):
for z in range(3):
T1 = T[x][y]
T2 = set()
k1 = len(T1)
for m in range(k1)
T2 = set(T[T1[m]][z]).union(T2)
T3 = T[z][y]
```

```
T4 = set()
k2 = len(T3)
for n in range(k2):
T4 = set(T[T3[n]][x]).union(T4)
if T2 = = T4:
count += 1
while count = = 3**3:
print('{} is an LA-semihypergroup'.format(T))
break
```

Run: program 1

```
C:\Users\Think\Anaconda3\python.exe C:/Users/Think/PycharmProjects/1/program1.py
[ [[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]] ] is an LA-semihypergroup.
Process finished with exit code 0
```

Figure 1. The result of Python program 1.

Furthermore, we get

$$0 \in (0*0) \cap (0*0),\ 0 \in (0*0) \cap (0*0)$$

$$0 \in (0*0) \cap (0*0),\ 0 \in (1*0) \cap (0*1)$$

$$0 \in (0*0) \cap (0*0),\ 0 \in (2*0) \cap (0*2)$$

$$1 \in (1*1) \cap (1*1),\ 1 \in (1*1) \cap (1*1)$$

$$2 \in (2*2) \cap (2*2),\ 2 \in (2*2) \cap (2*2).$$

By Definition 11, (0, 0, 0), (0, 0, 1),(0, 0, 2) (1, 1, 1), (2, 2, 2) are all hyper neutrosophic-triplets and (L, ∗) is an NET-LA-semihypergroup. These results can also be verified by Python program 2 (please see Figure 2).

Python program 2 Verification of NET-LA-semihypergroup 1

```
T = [ [[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]] ]
test = []
for t in range(3):
for neut_t in range(3):
for anti_t in range(3):
S1 = set(T[t][neut_t])
S2 = set(T[t][anti_t])
S3 = set(T[neut_t][t])
S4 = set(T[anti_t][t])
S5 = set(list([t]))
S6 = set(list([neut_t]))
if S5.issubset(S1 & S3) and S6.issubset(S2 & S4):
```

```
13: test.append([t, neut_t, anti_t])
14: test2 = test
15: test1 = set([test2[i][0] for i in range(len(test2))])
16: if test1 == set([x for x in range(3)]):
17: print('{0} is an Net-LA-semihypergroup and hyper neutrosophic-triplet are {1}'.format(T, test2))
```

Run: programm 2

```
C:\Users\Think\Anaconda3\python.exe C:/Users/Think/PycharmProjects/1/program2.py
[ [[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]] ] is an Net-LA-semihypergroup and hyper neutrosophic-triplet are [[0,0,0], [0,0,1], [0,0,2], [1,1,1], [2,2,2]]
Process finished with exit code 0
```

Figure 2. The result of Python program 2.

Example 2. *Suppose R is the set of real numbers, the binary hypergroupoid (R, ∗) is as follows.*

$$x * y = \begin{cases} (x, y) & x < y, \\ (y, x) & y < x, \\ x & x = y. \end{cases}$$

for all x, y ∈ R, where (x, y) is the open interval.

When z < x < y,

$$(x*y)*z = \bigcup_{s\in(x*y)} (s * z) = \bigcup_{s\in(x,y)} (z, s) = (z, y)$$

$$(z*y)*x = \bigcup_{t\in(z*y)} (t * x) = \bigcup_{t\in(z,y)} (t * x) = [\bigcup_{t\in(z,x)} (t * x)] \bigcup [\bigcup_{t=x} (t * x)] \bigcup [\bigcup_{t\in(x,y)} (t * x)]$$

$$= [\bigcup_{t\in(z,x)} (t, x)] \bigcup (\{x\}) \bigcup [\bigcup_{t\in(x,y)} (x, t)] = (z, x) \bigcup (\{x\}) \bigcup (x, y) = (z, y) = (x*y)*z.$$

In the same way, we have

$$(x*y)*z = (z*y)*x,$$

for all x, y, z ∈ R. Hence (R, ∗) is an LA-semihypergroup. On the other hand, Since

$$x \in (x*x) \cap (x*x), x \in (x*x) \cap (x*x),$$

for any given x ∈ R, $x \in \{\}_{neut(x)}$, $x \in \{\}_{anti(x)_x}$. By Definition 11, (R, ∗) is an NET-LA-semihypergroup.

Example 3. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 2).*

Table 2. The binary hypergroupoid (*L*, ∗).

∗	0	1	2

0	*0*	*0*	*0*
1	*0*	*0*	*0*
2	*0*	*0*	*{0, 1}*

By Python program, (L, ∗) is an LA-semihypergroup. In addition, we get

$$1 \notin (0*1) \cap (1*0),\ 1 \notin (1*1) \cap (1*1),\ 1 \notin (2*1) \cap (1*2).$$

This shows that $\{\}_{neut(1)} = \phi$*. By Definition 11, (L, ∗) is not an NET-LA-semihypergroup.*

Remark 1. *Every NET-LA-semihypergroup is an LA-semihypergroup but not vice versa.*

Example 4. *Put L = {0, 1, 2, 3}, the binary hypergroupoid (L, ∗) is as follows(see Table 3).*

Table 3. The binary hypergroupoid (*L*, ∗).

∗	0	1	2	3
0	*0*	*0*	*0*	*{0,1,2,3}*
1	*0*	*0*	*0*	*{0,1,2,3}*
2	*0*	*0*	*{0,1}*	*{2,3}*
3	*{1,2,3}*	*{0,1,2,3}*	*{2,3}*	*{0,3}*

By Python program 3 and Python program 4, (L, ∗) is both an LA-semihypergroup(please see Figure 3) and an NET-LA-semihypergroup(please see Figure 4). In addition,

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 3), (0, 2, 3), (1, 3, 3), (2, 3, 3)$$

(3, 0, 1), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 1), (3, 3, 2), (3, 3, 3) are all hyper neutrosophic-triplets(please see Figure 4). Let M = {0, 1, 2 } $\subseteq$ *L, then (M, ∗) is a sub LA-semihypergroup of (L, ∗). From Example 3, (M, ∗) is not an NET-LA-semihypergroup.*

Python program 3 Verification of LA-semihypergroup 2

```
T = [ [[0],[0],[0],[0,1,2,3]], [[0], [0], [0],[0,1,2,3]], [[0], [0], [0,1],[2,3]], [[1,2,3],[0,1,2,3],[2,3],[0,3]] ]
count = 0
for x in range(4):
for y in range(4):
for z in range(4):
T1 = T[x][y]
T2 = set()
k1 = len(T1)
for m in range(k1)
T2 = set(T[T1[m]][z]).union(T2)
T3 = T[z][y]
T4 = set()
k2 = len(T3)
for n in range(k2):
T4 = set(T[T3[n]][x]).union(T4)
```

```
if T2 = = T4:
count += 1
while count = = 4**3:
print('( T,∗) is an LA-semihypergroup.')
break
```

Run: program 3

```
C:\Users\Think\Anaconda3\python.exe C:/Users/Think/PycharmProjects/1/program3.py
(T, ∗) is an LA-semihypergroup.
Process finished with exit code 0
```

Figure 3. The result of Python program 3.

Python program 4 Verification of NET-LA-semihypergroup 2

```
T = [ [[0],[0],[0],[0,1,2,3]], [[0], [0], [0],[0,1,2,3]], [[0], [0], [0,1],[2,3]], [[1,2,3],[0,1,2,3],[2,3],[0,3]] ]
test = []
for t in range(4):
for neut_t in range(4):
for anti_t in range(4):
S1 = set(T[t][neut_t])
S2 = set(T[t][anti_t])
S3 = set(T[neut_t][t])
S4 = set(T[anti_t][t])
S5 = set(list([t]))
S6 = set(list([neut_t]))
if S5.issubset(S1 & S3) and S6.issubset(S2 & S4):
test.append([t, neut_t, anti_t])
test2 = test
test1 = set([test2[i][0] for i in range(len(test2))])
if test1 == set([x for x in range(3)]):
print('(T,∗) is an NET-LA-semihypergroup and hyper neutrosophic-triplet are {}'.format(test2).
```

Run: program 4

```
C:\Users\Think\Anaconda 3\python.exe C:/Users/Think/PycharmProjects/1/proram4.py
(T,∗) is an NET-LA-semihypergroup and hyper neutrosopic-triplet are [[0,0,0], [0,0,1], [0,0,2], [0,2,3], [1,3,3], [2,3,3], [3,0,1], [3,0,3], [3,1,0], [3,1,1], [3,2,0], [3,2,1], [3,2,2], [3,3,0], [3,3,1], [3,3,2], [3,3,3]]
Process finished with exit code 0
```

Figure 4. The result of Python program 4.

Remark 2. *From Example 4, we know that for a certain t in an NET-LA-semihypergroup,* $|\{\}_{neut(x)}|$ *may be greater than or equal to one and for a certain* $p \in \{\}_{neut(x)}$, $|\{\}_{anti(x)_p}|$ *may be greater than or equal to one. According to the results of Example 4, we have*

$$\{\}_{neut(0)} = \{0, 1, 2\}, \{\}_{anti(0)_0} = \{0, 1, 2\}, \{\}_{anti(0)_1} = \{3\}, \{\}_{anti(0)_2} = \{3\}$$

$$\{\}_{neut(1)} = \{3\}, \{\}_{anti(1)_3} = \{3\}; \{\}_{neut(2)} = \{3\}, \{\}_{anti(2)_3} = \{3\}$$

$$\{\}_{neut(3)} = \{0, 1, 2, 3\}, \{\}_{anti(3)_0} = \{1, 3\}, \{\}_{anti(3)_1} = \{0, 1\}, \{\}_{anti(3)_2} = \{0, 1, 2\};$$
$$\{\}_{anti(3)_3} = \{0, 1, 2, 3\}$$

Definition 12. *(L, ∗) is said to be an NET-LA-hypergroup if it is both an LA-hypergroup(see Definition 6) and an NET-LA-semihypergroup.*

Proposition 1. *Every LA-hypergroup is a regular LA-semihypergroup.*

Proof. Since $(L, *)$ is an LA-hypergroup, to every $t \in L$, $t * L = L * t = L$. Thus

$$t \in L = L * t = t * L * t$$

By Definition 5, $(L, *)$ is a regular LA-semihypergroup. □

Example 5. *Put* $L = \{0, 1, 2\}$*, the binary hypergroupoid* $(L, *)$ *is as follows(see Table 4).*

Table 4. The binary hypergroupoid $(L, *)$.

∗	0	1	2
0	0	0	0
1	0	1	{0, 1}
2	0	{0, 1}	{2}

By Python program, $(L, *)$ *is an LA-semihypergroup. Furthermore, we have*

$$0 \in 0 * 0 * 0,\ 1 \in 1 * 2 * 1,\ 2 \in 2 * 2 * 2$$

By Definition 5, $(L, *)$ *is a regular LA-semihypergroup. But*

$$0 * L = 0 \neq L$$

By Definition 6, $(L, *)$ *is not an LA-hypergroup.*

Remark 3. *From Example 5, a regular LA-semihypergroup is not necessarily an LA-hypergroup.*

Proposition 2. *Every NET-LA-semihypergroup is a regular LA-semihypergroup.*

Proof. Suppose $(L, *)$ is an NET-LA-semihypergroup, then to any given $a \in L$, there are $p \in \{\}_{neut(a)} \subseteq L$ and $q \in \{\}_{anti(a)_p} \subseteq L$ such that

$$a \in (p * a) \cap (a * p)$$

$$p \in (q*a) \cap (a*q)$$

Hence

$$a \in (p*a) \text{ and } p \in (a*q)$$

that is

$$a \in p*a \in (a*q)*a$$

By Definition 5, *(L, ∗)* is a regular LA-semihypergroup. □

Example 6. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 5).*

Table 5. The binary hypergroupoid (*L*, ∗).

∗	0	1	2
0	0	0	0
1	0	2	2
2	0	{0,1,2}	{0,1,2}

By Python program, (L, ∗) is an LA-semihypergroup. Furthermore, we have

$$0 \in 0*0*0\,,\ 1 \in 1*2*1,\ 2 \in 2*1*2$$

By Definition 5, (L, ∗) is a regular LA-semihypergroup. But

$$1 \notin (0*1) \cap (1*0),\ 1 \notin (1*1) \cap (1*1),\ 1 \notin (2*1) \cap (1*2)$$

This shows that $\{\}_{neut(1)} = \phi$ *. By Definition 11, (L, ∗) is not an NET-LA-semihypergroup.*

Remark 4. *From Example 6, a regular LA-semihypergroup is not necessarily an NET-LA-semihypergroup.*

Example 7. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 6).*

Table 6. The binary hypergroupoid (*L*, ∗).

∗	0	1	2
0	0	0	0
1	0	1	0
2	0	0	{0 ,2}

By Python program, (L, ∗) is an LA-semihypergroup. Furthermore, we get

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (1, 1, 1), (2, 2, 2)$$

are all hyper neuromorphic-triplets. By Definition 11, (L, ∗) is an NET-LA-semihypergroup. But

$$0*L = 0 \neq L$$

By Definition 6, (L, ∗) is not an LA-hypergroup.

Example 8. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 7).*

Table 7. The binary hypergroupoid $(L, *)$.

$*$	**0**	**1**	**2**
0	0	{0,1,2}	{0,1,2}
1	0	{0 ,2}	{1 ,2}
2	{0,1,2}	{0 ,2}	{0,1,2}

By Python program, $(L, *)$ is an LA-semi hypergroup. Furthermore, we get

$$0 * L = L * 0 = L,\ 1 * L = L * 1 = L,\ 2 * L = L * 2 = L$$

By Definition 6, $(L, *)$ is an LA-hypergroup. But

$$1 \notin (0 * 1) \cap (1 * 0),\ 1 \notin (1 * 1) \cap (1 * 1),\ 1 \notin (2 * 1) \cap (1 * 2)$$

This shows that $\{\}_{neut(1)} = \phi$. By Definition 11, $(L, *)$ is not an NET-LA-semihypergroup.

Proposition 3. *Every regular LA-hypergroup is an NET-LA-hypergroup.*

Example 9. *Put L = {0, 1, 2}, the binary hypergroupoid $(L, *)$ is as follows(see Table 8).*

Table 8. The binary hypergroupoid $(L, *)$.

$*$	**0**	**1**	**2**
0	{1,2}	{0,1,2}	{0,1,2}
1	{0,1,2}	{0,2}	{0,2}
2	{0,1}	{1,2}	{0,1}

By Python program, $(L, *)$ is an LA-semihypergroup. Furthermore, we get

$$(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (1, 0, 0), (1, 0, 1), (2,1,0), (2, 1, 2)$$

are all hyper neutrosophic-triplets, and

$$0 * L = L * 0 = L,\ 1 * L = L * 1 = L,\ 2 * L = L * 2 = L$$

by Definition 12, $(L, *)$ is an NET-LA-hypergroup. But

$$0 \notin (0 * 0) \cap (0 * 0),\ 1 \notin (1 * 1) \cap (1 * 1),\ 2 \notin (2 * 2) \cap (2 * 2)$$

This shows that the identity of $(L, *)$ does not exist. By Definition 8, $(L, *)$ is not a regular LA-hypergroup.

Based on the above, the relationships of LA-semihypergroup, regular LA-semihypergroup, LA-hypergroup, NET-LA-semihypergroup, NET-LA-hypergroup and regular LA-hypergroup, can be represented by the flowing Figure 5.

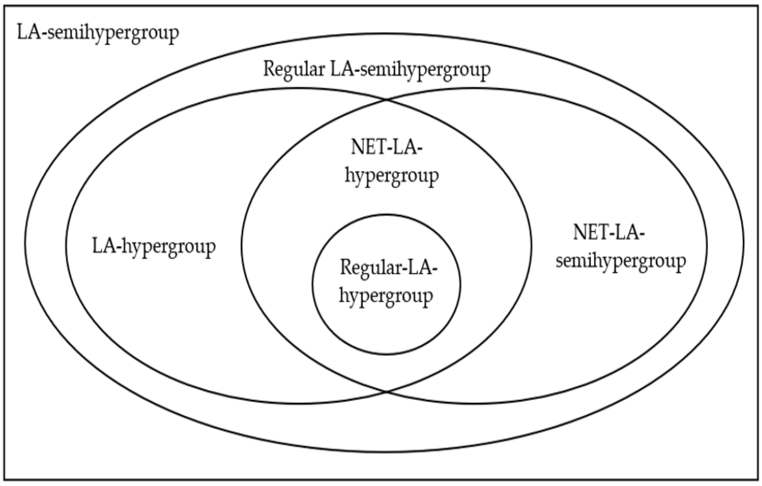

Figure 5. The relationships of various LA-semihypergroups.

Proposition 4. *An NET-LA-semihypergroup $(L, *)$ is both an LNET-LA-semihypergroup and a RNET -LA-semihypergroup.*

Proof. Since $(L, *)$ is an NET-LA-semihypergroup , to any given a ∈ L, there are $s \in \{\}_{neut(a)}$ and $t \in \{\}_{anti(a)_s}$ such that

$$a \in (s*a) \cap (a*s) \text{ and } s \in (t*a) \cap (a*t).$$

Hence $a \in (s*a)$ and $s \in (t*a)$, This shows

$$s \in \{\}_{lneut(a)} \text{ and } t \in \{\}_{lanti(a)_s}.$$

Thus $(L, *)$ is an LNET-LA-semihypergroup. In the same way, we can prove that $(L, *)$ is al so a RNET-LA-semihypergroup. □

Example 10. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 9).*

Table 9. The binary hypergroupoid $(L, *)$.

∗	0	1	2
0	0	0	0
1	0	2	2
2	0	{1,2}	{1,2}

By Python program, (L, ∗) is an LA-semihypergroup and

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (1, 2, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1), (2, 2, 2)$$

are all left-hyper neutrosophic-triplets;

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)$$

are all right-hyper neutrosophic-triplets;

$$(0, 0, 0),(0, 0, 1),(0, 0, 2),(2, 1, 2),(2, 2, 1),(2, 2, 2)$$

are all hyper neutrosophic-triplets. By Definition 11, (L, ∗) is an LNET-LA-semihypergroup but it is neither a RNET-LA-semihypergroup nor an NET-LA-semihypergroup.

Example 11. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 10).*

Table 10. The binary hypergroupoid (*L*, ∗).

∗	0	1	2
0	0	{0, 1, 2}	{0, 1, 2}
1	2	2	{ 1, 2}
2	{0, 1, 2}	{0, 2}	{0, 1, 2}

By Python program, (L, ∗) is an LA-semihypergroup and

$$(0, 0, 0),(0, 0, 2),(0, 2, 1),(0, 2, 2),(1, 0, 0),(1, 0, 2),(2, 0, 0)$$

$$(2, 0, 2),(2, 1, 0),(2, 1, 1),(2, 1, 2),(2, 2, 0),(2, 2, 1),(2, 2, 2)$$

are all left-hyper neutrosophic-triplets;

$$(0, 0, 0),(0, 0, 1)(0, 0, 2),(0, 1, 1),(0, 1, 2)(0, 2, 1),(0, 2, 2),(1, 2, 0),(1, 2, 1)$$

$$(1, 2, 2),(2, 0, 0),(2, 0, 1),(2, 0, 2),(2, 1, 0),(2, 1, 2),(2, 2, 0),(2, 2, 1),(2, 2, 2)$$

are all right-hyper neutrosophic-triplets; But

$$1 \notin (0*1) \cap (1*0), 1 \notin (1*1) \cap (1*1), 1 \notin (2*1) \cap (1*2)$$

This shows that $\{\}_{neut(1)} = \phi$. *By Definition 11, (L, ∗) is both an LNET-LA-semihypergroup and a RNET-LA-semihypergroup but not an NET-LA-semihypergroup. Moreover, from Example 11, we know that*

$$\{\}_{lneut(0)} = \{0, 2\}, \{\}_{lanti(0)_0} = \{0, 2\}, \{\}_{lanti(0)_2} = \{1, 2\}$$

$$\{\}_{lneut(1)} = \{0\}, \{\}_{lanti(1)_0} = \{0, 2\}$$

$$\{\}_{lneut(2)} = \{0, 1, 2\}, \{\}_{lanti(2)_0} = \{0, 2\}, \{\}_{lanti(2)_1} = \{0, 1, 2\}, \{\}_{lanti(2)_2} = \{0, 1, 2\}$$

These means that for a certain x in an LNET-LA-semihypergroup, $|\{\}_{lneut(x)}|$ *may be greater than or equal to one and for a certain* $p \in \{\}_{lneut(x)}$, $|\{\}_{lanti(x)_p}|$ *may be greater than or equal to one. There are similar conclusions in RNET-LA-semihypergroup. In addition, for a certain x in an LA-semihypergroup, if* $s \in \{\}_{lneut(x)}$ *(or* $s \in \{\}_{rneut(x)}$*), then s may be not in* $\{\}_{rneut(x)}$ *(or* $\{\}_{lneut(x)}$*). By Example 11, we have* $1 \in \{\}_{rneut(0)}$ *but* $1 \notin \{\}_{lneut(0)}$.

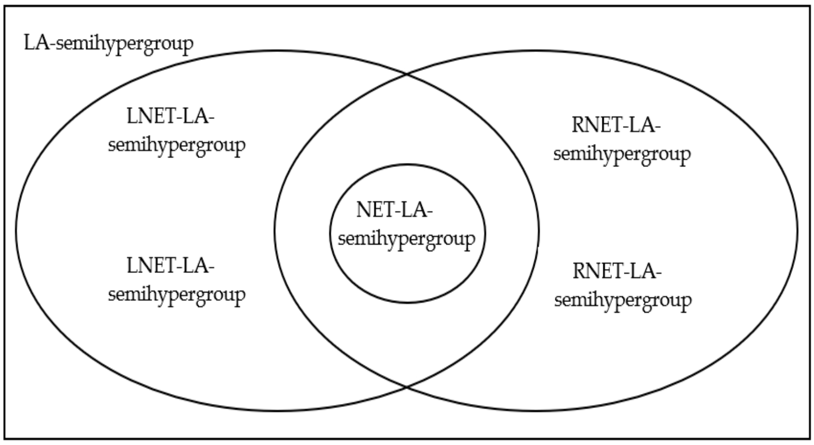

Figure 6. The relationships of various LA-semihypergroups.

Remark 5. *Non-LNET-LA-semihypergroup(or Non-RNET-LA-semihypergruop) is not an NET-LA-semihypergroup. (L, *) is both an LNET-LA-semihypergroup and a RNET-LA-semihypergroup but it is not necessarily an NET-LA-semihypergroup.*

Based on the above, the relationships of NET-LA-semihypergroup, RNET-LA-semihypergroup and LNET-LA-hypergroup, can be represented by Figure 6.

3. Strong Pure Neutrosophic Extended Triplet LA-Semihypergroups (SPNET-LA-Semihypergroups)

Definition 13. *An LA-semihypergroup (L, ∗) is said to be*

(a) a pure left neutrosophic extended triplet LA-semihypergroup (PLNET-LA-semihypergroup), if to any given $a \in L$, there are $p \in L$ and $q \in L$, in such a way that

$$a = p * a \text{ and } p = q * a$$

(b) a pure right neutrosophic extended triplet LA-semihypergroup (PRNET-LA-semihypergroup), if to any given $a \in L$, there are $s \in L$ and $t \in L$, in such a way that

$$a = a * s \text{ and } s = a * t$$

(c) a pure neutrosophic extended triplet LA-semihypergroup (PNET-LA-semihypergroup), if to any given $a \in L$, there are $m \in L$ and $n \in L$, in such a way that

$$a = (m * a) \cap (a * m) \text{ and } m = (n * a) \cap (a * n)$$

(d) a strong pure neutrosophic extended triplet LA-semihypergroup (SPNET-LA-semihypergroup), if to any given $a \in L$, there are $m \in L$ and $n \in L$, in such a way that

$$a = m * a = a * m \text{ and } m = n * a = a * n$$

Proposition 5. *Every SPNET-LA-semihypergroup is a PNET-LA-semihypergroup; Every PNET-LA-semihypergroup is an NET-LA-semihypergroup. Every PLNET-LA-semihypergroup is an LNET-LA-semihypergroup; Every PRNET-LA-semihypergroup is a RNET-LA-semihypergroup.*

Remark 6. *From Proposition 5, we know that the signs in the Definition 11 can still be used, such as*

$\{\}_{lneut(a)}$, $\{\}_{rneut(a)}$, $\{\}_{neut(a)}$, $\{\}_{lanti(a)_p}$, $\{\}_{ranti(a)_p}$, etc.

Proposition 6. *Every commutative PNET-LA-semihypergroup is an SPNET-LA-semihypergroup; Every commutative PLNET-LA-semihypergroup(or PRNET-LA-semihypergroup) is an SPNET-LA-semihypergroup.*

Proposition 7. *Suppose (L, ∗) is an SPNET-LA-semihypergroup, for any a, b, c ∈ L,*

(1) if $s \in \{\}_{neut(a)}$, *then s is unique and* $s*s = s$;

(2) if $s = neut(a)$, *then* $neut(s) = s$ *and* $s \in \{\}_{anti(s)_s}$;

(3) if $s = neut(a)$, $t \in \{\}_{anti(a)_s}$, $r \in \{\}_{anti(s)_s}$, *then* $r*t \subseteq \{\}_{lanti(a)_s}$;

(4) if $s = neut(a)$, $t \in \{\}_{anti(a)_s}$, *then* $s*t \subseteq \{\}_{lanti(a)_s}$;

(5) if $p = neut(a)$, $s = neut(b)$, $q \in \{\}_{anti(a)_p}$, $t \in \{\}_{anti(b)_s}$ *and* $|a*b| = |p*s| = 1$, *then*

$$neut(a*b) = p*s \text{ and } q*t \subseteq \{\}_{anti(a*b)_{p*s}}$$

(6) if $s = neut(a) = neut(b)$, $q \in \{\}_{anti(a)_s}$, $t \in \{\}_{anti(b)_s}$ *and* $|a*b| = 1$, *then*

$$neut(a*b) = s \text{ and } q*t \subseteq \{\}_{anti(a*b)_s}$$

(7) if $neut(a) = neut(b)$, *then* $a*b = b*a$;

(8) then $s*b = s*c$ *if* $b*a = c*a$, *where* $s = neut(a)$;

(9) if $s = neut(a)$, $q, t \in \{\}_{anti(a)_s}$, *then* $s*q = s*t$.

Proof. (1) Suppose there are $s, p \in \{\}_{neut(a)}$, $t \in \{\}_{anti(a)_s}$, $q \in \{\}_{anti(a)_p}$. (*L*, ∗) is an SPNET-LA-semihypergroup, hence

$$a = s*a = a*s,\ s = t*a = a*t$$

$$a = a*p = p*a,\ p = a*q = q*a$$

we get

$$s*p = (t*a)*p = (p*a)*t = a*t = s$$

$$p*s = (q*a)*s = (s*a)*q = a*q = p$$

$$s*p = (a*t)*(q*a) = (a*q)*(t*a) = p*s$$

Thus $p = s$, it implies s is unique and $s*s = s$.

(2) From (1), if $s = neut(a) \in L$, then $s*s = s*s = s$, This implies $neut(s) = s$ and $s \in \{\}_{anti(s)_s}$.

(3) For any given $a \in L$, if $s = neut(a)$, $t \in \{\}_{anti(a)_s}$, then

$$a = a*s = s*a,\ s = a*t = t*a$$

On the other hand, from $neut(s) = s$ and $r \in \{\}_{anti(s)_s}$, we get

$$s = s * s = s * s,\ s = r * s = s * r$$

Thus

$$\bigcup_{m \in r*t} (m * a) = (r * t) * a = (a * t) * r = s * r = s$$

where $m * a$ is a nonempty set, hence for any $m \in r * t$, $m * a = s$. This implies $m \in \{\}_{lanti(a)_s}$. In other words, $r * t \subseteq \{\}_{lanti(a)_s}$.

(4) By (2), (3), we can get (4).

(5) if $p = neut(a)$, $s = neut(b)$, $q \in \{\}_{anti(a)_p}$, $t \in \{\}_{anti(b)_s}$, then

$$(p * s) * (a * b) = (p * a) * (s * b) = a * b$$

$$(a * b) * (p * s) = (a * p) * (b * s) = a * b.$$

That is,

$$(p * s) * (a * b) = (a * b) * (p * s) = a * b. \tag{14}$$

On the other hand,

$$\bigcup_{l \in q*t} [(a * b) * l] = (a * b) * (q * t) = (a * q) * (b * t) = p * s,$$

where $(a * b) * l$ is a nonempty set, $|a * b| = 1$ and $|p * s| = 1$. Hence for any $l \in q * t$, $(a * b) * l = p * s$. In the same way, we can prove that for any $l \in q * t$, $l * (a * b) = p * s$. Thus for any $l \in q * t$,

$$l * (a * b) = (a * b) * l = p * s. \tag{15}$$

From (14), (15) and $|a * b| = |p * s| = 1$, we get $neut(a * b) = p * s$ and $q * t \subseteq \{\}_{anti(a*b)_{p*s}}$.

(6) Let $p = s$ *in* Proposition 7 (5), we can get the conclusion.

(7) $(L, *)$ is an SPNET-LA-semihypergroup, hence for any given $a, b \in L$, there are $neut(a) = s$, $neut(b) = p$, $t \in \{\}_{anti(a)_s}$, $q \in \{\}_{anti(b)_p}$ such that

$$a = a * s = s * a,\ s = a * t = t * a$$

$$b = b * p = p * b,\ p = b * q = q * b.$$

If $s = p$, then we have

$$a * b = (a * s) * (b * p) = (a * b) * (s * p) = (a * b) * (s * s) = (a * b) * s = (s * b) * a = (p * b) * a = b * a.$$

(8) Suppose that $b * a = c * a$ *for* $a, b, c \in L$. There are $s = neut(a) \in L$ and $t \in \{\}_{anti(a)_s}$. Multiply $b * a = c * a$ by t, we have

$$(b * a) * t = (c * a) * t$$

$$(t * a) * b = (t * a) * c$$

$$s * b = s * c$$

(9) For any given $a \in L$, there is $s = neut(a) \in L$, if $q, t \in \{\}_{anti(a)_s}$, then

$$s * q = (t * a) * q = (q * a) * t = s * t.$$

□

Theorem 1. *Suppose (L, ∗) is a PRNET-LA-semihypergroup, for any $x \in L$,*

a) if $p \in \{\}_{rneut(x)}$, $q \in \{\}_{ranti(x)_p}$ and $|p * p| = 1$, then

$$p * p \subseteq \{\}_{lneut(x)} \text{ and } p * q \subseteq \{\}_{lanti(x)_{p*p}}$$

and (L, ∗) is an PLNET-LA-semihypergroup.

b) if $p \in \{\}_{rneut(x)}$, $q \in \{\}_{ranti(x)_p}$, $p * p = p$ and $q \in p * q$, then

$$p = neut(x) \text{ and } q \in \{\}_{anti(x)_p}$$

and (L, ∗) is an SPNET-LA-semihypergroup.

Proof. (1) Since (*L*, ∗) is a PRNET-LA-semihypergroup, for any given $x \in L$, there are $p \in \{\}_{rneut(x)}$ and $q \in \{\}_{ranti(x)_p}$ such that

$$x = x * p,\ p = x * q$$

multiply $x = x * p$ by p, we have

$$x = x * p = (x * p) * p = (p * p) * x$$

In addition,

$$\bigcup_{s \in p*q} (s * x) = (p * q) * x = (x * q) * p = p * p$$

where $s * x$ is a nonempty set and $|p * p| = 1$. Thus for any $s \in p * q$, $s * x = p * p$. It means that for any $x \in L$, there are $p * p$, $s \in p * q$ such that

$$(p * p) * x = x,\ s * x = p * p$$

It shows that

$$p * p \subseteq \{\}_{lneut(x)},\ s \in p * q \subseteq \{\}_{lanti(x)_{p*p}}$$

By Definition 11, (*L*, ∗) is an LNET-LA-semihypergroup.

(2) By Theorem 1 (a),

$$p = p * p \in \{\}_{lneut(x)}$$

$$q \in p * q \subseteq \{\}_{lanti(x)_{p*p}} = \{\}_{lanti(x)_p}$$

It shows that for any given $x \in L$, there is $p \in L$ such that

$$p * x = x \text{ and } q * x = p$$

On the other hand, $p \in \{\}_{rneut(x)}$, $q \in \{\}_{ranti(x)_p}$, we get

$$x = x * p \text{ and } x * q = p$$

Based on the above, for any given $x \in L$, there are p and q such that

$$x = x * p = p * x$$

$$p = x * q = q * x$$

That is,

$$p \in \{\}_{neut(x)} \text{ and } q \in \{\}_{anti(x)_p}$$

By Definition 11, (L, $*$) is an SPNET-LA-semihypergroup. Applying Proposition *7 (1),* we get $p = neut(x)$. □

Example 12. *Put L = {0, 1, 2}, the binary hypergroupoid (L, ∗) is as follows(see Table 11).*

Table 11. The binary hypergroupoid (L, $*$).

∗	0	1	2
0	0	1	{0,1,2}
1	1	0	{0,1,2}
2	{0,1,2}	{0,1,2}	2

By Python program, (L, ∗) is an LA-semihypergroup. Furthermore, we have

$$rneut(0) = 0,\ rneut(1) = 0,\ rneut(2) = 2$$

$$ranti(0)_{rneut(0)=0} = 0,\quad ranti(1)_{rneut(1)=0} = 1,\ ranti(2)_{rneut(2)=2} = 2$$

$$0 * 0 = 0,\ 0 * 0 = 0,\ 2 * 2 = 2$$

$$0 \in 0 * 0,\ 1 \in 0 * 1,\ 2 \in 2 * 2$$

By Theorem 1 (b), we know that (L, ∗) is an SPNET-LA-semihypergroup.

Corollary 1. *A PRNET-LA-semihypergroup (L, ∗), which satisfies conditions of Theorem 1 (b), then* $neut(p * s) = neut(p) * neut(s)$ *if* $|p * s| = |neut(p) * neut(s)| = 1$, *where* $p, s \in L$.

Proof. It follows from Theorem 1 (b) and Proposition 7 (5). □

Corollary 2. *An idempotent PRNET-LA-semihypergroup is a PLNET-LA-semihypergroup.*

Proof. It follows from Theorem 1 (a). □

Proposition 8. *An idempotent PRNET-LA-semihypergroup with pure left identity is a commutative SPNET-LA-semihypergroup and its pure left identity is pure right identity.*

Proof. Put e is a pure left identity of (L, $*$). Then for any $t \in L$,

$$e * t = t,$$

by idempotent law, we get

$$t * e = (t * t) * e = (e * t) * t = t * t = t.$$

It shows that e is pure right identity of $(L\ *)$. Furthermore, for any $m, n \in L$,

$$m * n = (m * e) * n = (n * e) * m = n * m.$$

It follows that $(L, *)$ satisfies commutative law.

On the other hand, $(L, *)$ is a PRNET-LA-semihypergroup. Hence for any given $a \in L$, there are $s \in \{\}_{rneut(a)}$ and $t \in \{\}_{ranti(a)_s}$ such that

$$a = a * s,\ s = a * t.$$

Applying commutative law, we get

$$a = a * s = s * a,\ s = a * t = t * a.$$

Thus $(L, *)$ a commutative SPNET-LA-semihypergroup. □

Proposition 9. *Suppose $(L, *)$ is a PRNET-LA-semihypergroup(or a PLNET-LA-semihypergroup) with pure right identity, then pure right identity is pure left identity and $(L, *)$ is a commutative Net-semihypergroup.*

Proof. Put e is a pure right identity of $(L, *)$, Then for any given $t \in L$,

$$t * e = t,$$

we have

$$t = t * e = (t * e) * e = (e * e) * t = e * t.$$

This shows that e is pure left identity of $(L, *)$. Furthermore, for any $l, m, n \in L$,

$$m * n = (m * e) * n = (n * e) * m = n * m$$

$$(l * m) * n = (l * m) * (e * n) = (l * e) * (m * n) = l * (m * n).$$

It follows that $(L, *)$ satisfies commutative law and associative law. In addition, $(L, *)$ is a PRNET-LA-semihypergroup. Hence for any given $s \in L$, there are $p \in \{\}_{rneut(s)}$ *and* $q \in \{\}_{ranti(s)_p}$ such that

$$s = s * p,\ p = s * q.$$

Applying commutative law, we get

$$s = s * p = p * s,\ p = s * q = q * s.$$

By Definition 10, $(L, *)$ is a commutative NET-semihypergroup. □

Theorem 2. *Let $(L, *)$ be a PRNET-LA-semihypergroup, which satisfies the following conditions:*

(1) for any $t \in L$, there are $p \in \{\}_{rneut(t)}$, $q \in \{\}_{ranti(t)_p}$ such that

$$p * p = p,\ q = p * q; \tag{16}$$

By condition (1), for a certain q in (1), there are $r \in \{\}_{rneut(q)}$, $l \in \{\}_{ranti(q)_r}$ *such that*

$$r * r = r,\ l = r * l \tag{17}$$

(2) $|p * r| = 1$*, where p in (16) and r in (17);*

(3) for any m, n $\in$ *L, if neut(m) = neut(n), then* $|m * n| = 1$*.*

Define an equivalent relation φ *on L,*

$$m \varphi n \text{ if and only if } neut(m) = neut(n)$$

Then

(a) (To every t $\in$ *L, ([t], *) is a sub NET-LA-semihypergroup of (L, *), in which [t] is the equivalent class of t based on equivalent relation* φ*;*

(b) To every t $\in$ *L, ([t], *) is a regular LA-hypergroup.*

Proof. (a) Firstly, by Theorem 1 (b) and Theorem 2's condition (1), we know that $(L,*)$ is an SPNET-LA-semihypergroup. Suppose $m, n \in [t]$, by Theorem 2's condition (3), we have

$$neut(m) = neut(n) = neut(t) \text{ and } |m * n| = 1$$

Applying Proposition 7 (6), we get $neut(m * n) = neut(t)$. It shows that $m * n \in [t]$.

Secondly, applying Proposition 7 (2), we have

$$neut(neut(m)) = neut(neut(t)) = neut(t)$$

It means that for any $m \in [t]$, $neut(m) \in [t]$.

Lastly, by Theorem 2's condition (1) and Theorem 1 (b), for any $m \in [t] \subseteq L$, there is $q \in L$ such that

$$q = neut(m) * q \in \{\}_{anti(m)_{neut(m)}} \tag{18}$$

and for the *q in* (18), there are $r \in \{\}_{rneut(q)}$, $l \in \{\}_{ranti(q)_r}$ such that

$$r * r = r,\ l = r * l$$
$$\text{and } r = neut(q). \tag{19}$$

By Theorem 2's condition (2) and (19), we get

$$|neut(m) * r| = |neut(m) * neut\,(q)| = |neut(neut(m)) * neut\,(q)| = 1.$$

Applying Proposition 7 (5), we get

$$neut(neut(m) * q) = neut(neut(m)) * neut\,(q) = neut(m) * neut\,(q)$$
$$= neut(m * q) = neut(neut(m)) = neut(m) = neut(t).$$

This implies $q = neut(m) * q \in \{\}_{anti(m)_{neut(m)}} \in [t]$. Thus $([t], *)$ is a sub SPNET-LA-semihypergroup.

(b) Firstly, from (a), for any given $t \in L$, ($[t]$, $*$) is a sub-SPNET-LA-semihypergroup of (L, $*$). By the definition of φ, if $m \in [t]$, then for any $n \in [t]$, $neut(m) = neut(n) = neut(t)$. Applying Proposition 7 (7), we get

$$m * n = n * m.$$

That is $m * [t] = [t] * m$.

Secondly, for any $s \in [t]$, $s * m \in [t]$, hence $[t] * m \subseteq [t]$; On the other hand, by proof of (a), we know that for any $m \in [t]$, there is $q \in [t]$ such that

$$q = neut(m) * q \in \{\}_{anti(m)_{neut(m)}}$$

hence for any $s \in [t]$, $s * q \in [t]$. Thus

$$s = neut(s) * s = neut(m) * s = (m * q) * s = (s * q) * m \subseteq [t] * m.$$

That is, $[t] \subseteq [t] * m$. Thus $[t] = [t] * m = m * [t]$. It implies that ($[t]$, $*$) is a LA-hypergroup.

Lastly, it can be easily proved that $neut(t)$ is a scalar identity of ($[t]$, $*$) and for every $l \in [t]$ has at least one inverse. By Definition 8, ($[t]$, $*$) is a regular LA-hypergroup. □

Corollary 3. *If a PRNET-LA-semihypergroup (L, ∗) which satisfies conditions of Theorem 2 and φ is the equivalence relation on L defined in Theorem 2, then L/φ is the partition of set L.*

Example 13. *Put L = {0, 1, 2, 3, 4}, the binary hypergroupoid (L, ∗) is as follows(see Table 12).*

Table 12. The binary hypergroupoid (L, $*$).

∗	0	1	2	3	4
0	0	1	{0, 1, 2}	0	4
1	1	0	{0, 1, 2}	1	4
2	{0, 1, 2}	{0, 1, 2}	2	{0, 1, 2}	4
3	0	1	{0, 1, 2}	3	4
4	4	4	4	4	4

By Python program, (L, ∗) is LA-semihypergroup. Firstly, we have

$$rneut(0) = 0,\ rneut(1) = 0,\ rneut(2) = 2,\ rneut(3) = 3,\ rneut(4) = 4$$

$$ranti(0)_{rneut(0)=0} = 0,\quad ranti(1)_{rneut(1)=0} = 1,\ ranti(2)_{rneut(2)=2} = 2,\ ranti(3)_{rneut(3)=3} = 3,$$
$$ranti(4)_{rneut(4)=4} = 4$$

$$0 * 0 = 0,\ 0 * 0 = 0,\ 2 * 2 = 2,\ 3 * 3 = 3,\ 4 * 4 = 4$$

$$0 = 0 * 0,\ 1 = 0 * 1,\ 2 = 2 * 2,\ 3 = 3 * 3,\ 4 = 4 * 4.$$

These means that Theorem 2's condition 1) hold; Secondly, we get

$$|rneut(0) * rneut(ranti(0)_{rneut(0)=0})| = |rneut(0) * rneut(0)| = |0| = 1$$

$$|rneut(1) * rneut(ranti(1)_{rneut(1)=0})| = |rneut(1) * rneut(1)| = |0| = 1$$

$$|rneut(2) * rneut(ranti(2)_{rneut(2)=2})| = |rneut(2) * rneut(2)| = |2| = 1$$

$$|rneut(3)*rneut(ranti(3)_{rneut(3)=3})| = |rneut(3)*rneut(3)| = |3| = 1$$

$$|rneut(4)*rneut(ranti(4)_{rneut(4)=4})| = |rneut(4)*rneut(4)| = |4| = 1$$

These means that Theorem 2's condition (2) hold. Lastly,

$$rneut(0) = rneut(1) = 0, |0*1| = 1$$

These means that Theorem 2's condition (3) hold. By Theorem 1 and Theorem 2, we know that (L, ∗) is an SPNET-LA-semihypergroup and

$$L_1 = \{0, 1\} = [0] = [1], L_2 = \{2\} = [2], L_3 = \{3\} = [3], L_4 = \{4\} = [4]$$

$$L = L_1 \cup L_2 \cup L_3 \cup L_4$$

where $(L_1, *), (L_2, *), (L_3, *), (L_4, *)$ *are all regular LA-hypergroups.*

Definition 14. *An NET-LA-semihypergroup (L, ∗) satisfies weak commutative law, if for any* $y \in L$,

$$p*y = y*p,\ q*x = x*q$$

where x is any element of set L, $p \in \{\}_{neut(x)}$, $q \in \{\}_{anti(x)_p}$.

Proposition 10. *An SPNET-LA-semihypergroup (L, ∗) satisfies weak commutative law if and only if it is a commutative.*

Proof. If (*L*, ∗) is a weak commutative, then for any $x, y \in L$, $l \in \{\}_{neut(x)}$, $m \in \{\}_{neut(y)}$, we have

$$x*y = (x*l)*(y*m) = (l*x)*(y*m) = (l*y)*(x*m) = (y*l)*(m*x) = (y*m)*(l*x) = y*x$$

That is, (*L*, ∗) is commutative. □

4. Conclusions

In this study, we give the new notions of NET-LA-semihypergroup, NET-LA-hypergroup, LNET-LA-semihypergroup, RNET-LA-semihypergroup, PLNET-LA-semihypergroup, PRNET-LA-semihypergroup, PNET-LA-semihypergroup, SPNET-LA-semihypergroup, discuss the relationships of them(see Figures 5 and 6), get some special properties of SPNET-LA-semihypergroup(see Proposition 7). In particular, we prove that a RNET-LA-semihypergroup which satisfies certain conditions(the condition of asymmetry) be an SPNET-LA-semihypergroup and this SPNET-LA-semihypergroup is the union of some disjoint regular hypergroups, where every regular hypergroup is its subhypergroup(see Theorem 2). At last, we discuss the relationships of various NET-LA-semihypergroups(see Figure 7).

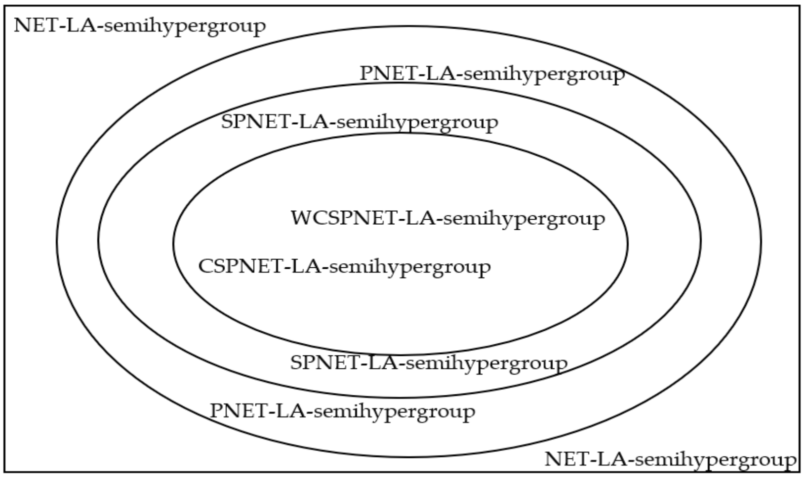

Figure 7. The relationships of various NET-LA-semihypergroups.

These studies help us to enhence the understanding of this hyperalgebraic structure about NET and tell us this hyper algebraic structure is a complex and unique structure. There is still a lot of unknown knowledge in this field to explore. In the future, we will discuss properties of NET-CA-semihypergroup.

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