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Abstract

The aim of this paper is to introduce and study some new neutrosophic fuzzy pairwise notions via neutrosophic fuzzy ideals. Relationships between the above new neutrosophic fuzzy pairwise notions and there other relevant classes are investigated.

Keywords: Neutrosophic set; Neutrosophic topology; Neutrosophic ideal; Neutrosophic ideal open set; Neutrosophic closed set

Introduction

The neutrosophic set was introduced by Smarandache in [3,4,7] and Salama introduced the neutrosophic crisp set and neutrosophic topological spaces and many applications in computer science and system [5,6,8,10,11,12,13,14,15], information systems. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts[1,2,3,4,5,7], such as a neutrosophic set theory, in this paper is to introduce and study some new neutrosophic fuzzy pairwise notion via neutrosophic fuzzy pairwise ideals. We, also generalize the notion of PL-open sets due to Abd El-Monsef, et. al [1,2]. In addition to generalize the concept of PL-closed sets, NPL-continuity and NPL-open functions. A neutrosophic fuzzy quasi pairwise L-openness and neutrosophic fuzzy quasi pairwise L-continuity are considered as a generalization of a fuzzy pairwise PL-openness and fuzzy pairwise PL-continuity which are known before.

1. Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [3,4,7] and Salama et al. [5, 6,10,14,15].

The following table represents abbreviations and terms:

Abbreviations	Terms
NFPL-continuous	Neutrosophic Fuzzy pairwise L-continuous Functions
NPL-open	Neutrosophic Pairwise L-open
NFBT	Neutrosophic Fuzzy Bitopological Space
PL	Pairwise L

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2. Neutrosophic Fuzzy pairwise L-continuous Functions.

By utilizing the notian of NFPL-open sets, we establish in this article a class of neutrosophic fuzzy NFPL-continuous function which contained in the class of neutrosophic fuzzy pairwise precontinuous function .

Each of neutrosophic NFPL-continuous and neutrosophic fuzzy pairwise continuous function may be independent concepts. Many characterizations and properties of this concept are investigated.

Definition 2.1: A Neutrosophic fuzzy pairwise function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L on X is said to be neutrosophic fuzzy NFPL-continuous if for every $<\xi, \rho, \theta>$ in T, $f^{-1}(<\xi, \rho, \theta>)$ in $NFPLO(X)$.

Remark 2.1: Every neutrosophic fuzzy NFPL-continuity is neutrosophic fuzzy pairwise precontinuity but the converse not true in general as seen by the following example.

Example 2.1: Let X=Y={x}, τ_i , $i \in \{1,2\}$ may be neutrosophic fuzzy pairwise indiscrete bitopological , σ may be neutrosophic fuzzy pairwise discrete bitopology and L=$\{0_N, , <\mu, \sigma, u>\} \vee \{x_{\varepsilon=<\alpha,\gamma,\beta>} : \varepsilon \leq X_{<0.2,0.5,0.8>}\}$ $<\mu, \sigma, u>(x) = <0.2,0.5,0.8>$. The neutrosophic fuzzy pairwise identity function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ may be neutrosophic fuzzy pairwise precontinuous but not neutrosophic fuzzy NFPL-continuous, since $<\mu, \sigma, u>$ in T while $f^{-1}(<\mu, \sigma, u>) \notin NFPLO(X)$.

Theorem 2.1: For a function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L on X the following are equivalent

(i.) f may be neutrosophic fuzzy NFPL-continuous.

(ii.) For $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in X and each $<\xi, \rho, \theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$, there exists $<\mu, \sigma, u>$ in $NFPLO(X)$ containing $x_{\varepsilon=<\alpha,\gamma,\beta>}$ such that $(<\mu, \sigma, u>)$ in T.

(iii.) For each neutrosophic fuzzy pairwise point $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in X and $<\xi, \rho, \theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$, $(f^{-1}(\zeta))^*$ is neutrosophic fuzzy pairwise npbd of x_ε.

(iv.) The inverse image of each neutrosophic fuzzy pairwise closed set in Y is a neutrosophic fuzzy NPL-closed.

Proof: (i.)⟶(ii.). Since $<\xi, \rho, \theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$, then by (i), $f^{-1}(<\xi, \rho, \theta>)$ in $NPLO(X)$, by putting $<\mu, \sigma, u> = f^{-1}(<\xi, \rho, \theta>)$ which containing $x_{\varepsilon=<\alpha,\gamma,\beta>}$, we have $f(<\mu, \sigma, u>)$ in T.

(ii.) ⟶(iii.). Let $<\xi, \rho, \theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$. Then by (ii) there exists $<\mu, \sigma, u>$ in $NFPLO(X)$containing $x_{\varepsilon=<\alpha,\gamma,\beta>}$ such that $(<\mu, \sigma, u>) \leq \sigma$, so $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in $<\mu, \sigma, u> \leq Nint <\mu, \sigma, u>^* \leq Nint(f^{-1}(<\xi, \rho, \theta>))^* \leq (f^{-1}(<\xi, \rho, \theta>))^*$. Hence $(f^{-1}(<\xi, \rho, \theta>))^*$ is a neutrosophic fuzzy npbd of $x_{\varepsilon=<\alpha,\gamma,\beta>}$.

(iii.)→(i.) Let $<\xi, \rho, \theta>$ in T, since $(f^{-1}(<\xi, \rho, \theta>))$ is a neutrosophic fuzzy pairwise npbd of any point $f^{-1}(<\xi, \rho, \theta>)$, every point $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in $(f^{-1}(<\xi, \rho, \theta>))^*$ may be a neutrosophic fuzzy pairwise interior point of $f^{-1}(<\xi, \rho, \theta>)^*$.Then $f^{-1}(<\xi, \rho, \theta>) \leq Nint(f^{-1}(\zeta <\xi, \rho, \theta>))^*$ and hence f is a neutrosophic fuzzy pairwise NFPL-continuous.

(i.) →(iv.) Let $<\xi, \rho, \theta>$ in T be a neutrosophic fuzzy pairwise closed set. Then $<\xi, \rho, \theta>^c$ may be neutrosophic fuzzy pairwise open set, by $f^{-1}(<\xi, \rho, \theta>^c) = f^{-1}(<\xi, \rho, \theta>)^c$ in $NFPLO(X)$. Thus $f^{-1}(<\xi, \rho, \theta>)$ is a neutrosophic fuzzy pairwise NPL-closed set.

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The following theorem establish the relationship between neutrosophic fuzzy pairwise NPL-continuous and neutrosophic fuzzy pairwise continuous by using the previous neutrosophic fuzzy pairwise notions.

Theorem 2.2: Given $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ may be a function with neutrosophic fuzzy ideal L on X then we have. If f may be neutrosophic fuzzy pairwise NFPL-continuous of each neutrosophic fuzzy pairwise*-perfect set in X, then f is a neutrosophic fuzzy pairwise continuous.

Proof: Obvious.

Corollary 2.1: Given a function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ and each member of X is a neutrosophic fuzzy pairwise*-dense-in-itself.
Then we have:-
(i.) Every neutrosophic fuzzy pairwise continuous function is a neutrosophic fuzzy pairwise NFPL-continuous.
(ii.) Each of neutrosophic fuzzy pairwise precontinuous function and neutrosophic fuzzy pairwise NFPL-continuous are equivalent.

Proof: It's clear.

3. Neutrosopic Fuzzy quasi pairwise NPL-open and Neutrosophic Fuzzy quasi pairwise NPL-continuity.

Definition 3.1: In a neutrosophic fuzzy bitopological spaces $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X, μ, σ, u in I^x is said to be neutrosophic fuzzy quasi pairwise NPL-open if $< \mu, \sigma, u > \leq \mathrm{Ncl}(\mathrm{Nint}(< \mu, \sigma, u >^*))$, $< \mu, \sigma, u >c$ may be called neutrosophic fuzzy quasi pairwise NPL-closed set. The collection of all neutrosophic fuzzy quasi pairwise NPL-open sets of $(X,\tau_i), i\epsilon\{1,2\}$ will denoted by $\mathrm{NFQPLO}(X,\tau_i), i\epsilon\{1,2\}$.

The connection between neutrosophic fuzzy quasi pairwise NFPL-openness with some other corresponding types have been given throughout the following implication

NFPL-open $\longrightarrow$ NF quasi NFPL-open

Proposition 3.1: Arbitrary union of neutrosophic fuzzy quasi pairwise NFPL-open sets is a neutrosophic fuzzy quasi pairwise NFPL-open.

Proof: Let $(X,\tau_i), i\epsilon\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L on X and $< \mu, \sigma, u >_J$ in $\mathrm{NFQPLO}(X)$ this means that for each for $i \in N$, $< \mu, \sigma, u >_J \leq \mathrm{Ncl}(\mathrm{Nint}(< \mu, \sigma, u >^*))$and so, $\bigvee_{J\in HN} < \mu, \sigma, u >_J \leq \bigvee_{J\in HN} \mathrm{Ncl}\left(\mathrm{Nint}\left(< \mu, \sigma, u >^*{}_J\right)\right) \leq \mathrm{Ncl}\left(\vee\left(\mathrm{Nint}(< \mu, \sigma, u >^*)\right)\right) \leq \mathrm{Ncl}(\mathrm{Nint}(\vee< \mu, \sigma, u >)^*)$. Hence $\bigvee_{J\in HN} < \mu, \sigma, u >_J \in \mathrm{NFQPLO}(X)$.

Above two results are useful to obtained the following theorem.

Theorem 3.1: For a neutrosophic bitopological space $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n, the class FQPLO(X) forms a neutrosophic fuzzy pairwise suprabitopological.

Proof: Follows by the fact 0*N=0N and both of the fact $1_{N=<\eta,\delta,\theta>} = 1^*{}_{N=<\eta,\delta,\theta>}$, and proposition 3.1.

Remark 3.1: A finite neutrosophic fuzzy intersection pairwise of neutrosophic fuzzy quasi pairwise NFPL-open is a neutrosophic fuzzy quasi pairwise NFPL-open.

Theorem 3.2: $\mathrm{NFQPLO}(X,\tau_i), i\epsilon\{1,2\}$ from a neutrosophic fuzzy bitopological.

Proof: Follows directly from Theorem 3.1 and Remark 3.1.

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Theorem 3.3: For a neutrosophic bitopological space $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X. The following statements are verified.

(i) If L=$\{0_{N=<\eta,\delta,\theta>}\}$ then $NFQPLO(X,\tau_i) = NFP\beta O(X,\tau_i), i\epsilon\{1,2\}$.

(ii) If L=$\{I^x\}$ then $NFQPLO(X,\tau_i) = NFPLO(X,\tau_i), i\epsilon\{1,2\}$.

(iii) If L neutrosophic fuzzy ideal on X, each neutrosophic fuzzy quasi pairwise NFPL-open(resp. neutrosophic fuzzy semi pairwise open) which it is neutrosophic fuzzy pairwise-closed (resp.PN*- dense – in itself) is a neutrosophic fuzzy semi pairwise open (resp. neutrosophic fuzzy quasi pairwise NFPL-open).

Proof: Obvious.

Theorem 3.4: In a neutrosophic bitopological space $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n on X, if $<\mu,\sigma,u>$ in $NFQPLO(X,\tau_i), i\epsilon\{1,2\}$, then it is a neutrosophic fuzzy semi pairwise open.

Hence we can deduce that an neutrosophic fuzzy quasi pairwise NFPL-open set which is neutrosophic fuzzy pairwise*-closed for any $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L may be equivalent with the neutrosophic fuzzy quasi pairwise NFPL-openness in $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n which a useful to obtain the following.

Proposition 3.2: In a neutrosophic bitopological space $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X, any neutrosophic fuzzy pairwise preclosed set μ,σ,u in $I^{x=\eta,\delta,\theta}$ is neutrosophic fuzzy pairwise regular closed if one of the following hold:

(i) $<\mu,\sigma,u>$ is a FPN*-closed and neutrosophic fuzzy quasi pairwise NFPL-open.

(ii) $<\mu,\sigma,u>$ in $NFQPLO(X,\tau_i), i\epsilon\{1,2\}$ with a neutrosophic fuzzy ideal L_n.

Definition 3.2: A function $f:(X,\tau_i) \longrightarrow (Y,T), i \in \{1,2\}$ with neutrosophic fuzzy ideal Lon X is called neutrosophic fuzzy quasi pairwise NPL-continuous if for every μ,σ,u in T, $f^{-1}(<\mu,\sigma,u>)$in $NFQPLO(X,\tau_i), i \in \{1,2\}$.

The relationships between this new class of functions and some types of known continuous ones are obtained as follows.

NFPL-continuity $\longrightarrow$ NF quasi pairwise PFL-continuity

Proposition 3.3: The following are equivalents

(i) $f:(X,\tau_i) \longrightarrow (Y,T), i \in \{1,2\}$ with neutrosophic fuzzy deal L=$\{0_{N=<\eta,\delta,\theta>}\}$, may be neutrosophic fuzzy quasi pairwise NFPL-continuous iff it is a neutrosophic fuzzy pairwise NFPβ-continuous.

(ii) $f:(X,\tau_i) \longrightarrow (Y,T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L=$\{I^x\}$, may be neutrosophic fuzzy quasi pairwise NFPL-continuous iff it is a NFPL-continuous.

Theorem 3.5: For a function $f:(X,\tau_i) \longrightarrow (Y,T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L on X, the following are equivalent:

(i.) f is a neutrosophic fuzzy quasi pairwise NFPL-continuouis.

(ii.) The inverse image of each neutrosophic fuzzy pairwise closed set in (Y,T) is a neutrosophic fuzzy quasi pairwise NFPL-closed.

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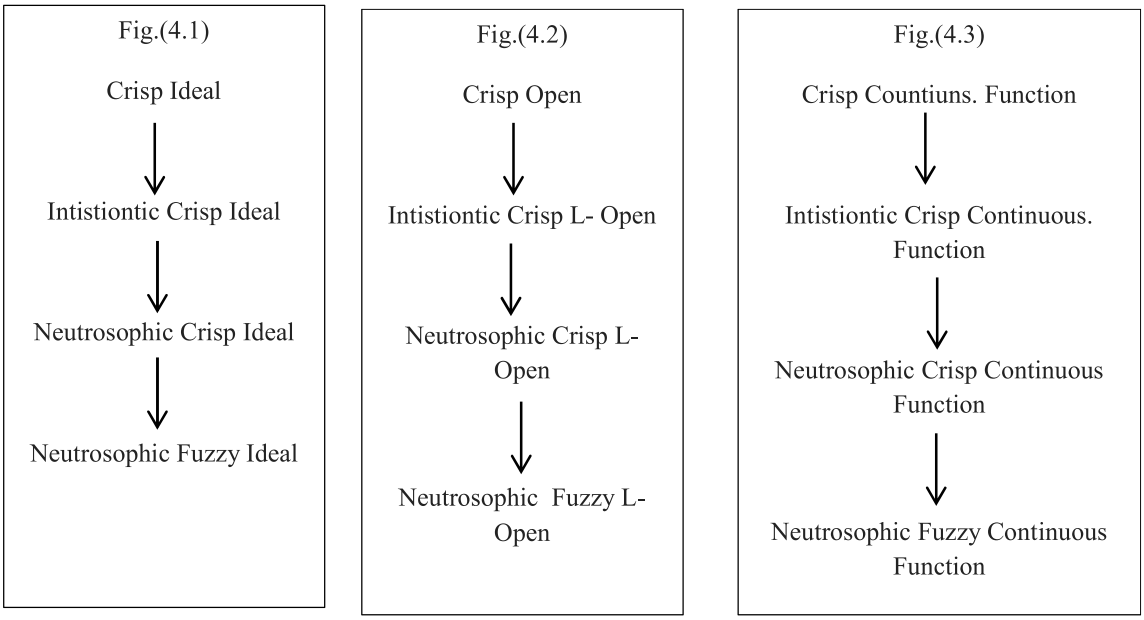
(iii.) For each x in X and each μ, σ, u in T containing f (x). There exists $< \lambda, \omega, \kappa >$ in $\mathrm{NFQPLO}(X, \tau_i), i \in \{1,2\}$ containing x such tha $(< \lambda, \omega, \kappa >) << \mu, \sigma, u >$.

Proposition 3.4: For a function $f{:}(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L on X, the following are true.

(i.) A neutrosophic fuzzy quasi pairwise NFPL_n- continuous function is a neutrosophic fuzzy pairwise semi continuous.

(ii) A neutrosophic fuzzy quasi pairwise NFPL-continuous (resp. Neutrosophic fuzzy pairwise semi-continuous) and for each μ, σ, u in T, $f^{-1}(< \mu, \sigma, u >)$ may be FPN*-closed (resp. FPN*dense-in-itself) then f is a neutrosophic fuzzy pairwise semi continuous (resp. Neutrosophic fuzzy pairwise NFPL-continuous).

4. Relations: The following Graph represent the relation betwen the concepts



Conclusions

The notions of the sets and functions in neutrosophic fuzzy bitopological spaces are highly developed and several characterizations have already been obtained. These are used extensively in many practical and engineering problems, computational bitopology for geometric design, computer-aided geometric design, engineering design research, Geographic Information System (GIS) and mathematical sciences. As the works of professor Florentin Smarandache indicates that neutrosophc fuzzy bitopology may be relevant to quantum physics particularly in connection with string theory and neutrosophic fuzzy bitopology may be used to provide information about the elementary particles content of the standard model of high energy physics, the notions and results given in this paper may turn out to be useful in quantum physics. Several characterizations of neutrosophic fuzzy sets and several generalizations of neutrosophic fuzzy continuous functions.

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