On Studying Certain Fuzzy Neutrosophic Matrices and Operators

R.Sophia Porchelvi\(^1\) and V.Jayapriya\(^2\)

\(^1\)Department of Mathematics, ADM College for Women (Autonomous) 
Nagapattinam, Tamilnadu, India. 
Email: sophiaporchelvi@gmail.com

\(^2\)Department of Mathematics, Idhaya College for Women, Kumbakonam, Tamilnadu 
India. Email: vaishnamurugan@gmail.com

Received 18 December 2018; accepted 28 January 2019

ABSTRACT

In this paper, some types of neutrosophic fuzzy matrices have been detailed and the following operators \(\oplus, \odot, \ominus\) are defined for this neutrosophic fuzzy matrices. Certain Properties based on these operators \(\oplus, \odot, \ominus\) have been presented. In addition, some results on the existing operators like \(\lor, \land\) along with these operators are also put forth. Numerical examples are also provided.

Keywords: Neutrosophic set; Neutrosophic fuzzy matrix

Mathematical Subject Classification (2010): 15B15

1. Introduction

Neutrosophic set is a new mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 1988, Smarandache introduce the concept of a neutrosophic set from a philosophical point of view. The neutrosophic set is a powerful general framework that generalizes the concept of fuzzy set and intuitionistic fuzzy set. Each element had three associated defining functions, namely the membership function (T), indeterminacy function (I), the non-membership function (F) defined on the universe of discourse X, the three functions are completely independent. The theory has been found extensive application in various fields for dealing with indeterminate and inconsistent information in real world.

Fuzzy matrices play an important role in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In [7] Thomasan introduced fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In 2004, Kandasamy and Smarandache [5] introduced fuzzy relational maps and neutrosophic relational maps. In 2014, Smarandache [6] introduced a type of neutrosophic matrices, called Square neutrosophic matrices, whose entries are of the form \(a+Ib\) (neutrosophic number), where \(a,b\) are the elements of \([0,1]\) and \(I\) is an indeterminate such that \(I^n = I\), \(n\) being a positive integer.
2. Preliminaries

**Definition 2.1. (Neutrosophic set)** Let X be a non-empty set. A neutrosophic set A is an object having the form \( A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \} \) where the functions \( \mu_A : X \to [0, 1^+] \), \( \sigma_A : X \to [0, 1^+] \), \( \nu_A : X \to [0, 1^+] \) denote the degree of membership function, degree of indeterminacy function, degree of non-membership function respectively of each element \( x \in X \) to the set \( A \) and \( 0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+ \), for each \( x \in X \).

**Definition 2.2. (Neutrosophic fuzzy set)** Let X be a non-empty set. A neutrosophic fuzzy set A is an object having the form \( A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \} \) where the functions \( \mu_A : X \to [0, 1^+] \), \( \sigma_A : X \to [0, 1^+] \), \( \nu_A : X \to [0, 1^+] \) denote the degree of membership function, degree of indeterminacy function, degree of non-membership function respectively of each element \( x \in X \) to the set \( A \) and \( 0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+ \), for each \( x \in X \).

**Definition 2.3. (Complement of a neutrosophic fuzzy set)** Let X be a non-empty set and let \( A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \} \) and \( B = \{ (x, \mu_B(x), \sigma_B(x), \nu_B(x)) : x \in X \} \) two neutrosophic fuzzy sets on X then
(i) \( A \subseteq B \) iff \( \mu_A(x) \leq \mu_B(x) \), \( \sigma_A(x) \leq \sigma_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \).
(ii) \( A = B \) iff \( A \subseteq B \) and \( B \subseteq A \).
(iii) \( \tilde{A} = \{ (x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)) : x \in X \} \).
(iv) \( A \oplus B = \{ (x, \mu_A(x)) \land \nu_B(x), \sigma_A(x) \land (1 - \sigma_B(x)), \nu_A(x) \lor \mu_B(x) \} \).

**Definition 2.4. (Empty neutrosophic fuzzy set)** A Neutrosophic fuzzy set A over the universe X is said to be null or empty neutrosophic fuzzy set if \( \mu_A(x) = 0, \sigma_A(x) = 0, \nu_A(x) = 1 \) for all \( x \in X \). It is denoted by \( 0_N \).

**Definition 2.5. (Absolute neutrosophic fuzzy set)** A Neutrosophic fuzzy set A over the universe X is said to be absolute neutrosophic fuzzy set if \( \mu_A(x) = 1, \sigma_A(x) = 1, \nu_A(x) = 0 \) for all \( x \in X \). It is denoted by \( 1_N \).

**Definition 2.6. (Union and Intersection of neutrosophic fuzzy sets)** Let X be a non-empty set and let \( A_i \) \( i \in I \) be an arbitrary family of Neutrosophic fuzzy sets in X where \( A_i = \{ (x, \mu_{A_i}(x), \sigma_{A_i}(x), \nu_{A_i}(x)) : x \in X \} \), then
(i) \( \bigcup A_i = \{ (x, \lor_{i \in I} \mu_{A_i}(x), \land_{i \in I} \sigma_{A_i}(x), \lor_{i \in I} \nu_{A_i}(x)) \} \)
\[(ii) \quad \text{\(\bigcap A_i = \{\langle x, \bigwedge_{i \in J} \mu_A(x), \bigwedge_{i \in J} \sigma_A(x), \bigvee_{i \in J} \nu_A(x)\}\)\]}

3. Fundamental neutrosophic fuzzy matrices

**Definition 3.1. (Neutrosophic fuzzy matrix)** Neutrosophic fuzzy matrix of order \(m \times n\) is defined as \(A = (a_{ij})_{m \times n}\) where \(a_{ij} = (\mu_A(x), \sigma_A(x), \gamma_A(x))\) is the \(ij\)th element of \(A\) where \(\mu_A(x)\) denote the degree of membership function, \(\sigma_A(x)\) denote the degree of indeterminacy function and \(\gamma_A(x)\) denote the degree of non membership function respectively.

**Definition 3.2. (Null neutrosophic fuzzy matrix)** Neutrosophic fuzzy matrix is said to be Null Neutrosophic fuzzy matrix if all its entries are zero, i.e., all elements are \((0,0,1)\).

**Definition 3.3. (Unit neutrosophic fuzzy matrix)** A square Neutrosophic fuzzy matrix is said to be unit Neutrosophic fuzzy matrix if \(a_{ii} = (1,1,0)\) and \(a_{ij} = (0,0,1)\), \(i \neq j\) for all \(i, j\). It is denoted by \(I\).

**Definition 3.4 (Symmetric neutrosophic fuzzy matrix)** A square Neutrosophic fuzzy matrix is said to be symmetric Neutrosophic fuzzy matrix if \(a_{ij} = a_{ji}\).

**Definition 3.5. (Triangular neutrosophic fuzzy matrix)** A square Neutrosophic fuzzy matrix is said to be triangular Neutrosophic fuzzy matrix if either \(a_{ij} = (0,0,1)\) for all \(i > j\) or \(a_{ij} = (0,0,1)\) for all \(i < j\). A square Neutrosophic fuzzy matrix is said to be upper triangular Neutrosophic fuzzy matrix if either \(a_{ij} = (0,0,1)\) for all \(i > j\) and is said to be lower triangular Neutrosophic fuzzy matrix if \(a_{ij} = (0,0,1)\) for all \(i < j\).

3.1. Special neutrosophic fuzzy matrices

**Definition 3.1.1. Vandermonde neutrosophic fuzzy matrix**
A Vandermonde Neutrosophic fuzzy matrix of order \(n\) is of the form
\[
\begin{bmatrix}
(0,0,1) & a_1 & a_1^2 & \cdots & a_1^{n-1} \\
(0,0,1) & a_2 & a_2^2 & \cdots & a_2^{n-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
(0,0,1) & a_n & a_n^2 & \cdots & a_n^{n-1}
\end{bmatrix}
\]
A Vandermonde Neutrosophic fuzzy matrix is also called an alternate neutrosophic fuzzy matrix. Sometimes, the transpose of an alternate fuzzy matrix is known as the Vandermonde defuzzy matrix.

**Definition 3.1.2. Moore neutrosophic fuzzy matrix**
A moore neutrosophic fuzzy matrix is a Neutrosophic fuzzy matrix of the form
Definition 3.1.3. Anti –diagonal neutrosophic fuzzy matrix

Anti – diagonal is the diagonal of a fuzzy matrix starting from the lower left corner to the upper right corner of the matrix. An anti-diagonal neutrosophic fuzzy matrix is a Neutrosophic fuzzy matrix where all the entries are zero except the anti – diagonal.

\[
A = \begin{bmatrix}
  a & (0,0,1) & (0,0,1) \\
  (0,0,1) & b & (0,0,1) \\
  (0,0,1) & (0,0,1) & c
\end{bmatrix}
\]

Neutrosophic fuzzy matrix is a square Neutrosophic fuzzy matrix which has zeros in all entries except the first row, first column, and the main diagonal. The general form of an Arrow head matrix is

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} \\
  a_{21} & a_{22} & (0,0,1) & (0,0,1) \\
  a_{31} & (0,0,1) & a_{33} & (0,0,1) \\
  a_{41} & (0,0,1) & (0,0,1) & a_{44}
\end{bmatrix}
\]

Definition 3.1.5. Band neutrosophic fuzzy matrix

A band Neutrosophic fuzzy matrix is a sparse Neutrosophic fuzzy matrix whose non-zero entries are confined to a diagonal band, comprising the main diagonal and zero or more diagonals on either side. The general form of a Band matrix is

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & (0,0,1) & (0,0,1)(0,0,1) \\
  a_{21} & a_{22} & a_{23} & (0,0,1)(0,0,1) \\
  a_{31} & (0,0,1) & a_{33} & a_{34} (0,0,1) \\
  a_{41} (0,0,1) & (0,0,1) & a_{43} & a_{44} a_{45} \\
  a_{51} (0,0,1) & (0,0,1) & a_{53} a_{54} & a_{55}
\end{bmatrix}
\]

Let us consider a \( n \times n \) fuzzy matrix \( A = (a_{ij}) \). If all the elements of the matrix are zero outside a diagonally bordered band whose range is determined by the constants \( p \) and \( q \) as follows:

\[ a_{ij} = 0 \text{ if } j < i - p \text{ or } j > i + q, \; p, q \geq 0, \text{ then the quantities } p \text{ and } q \text{ are called the lower and upper bandwidth, respectively. The bandwidth of the matrix is the maximum of } p \text{ and } q, \text{ that is it is the number } r \text{ such that } a_{ij} = 0 \text{ if } |i - j| > r. \]

A Neutrosophic fuzzy matrix is called a banded neutrosophic fuzzy matrix if its bandwidth is reasonably small. A band Neutrosophic fuzzy matrix with \( p = q = 0 \) is a diagonal Neutrosophic fuzzy matrix. A band Neutrosophic fuzzy matrix with \( p = q = 1 \) is a tridiagonal Neutrosophic fuzzy matrix. When \( p = q = 2 \), it is a pentadiagonal Neutrosophic fuzzy matrix. If \( p = 0, \; q = n-1 \), we get an upper triangular Neutrosophic fuzzy matrix. Similarly, for \( p = n-1, q = 0 \), we obtain a lower triangular Neutrosophic fuzzy matrix.
On Studying Certain Fuzzy Neutrosophic Matrices and Operators

**Definition 3.1.6. Binary neutrosophic fuzzy matrix**
A Binary Neutrosophic fuzzy matrix or a Boolean neutrosophic matrix is a being Neutrosophic fuzzy matrix with entries either zero or one.

**Definition 3.1.7. Permutation neutrosophic fuzzy matrix**
A Permutation Neutrosophic fuzzy matrix is a square fuzzy neutrosophic matrix obtained from the same size identity matrix by a permutation of rows. Every row and column has a single \((1,1,0)\)’s everywhere else.

**Definition 3.1.8. Exchange neutrosophic fuzzy matrix**
The exchange fuzzy neutrosophic matrix is an anti-diagonal fuzzy neutrosophic matrix in which all the entries in the anti-diagonal are \((1,1,0)\) and all the other elements are \((0,0,1)\). It is also a special case of permutation Neutrosophic fuzzy matrix.

**Definition 3.1.9. Bisymmetric neutrosophic fuzzy matrix**
A bisymmetric Neutrosophic fuzzy fuzzy matrix is a square fuzzy neutrosophic matrix which is symmetric about both of its main diagonals. More precisely an \(n \times n\) matrix \(B\) is bisymmetric Neutrosophic fuzzy fuzzy matrix if it satisfies both \(B = B^T\) and \(BE = EB\) where \(E\) is the \(n \times n\) exchange fuzzy neutrosophic matrix.

\[
B = \begin{bmatrix}
1 & a & b & c & d \\
1 & e & f & g & h \\
1 & i & j & k & l \\
1 & m & n & o & p \\
1 & q & r & s & t
\end{bmatrix}
\]

**Definition 3.1.10. Centro symmetric neutrosophic fuzzy matrix**
A matrix which is symmetric about its center is called centro symmetric fuzzy neutrosophic matrix. More precisely \(A = [A_{ij}]\) is centro symmetric when its entries satisfy \(A_{ij} = A_{m1-i+1,m-j+1}\) for \(1 \leq i,j \leq m\). \((i.e.)\) If \(E\) denotes the \(m \times m\) matrix with all the counterdiagonal elements as \((1,1,0)\) and \((0,0,1)\) elsewhere, then a matrix \(A\) is centrosymmetric if and only if \(AE = EA\). The matrix \(E\) sometimes named as the exchange matrix.

![Diagram](image)

**Definition 3.1.11. Persymmetric neutrosophic fuzzy matrix**
A persymmetric fuzzy neutrosophic matrix is a square fuzzy neutrosophic matrix which is symmetric in the northeast-to-southeast diagonal or a square fuzzy neutrosophic matrix such that the values on each line perpendicular to the main diagonal are the same for a given line.
For example,
\[
A_{\text{Block}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]
where A,B,C,D are fuzzy neutrosophic matrices, is a Block Neutrosophic fuzzy matrix.

**Definition 3.1.12. Block neutrosophic fuzzy matrix**
A Block Neutrosophic fuzzy matrix is a fuzzy neutrosophic matrix which is obtained using smaller matrices called blocks. It is also called partitioned fuzzy neutrosophic matrices.

**Definition 3.1.13. Sparseneutrosophic fuzzy matrix**
A Neutrosophic fuzzy matrix in which most of its elements are zero is called a sparse fuzzy neutrosophic matrix. Also, if most of the elements are nonzero, then the neutrosophic fuzzy matrix is said to be dense.

**Definition 3.1.14. Toeplitzung neutrosophic fuzzy matrix**
A n × n Neutrosophic fuzzy matrix in which the negative sloping diagonal elements are constants is called a Toeplitzung neutrosophic fuzzy matrix. Its of the form

\[
T = \begin{bmatrix}
x_0 & x_{-1} & x_{-2} & \cdots & x_{-(n-1)} \\
x_1 & x_0 & x_{-1} & \cdots & x_{-(n-2)} \\
x_2 & x_1 & x_0 & \cdots & x_{-(n-3)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{n-1} & \cdots & \cdots & \cdots & x_0
\end{bmatrix}
\]

**4. Operators on neutrosophic fuzzy matrices**
Let \( M = (M_{ij}) \) and \( N = (N_{ij}) \) be two Neutrosophic fuzzy matrices of order \( n \times n \) then
(i) \( M \oplus N = [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})] \)
(ii) \( M \odot N = [m_{ij} \cdot n_{ij}] \)
(iii) \( M \ominus N = [m_{ij} - n_{ij}] \)
(iv) \( M' = [m_{ij}] \)
(v) \( M \land N = [m_{ij} \lor n_{ij}] \)
(vi) \( M \lor N = [m_{ij} \lor n_{ij}] \)

**Property 1.** If \( M \) and \( N \) are symmetric Neutrosophic fuzzy matrices then \( M \oplus N, M \odot N, M \ominus N, N \ominus M \) are symmetric Neutrosophic fuzzy matrices.

**Proof:** Let \( M = [m_{ij}] \) and \( N = [n_{ij}] \) be two symmetric Neutrosophic fuzzy matrices. Therefore \( m_{ij} = m_{ji} \) and \( n_{ij} = n_{ji} \)

Let \( c_{ij} \) be the \( ij \)th element of \( M \oplus N \)

\[
c_{ij} = (m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij}) = (m_{ji} + n_{ji}) - (m_{ji} \cdot n_{ji}) = c_{ji}
\]
On Studying Certain Fuzzy Neutrosophic Matrices and Operators

Hence $M \oplus N$ is symmetric Neutrosophic fuzzy matrices
Let $d_{ij}$ be the $ij$th element of $M \ominus N$

d_{ij} = m_{ij} \cdot n_{ij} = m_{ji} \cdot n_{ji} = d_{ji}

Hence $M \ominus N$ is symmetric Neutrosophic fuzzy matrix.

Property 2. Let $M, N$ be any two Neutrosophic fuzzy matrices, then
(i) $(M \odot N)' = M' \odot N'$
(ii) $(M \oplus N)' = M' \oplus N'$

Proof:
(i) Let $c_{ij}$ and $d_{ij}$ be the $ij$th element of $M \odot N$ and $M' \odot N'$ respectively.

$e_{ij} = c_{ij}$ is the $ij$th element of $(M \odot N)'$. Then $c_{ij} = m_{ij} \cdot n_{ij}$. Thus $c_{ij} = m_{ji} \cdot n_{ji}$.

Hence $M (M \odot N)' = M' \odot N'$.

(ii) Let $c_{ij}$ and $d_{ij}$ be the $ij$th element of $M \oplus N$ and $M' \oplus N'$ respectively.

Then, $c_{ij} = (m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})$ and $d_{ij} = (m_{ij} + n_{ij}) - (m_{ji} \cdot n_{ji})$

$e_{ij} = (m_{ij} + n_{ij}) - (m_{ji} \cdot n_{ji}) = d_{ij}$.

Hence $(M \oplus N)' = M' \oplus N'$.

Property 3. Let $M, N, P$ be three Neutrosophic fuzzy matrices, then
(i) $M \oplus N = N \oplus M$
(ii) $(M \odot N) \odot P = M \odot (N \odot P)$
(iii) $(M \oplus N) \oplus P = M \oplus (N \oplus P)$

Proof:
(i) $M \oplus N = [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})] = [(m_{ij} + m_{ij}) - (n_{ij} \cdot m_{ij})] = N \oplus M$

(ii) $M \odot N = m_{ij} \cdot n_{ij} = n_{ij} \cdot m_{ij} = N \odot M$

(iii) $(M \oplus N) \odot P = [(r_{ij} + p_{ij}) - (r_{ij} \cdot p_{ij})] = M \odot (N \oplus P)$

(iv) $(M \oplus N) \oplus P = (m_{ij} \cdot n_{ij}) p_{ij} = m_{ij} \cdot n_{ij} \cdot p_{ij} = m_{ij} \cdot (n_{ij} \cdot p_{ij}) = M \odot (N \oplus P)$

Property 4. If $M$ is a symmetric Neutrosophic fuzzy matrix then $I_3 \odot (M \oplus M')$ is symmetric Neutrosophic fuzzy matrix.

Proof: $M \oplus M' = [(m_{ij} + m_{ji}) - (m_{ij} \cdot m_{ji})] \text{ and } I_3 \odot (M \oplus M') = t_{ij}$, where $t_{ii} = 1$
and $t_{ij} = [(m_{ij} + m_{ji}) - (m_{ij} \cdot m_{ji})]$ for $i \neq j$.  

69
R. Sophia Porchelvi and V. Jayapriya

Now, \( t_{ij} = [(m_{ij} + m_{ji}) - (m_{ij} \cdot m_{ji})] = t_{ij}. \) That is each diagonal elements of \( I_n \oplus (M \oplus M') \) is 1 and all non diagonal elements are \((m_{ij} + m_{ji}) - (m_{ij} \cdot m_{ji}). \)

Hence \( I_n \oplus (M \oplus M') \) is symmetric Neutrosophic fuzzy matrix. 

**Property 5.** For any two Neutrosophic fuzzy matrices M and N, \((M \vee N) \oplus (M \ominus N) = (M \vee N). \)

**Proof:** Let \( c_{ij}, d_{ij}, e_{ij} \) and \( f_{ij} \) be the ijth elements of \((M \vee N), (M \ominus N)\) and \((M \vee N) \oplus (M \ominus N)\) respectively.

Let \( c_{ij} = (m_{ij} \vee n_{ij}) = \{ \mu_{m_{ij}} \vee \mu_{n_{ij}}, \sigma_{m_{ij}} \vee \sigma_{n_{ij}}, \gamma_{m_{ij}} \wedge \gamma_{n_{ij}} \} \)

\( d_{ij} = \{ \mu_{m_{ij}} \wedge \gamma_{n_{ij}}, \sigma_{m_{ij}} \wedge (1 - \sigma_{n_{ij}}), \gamma_{m_{ij}} \vee \mu_{n_{ij}} \} \)

\( e_{ij} = \{ \mu_{m_{ij}} \vee \mu_{n_{ij}}, \sigma_{m_{ij}} \vee \sigma_{n_{ij}}, \gamma_{m_{ij}} \wedge \gamma_{n_{ij}} \} \wedge (\mu_{m_{ij}} \wedge \gamma_{n_{ij}}, \sigma_{m_{ij}} \wedge (1 - \sigma_{n_{ij}}), \gamma_{m_{ij}} \vee \mu_{n_{ij}}) = c_{ij}. \)

Therefore \((M \vee N) \oplus (M \ominus N) = (M \vee N). \)

**Property 6.** Let M, N and P be three Neutrosophic fuzzy matrices, then \( M \oplus (N \vee P) = (M \oplus N) \oplus (N \vee P). \)

**Proof:** Let \( c_{ij}, d_{ij}, e_{ij}, f_{ij} \) and \( g_{ij} \) be the ijth elements of \((N \vee P), (M \oplus N), (M \oplus P), M \oplus (N \vee P)\) and \((M \oplus N) \vee (M \oplus P)\) respectively.

Let \( c_{ij} = \max \{ n_{ij} \vee p_{ij} \}, d_{ij} = [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})], e_{ij} = [(n_{ij} + p_{ij}) - (n_{ij} \cdot p_{ij})] \)

\( f_{ij} = m_{ij} \oplus \max \{ n_{ij} \vee p_{ij} \} = \begin{cases} [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})], & n_{ij} > p_{ij} \\ [(m_{ij} + p_{ij}) - (m_{ij} \cdot p_{ij})], & n_{ij} \leq p_{ij} \end{cases} \)

\( g_{ij} = \max \{ d_{ij} \vee e_{ij} \} = \begin{cases} [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})], & d_{ij} > e_{ij} \\ [(m_{ij} + p_{ij}) - (m_{ij} \cdot p_{ij})], & d_{ij} \leq e_{ij} \end{cases} \)

Therefore, \( M \oplus (N \vee P) = (M \oplus N) \vee (M \oplus P). \)

5. Numerical examples

**Example 5.1.**

\[
M = \begin{bmatrix}
0.2,0.3,0.5 & 0.1,0.3,0.5 \\
0.1,0.3,0.5 & 0.4,0.3,0.3
\end{bmatrix}, \quad N = \begin{bmatrix}
0.1,0.4,0.5 & 0.3,0.6,0.7 \\
0.3,0.6,0.7 & 0.2,0.4,0.5
\end{bmatrix}
\]

are symmetric Neutrosophic fuzzy matrices.

(i) \( M \oplus N = \begin{bmatrix}
0.27,0.58,0.75 & 0.27,0.72,0.85 \\
0.27,0.72,0.85 & 0.27,0.58,0.75
\end{bmatrix} \) is symmetric neutrosophic fuzzy matrix.

(ii) \( M \ominus N = \begin{bmatrix}
0.02,0.12,0.25 & 0.03,0.18,0.35 \\
0.03,0.18,0.35 & 0.02,0.12,0.25
\end{bmatrix} \) is symmetric neutrosophic fuzzy matrix.
On Studying Certain Fuzzy Neutrosophic Matrices and Operators

(iii) \( M \odot N = \begin{bmatrix} (0.2,0.3,0.5) & (0.1,0.3,0.5) \\ (0.1,0.3,0.5) & (0.4,0.3,0.3) \end{bmatrix} \) is symmetric neutrosophic fuzzy matrix.

(iv) \( N \odot M = \begin{bmatrix} (0.1,0.4,0.5) & (0.3,0.6,0.7) \\ (0.3,0.6,0.7) & (0.2,0.4,0.5) \end{bmatrix} \) is symmetric neutrosophic fuzzy matrix.

Example 5.2.
\[
M = \begin{bmatrix} (0.3,0.2,0.5) & (0.4,0.3,0.3) \\ (0.3,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix}, \quad N = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.4,0.3) \\ (0.4,0.5,0.1) & (0.2,0.4,0.4) \end{bmatrix}
\]
\[
M' = \begin{bmatrix} (0.3,0.2,0.5) & (0.3,0.1,0.6) \\ (0.4,0.3,0.3) & (0.2,0.3,0.5) \end{bmatrix}, \quad N' = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.4,0.3) \\ (0.2,0.4,0.4) \end{bmatrix}
\]

(i) \( (M \odot N)' = \begin{bmatrix} (0.06,0.02,0.35) & (0.12,0.12,0.09) \\ (0.12,0.05,0.06) & (0.04,0.12,0.20) \end{bmatrix} = M' \odot N' \)

(ii) \( (M \oplus N)' = \begin{bmatrix} (0.44,0.28,0.85) & (0.58,0.58,0.51) \\ (0.58,0.45,0.54) & (0.36,0.58,0.7) \end{bmatrix} = M' \oplus N' \)

Example 5.3. Consider \( M \) and \( N \) in Example 5.2
\[
M \vee N = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.3,0.3) \\ (0.3,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix}, \quad M \ominus N = \begin{bmatrix} (0.3,0.2,0.2) & (0.3,0.3,0.3) \\ (0.1,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix}
\]
\[
(M \vee N) \vee (M \ominus N) = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.3,0.3) \\ (0.3,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix} = (M \vee N).
\]

6. Conclusion
Up to this point we introduce Neutrosophic fuzzy matrix and some special types Neutrosophic fuzzy matrix and also we investigated the properties of Neutrosophic fuzzy matrix based on these operators \( \oplus, \odot, \ominus \). Further some future works are necessary to deal with determinants and properties of determinants of such kinds of matrices.

Conflict of Interests
The authors have declared that no Conflict of Interest exists.

REFERENCES