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On Q-neutrosophic subring

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Abstract. In recent years, the theory of neutrosophic set and its hybrid models have captured the attention of many researchers. The concept of Q-neutrosophic set was introduced as an extension of neutrosophic set to handle two-dimensional uncertain, indeterminate and inconsistent information. This paper focuses on developing the algebraic structure pertaining to subring for the Q-neutrosophic set model. Properties of Q-neutrosophic subring are proposed and results are discussed.

1. Introduction

The theory of fuzzy set theory founded by Zadeh [1] is an important aspect in the study of uncertainty. This idea has brought about many extensions of fuzzy set such as the intuitionistic fuzzy set [2], interval-valued fuzzy set [3], vague set [4], and hesitant fuzzy set [5]. Smarandache [6, 7] established a new model called neutrosophic set theory which refers to neutral knowledge. It was further extended to Q-neutrosophic set [8] in order to handle two-dimensional neutral knowledge. Henceforth, the theory of fuzzy set and its hybrid models became an energetic area of research [9-29]. The fuzzy algebraic structures play a noted role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, group theory, real analysis and others. The study of the fuzzy algebraic structures started with fuzzy subgroups and fuzzy ideals by Rosenfeld [30]. Solairaju and Nagarajan [31] introduced the notion of Q-fuzzy groups. Thiruveni and Solairaju defined the concept of neutrosophic Q-fuzzy subgroups [32], while Rasuli [33] established the notion of Q-fuzzy subrings and anti Q- fuzzy subrings. Recently, many researchers have applied different hybrid models of fuzzy set to multi-fuzzy sets [34-36], several algebraic structures such as groups, semigroups, rings, fields and BCK/BCI-algebras [37-39], neutrosophy [40], abstract algebra [41] and genetics [42-44]. Research on decision making are no longer limited to programming [45-52] or discrete forms [53-65]. In this paper we define the notion of Q-neutrosophic subring and investigate some of its properties.

2. Preliminaries

In this section, we recall the notions of neutrosophic set and Q-neutrosophic set with some basic properties which are relevant to this paper.

Definition 1. [6] A neutrosophic set Γ on the universe X is defined as

$$\Gamma = \{\langle x, (T_\Gamma(x),\ I_\Gamma(x),\ F_\Gamma(x))\rangle : x \in X\},$$

where $T, I, F: X \to]0^-, 1^+[$ and $0^- \leq T_\Gamma(x) + I_\Gamma(x) + F_\Gamma(x) \leq 3^+$.

Definition 2. [40] Let Γ and Ψ be two neutrosophic sets. Then we say that
1) $\Gamma \subseteq \Psi$ if and only if $T_\Gamma(x) \leq T_\Psi(x), I_\Gamma(x) \geq I_\Psi(x)$ and $F_\Gamma(x) \geq F_\Psi(x)$ for all $x \in X$.
2) $\Gamma \cup \Psi = \{\langle x, (\max\{T_\Gamma(x), T_\Psi(x)\}, \min\{I_\Gamma(x), I_\Psi(x)\}, \min\{F_\Gamma(x), F_\Psi(x)\})\rangle : x \in X\}$.
3) $\Gamma \cap \Psi = \{\langle x, (\min\{T_\Gamma(x), T_\Psi(x)\}, \max\{I_\Gamma(x), I_\Psi(x)\}, \max\{F_\Gamma(x), F_\Psi(x)\})\rangle : x \in X\}$.

Definition 3. [40] The complement of a neutrosophic set Γ in the universe X is denoted by Γ^c, where

$$\Gamma^c = \left\{\langle x, \left(1 - T_\Gamma(x), 1 - I_\Gamma(x), 1 - F_\Gamma(x)\right)\rangle : x \in X\right\}.$$

The neutrosophic empty set Γ_0 in the universe X is $\Gamma_0 = \{\langle x, (0, 1, 1)\rangle\}$.

Definition 4. [8] Let X be a universal set and Q be a nonempty set. A Q-neutrosophic set Γ_Q in X and Q is an object of the form $\Gamma_Q = \left\{\langle x, \left(T_{\Gamma_Q}(x,q), I_{\Gamma_Q}(x,q), F_{\Gamma_Q}(x,q)\right)\rangle : x \in X, q \in Q\right\}$,

where $T_{\Gamma_Q}, I_{\Gamma_Q}, F_{\Gamma_Q} : X \times Q \to]0^-, 1^+[$ are the true membership function, indeterminacy membership function and false membership function, respectively with $0^- \leq T_{\Gamma_Q} + I_{\Gamma_Q} + F_{\Gamma_Q} \leq 3^+$.

3. Q-neutrosophic subring

In this section, we define the notion of Q-neutrosophic subring and investigate some of its properties. Throughout this paper, we will denote the ring $(R, +, .)$ simply as R.

Definition 5. Let R be a ring and Γ_Q be a Q-neutrosophic subset of R. Then Γ_Q is called a Q-neutrosophic subring of R if for all $x, y \in R$ and $q \in Q$ the following conditions are satisfied:
1) $T_{\Gamma_Q}(x+y,q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\}$, $I_{\Gamma_Q}(x+y,q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\}$, $F_{\Gamma_Q}(x+y,q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\}$.
2) $T_{\Gamma_Q}(-x,q) \geq T_{\Gamma_Q}(x,q)$, $I_{\Gamma_Q}(-x,q) \leq I_{\Gamma_Q}(x,q)$, $F_{\Gamma_Q}(-x,q) \leq F_{\Gamma_Q}(x,q)$.
3) $T_{\Gamma_Q}(xy,q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\}$, $I_{\Gamma_Q}(xy,q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\}$, $F_{\Gamma_Q}(xy,q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\}$.

Example 1. Let $\mathbb{Z}$ be a ring. Let Γ_Q be a Q-neutrosophic set in $\mathbb{Z}$ defined by

$$T_{\Gamma_Q}(x,q) = \begin{cases} 0, & if\ x = 2k-1, \exists k \in \mathbb{Z} \\ \frac{1}{2}, & if\ x = 2k, \exists k \in \mathbb{Z}, \end{cases}$$

$$I_{\Gamma_Q}(x,q) = \begin{cases} \frac{1}{2}, & if\ x = 2k-1, \exists k \in \mathbb{Z} \\ 0, & if\ x = 2k, \exists k \in \mathbb{Z}, \end{cases}$$

$$F_{\Gamma_Q}(x,q) = \begin{cases} \frac{1}{2}, & if\ x = 2k-1, \exists k \in \mathbb{Z} \\ 0, & if\ x = 2k, \exists k \in \mathbb{Z}, \end{cases}$$

for all $q \in Q$. By routine calculations we know that Γ_Q is a Q-neutrosophic subring of $\mathbb{Z}$.

Lemma 1. Let Γ_Q be a Q-neutrosophic subring of R. Then for all $x, y \in R$ and $q \in Q$ we have $T_{\Gamma_Q}(0,q) \geq T_{\Gamma_Q}(x,q)$, $I_{\Gamma_Q}(0,q) \leq I_{\Gamma_Q}(x,q)$ and $F_{\Gamma_Q}(0,q) \leq F_{\Gamma_Q}(x,q)$.

Proof. For all $x \in R$, we have $T_{\Gamma_Q}(0,q) = T_{\Gamma_Q}(\mathrm{x}-\mathrm{x},q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(x,q)\right\} = T_{\Gamma_Q}(x,q)$. Now, $I_{\Gamma_Q}(0,q) = I_{\Gamma_Q}(\mathrm{x}-\mathrm{x},q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(x,q)\right\} = I_{\Gamma_Q}(x,q)$. Similarly, we can obtain $F_{\Gamma_Q}(0,q) \leq F_{\Gamma_Q}(x,q)$.

Theorem 1. Let Γ_Q be a Q-neutrosophic subset of R. Then Γ_Q is called a Q-neutrosophic subring of R if and only if for all $x, y \in R$ and $q \in Q$, the following conditions hold:

i. $T_{\Gamma_Q}(x-y,q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\}$, $I_{\Gamma_Q}(x-y,q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\}$, $F_{\Gamma_Q}(x-y,q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\}$.

ii. $T_{\Gamma_Q}(xy,q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\}$, $I_{\Gamma_Q}(xy,q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\}$, $F_{\Gamma_Q}(xy,q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\}$.

Proof. Suppose that Γ_Q is a Q-neutrosophic subring of R. Then for $x, y \in R$ and $q \in Q$, by Definition 5 we have

$$T_{\Gamma_Q}(x-y,q) = T_{\Gamma_Q}(x+(-y),q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(-y,q)\right\} \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\},$$
$$I_{\Gamma_Q}(x-y,q) = I_{\Gamma_Q}(x+(-y),q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(-y,q)\right\} \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\},$$
$$F_{\Gamma_Q}(x-y,q) = F_{\Gamma_Q}(x+(-y),q) \leq \max\left\{\mathrm{F}(x,q), F_{\Gamma_Q}(-y,q)\right\} \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\}.$$

Therefore, (i) holds.

Now, since Γ_Q is a Q-neutrosophic subring of R, then (ii) automatically holds.

Conversely, suppose that (i) and (ii) hold. Then, from Lemma 1 we have $T_{\Gamma_Q}(0,q) \geq T_{\Gamma_Q}(x,q)$, $I_{\Gamma_Q}(0,q) \leq I_{\Gamma_Q}(x,q)$ and $F_{\Gamma_Q}(0,q) \leq F_{\Gamma_Q}(x,q)$ for all $x, y \in R$ and $q \in Q$.

Now,

$$T_{\Gamma_Q}(-x,q) = T_{\Gamma_Q}(0-x,q) \geq \min\left\{T_{\Gamma_Q}(0,q), T_{\Gamma_Q}(\mathrm{x},q)\right\} \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(x,q)\right\} = T_{\Gamma_Q}(x,q),$$
$$I_{\Gamma_Q}(-x,q) = I_{\Gamma_Q}(0-x,q) \leq \max\left\{I_{\Gamma_Q}(0,q), I_{\Gamma_Q}(\mathrm{x},q)\right\} \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(x,q)\right\} = I_{\Gamma_Q}(x,q),$$
$$F_{\Gamma_Q}(-x,q) = F_{\Gamma_Q}(0-x,q) \leq \max\left\{F_{\Gamma_Q}(0,q), F_{\Gamma_Q}(\mathrm{x},q)\right\} \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(x,q)\right\} = F_{\Gamma_Q}(x,q).$$

Also,

$$T_{\Gamma_Q}(x+y,q) = T_{\Gamma_Q}(x-(-y),q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(-\mathrm{y},q)\right\} \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(\mathrm{y},q)\right\},$$
$$I_{\Gamma_Q}(x+y,q) = I_{\Gamma_Q}(x-(-y),q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(-\mathrm{y},q)\right\} \geq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(\mathrm{y},q)\right\},$$
$$F_{\Gamma_Q}(x+y,q) = F_{\Gamma_Q}(x-(-y),q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(-\mathrm{y},q)\right\} \geq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(\mathrm{y},q)\right\}.$$

Thus, conditions (1) and (2) of Definition 5 hold. Now, condition (3) of Definition 5 automatically holds using (ii). Hence, Γ_Q is a Q-neutrosophic subring of R.

Theorem 2. The intersection of any two Q-neutrosophic subrings of R is also a Q- neutrosophic subring.

Proof. Let Γ_Q and Ψ_Q be two Q-neutrosophic subrings of R. Let $\Lambda_Q = \Gamma_Q \cap \Psi_Q, x, y \in R$ and $q \in Q$. Then

$$\Lambda_Q(x-y,q) = \Gamma_Q(x-y,q) \cap \Psi_Q(x-y,q)$$
$$= \left\{(x-y,q), T_{\Lambda_Q}(x-y,q), I_{\Lambda_Q}(x-y,q), F_{\Lambda_Q}(x-y,q)\right\}.$$

Now,

$$\begin{aligned} T_{\Lambda_Q}(x-y,q) &= \min\left\{T_{\Gamma_Q}(x-y,q), T_{\Psi_Q}(x-y,q)\right\} \\ &\geq \min\left\{\min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\}, \min\left\{T_{\Psi_Q}(x,q), T_{\Psi_Q}(y,q)\right\}\right\} \\ &\geq \min\left\{\min\left\{T_{\Gamma_Q}(x,q), T_{\Psi_Q}(x,q)\right\}, \min\left\{T_{\Gamma_Q}(y,q), T_{\Psi_Q}(y,q)\right\}\right\} \\ &= \min\left\{T_{\Lambda_Q}(x,q), T_{\Lambda_Q}(y,q)\right\} \end{aligned}$$

and

$$\begin{aligned} I_{\Lambda_Q}(x-y,q) &= \max\left\{I_{\Gamma_Q}(x-y,q), I_{\Psi_Q}(x-y,q)\right\} \\ &\leq \max\left\{\max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\}, \max\left\{I_{\Psi_Q}(x,q), I_{\Psi_Q}(y,q)\right\}\right\} \\ &\leq \max\left\{\max\left\{I_{\Gamma_Q}(x,q), T_{\Psi_Q}(x,q)\right\}, \max\left\{I_{\Gamma_Q}(y,q), I_{\Psi_Q}(y,q)\right\}\right\} \\ &= \max\left\{I_{\Lambda_Q}(x,q), I_{\Lambda_Q}(y,q)\right\}. \end{aligned}$$

Similarly, we can show that $F_{\Lambda_Q}(x-y,q) \leq \max\left\{F_{\Lambda_Q}(x,q), F_{\Lambda_Q}(y,q)\right\}$. Also,

$$\begin{aligned} \Lambda_Q(xy,q) &= \Gamma_Q(xy,q) \cap \Psi_Q(xy,q) \\ &= \left\{(xy,q), T_{\Lambda_Q}(xy,q), I_{\Lambda_Q}(xy,q), F_{\Lambda_Q}(xy,q)\right\}. \end{aligned}$$

Now,

$$\begin{aligned} T_{\Lambda_Q}(xy,q) &= \min\left\{T_{\Gamma_Q}(xy,q), T_{\Psi_Q}(xy,q)\right\} \\ &\geq \min\left\{\min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\}, \min\left\{T_{\Psi_Q}(x,q), T_{\Psi_Q}(y,q)\right\}\right\} \\ &\geq \min\left\{\min\left\{T_{\Gamma_Q}(x,q), T_{\Psi_Q}(x,q)\right\}, \min\left\{T_{\Gamma_Q}(y,q), T_{\Psi_Q}(y,q)\right\}\right\} \\ &= \min\left\{T_{\Lambda_Q}(x,q), T_{\Lambda_Q}(y,q)\right\} \end{aligned}$$

and

$$\begin{aligned} I_{\Lambda_Q}(xy,q) &= \max\left\{I_{\Gamma_Q}(xy,q), I_{\Psi_Q}(xy,q)\right\} \\ &\leq \max\left\{\max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\}, \max\left\{I_{\Psi_Q}(x,q), I_{\Psi_Q}(y,q)\right\}\right\} \\ &\leq \max\left\{\max\left\{I_{\Gamma_Q}(x,q), T_{\Psi_Q}(x,q)\right\}, \max\left\{I_{\Gamma_Q}(y,q), I_{\Psi_Q}(y,q)\right\}\right\} \\ &= \max\left\{I_{\Lambda_Q}(x,q), I_{\Lambda_Q}(y,q)\right\}. \end{aligned}$$

Similarly, we can show that $F_{\Lambda_Q}(x-y,q) \leq \max\left\{F_{\Lambda_Q}(x,q), F_{\Lambda_Q}(y,q)\right\}$. This completes the proof.

Definition 6. Let Γ_Q be a Q-neutrosophic subset of R. Let $\alpha, \beta, \gamma \in [0,1]$ with $\alpha + \beta + \gamma \leq 3$. Then $[\Gamma_Q]_{(\alpha,\beta,\gamma)}$ is a Q-level subset of Γ_Q defined by $[\Gamma_Q]_{(\alpha,\beta,\gamma)} = \left\{x \in X, q \in Q: T_{\Gamma_Q}(x,q) \geq \alpha, I_{\Gamma_Q}(x,q) \leq \beta, F_{\Gamma_Q}(x,q) \leq \gamma\right\}$.

Theorem 3. [41] Let R be a ring. A non-empty subset I of R is a subring of R if and only if $x - y \in I$ and $xy \in I$ for all $x, y \in I$.

Theorem 4. Let Γ_Q be a Q-neutrosophic subring of R. Then for all $\alpha, \beta, \gamma \in [0,1]$, $[\Gamma_Q]_{(\alpha,\beta,\gamma)} \neq \emptyset$ is a subring of R.

Proof. Let $x, y \in R$ and $q \in Q$. Then

$$T_{\Gamma_Q}(x-y,q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\} \geq \{\alpha,\alpha\} = \alpha,$$

$$I_{\Gamma_Q}(x-y,q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\} \leq \{\beta,\beta\} = \beta,$$

$$F_{\Gamma_Q}(x-y,q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\} \leq \{\gamma,\gamma\} = \gamma.$$

Hence, $x - y \in [\Gamma_Q]_{(\alpha,\beta,\gamma)}$. Also,

$$T_{\Gamma_Q}(xy,q) \geq \min\left\{T_{\Gamma_Q}(x,q), T_{\Gamma_Q}(y,q)\right\} \geq \{\alpha,\alpha\} = \alpha,$$

$$I_{\Gamma_Q}(xy,q) \leq \max\left\{I_{\Gamma_Q}(x,q), I_{\Gamma_Q}(y,q)\right\} \leq \{\beta,\beta\} = \beta,$$

$$F_{\Gamma_Q}(xy,q) \leq \max\left\{F_{\Gamma_Q}(x,q), F_{\Gamma_Q}(y,q)\right\} \leq \{\gamma,\gamma\} = \gamma.$$

Therefore, $x - y \in [\Gamma_Q]_{(\alpha,\beta,\gamma)}$. Thus $[\Gamma_Q]_{(\alpha,\beta,\gamma)}$ is a subring of R by Theorem 3.

4. Conclusion

We have introduced the notion of Q-neutrosophic subring and developed its algebraic structure. Properties of the Q-neutrosophic subring were proposed and discussed.

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