

Article

On Some NeutroHyperstructures

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Abstract: Neutrosophy, the study of neutralities, is a new branch of Philosophy that has applications in many different fields of science. Inspired by the idea of Neutrosophy, Smarandache introduced NeutroAlgebraicStructures (or NeutroAlgebras) by allowing the partiality and indeterminacy to be included in the structures' operations and/or axioms. The aim of this paper is to combine the concept of Neutrosophy with hyperstructures theory. In this regard, we introduce NeutroSemihypergroups as well as Neutro H_v -Semigroups and study their properties by providing several illustrative examples.

Keywords: NeutroHypergroupoid; NeutroSemihypergroup; Neutro H_v -semigroup; NeutroHyperideal; NeutroStrongIsomorphism

MSC: 03A99; 03G99; 20N20



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1. Introduction

In 1995 and inspired by the existence of neutralities, Smarandache introduced Neutrosophy as a new branch of Philosophy that deals with indeterminacy. During the past, ideas were viewed as “True” or “False”; however, if we view an idea from a neutrosophic point of view, it will be “True”, “False”, or “Indeterminate”. The indeterminacy is the key that distinguishes Neutrosophy from other approaches. In the past twenty years, this field demonstrated important progress in which it grabbed the attention of many researchers and different works were done from both a theoretical point of view and from an applicative view. Unlike our real world that is full of imperfections and partialities, abstract systems are constructed on a given perfect space (set), where the operations are totally well-defined and the axioms are totally true for all spacial elements. Starting from the latter idea, Smarandache [1–3] introduced NeutroAlgebra, whose operations are partially well-defined, partially indeterminate, and partially outer-defined, and the axioms are partially true, partially indeterminate, and partially false. Many researchers worked on special types of NeutroAlgebras by applying them to different types of algebraic structures such as groups, rings, *BE*-Algebras, *BCK*-Algebras, etc. For more details, we refer to [4–10].

On the other hand, hyperstructure theory is a generalization of classical algebraic structures and was introduced in 1934 at the eighth Congress of Scandinavian Mathematicians by Marty [11]. Marty generalized the notion of groups by defining hypergroups. The class of algebraic hyperstructures is larger than that of algebraic structures where the operation on two elements in the latter is again an element, whereas the hyperoperation of two elements in the first class is a non-void set. For details about hyperstructure theory and its applications, we refer to the articles [12–15] and the books [16–18]. A generalization of algebraic hyperstructures, known as weak hyperstructures (H_v -structures), was introduced

in 1994 by Vougiouklis [19]. The axioms in the latter are weaker than that of algebraic hyperstructures. For details about H_v -structures, we refer to [19–22].

As a natural extension of NeutroAlgebraicStructure, NeutroHyperstructure was defined recently [23,24] where Ibrahim and Agboola [23] defined NeutroHypergroups and studied a special type. Our paper is concerned about some NeutroHyperstructures and is organized as follows: Section 2 presents some basic preliminaries related to hyperstructure theory. Section 3 defines NeutroSemihypergroups, Neutro H_v -Semigroups, and some related new concepts and illustrates these new concepts via examples. Moreover, we study some properties of their subsets under NeutroStrongHomomorphism.

2. Algebraic Hyperstructures

In this section, we present some definitions and examples about (weak) algebraic hyperstructures that are used throughout the paper. For more details about hyperstructure theory, we refer to [16–20].

Definition 1 ([16]). *Let H be a non-empty set and $\mathcal{P}^*(H)$ be the family of all non-empty subsets of H . Then, a mapping $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ is called a binary hyperoperation on H . The couple (H, \circ) is called a hypergroupoid.*

If A and B are two non-empty subsets of H and $h \in H$, then we define:

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b, \quad h \circ A = \{h\} \circ A \text{ and } A \circ h = A \circ \{h\}.$$

A hypergroupoid (H, \circ) is called a *semihypergroup* if the associative axiom is satisfied. i.e., for every $x, y, z \in H$, $x \circ (y \circ z) = (x \circ y) \circ z$. In other words,

$$\bigcup_{u \in y \circ z} x \circ u = \bigcup_{v \in x \circ y} v \circ z.$$

An element h in a hypergroupoid (H, \circ) is called *idempotent* if $h \circ h = h$.

Example 1. *Let H be any non-empty set and define “ \star ” on H as follows. For all $x, y \in H$, $x \star y = \{x, y\}$. Then (H, \star) is a semihypergroup.*

Example 2. *Let $H_0 = \{e, b, c\}$ and $(H_0, +)$ be defined by the following table.*

+	e	b	c
e	e	$\{e, b\}$	$\{e, c\}$
b	e	$\{e, b\}$	$\{e, c\}$
c	e	$\{e, b\}$	$\{e, c\}$

Then $(H_0, +)$ is a semihypergroup and e is an idempotent element in H_0 .

As a generalization of algebraic hyperstructures, Vougiouklis [19,20] introduced H_v -structures. Weak axioms in H_v -structures replace some axioms of classical algebraic hyperstructures.

Definition 2 ([19,20]). *A hypergroupoid (H, \circ) is called an H_v -semigroup if the weak associative axiom is satisfied. i.e., $(x \circ (y \circ z)) \cap ((x \circ y) \circ z) \neq \emptyset$ for all $x, y, z \in H$.*

Example 3. *Let $H_1 = \{0, 1, 2, 3\}$ and “+” be the hyperoperation on H_1 defined by the following table.*

+	0	1	2	3
0	0	1	$\{0, 2\}$	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Then $(H_1, +)$ is an H_v -semigroup.

Remark 1. Every semigroup is a semihypergroup and every semihypergroup is an H_v -semigroup.

Definition 3 ([17]). Let (H, \circ) be a semihypergroup (H_v -semigroup) and $M \neq \emptyset \subseteq H$. Then M is a

1. subsemihypergroup (H_v -subsemigroup) of H if (M, \circ) is a semihypergroup (H_v -semigroup).
2. left hyperideal of H if M is a subsemihypergroup (H_v -subsemigroup) of H and $h \circ a \subseteq M$ for all $h \in H$.
3. right hyperideal of H if M is a subsemihypergroup (H_v -subsemigroup) of H and $a \circ h \subseteq M$ for all $h \in H$.
4. hyperideal of H if M is both: a left hyperideal of H and a right hyperideal of H .

Remark 2. Let (H, \circ) be a semihypergroup (H_v -semigroup) and $M \neq \emptyset \subseteq H$. To prove that M is subsemihypergroup (H_v -subsemigroup) of H , it suffices to show that $a \circ b \subseteq M$ for all $a, b \in M$.

3. NeutroHyperstructures

In this section, we define NeutroSemihypergroups and Neutro H_v -Semigroups, present some illustrative examples, and study several properties of some important subsets of NeutroSemihypergroups and Neutro H_v -Semigroups.

Definition 4. Let A be any non-empty set and “ \cdot ” be a hyperoperation on A . Then “ \cdot ” is called a NeutroHyperoperation on A if some (or all) of the following conditions hold in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. There exist $x, y \in A$ with $x \cdot y \subseteq A$. (This condition is called degree of truth, “T”).
2. There exist $x, y \in A$ with $x \cdot y \not\subseteq A$. (This condition is called degree of falsity, “F”).
3. There exist $x, y \in A$ with $x \cdot y$ is indeterminate in A . (This condition is called degree of indeterminacy, “I”).

Definition 5. Let A be any non-empty set and “ \cdot ” be a hyperoperation on A . Then “ \cdot ” is called an AntiHyperoperation on A if $x \cdot y \not\subseteq A$ for all $x, y \in A$.

Definition 6. Let A be any non-empty set and “ \cdot ” be a hyperoperation on A . Then “ \cdot ” is called NeutroAssociative on A if there exist $x, y, z, a, b, c, e, f, g \in A$ satisfying some (or all) of the following conditions in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$; (This condition is called degree of truth, “T”).
2. $a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$; (This condition is called degree of falsity, “F”).
3. $e \cdot (f \cdot g)$ is indeterminate or $(e \cdot f) \cdot g$ is indeterminate or we cannot find if $e \cdot (f \cdot g)$ and $(e \cdot f) \cdot g$ are equal. (This condition is called degree of indeterminacy, “I”).

Definition 7. Let A be any non-empty set and “ \cdot ” be a hyperoperation on A . Then “ \cdot ” is called AntiAssociative on A if $a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$ for all $a, b, c \in A$.

Definition 8. Let A be any non-empty set and “ \cdot ” be a hyperoperation on A . Then “ \cdot ” is called a NeutroWeakAssociative on A if there exist $x, y, z, a, b, c, e, f, g \in A$ satisfying some (or all) of the following conditions in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $[x \cdot (y \cdot z)] \cap [(x \cdot y) \cdot z] \neq \emptyset$; (This condition is called degree of truth, “T”).
2. $[a \cdot (b \cdot c)] \cap [(a \cdot b) \cdot c] = \emptyset$; (This condition is called degree of falsity, “F”).
3. $e \cdot (f \cdot g)$ is indeterminate or $(e \cdot f) \cdot g$ is indeterminate or we cannot find if $e \cdot (f \cdot g)$ and $(e \cdot f) \cdot g$ have common elements. (This condition is called degree of indeterminacy, “I”).

Definition 9. Let A be a non-empty set and “ \cdot ” be a hyperoperation on A . Then (A, \cdot) is called a

1. NeutroHypergroupoid if “ \cdot ” is a NeutroHyperoperation.
2. NeutroSemihypergroup if “ \cdot ” is NeutroAssociative but not an AntiHyperoperation.

3. *NeutroH_v-Semigroup if “.” is NeutroWeakAssociative but not an AntiHyperoperation.*

Example 4. Let $A = \{0, 1\}$ and $(A, +)$ be defined by the following table.

+	0	1
0	{0, 1}	0
1	1	0

Then $(A, +)$ is a NeutroSemihypergroup and NeutroH_v-Semigroup. This is clear as

$$0 + (0 + 0) = \{0, 1\} = (0 + 0) + 0 \text{ and } (1 + 1) + 1 = 0 \neq 1 = 1 + (1 + 1).$$

Example 5. Let \mathbb{R} be the set of real numbers and define “ \star ” on \mathbb{R} as follows.

$$x \star y = \begin{cases} [x, y] & \text{if } x < y; \\ [y, x] & \text{if } y < x; \\ 0 & \text{if } x = y = 0; \\ \frac{1}{x} & \text{if } x = y \neq 0. \end{cases}$$

Then (\mathbb{R}, \star) is a NeutroSemihypergroup. This is clear as $(1 \star 1) \star 1 = 1 = 1 \star (1 \star 1)$ and $(1 \star 2) \star 2 = \{\frac{1}{2}\} \cup [1, 2] \neq [\frac{1}{2}, 1] = 1 \star (2 \star 2)$.

Example 6. Let $M = \{m, a, d\}$ and (M, \cdot) be defined by the following table.

·	m	a	d
m	m	m	m
a	m	{m, a}	d
d	m	d	d

Then (M, \cdot) is a NeutroSemihypergroup. This is clear as $m \cdot (m \cdot m) = m = (m \cdot m) \cdot m$ and $a \cdot (a \cdot d) = d \neq \{m, d\} = (a \cdot a) \cdot d$.

Remark 3. It is well known in classical algebraic hyperstructures that every semihypergroup is a hypergroupoid. This may fail to occur in NeutroHyperstructures. In Example 6, (M, \cdot) is a NeutroSemihypergroup that is not a NeutroHypergroupoid.

Proposition 1. Every H_v-semigroup that is not a semihypergroup and has an idempotent element is a NeutroSemihypergroup.

Proof. Let (H, \circ) be an H_v-semigroup with $h^2 = h$ for some $h \in H$. Then $h \circ (h \circ h) = h = (h \circ h) \circ h$. Since (H, \circ) is not a semihypergroup, it follows that there exist $x, y, z \in H$ with $x \circ (y \circ z) \neq (x \circ y) \circ z$. Therefore, (H, \circ) is a NeutroSemihypergroup. □

Example 7. Let $M = \{m, a, d\}$ and (M, \diamond) be defined by the following table.

◇	m	a	d
m	m	{a, d}	d
a	{a, d}	d	m
d	d	m	a

Then (M, \diamond) is an H_v-semigroup having m as an idempotent element and hence, it is a NeutroSemihypergroup.

Remark 4. It is well known in algebraic hyperstructures that every semihypergroup is an H_v -semigroup. This may not hold in NeutroHyperstructures. i.e., A NeutroSemihypergroup may not be a Neutro H_v -Semigroup.

The H_v -semigroup (M, \diamond) in Example 7 is a NeutroSemihypergroup that is not Neutro H_v -Semigroup.

Example 8. Let \mathbb{Z} be the set of integers and define “ \oplus ” on \mathbb{Z}^2 as follows. For all $m, n, p, q \in \mathbb{Z}$,

$$(m, 0) \oplus (0, 0) = (0, 0) \oplus (m, 0) = \{(0, 0), (m, 0)\},$$

$$(0, n) \oplus (0, 0) = (0, 0) \oplus (0, n) = \{(0, 0), (0, n)\},$$

and if $(n, p, q) \neq (0, 0, 0), (m, p, q) \neq (0, 0, 0)$

$$(m, n) \oplus (p, q) = (p, q) \oplus (m, n) = (m + p, n + q).$$

Then (\mathbb{Z}^2, \oplus) is a NeutroSemihypergroup. This is clear as

$$[(1, 2) \oplus (1, 3)] \oplus (1, 4) = (3, 9) = (1, 2) \oplus [(1, 3) \oplus (1, 4)]$$

and

$$[(1, 0) \oplus (1, 0)] \oplus (0, 0) = \{(2, 0), (0, 0)\} \neq \{(2, 0), (1, 0), (0, 0)\} = (1, 0) \oplus [(1, 0) \oplus (0, 0)].$$

Example 9. Let \mathbb{Z} be the set of integers and define “ \odot ” on \mathbb{Z}^2 as follows. For all $m, n, p, q \in \mathbb{Z}$,

$$(m, n) \odot (p, q) = \begin{cases} (mp, nq) & \text{if } (m, n) \neq (1, 1) \text{ and } (p, q) \neq (1, 1); \\ \{(p, q), (1, 1)\} & \text{if } (m, n) = (1, 1); \\ \{(m, n), (1, 1)\} & \text{if } (p, q) = (1, 1). \end{cases}$$

Then (\mathbb{Z}^2, \odot) is a NeutroSemihypergroup. This is clear as

$$[(1, 2) \odot (1, 3)] \odot (1, 4) = (1, 24) = (1, 2) \odot [(1, 3) \odot (1, 4)]$$

and

$$(1, 1) \odot [(2, 2) \odot (3, 3)] = \{(1, 1), (6, 6)\} \neq \{(1, 1), (3, 3), (6, 6)\} = [(1, 1) \odot (2, 2)] \odot (3, 3).$$

Example 10. Let \mathbb{Z}_6 be the set of integers under addition modulo 6 and define “ \boxplus ” on \mathbb{Z}_6 as follows.

$$x \boxplus y = (x + y) \pmod 6 \text{ for all } (x, y) \notin \{(\bar{0}, \bar{3}), (\bar{0}, \bar{5})\},$$

$$\bar{0} \boxplus \bar{3} = \{\bar{0}, \bar{3}\}, \text{ and } \bar{0} \boxplus \bar{5} = \{\bar{0}, \bar{5}\}.$$

Then (\mathbb{Z}_6, \boxplus) is a NeutroSemihypergroup. This is clear as $\bar{0} \boxplus (\bar{0} \boxplus \bar{0}) = \bar{0} = (\bar{0} \boxplus \bar{0}) \boxplus \bar{0}$ and $\bar{0} \boxplus (\bar{1} \boxplus \bar{2}) = \{\bar{0}, \bar{3}\} \neq \bar{3} = (\bar{0} \boxplus \bar{1}) \boxplus \bar{2}$.

Example 11. Let $M = \{m, a, d\}$ and (M, \bullet) be defined by the following table.

\bullet	m	a	d
m	a	a	d
a	$\{m, a\}$	m	d
d	d	d	m

Then (M, \bullet) is a Neutro H_v -Semigroup. This is clear as

$$[m \bullet (m \bullet m)] \cap [(m \bullet m) \bullet m] = \{a\} \cap \{m, a\} \neq \emptyset$$

and

$$[m \bullet (d \bullet d)] \cap [(m \bullet d) \bullet d] = \{a\} \cap \{m\} = \emptyset.$$

Moreover, (M, \bullet) is a NeutroSemihypergroup as $d \bullet (d \bullet d) = (d \bullet d) \bullet d$.

Remark 5. Every NeutroSemigroup is both: a NeutroSemihypergroup and a NeutroH_v-Semigroup. So, the results related to NeutroSemihypergroups (NeutroH_v-Semigroups) are more general than that related to NeutroSemigroups and as a result, we can deal with NeutroSemigroups as a special case of NeutroSemihypergroups (NeutroH_v-Semigroups).

Example 12. Let $S_1 = \{s, a, m\}$ and (S_1, \cdot_1) be defined by the following table.

\cdot_1	s	a	m
s	s	m	s
a	m	a	m
m	m	m	m

In [6], Al-Tahan et al. proved that (S_1, \cdot_1) is a NeutroSemigroup. Thus, (S_1, \cdot_1) is a NeutroSemihypergroup.

Theorem 1. Let (H, \circ) be a NeutroSemihypergroup (NeutroH_v-Semigroup) and “ \star ” be defined on H as $x \star y = y \circ x$ for all $x, y \in H$. Then (H, \star) is a NeutroSemihypergroup (NeutroH_v-Semigroup).

Proof. The proof is straightforward. \square

Example 13. Let $M = \{m, a, d\}$ and (M, \bullet) be the NeutroSemihypergroup defined in Example 11. By applying Theorem 1, we get that (M, \otimes) defined in the following table is a NeutroSemihypergroup and a NeutroH_v-Semigroup.

\otimes	m	a	d
m	a	$\{m, a\}$	d
a	a	m	d
d	d	d	m

Definition 10. Let (H, \circ) be a NeutroSemihypergroup (NeutroH_v-Semigroup) and $S \neq \emptyset \subseteq H$. Then S is a NeutroSubsemihypergroup (NeutroH_v-Subsemigroup) of H if (S, \circ) is a NeutroSemihypergroup (NeutroH_v-Semigroup).

Remark 6. Let (H, \circ) be a NeutroSemihypergroup (NeutroH_v-Semigroup) and $S \neq \emptyset \subseteq H$. Unlike the case in algebraic hyperstructures (Remark 2), proving that $a \circ b \subseteq S$ for all $a, b \in S$ does not imply that S is a NeutroSubsemihypergroup (NeutroH_v-Subsemigroup) of H .

As an illustration of Remark 6, $0 \star 0 = \{0\} \subseteq \{0\}$ in Example 5 but $\{0\}$ is not a NeutroSubsemihypergroup of \mathbb{R} .

Definition 11. Let (H, \circ) be a NeutroSemihypergroup (NeutroH_v-Semigroup) and $S \neq \emptyset \subseteq H$ be a NeutroSubsemihypergroup (NeutroH_v-Subsemigroup). Then

- (1) S is a NeutroLeftHyperideal of H if there exists $x \in S$ such that $r \circ x \subseteq S$ for all $r \in H$.
- (2) S is a NeutroRightHyperideal of S if there exists $x \in S$ such that $x \circ r \subseteq S$ for all $r \in H$.
- (3) S is a NeutroHyperideal of H if there exists $x \in S$ such that $r \circ x \subseteq S$ and $x \circ r \subseteq S$ for all $r \in H$.

A NeutroSemihypergroup (NeutroH_v-Semigroup) is called simple if it has no proper NeutroSubsemihypergroups (NeutroH_v-Subsemigroups).

Example 14. Let $(A, +)$ be the NeutroSemihypergroup defined in Example 4. Then A is simple. This is clear as $\{0\}$ and $\{1\}$ are the only options for any possible proper NeutroSubsemihypergroup and $(\{0\}, +)$ and $(\{1\}, +)$ are AntiHypergroupoids.

Example 15. Let (M, \bullet) be the NeutroSemihypergroup defined in Example 11. Then $\{m, a\}$ is a NeutroSubsemihypergroup of M .

Example 16. Let (\mathbb{Z}^2, \oplus) be the NeutroSemihypergroup defined in Example 8, $M_1 = \{(x, 0) : x \in \mathbb{Z}\}$, and $M_2 = \{(0, x) : x \in \mathbb{Z}\}$. Then M_1, M_2 are NeutroSubsemihypergroups of \mathbb{Z}^2 .

Remark 7. The intersection of NeutroSubsemihypergroups may fail to be a NeutroSubsemihypergroup. This is clear from Example 16 as $\{(0, 0)\} = M_1 \cap M_2$ is not a NeutroSubsemihypergroup of \mathbb{Z}^2 .

Lemma 1. Let (H, \circ) be a NeutroSemihypergroup (Neutro H_v -Semigroup) and A, B be hypergroupoids. If A, B are NeutroSubsemihypergroups (Neutro H_v -Subsemigroups) of H then $A \cup B$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H .

Proof. Let A, B be NeutroSubsemihypergroups. Since A and B are hypergroupoids, it follows that “ \circ ” is NeutroAssociative on both of A and B . The latter implies that there exist $x, y, z, a, b, c, e, f, g \in A \subseteq A \cup B$ satisfying some (or all) of the following conditions in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $T: x \circ (y \circ z) = (x \circ y) \circ z$;
2. $F: a \circ (b \circ c) \neq (a \circ b) \circ c$;
3. $I: e \circ (f \circ g)$ is indeterminate or $(e \circ f) \circ g$ is indeterminate or we cannot find if $e \circ (f \circ g)$ and $(e \circ f) \circ g$ are equal.

Therefore, $A \cup B$ is a NeutroSubsemihypergroup of H . The proof of (Neutro H_v -Subsemigroup) is done similarly. \square

Example 17. Let (\mathbb{Z}^2, \odot) be the NeutroSemihypergroup defined in Example 9, $N_1 = \{(x, y) \in \mathbb{Z}^2 : x, y \geq 1\} \cup \{(0, 0)\}$, and $N_2 = \{(x, y) \in \mathbb{Z}^2 : x, y \leq 1\} \cup \{(0, 0)\}$. Then N_1, N_2 are NeutroHyperideals of \mathbb{Z}^2 . We show that N_1 is a NeutroHyperideal of \mathbb{Z}^2 and N_2 may be done similarly. Since

$$[(1, 2) \odot (1, 3)] \odot (1, 4) = (1, 24) = (1, 2) \odot [(1, 3) \odot (1, 4)]$$

and

$$(1, 1) \odot [(2, 2) \odot (3, 3)] = \{(1, 1), (6, 6)\} \neq \{(1, 1), (3, 3), (6, 6)\} = [(1, 1) \odot (2, 2)] \odot (3, 3),$$

it follows that N_1 is a NeutroSubsemihypergroup of \mathbb{Z}^2 . Having $(0, 0) \in N_1$ and for all $(r, s) \in \mathbb{Z}^2$,

$$(r, s) \odot (0, 0) = (0, 0) \odot (r, s) = \begin{cases} (0, 0) & \text{if } (r, s) \neq (1, 1); \\ \{(0, 0), (1, 1)\} & \text{otherwise.} \end{cases} \subseteq N_1$$

implies that N_1 is a NeutroHyperideal of \mathbb{Z}^2 .

Remark 8. The intersection of NeutroHyperideals may fail to be a NeutroHyperideal. This is clear from Example 17 as $\{(0, 0), (1, 1)\} = N_1 \cap N_2$ is not a NeutroHyperideal of \mathbb{Z}^2 .

Lemma 2. Let (H, \circ) be a NeutroSemihypergroup (Neutro H_v -Semigroup) and A, B be hypergroupoids. If A, B are NeutroLeftHyperideals (NeutroRightHyperideals or NeutroHyperideals) of H . Then $A \cup B$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H .

Proof. Let A, B be NeutroLeftHyperideals of H . Lemma 1 asserts that $A \cup B$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H . Since A is a NeutroLeftHyperideal

of H , it follows that there exists $a \in A$ such that $r \circ a \subseteq A$ for all $r \in H$. The latter implies that there exists $a \in A \cup B$ such that $r \circ a \subseteq A \cup B$ for all $r \in H$. Thus, $A \cup B$ is a NeutroLeftHyperideal of H . \square

Definition 12. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi : H \rightarrow H'$ be a function. Then

- (1) ϕ is called NeutroHomomorphism if $\phi(x \circ y) = \phi(x) \star \phi(y)$ for some $x, y \in A$.
- (2) ϕ is called NeutroIsomomorphism if ϕ is a bijective NeutroHomomorphism.
- (3) ϕ is called NeutroStrongHomomorphism if for all $x, y \in A$, $\phi(x \circ y) = \phi(x) \star \phi(y)$ when $x \circ y \subseteq H$, $\phi(x) \star \phi(y) \not\subseteq H'$ when $x \circ y \not\subseteq H$, and $\phi(x) \star \phi(y)$ is indeterminate when $x \circ y$ is indeterminate.
- (4) ϕ is called NeutroStrongIsomomorphism if ϕ is a bijective NeutroOrderedStrongHomomorphism. In this case we say that $(H, \circ) \cong_{SI} (H', \star)$.

Example 18. Let (M, \bullet) and (M, \otimes) be the NeutroSemihypergroups defined in Examples 11 and 13, respectively. Then $(M, \bullet) \cong_{SI} (M, \otimes)$ as $\phi : (M, \bullet) \rightarrow (M, \otimes)$ is a NeutroStongIsomorphism. Here,

$$\phi(m) = a, \phi(a) = m, \text{ and } \phi(d) = d.$$

Theorem 2. The relation “ \cong_{SI} ” is an equivalence relation on the set of NeutroSemihypergroups (Neutro H_v -Semigroups).

Proof. By taking the identity map, we can easily prove that “ \cong_{SI} ” is a reflexive relation. Let $A \cong_{SI} B$. Then there exists a NeutroStrongIsomorphism $\phi : (A, \star) \rightarrow (B, \otimes)$. We prove that the inverse function $\phi^{-1} : B \rightarrow A$ of ϕ is a NeutroStrongIsomorphism. For all $b_1, b_2 \in B$, there exist $a_1, a_2 \in A$ with $\phi(a_1) = b_1$ and $\phi(a_2) = b_2$. We have

$$\phi^{-1}(b_1 \otimes b_2) = \phi^{-1}(\phi(a_1) \otimes \phi(a_2))$$

We consider the following cases for $\phi(a_1) \otimes \phi(a_2)$.

Case $\phi(a_1) \otimes \phi(a_2) \subseteq B$. Having ϕ a NeutroStrongIsomorphism and $\phi(a_1) \otimes \phi(a_2) \subseteq B$ imply that $a_1 \star a_2 \subseteq A$ and hence,

$$\phi^{-1}(b_1 \otimes b_2) = \phi^{-1}(\phi(a_1) \otimes \phi(a_2)) = \phi^{-1}(\phi(a_1 \star a_2)) = a_1 \star a_2 = \phi^{-1}(b_1) \star \phi^{-1}(b_2).$$

Case $\phi(a_1) \otimes \phi(a_2) \not\subseteq B$. Suppose, to get contradiction, that $\phi^{-1}(\phi(a_1)) \star \phi^{-1}(\phi(a_2)) = a_1 \star a_2 \subseteq A$ or indeterminate. Then by using our hypothesis that ϕ is NeutroStrongIsomorphism, we get that $\phi(a_1) \otimes \phi(a_2) \subseteq B$ or indeterminate.

Case $\phi(a_1) \otimes \phi(a_2)$ is indeterminate. Suppose, to get contradiction, that $\phi^{-1}(\phi(a_1)) \star \phi^{-1}(\phi(a_2)) = a_1 \star a_2 \subseteq A$ or $a_1 \star a_2 \not\subseteq A$. Then by using our hypothesis that ϕ is NeutroStrongIsomorphism, we get that $\phi(a_1) \otimes \phi(a_2) \subseteq B$ or $\phi(a_1) \otimes \phi(a_2) \not\subseteq B$.

Thus, $B \cong_{SI} A$ and hence, “ \cong_{SI} ” is a symmetric relation. Let $A \cong_{SI} B$ and $B \cong_{SI} C$. Then there exist NeutroStrongIsomorphisms $\phi : A \rightarrow B$ and $\psi : B \rightarrow C$. One can easily see that the composition function $\psi \circ \phi : A \rightarrow C$ of ψ and ϕ is a NeutroStrongIsomorphism. Thus, $A \cong_{SI} C$ and hence, “ \cong_{SI} ” is a transitive relation. \square

Lemma 3. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi : H \rightarrow H'$ be an injective NeutroStrongHomomorphism. If $M \subset H$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H then $\phi(M)$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H' .

Proof. Let M be a NeutroSubsemihypergroup of H . If “ \circ ” is NeutroHyperoperation on M then it is clear that “ \star ” is NeutroHyperoperation on $\phi(M)$. If “ \circ ” is NeutroAssociative

then there exist $x, y, z, a, b, c, d, e, f \in M$ satisfying some (or all) of the following conditions in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $T: x \circ (y \circ z) = (x \circ y) \circ z;$
2. $F: a \circ (b \circ c) \neq (a \circ b) \circ c;$
3. $I: e \circ (f \circ g)$ is indeterminate or $(e \circ f) \circ g$ is indeterminate or we cannot find if $e \circ (f \circ g)$ and $(e \circ f) \circ g$ are equal.

The latter and having ϕ an injective NeutroStrongHomomorphism imply that some (or all) of the following conditions are satisfied in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $T: \phi(x) \star (\phi(y) \star \phi(z)) = (\phi(x) \star \phi(y)) \star \phi(z);$
2. $F: \phi(a) \star (\phi(b) \star \phi(c)) \neq (\phi(a) \star \phi(b)) \star \phi(c);$
3. $I: \phi(e) \star (\phi(f) \star \phi(g))$ is indeterminate or $(\phi(e) \star \phi(f)) \star \phi(g)$ is indeterminate or we cannot find if $\phi(e) \star (\phi(f) \star \phi(g))$ and $(\phi(e) \star \phi(f)) \star \phi(g)$ are equal.

Thus, $\phi(M)$ is a NeutroSubsemihypergroup. The proof that $\phi(M)$ is a Neutro H_v -Subsemigroup of H' is done similarly. \square

Example 19. Let (M, \bullet) and (M, \otimes) be the NeutroSemihypergroups defined in Examples 11 and 13, respectively. Example 15 asserts that $\{m, a\}$ is a NeutroSubsemihypergroup of (M, \bullet) . Using Example 18 and Lemma 3, we get that $\{a, m\} = \{\phi(m), \phi(a)\}$ is a NeutroSubsemihypergroup of (M, \otimes) .

Lemma 4. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi: H \rightarrow H'$ be a NeutroStrongIsomorphism. If $N \subseteq H'$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H' then $\phi^{-1}(N)$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H .

Proof. Let $N \subset H'$ be a NeutroSubsemihypergroup of H' . If “ \star ” is NeutroHyperoperation on N then it is clear that “ \circ ” is NeutroHyperoperation on $\phi^{-1}(N)$. Let “ \star ” be NeutroAssociative. Having ϕ is an onto NeutroStrongHomomorphism implies that there exist $\phi(x), \phi(y), \phi(z), \phi(a), \phi(b), \phi(c), \phi(d), \phi(e), \phi(f) \in N$ satisfying some (or all) of the following conditions in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $T: \phi(x) \star (\phi(y) \star \phi(z)) = (\phi(x) \star \phi(y)) \star \phi(z);$
2. $F: \phi(a) \star (\phi(b) \star \phi(c)) \neq (\phi(a) \star \phi(b)) \star \phi(c);$
3. $I: \phi(e) \star (\phi(f) \star \phi(g))$ is indeterminate or $(\phi(e) \star \phi(f)) \star \phi(g)$ is indeterminate or we cannot find if $\phi(e) \star (\phi(f) \star \phi(g))$ and $(\phi(e) \star \phi(f)) \star \phi(g)$ are equal.

Having ϕ be an injective NeutroStrongHomomorphism implies that there exist $x, y, z, a, b, c, d, e, f \in \phi^{-1}(N)$ satisfying some (or all) of the following conditions in a way that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

1. $T: x \circ (y \circ z) = (x \circ y) \circ z;$
2. $F: a \circ (b \circ c) \neq (a \circ b) \circ c;$
3. $I: e \circ (f \circ g)$ is indeterminate or $(e \circ f) \circ g$ is indeterminate or we cannot find if $e \circ (f \circ g)$ and $(e \circ f) \circ g$ are equal.

Thus, $\phi^{-1}(N)$ is a NeutroSubsemihypergroup of H . The proof that $\phi^{-1}(N)$ is a Neutro H_v -Subsemigroup of H may be done similarly. \square

Theorem 3. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi: H \rightarrow H'$ be a NeutroStrongIsomorphism. Then $M \subseteq H$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H if and only if $\phi(M)$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H' .

Proof. The proof follows from Theorem 2 and Lemmas 3 and 4. \square

Corollary 1. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi: H \rightarrow H'$ be a NeutroStrongIsomorphism. Then H is simple if and only if H' is simple.

Proof. The proof follows from Theorem 3. \square

Lemma 5. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi : H \rightarrow H'$ be a NeutroStrongIsomorphism. If $M \subseteq H$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H then $\phi(M)$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H' .

Proof. Let $M \subseteq H$ be a NeutroLeftHyperideal of H . Lemma 3 asserts that $\phi(M)$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H' . Having M a NeutroLeftHyperideal of H implies that there exists $x \in M$ such that $r \circ x \subseteq M$ for all $r \in H$. Having ϕ an onto NeutroStrongHomomorphism implies that $\phi(r) \star \phi(x) \subseteq \phi(M)$ for all $s = \phi(r) \in H'$. Thus, $\phi(M)$ is a NeutroLeftHyperideal of H' . The proofs of NeutroRightHyperideal and NeutroHyperideal are done similarly. \square

Lemma 6. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi : H \rightarrow H'$ be a NeutroStrongIsomorphism. If $N \subseteq H'$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H' then $\phi^{-1}(N)$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H .

Proof. Let $N \subseteq H'$ be a NeutroLeftHyperideal of H' . Lemma 3 asserts that $\phi^{-1}(N)$ is a NeutroSubsemihypergroup (Neutro H_v -Subsemigroup) of H . Having N a NeutroLeftHyperideal of H' implies that there exists $y \in N$ such that $s \star y \subseteq N$ for all $s \in H'$. Since ϕ is an NeutroStrongHomomorphism, it follows that $\phi(r \circ x) \subseteq N$ for all $r \in H$ where $y = \phi(x)$. The latter implies that there exists $x \in \phi^{-1}(N)$ with $r \circ x \subseteq \phi^{-1}(N)$ for all $r \in H$. Thus, $\phi^{-1}(N)$ is a NeutroLeftHyperideal of H . The proofs of NeutroRightHyperideal and NeutroHyperideal are done similarly. \square

Theorem 4. Let $(H, \circ), (H', \star)$ be NeutroSemihypergroups (Neutro H_v -Semigroups) and $\phi : H \rightarrow H'$ be a NeutroStrongIsomorphism. Then $M \subseteq H$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H if and only if $\phi(M)$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H' .

Proof. The proof follows from Theorem 2, Lemmas 5 and 6. \square

Let H_α be any non-empty set for all $\alpha \in \Gamma$ and " \cdot_α " be a hyperoperation on H_α . We define " \circ " on $\prod_{\alpha \in \Gamma} H_\alpha$ as follows: For all $(x_\alpha), (y_\alpha) \in \prod_{\alpha \in \Gamma} H_\alpha$, $(x_\alpha) \circ (y_\alpha) = \{(t_\alpha) : t_\alpha \in x_\alpha \cdot_\alpha y_\alpha\}$.

Theorem 5. Let (H_1, \circ_1) and (H_2, \circ_2) be hypergroupoids. Then $(H_1 \times H_2, \circ)$ is a NeutroSemihypergroup (Neutro H_v -Semigroup) if and only if either (H_1, \circ_1) is a NeutroSemihypergroup (Neutro H_v -Semigroup) or (H_2, \circ_2) is a NeutroSemihypergroup (Neutro H_v -Semigroup) or both are NeutroSemihypergroups (Neutro H_v -Semigroups).

Proof. The proof is straightforward. \square

Example 20. Let (\mathbb{R}, \ast) be the semihypergroup defined as: $x \ast y = \{x, y\}$ for all $x, y \in \mathbb{R}$ and (M, \cdot) be the NeutroSemihypergroup defined in Example 6. Then the following are true.

1. $(\mathbb{R} \times M, \circ)$ is a NeutroSemihypergroup,
2. $(M \times \mathbb{R}, \circ)$ is a NeutroSemihypergroup, and
3. $(M \times M, \circ)$ is a NeutroSemihypergroup.

In what follows, we present a way to construct a new NeutroSemihypergroup (Neutro H_v -Semigroup) from an existing one. This tool is of great importance to prove that for any positive integer $n \geq 2$, there exists at least one NeutroSemihypergroup (Neutro H_v -Semigroup) of order n .

Let (H, \circ) be a NeutroSemihypergroup (Neutro H_v -Semigroup) and J be any non-empty set such that $H \cap J = \emptyset$ and $(H \circ H) \cap J = \emptyset$. The extension $H[J]$ of H by J is given as $H[J] = H \cup J$. We define the hyperoperation “ \odot ” on $H[J]$ as follows.

$$x \odot y = \begin{cases} x \circ y & \text{if } x, y \in H; \\ H \cup J & \text{otherwise.} \end{cases}$$

Theorem 6. Let (H, \circ) be a NeutroSemihypergroup (Neutro H_v -Semigroup) and J be any non-empty set such that $H \cap J = \emptyset$ and $(H \circ H) \cap J = \emptyset$. Then $(H[J], \odot)$ is a NeutroSemihypergroup (Neutro H_v -Semigroup).

Proof. Let (H, \circ) be a NeutroSemihypergroup. If “ \circ ” is a NeutroHyperoperation then there exist $u, v, w, x, y, z \in H$ with $u \circ v \subseteq H$ representing “T”, $w \circ x \not\subseteq H$ representing “F”, $y \circ z$ is indeterminate representing “I”. Where $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$. Since $(H \circ H) \cap J = \emptyset$, it follows that there exist $u, v, w, x, y, z \in H$ with $u \circ v \subseteq H[J]$ representing “T”, $w \circ x \not\subseteq H[J]$ representing “F” (as $w \circ x \not\subseteq H$ and $w \circ x \not\subseteq J$), $y \circ z$ is indeterminate representing “I”. Where $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$. Thus, “ \odot ” is NeutroHyperoperation on $H[J]$. If “ \circ ” is NeutroAssociative on H then it is clear that “ \odot ” is NeutroAssociative on $H[J]$. Therefore, $(H[J], \odot)$ is a NeutroSemihypergroup. The case $(H[J], \odot)$ is a Neutro H_v -Semigroup is done similarly. \square

Example 21. Let (M, \cdot) be the NeutroSemihypergroup defined in Example 6 and $N = \{n\}$. Then $M[N] = \{m, a, d, n\}$ and $(M[N], \odot)$ is the NeutroSemihypergroup defined by the following table.

\odot	m	a	d	n
m	m	m	m	$\{m, a, d, n\}$
a	m	$\{m, a\}$	d	$\{m, a, d, n\}$
d	m	d	d	$\{m, a, d, n\}$
n	$\{m, a, d, n\}$	$\{m, a, d, n\}$	$\{m, a, d, n\}$	$\{m, a, d, n\}$

Theorem 7. Let $n \geq 2$ be an integer. Then there is at least one NeutroSemihypergroup of order n .

Proof. The proof follows from Example 4 and Theorem 6. \square

Corollary 2. There are infinitely many NeutroSemihypergroups up to NeutroStrongIsomorphism.

Proof. The proof follows from Theorem 7. \square

Theorem 8. Let $n \geq 2$ be any integer. Then there is at least one Neutro H_v -Semigroup of order n .

Proof. The proof follows from Example 4 and Theorem 6. \square

Corollary 3. There are infinitely many Neutro H_v -Semigroups up to NeutroStrongIsomorphism.

Proof. The proof follows from Theorem 8. \square

4. Conclusions

In this paper, we discussed the properties of some NeutroHyperstructures. More precisely, we introduced NeutroSemihypergroups (Neutro H_v -Semigroups), constructed several examples, and studied some of their important subsets under NeutroStrongIsomorphism. It was shown through examples that some of the well known results for algebraic hyperstructures do not hold for NeutroHyperstructures. Moreover, it was proved that there is at least one NeutroSemihypergroup (Neutro H_v -Semigroups) of order n where n is any integer greater than one. The results in this paper may be considered as a base for any possible study in the field of NeutroHyperstructures.

For future research, we raise the following ideas.

1. Find all NeutroSemihypergroups (Neutro H_v -Semigroups) of small order (up to NeutroStrongIsomorphism).
2. Find bounds for the number of finite NeutroSemihypergroups (Neutro H_v -Semigroups) of arbitrary order n (up to NeutroStrongIsomorphism).
3. Classify simple NeutroSemihypergroups (Neutro H_v -Semigroups) up to NeutroStrongIsomorphism.
4. Define other NeutroHyperstructures such as NeutroPolygroup, NeutroHyperring, etc.
5. Find applications of NeutroHyperstructures in some fields like Biology, Physics, Chemistry, etc.

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