# On Neutrosophic Semi-Supra Open Set and Neutrosophic Semi-Supra Continuous Functions 

R. Dhavaseelan ${ }^{1}$, M. Parimala ${ }^{2}$, S. Jafari ${ }^{3}$, F. Smarandache ${ }^{4}$<br>${ }^{1}$ Department of Mathematics, Sona College of Technology, Salem-636005,Tamil Nadu,India. E-mail dhavaseelan.r@gmail.com<br>${ }^{2}$ Department of Mathematics, Bannari Amman Institute of Technology Sathyamangalam-638401,Tamil Nadu, India . E-mail: rishwanthpari@gmail.com<br>${ }^{3}$ Department of Mathematics, College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark. E-mail: jafaripersia@gmail.com<br>${ }^{4}$ Mathematics \& Science Department, University of New Maxico, 705 Gurley Ave, Gallup, NM 87301, USA. E-mail: fsmarandache@gmail.com

Abstract: In this paper, we introduce and investigate a new class
of sets and functions between topological space called neutrosophic
semi-supra open set and neutrosophic semi-supra open continuous functions respectively.

Keywords: Supra topological spaces; neutrosophic supra-topological spaces; neutrosophic semi-supra open set.

## 1 Introduction and Preliminaries

Intuitionistic fuzzy set is defined by Atanassov [2] as a generalization of the concept of fuzzy set given by Zadesh [14]. Using the notation of intuitionistic fuzzy sets, Coker [3] introduced the notion of an intuitionistic fuzzy topological space. The supra topological spaces and studied $s$-continuous functions and $s^{*}$ continuous functions were introduced by A. S. Mashhour [6] in 1993. In 1987, M. E. Abd El-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and obtained some properties and characterizations. In 1996, Keun Min [13] introduced fuzzy $s$-continuous, fuzzy $s$-open and fuzzy $s$-closed maps and established a number of characterizations. In 2008, R. Devi et al. [4] introduced the concept of supra $\alpha$-open set, and in 1983, A. S. Mashhour et al. introduced the notion of supra-semi open set, supra semicontinuous functions and studied some of the basic properties for this class of functions. In 1999, Necla Turan [11] introduced the concept of intuitionistic fuzzy supra topological space. The concept of intuitionistic fuzzy semi-supra open set was introduced by Parimala and Indirani [7]. After the introduction of the concepts of neutrosophy and a neutrosophic se by F. Smarandache [[9], [10]], A. A. Salama and S. A. Alblowi[8] introduced the concepts of neutrosophic crisp set and neutrosophic topological spaces.

The purpose of this paper is to introduce and investigate a new class of sets and functions between topological space called neutrosophic semi-supra open set and neutrosophic semi-supra open continuous functions, respectively.

Definition 1.1. Let $T, I, F$ be real standard or non standard subsets of $] 0^{-}, 1^{+}$, with $\sup _{T}=t_{\text {sup }}$, inf $f_{T}=t_{\text {inf }}$
sup $_{I}=i_{\text {sup }}$, inf $_{I}=i_{\text {inf }}$
$s u p_{F}=f_{\text {sup }}, i n f_{F}=f_{\text {inf }}$
$n-$ sup $=t_{\text {sup }}+i_{\text {sup }}+f_{\text {sup }}$
$n-i n f=t_{i n f}+i_{i n f}+f_{\text {inf }} . T, I, F$ are neutrosophic components.

Definition 1.2. Let $X$ be a nonempty fixed set. A neutrosophic set [briefly NS] $A$ is an object having the form $A=$ $\left\{\left\langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x)\right\rangle: x \in X\right\}$, where $\mu_{A}(x), \sigma_{A}(x)$ and $\gamma_{A}(x)$ represent the degree of membership function (namely $\mu_{A}(x)$ ), the degree of indeterminacy (namely $\sigma_{A}(x)$ ) and the degree of nonmembership (namely $\gamma_{A}(x)$ ) respectively of each element $x \in X$ to the set $A$.

Remark 1.1. (1) A neutrosophic set $A=$ $\left\{\left\langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x)\right\rangle: x \in X\right\}$ can be identified to an ordered triple $\left\langle\mu_{A}, \sigma_{A}, \gamma_{A}\right\rangle$ in $] 0^{-}, 1^{+}[$on $X$.
(2) For the sake of simplicity, we shall use the symbol $A=\left\langle\mu_{A}, \sigma_{A}, \gamma_{A}\right\rangle$ for the neutrosophic set $A=$ $\left\{\left\langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x)\right\rangle: x \in X\right\}$.

Definition 1.3. Let $X$ be a nonempty set and the neutrosophic sets $A$ and $B$ in the form
$A=\left\{\left\langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x)\right\rangle: x \in X\right\}, \quad B=$ $\left\{\left\langle x, \mu_{B}(x), \sigma_{B}(x), \gamma_{B}(x)\right\rangle: x \in X\right\}$. Then
(a) $A \subseteq B$ iff $\mu_{A}(x) \leq \mu_{B}(x), \sigma_{A}(x) \leq \sigma_{B}(x)$ and $\gamma_{A}(x) \geq$ $\gamma_{B}(x)$ for all $x \in X$;
(b) $A=B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) $\bar{A}=\left\{\left\langle x, \gamma_{A}(x), \sigma_{A}(x), \mu_{A}(x)\right\rangle: x \in X\right\}$; [Complement of $A$ ]
(d) $A \cap B=\left\{\left\langle x, \mu_{A}(x) \wedge \mu_{B}(x), \sigma_{A}(x) \wedge \sigma_{B}(x), \gamma_{A}(x) \vee\right.\right.$ $\left.\left.\gamma_{B}(x)\right\rangle: x \in X\right\} ;$
(e) $A \cup B=\left\{\left\langle x, \mu_{A}(x) \vee \mu_{B}(x), \sigma_{A}(x) \vee \sigma_{B}(x), \gamma_{A}(x) \wedge\right.\right.$ $\left.\left.\gamma_{B}(x)\right\rangle: x \in X\right\} ;$
(f) []$A=\left\{\left\langle x, \mu_{A}(x), \sigma_{A}(x), 1-\mu_{A}(x)\right\rangle: x \in X\right\} ;$
(g) $\left\rangle A=\left\{\left\langle x, 1-\gamma_{A}(x), \sigma_{A}(x), \gamma_{A}(x)\right\rangle: x \in X\right\}\right.$.

Definition 1.4. Let $\left\{A_{i}: i \in J\right\}$ be an arbitrary family of neutrosophic sets in $X$. Then
(a) $\cap A_{i}=\left\{\left\langle x, \wedge \mu_{A_{i}}(x), \wedge \sigma_{A_{i}}(x), \vee \gamma_{A_{i}}(x)\right\rangle: x \in X\right\}$;
(b) $\bigcup A_{i}=\left\{\left\langle x, \vee \mu_{A_{i}}(x), \vee \sigma_{A_{i}}(x), \wedge \gamma_{A_{i}}(x)\right\rangle: x \in X\right\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets $0_{N}$ and $1_{N}$ in X as follows:

Definition 1.5. $0_{N}=\{\langle x, 0,0,1\rangle: x \in X\}$ and $1_{N}=$ $\{\langle x, 1,1,0\rangle: x \in X\}$.

Definition 1.6. [5] A neutrosophic topology (NT) on a nonempty set $X$ is a family $T$ of neutrosophic sets in $X$ satisfying the following axioms:
(i) $0_{N}, 1_{N} \in T$,
(ii) $G_{1} \cap G_{2} \in T$ for any $G_{1}, G_{2} \in T$,
(iii) $\cup G_{i} \in T$ for arbitrary family $\left\{G_{i} \mid i \in \Lambda\right\} \subseteq T$.

In this case the ordered pair $(X, T)$ or simply $X$ is called a neutrosophic topological space (NTS) and each neutrosophic set in $T$ is called a neutrosophic open set (NOS). The complement $\bar{A}$ of a NOS $A$ in $X$ is called a neutrosophic closed set (NCS) in $X$.

Definition 1.7. [5] Let $A$ be a neutrosophic set in a neutrosophic topological space $X$. Then
$\operatorname{Nint}(A)=\bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of $A$;
$\operatorname{Ncl}(A)=\bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of $A$.

Definition 1.8. Let $X$ be a nonempty set. If $r, t, s$ be real standard or non standard subsets of $] 0^{-}, 1^{+}[$, then the neutrosophic set $x_{r, t, s}$ is called a neutrosophic point(in short NP )in $X$ given by

$$
x_{r, t, s}\left(x_{p}\right)= \begin{cases}(r, t, s), & \text { if } x=x_{p} \\ (0,0,1), & \text { if } x \neq x_{p}\end{cases}
$$

for $x_{p} \in X$ is called the support of $x_{r, t, s}$, where $r$ denotes the degree of membership value, $t$ denotes the degree of indeterminacy and $s$ is the degree of non-membership value of $x_{r, t, s}$.

Now we shall define the image and preimage of neutrosophic sets. Let $X$ and $Y$ be two nonempty sets and $f: X \rightarrow Y$ be a function.

Definition 1.9. [5]
(a) If $B=\left\{\left\langle y, \mu_{B}(y), \sigma_{B}(y), \gamma_{B}(y)\right\rangle: y \in Y\right\}$ is a neutrosophic set in $Y$, then the preimage of $B$ under $f$, denoted by $f^{-1}(B)$, is the neutrosophic set in $X$ defined by $f^{-1}(B)=\left\{\left\langle x, f^{-1}\left(\mu_{B}\right)(x), f^{-1}\left(\sigma_{B}\right)(x), f^{-1}\left(\gamma_{B}\right)(x)\right\rangle:\right.$ $x \in X\}$.
(b) If $A=\left\{\left\langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x)\right\rangle: x \in X\right\}$ is a neutrosophic set in $X$, then the image of $A$ under $f$, denoted by $f(A)$, is the neutrosophic set in $Y$ defined by
$f(A)=\left\{\left\langle y, f\left(\mu_{A}\right)(y), f\left(\sigma_{A}\right)(y),\left(1-f\left(1-\gamma_{A}\right)\right)(y)\right\rangle:\right.$ $y \in Y\}$. where

$$
\begin{gathered}
f\left(\mu_{A}\right)(y)= \begin{cases}\sup _{x \in f^{-1}(y)} \mu_{A}(x), & \text { if } f^{-1}(y) \neq \emptyset, \\
0, & \text { otherwise },\end{cases} \\
f\left(\sigma_{A}\right)(y)= \begin{cases}\sup _{x \in f^{-1}(y)} \sigma_{A}(x), & \text { if } f^{-1}(y) \neq \emptyset, \\
0, & \text { otherwise, },\end{cases} \\
\left(1-f\left(1-\gamma_{A}\right)\right)(y)= \begin{cases}\inf _{x \in f^{-1}(y)} \gamma_{A}(x), & \text { if } f^{-1}(y) \neq \emptyset, \\
1, & \text { otherwise },\end{cases}
\end{gathered}
$$

For the sake of simplicity, let us use the symbol $f_{-}\left(\gamma_{A}\right)$ for $1-f\left(1-\gamma_{A}\right)$.

Corollary 1.1. [5] Let $A, A_{i}(i \in J)$ be neutrosophic sets in $X, B, B_{i}(i \in K)$ be neutrosophic sets in $Y$ and $f: X \rightarrow Y$ a function. Then
(a) $A_{1} \subseteq A_{2} \Rightarrow f\left(A_{1}\right) \subseteq f\left(A_{2}\right)$,
(b) $B_{1} \subseteq B_{2} \Rightarrow f^{-1}\left(B_{1}\right) \subseteq f^{-1}\left(B_{2}\right)$,
(c) $A \subseteq f^{-1}(f(A))\left\{\right.$ If f is injective,then $\left.A=f^{-1}(f(A))\right\}$,
(d) $f\left(f^{-1}(B)\right) \subseteq B\left\{\right.$ If f is surjective, then $\left.f\left(f^{-1}(B)\right)=B\right\}$,
(e) $f^{-1}\left(\bigcup B_{j}\right)=\bigcup f^{-1}\left(B_{j}\right)$,
(f) $f^{-1}\left(\bigcap B_{j}\right)=\bigcap f^{-1}\left(B_{j}\right)$,
(g) $f\left(\bigcup A_{i}\right)=\bigcup f\left(A_{i}\right)$,
(h) $f\left(\cap A_{i}\right) \subseteq \bigcap f\left(A_{i}\right)\left\{\right.$ If f is injective, then $f\left(\cap A_{i}\right)=$ $\left.\bigcap f\left(A_{i}\right)\right\}$,
(i) $f^{-1}\left(1_{N}\right)=1_{N}$,
(j) $f^{-1}\left(0_{N}\right)=0_{N}$,
(k) $f\left(1_{N}\right)=1_{N}$, if f is surjective
(l) $f\left(0_{N}\right)=0_{N}$,
(m) $\overline{f(A)} \subseteq f(\bar{A})$, if f is surjective,
(n) $f^{-1}(\bar{B})=\overline{f^{-1}(B)}$.

## 2 Main Results

Definition 2.1. A neutrosophic set $A$ in a neutrosophic topological space $(X, T)$ is called

1) a neutrosophic semiopen set (NSOS) if $A \subseteq$ $\operatorname{Ncl}(\operatorname{Nint}(A))$.
2) a neutrosophic $\alpha$ open set $(N \alpha O S)$ if $A \subseteq$ $\operatorname{Nint}(\operatorname{Ncl}(\operatorname{Nint}(A)))$.
3) a neutrosophic preopen set (NPOS) if $A \subseteq \operatorname{Nint}(N c l(A))$.
4) a neutrosophic regular open set (NROS) if $A=$ $\operatorname{Nint}(N c l(A))$.
5) a neutrosophic semipre open or $\beta$ open set $(N \beta O S)$ if $A \subseteq$ $\operatorname{Ncl}(\operatorname{Nint}(N c l(A)))$.

A neutrosophic set $A$ is called a neutrosophic semiclosed set, neutrosophic $\alpha$ closed set, neutrosophic preclosed set, neutrosophic regular closed set and neutrosophic $\beta$ closed set, respectively (NSCS, N $\alpha$ CS, NPCS, NRCS and $\mathrm{N} \beta \mathrm{CS}$, resp), if the complement of $A$ is a neutrosophic semiopen set, neutrosophic $\alpha$-open set, neutrosophic preopen set, neutrosophic regular open set, and neutrosophic $\beta$-open set, respectively.

Definition 2.2. Let $(X, T)$ ba a neutrosophic topological space. A neutrosophic set $A$ is called a neutrosophic semi-supra open set (briefly NSSOS) if $A \subseteq s-N c l(s-N i n t(A))$. The complement of a neutrosophic semi-supra open set is called a neutrosophic semisupra closed set.

Proposition 2.1. Every neutrosophic supra open set is neutrosophic semi-supra open set.

Proof. Let $A$ be a neutrosophic supra open set in $(X, T)$. Since $A \subseteq s-N c l(A)$, we get $A \subseteq s-N c l(s-N i n t(A))$. Then $s-N \operatorname{int}(A) \subseteq s-N c l(s-N i n t(A))$. Hence $A \subseteq s-N c l(s-$ Nint(A)).

The converse of Proposition 2.1., need not be true as shown in Example 2.1.

Example 2.1. Let $X=\{a, b\}$. Define the neutrosophic sets $A$, $B$ and $C$ in $X$ as follows:
$A=\left\langle x,\left(\frac{a}{0.2}, \frac{b}{0.4}\right),\left(\frac{a}{0.2}, \frac{b}{0.4}\right),\left(\frac{a}{0.5}, \frac{b}{0.6}\right)\right\rangle, \quad B \quad=$ $\left\langle x,\left(\frac{a}{0.6}, \frac{b}{0.2}\right),\left(\frac{a}{0.6}, \frac{b}{0.2}\right),\left(\frac{a}{0.3}, \frac{b}{0.4}\right)\right\rangle$
and $C=\left\langle x,\left(\frac{a}{0.3}, \frac{b}{0.4}\right),\left(\frac{a}{0.3}, \frac{b}{0.4}\right),\left(\frac{a}{0.4}, \frac{b}{0.4}\right)\right\rangle$. Then the families $T=\left\{0_{N}, 1_{N}, A, B, A \cup B\right\}$ is neutrosophic topology on $X$. Thus, $(X, T)$ is a neutrosophic topological space. Then $C$ is called neutrosophic semi-supra open but not neutrosophic supra open set.

Proposition 2.2. Every neutrosophic $\alpha$-supra open is neutrosophic semi-supra open

Proof. Let $A$ be a neutrosophic $\alpha$-supra open in $(X, T)$, then $A \subseteq s-N i n t(s-N c l(s-N i n t(A)))$. It is obvious that $s-N i n t(s-$ $N c l(s-N i n t(A))) \subseteq s-N c l(s-N i n t(A))$. Hence $A \subseteq s-N c l(s-$ $\operatorname{Nint}(A))$.

The converse of Proposition 2.2., need not be true as shown in Example 2.2.

Example 2.2. Let $X=\{a, b\}$. Define the neutrosophic sets $A$, $B$ and $C$ in $X$ as follows:
$A=\left\langle x,\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.5}, \frac{b}{0.3}\right)\right\rangle, \quad B \quad=$ $\left\langle x,\left(\frac{a}{0.1}, \frac{b}{0.2}\right),\left(\frac{a}{0.1}, \frac{b}{0.2}\right),\left(\frac{a}{0.6}, \frac{b}{0.5}\right)\right\rangle$
and $C=\left\langle x,\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.2}, \frac{b}{0.3}\right)\right\rangle$. Then the families $T=\left\{0_{N}, 1_{N}, A, B, A \cup B\right\}$ is neutrosophic topology on $X$.Thus, $(X, T)$ is a neutrosophic topological space. Then $C$ is called neutrosophic semi-supra open but not neutrosophic $\alpha$-supra open set.

Proposition 2.3. Every neutrosophic regular supra open set is neutrosophic semi-supra open set

Proof. Let $A$ be a neutrosophic regular supra open set in $(X, T)$. Then $A \subseteq(s-N c l(A))$. Hence $A \subseteq s-N c l(s-N i n t(A))$.
The converse of Proposition 2.3., need not be true as shown in Example 2.3.

Example 2.3. Let $X=\{a, b\}$. Define the neutrosophic sets $A$, $B$ and $C$ in $X$ as follows:
$A \quad=\quad\left\langle x,\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.5}, \frac{b}{0.3}\right)\right\rangle, \quad B \quad=$ $\left\langle x,\left(\frac{a}{0.1}, \frac{b}{0.2}\right),\left(\frac{a}{0.1}, \frac{b}{0.2}\right),\left(\frac{a}{0.6}, \frac{b}{0.5}\right)\right\rangle$
and $C=\left\langle x,\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.2}, \frac{b}{0.3}\right),\left(\frac{a}{0.2}, \frac{b}{0.3}\right)\right\rangle$. Then the families $T=\left\{0_{N}, 1_{N}, A, B, A \cup B\right\}$ is neutrosophic topology on X . Thus, $(X, T)$ is a neutrosophic topological space. Then $C$ is neutrosophic semi-supra open but not neutrosophic regular-supra open set.

Definition 2.3. The neutrosophic semi-supra closure of a set $A$ is denoted by semi-s-Ncl $(A)=\bigcup\{\mathrm{G}: \mathrm{G}$ is aneutrosophic semisupra open set in $X$ and $G \subseteq A\}$ and the neutrosophic semisupra interior of a set $A$ is denoted by $\operatorname{semi}-s-\operatorname{Nint}(A)=\bigcap\{\mathrm{G}$ :G is a neutrosophic semi-supra closed set in $X$ and $G \supseteq A\}$.

Remark 2.1. It is clear that $\operatorname{semi-s}-\operatorname{Nint}(A)$ is a neutrosophic semi-supra open set and semi-s- $N c l(A)$ is a neutrosophic semisupra closed set.

Proposition 2.4. i) $\overline{\operatorname{semi}-s-\operatorname{Nint}(A)}=\operatorname{semi} s-N c l(\bar{A})$
ii) $\overline{\operatorname{semi}-s-N \operatorname{cl}(A)}=\operatorname{semi}$ s-int $(\bar{A})$
iii) if $A \subseteq B$ then $\operatorname{semi}-s-N c l(A) \subseteq \operatorname{semi}-s-N c l(B)$ and semi-s-Nint $(A) \subseteq \operatorname{semi-s-Nint}(B)$

Proof. It is obvious.
Proposition 2.5. (i) The intersection of a neutrosophic supra open set and a neutrosophic semi-supra open set is a neutrosophic semi- supra open set.
(ii) The intersection of a neutrosophic semi-supra open set and aneutrosophic pre-supra open set is a neutrosophic pre-supra open set.

Proof. It is obvious.
Definition 2.4. Let $(X, T)$ and $(Y, S)$ be two neutrosophic semisupra open sets and $R$ be a associated supra topology with $T$. A map $f:(X, T) \rightarrow(Y, S)$ is called neutrosophic semi- supra continuous map if the inverse image of each neutrosophic open set in $Y$ is a neutrosophic semi- supra open in $X$.

Proposition 2.6. Every neutrosophic supra continuous map is neutrosophic semi-supra continuous map.

Proof. Let $f:(X, T) \rightarrow(Y, S)$ be a neutrosophic supra continuous map and $A$ is a neutrosophic open set in $Y$. Then $f^{-1}(A)$ is a neutrosophic open set in $X$. Since $R$ is associated with $T$. Then $T \subseteq R$. Therefore $f^{-1}(A)$ is a neutrosophic supra open set in $X$ which is a neutrosophic supra open set in $X$. Hence $f$ is aneutrosophic semi-supra continuous map.

Remark 2.2. Every neutrosophic semi-supra continuous map need not be neutrosophic supra continuous map.

Proposition 2.7. Let $(X, T)$ and $(Y, S)$ be two neutrosophic topological spaces and $R$ be a associated neutrosophic supra topology with $T$. Let $f$ be a map from $X$ into $Y$. Then the following are equivalent.
i) $f$ is a neutrosophic semi-supra continuous map.
ii) The inverse image of a neutrosophic closed sets in $Y$ is a neutrosophic semi closed set in $X$.
iii) $\operatorname{Semi-s-Ncl}\left(f^{-1}(A)\right) \subseteq f^{-1}(\operatorname{Ncl}(A))$ for every neutrosophic set $A$ in $Y$.
iv) $f($ semi-s- $\operatorname{Ncl}(A)) \subseteq \operatorname{Ncl}(f(A))$ for every neutrosophic set A in X .
v) $f^{-1}(\operatorname{Nint}(B)) \subseteq \operatorname{semi-s-Nint}\left(f^{-1}(B)\right)$ for every neutrosophic set $B$ in $Y$.

Proof. $(i) \Rightarrow(i i)$ : Let $A$ be a neutrosophic closed set in $Y$. Then $\bar{A}$ is neutrosophic open in $Y$, Thus $f^{-1}(\bar{A})=\overline{f^{-1}(A)}$ is neutrosophic semi-open in $X$. It follows that $f^{-1}(A)$ is a neutrosophic semi-s closed set of $X$.
$(i i) \Rightarrow(i i i):$ Let $A$ be any subset of $X$. Since $N c l(A)$ is neutrosophic closed in $Y$ then it follows that $f^{-1}(\operatorname{Ncl}(A))$ is neutrosophic semi-s closed in $X$. Therefore, $f^{-1}(N c l(A))=s e m i-s$ $N c l\left(f^{-1}(N c l(A)) \supseteq \operatorname{semi-s}-N c l\left(f^{-1}(A)\right)\right.$
(iii) $\Rightarrow(i v)$ : Let $A$ be any subset of $X$. By (iii) we obtain $f^{-1}\left(N c l(f((A))) \supseteq \operatorname{semi}-s-N c l\left(f^{-1}(f(A))\right) \supseteq\right.$ semi-s$\operatorname{Ncl}(A)$ and hence $f(\operatorname{semi}-s-N c l(A)) \subseteq N c l(f(A))$.
$(i v) \Rightarrow(v)$ : Let $f(\operatorname{semi}-s-\operatorname{Ncl}(A)) \subseteq f(\operatorname{Ncl}(A)$ for every neutrosophic set $A$ in $X$. Then $\operatorname{semi-s-Ncl(A))\subseteq }$ $f^{-1}\left(\operatorname{Ncl}(f(A)) . \quad \overline{s e m i}-s-\operatorname{Ncl}(A) \supseteq \overline{f^{-1}(\operatorname{Ncl}(f(A)))}\right.$
and semi-s-Nint $(\bar{A}) \supseteq f^{-1}(\operatorname{Nint}(\overline{f(A)}))$. Then semi-s-$\operatorname{Nint}\left(f^{-1}(B)\right) \supseteq f^{-1}(\operatorname{Nint}(B))$. Therefore $f^{-1}(\operatorname{Nint}(B)) \subseteq$ $s-\operatorname{Nint}\left(f^{-1}(B)\right)$ for every $B$ in $Y$.
$(v) \Rightarrow(i) \quad$ Let $A$ be a neutrosophic open set in $Y$. Therefore $f^{-1}(\operatorname{Nint}(A)) \subseteq \operatorname{semi-s}-\operatorname{Nint}\left(f^{-1}(A)\right)$, hence $f^{-1}(A) \subseteq \operatorname{semi}-s-\operatorname{Nint}\left(f^{-1}(A)\right)$. But we know that semi-$s-\operatorname{Nint}\left(f^{-1}(A)\right) \subseteq f^{-1}(A)$, then $f^{-1}(A)=$ semi-s-$\operatorname{Nint}\left(f^{-1}(A)\right)$. Therefore $f^{-1}(A)$ is a neutrosophic semi-sopen set.

Proposition 2.8. If a map $f:(X, T) \rightarrow(Y, S)$ is a neutrosophic semi-s-continuous and $g:(Y, S) \rightarrow(Z, R)$ is neutrosophic continuous, Then $g \circ f$ is neutrosophic semi-s-continuous.

Proof. Obvious.
Proposition 2.9. Let a map $f:(X, T) \rightarrow(Y, S)$ be a neutrosophic semi-supra continuous map, then one of the following holds
i) $f^{-1}(\operatorname{semi-s}-\operatorname{Nint}(A)) \subseteq \operatorname{Nint}\left(f^{-1}(A)\right)$ for every neutrosophic set $A$ in $Y$.
ii) $\operatorname{Ncl}\left(f^{-1}(A)\right) \subseteq f^{-1}(\operatorname{semi}-s-\operatorname{Ncl}(A))$ for every neutrosophic set $A$ in $Y$.
iii) $f(N c l(B)) \subseteq \operatorname{semi}-s-N c l(f(B))$ for every neutrosophic set $B$ in $X$.

Proof. Let $A$ be any neutrosophic open set of $Y$, then condition (i) is satisfied, then $f^{-1}(\operatorname{semi}-s-\operatorname{Nint}(A)) \subseteq \operatorname{Nint}\left(f^{-1}(A)\right)$. We get, $f^{-1}(A) \subseteq \operatorname{Nint}\left(f^{-1}(A)\right)$. Therefore $f^{-1}(A)$ is a neutrosophic supra open set. Every neutrosophic supra open set is a neutrosophic semi supra open set. Hence $f$ is a neutrosophic semi-s-continuous function. If condition (ii) is satisfied, then we can easily prove that $f$ is a neutrosophic semi -s continuous function if condition (iii) is satisfied, and $A$ is any neutrosophic open set of $Y$, then $f^{-1}(A)$ is a set in $X$ and $f\left(N c l\left(f^{-1}(A)\right) \subseteq\right.$ semi-$s-N c l\left(f\left(f^{-1}(A)\right)\right)$. This implies $f\left(N c l\left(f^{-1}(A)\right)\right) \subseteq$ semi-s$\operatorname{Ncl}(A)$. This is nothing but condition (ii). Hence $f$ is a neutrosophic semi-s-continuous function.

## References

[1] M.E. Abd El-monsef and A. E. Ramadan, On fuzzy supra topological spaces, Indian J. Pure and Appl.Math.no.4, 18(1987), 322-329
[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1986), 87-96.
[3] D. Coker, An introduction to Intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88 (1997) 81-89
[4] R. Devi, S. Sampathkumar and M. Caldas, On supra $\alpha$-open sets and supra $\alpha$-continuous functions, General Mathematics, Vol.16, Nr.2(2008),77-84.
[5] R.Dhavaseelan and S. Jafari, Generalized Neutrosophic closed sets (submitted).
[6] A. S. Mashhour, A. A. Allam, F. H. Khedr, On supra topological spaces, Indian J. Pure and Appl. Math. no.4, 14 (1983), 502-510
[7] M. Parimala and C. Indirani, On Intuitionistic Fuzzy semisupra open set and intuitionistic fuzzy semi-supra continuous functions, Procedia Computer Science, 47 ( 2015 ) 319-325.
[8] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics, Volume 3, Issue 4 (Sep-Oct. 2012), 31-35
[9] F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002), smarand@unm.edu
[10] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.
[11] N. Turanl, On intuitionistic fuzzy supra topological spaces, International conference on modeling and simulation, spain, vol II, (1999) 69-77.
[12] N. Turanl, An overview of intuitionistic fuzzy supra topological spaces, Hacettepe Journal of mathematics and statistics, Volume 32 (2003), 17-26.
[13] Won Keun Min,On fuzzy s-continuous functions, Kangweon-Kyungki Math.J. no.1, 4(1996),77-82.
[14] L.A. Zadeh, Fuzzy sets, Information and control, 8 (1965), 338-353.

Received: May 11, 2017. Accepted: May 29, 2017.

