emerald insight



Kybernetes

On various neutrosophic topologies Francisco Gallego Lupiáñez,

Article information:

To cite this document: Francisco Gallego Lupiáñez, (2009) "On various neutrosophic topologies", Kybernetes, Vol. 38 Issue: 6, pp.1005-1009, <u>https://doi.org/10.1108/03684920910973207</u> Permanent link to this document: <u>https://doi.org/10.1108/03684920910973207</u>

Downloaded on: 27 April 2018, At: 08:17 (PT) References: this document contains references to 25 other documents. To copy this document: permissions@emeraldinsight.com The fulltext of this document has been downloaded 140 times since 2009*

Users who downloaded this article also downloaded:

(2009),"Interval neutrosophic sets and topology", Kybernetes, Vol. 38 Iss 3/4 pp. 621-624 https://doi.org/10.1108/03684920910944849

(2008),"On neutrosophic topology", Kybernetes, Vol. 37 Iss 6 pp. 797-800 https://doi.org/10.1108/03684920810876990

Access to this document was granted through an Emerald subscription provided by

For Authors

If you would like to write for this, or any other Emerald publication, then please use our Emerald for Authors service information about how to choose which publication to write for and submission guidelines are available for all. Please visit www.emeraldinsight.com/authors for more information.

About Emerald www.emeraldinsight.com

Emerald is a global publisher linking research and practice to the benefit of society. The company manages a portfolio of more than 290 journals and over 2,350 books and book series volumes, as well as providing an extensive range of online products and additional customer resources and services.

Emerald is both COUNTER 4 and TRANSFER compliant. The organization is a partner of the Committee on Publication Ethics (COPE) and also works with Portico and the LOCKSS initiative for digital archive preservation.

*Related content and download information correct at time of download.



The current issue and full text archive of this journal is available at www.emeraldinsight.com/0368-492X.htm

On various neutrosophic topologies

Francisco Gallego Lupiáñez

Department of Mathematics, University Complutense, Madrid, Spain

On various neutrosophic topologies

1005

Received 24 March 2009

Abstract

Purpose – Recently, F. Smarandache generalized the Atanassov's intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs) and also defined various notions of neutrosophic topologies on the non-standard interval. One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and neutrosophic topologies on the non-standard interval. The purpose of this paper is to show that this is false.

Design/methodology/approach – The possible relations between the intuitionistic fuzzy topology and neutrosophic topologies are studied.

Findings - Relations on IFT and neutrosophic topologies.

Research limitations/implications - Clearly, the paper is confined to IFSs and NSs.

Practical implications - The main applications are in the mathematical field.

Originality/value - The paper shows original results on fuzzy sets and topology.

Keywords Logic, Fuzzy logic, Topology, Set theory

Paper type Research paper

1. Introduction

In various recent papers, Smarandache (1998, 2002, 2003, 2005a, b) generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In Smarandache (2005a, b), some distinctions between NSs and IFSs are underlined.

The notion of IFS defined by Atanassov (1983) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces (IFTSs). This concept has been developed by many authors (Bayhan and Çoker, 2003; Çoker, 1996, 1997; Çoker and Eş, 1995; Eş and Çoker, 1996; Gürçay *et al.*, 1997; Hanafy, 2003; Hur *et al.*, 2004); Lee and Lee, 2000; Lupiáñez, 2004a, b, 2006a, b, 2007, 2008; Turanh and Çoker, 2000).

Smarandache (2002, 2005b) also defined various notions of neutrosophic topologies on the non-standard interval.

One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and the neutrosophic topology. We show in this paper that this is false. Indeed, the union and the intersection of IFSs do not coincide with the corresponding operations for NSs, and an IFT is not necessarily a neutrosophic topology on the non-standard interval, in the various senses defined by Smarandache.

2. Basic definitions

First, we present some basic definitions.

Definition 1. Let *X* be a non-empty set. An IFS *A*, is an object having the form $A = \{ \langle x, \mu_A, \gamma_A \rangle | x \in X \}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set *A*, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$ (Atanassov, 1983).



Kybernetes Vol. 38 No. 6, 2009 pp. 1005-1009 © Emerald Group Publishing Limited 0368-492X DOI 10.1108/03684920910973207 K 38,6

1006

Definition 2. Let X be a non-empty set, and the IFSs $A = \{ \langle x, \mu_A, \gamma_A \rangle | x \in X \}, B = \{ \langle x, \mu_B, \gamma_B \rangle | x \in X \}$. Let (Atanassov, 1988):

$$A = \{ \langle x, \gamma_A, \mu_A \rangle | x \in X \}$$
$$A \cap B = \{ \langle x, \mu_A \land \mu_B, \gamma_A \lor \gamma_B \rangle | x \in X \}$$

$$A \cup B = \{ \langle x, \mu_A \lor \mu_B, \gamma_A \land \gamma_B \rangle | x \in X \}.$$

Definition 3. Let X be a non-empty set. Let $0_{\sim} = \{ < x, 0, 1 > | x \in X \}$ and $1_{\sim} = \{ < x, 1, 0 > | x \in X \}$ (Çoker, 1997).

Definition 4. An IFT on a non-empty set X is a family τ of IFSs in X satisfying:

- $0_{\sim}, 1_{\sim} \in \tau;$
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$; and
- $\cup G_i \in \tau$ for any family $\{G_i | j \in J\} \subset \tau$.

In this case, the pair (X, τ) is called an IFTS and any IFS in τ is called an intuitionistic fuzzy open set in X (Coker, 1997).

Definition 5. Let T, I, F be real standard or non-standard subsets of the non-standard unit interval $]^{-}0, 1^{+}[$, with:

$$\sup T = t_{\sup}, \quad \inf T = t_{\inf}$$
$$\sup I = i_{\sup}, \quad \inf I = i_{\inf}$$

 $\sup F = f_{\sup}, \quad \inf F = f_{\inf} \quad and \quad n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup} \quad n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf},$

T, *I*, *F* are called neutrosophic components. Let *U* be an universe of discourse, and *M* a set included in *U*. An element *x* from *U* is noted with respect to the set *M* as x(T, I, F) and belongs to *M* in the following way: it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where *t* varies in *T*, *i* varies in *I*, *f* varies in *F*. The set *M* is called a NS (Smarandache, 2005a).

Remark. All IFS is a NS.

Definition 6. Let $J \in \{T, I, F\}$ be a component. Most known N-norms are:

- The algebraic product N-norm: $N_{n-a \lg \text{ebraic}} J(x, y) = x \cdot y$.
- The bounded N-norm: $N_{n-bounded} J(x, y) = \max\{0, x + y 1\}.$
- The default (min) N-norm: $N_{n-\min}J(x, y) = \min\{x, y\}.$
- N_n represent the intersection operator in NS theory. Indeed, $x \wedge y = (T_{\wedge}, I_{\wedge}, F_{\wedge})$ (Smarandache, 2005b).

Definition 7. Let $J \in \{T, I, F\}$ be a component. Most known N-conorms are:

- The algebraic product N-conorm: $N_{c-a \lg \text{ebraic}} J(x, y) = x + y x \cdot y$.
- The bounded N-conorm: $N_{c-bounded} J(x, y) = \min\{1, x + y\}.$
- The default (max) N-conorm: $N_{c-\max}J(x, y) = \max\{x, y\}$.
- N_c represent the union operator in NS theory. Indeed, $x \lor y = (T_{\lor}, I_V, F_{\lor})$ (Smarandache, 2005b).

3. Results

Proposition 1. Let *A* and *B* be two IFSs in *X*, and j(A) and j(B) be the corresponding NSs. We have that $j(A) \cup j(B)$ is not necessarily $j(A \cup B)$, and $j(A) \cap j(B)$ is not necessarily $j(A \cap B)$, for any of three definitions of intersection of NSs.

Proof. Let $A = \langle x, 1/2, 1/3 \rangle$ and $B = \langle x, 1/2, 1/2 \rangle$ (i.e. μ_A , ν_A , μ_B , ν_B are constant maps).

Then, $A \cup B = \langle x, \mu_A \lor \mu_B, \gamma_A \land \gamma_B \rangle = \langle x, 1/2, 1/3 \rangle$ and $x(1/2, 1/6, 1/3) \in j(A \cup B)$. On the other hand, $x(1/2, 1/6, 1/3) \in j(A), x(1/2, 0, 1/2) \in j(B)$.

- Then, we have that:
 - (1) for the union operator defined by the algebraic product N-conorm $x(3/4, 1/6, 2/3) \in j(A) \cup j(B);$
 - (2) for the union operator defined by the bounded N-conorm $x(1, 1/6, 5/6) \in j(A) \cup j(B)$; and
 - (3) for the union operator defined by the default (max) N-conorm $x(1/2, 1/6, 1/2) \in j(A) \cup j(B)$.

Thus, $j(A \cup B) \neq j(A) \cup j(B)$, with the three definitions.

Analogously, $A \cap B = < x, \mu_A \land \mu_B, \gamma_A \lor \gamma_B > = < x, 1/2, 1/2 > \text{and } x(1/2, 0, 1/2) \in j(A \cap B).$

And, we have that:

- (1) for the intersection operator defined by the algebraic product N-norm $x(1/4, 0, 1/6) \in j(A) \cap j(B);$
- (2) for the intersection operator defined by the bounded N-norm $x(0,0,0) \in j(A) \cap j(B)$; and
- (3) for the intersection operator defined by the default (min) N-norm $x(1/2, 0, 1/3) \in j(A) \cap j(B)$.

Thus, $j(A \cap B) \neq j(A) \cap j(B)$, with the three definitions.

Definition 8. Let us construct a neutrosophic topology on $NT =]^{-0}, 1^{+}[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space. There exist various notions of neutrosophic topologies on NT, defined by using various N-norm/N-conorm operators (Smarandache, 2002, 2005b).

Proposition 2. Let (X, τ) be an IFTS. Then, the family $\{j(U)|U \in \tau\}$ is not necessarily a neutrosophic topology on NT (in the three defined senses).

Proof. Let $\tau = \{1_{\sim}, 0_{\sim}, A\}$ where $A = \langle x, 1/2, 1/2 \rangle$ then $x(1, 0, 0) \in j(1_{\sim})$, $x \in (0, 0, 1) \in j(0_{\sim})$ and $x(1/2, 0, 1/2) \in j(A)$. Thus, $\tau^* = \{j(1_{\sim}), j(0_{\sim}), j(A)\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, for any neutrosophic topology on *NT*. Indeed:

- For the intersection defined by the algebraic product N-norm, we have that $x(1/2, 0, 0) \in j(1_{\sim}) \cap j(A)$, and this NS is not in the family τ^* .
- For the intersection defined by the bounded N-norm, we have also that $x(1/2, 0, 0) \in j(1_{\sim}) \cap j(A)$, and this NS is not in the family τ^* .

On various neutrosophic topologies

 \square

K 38,6

1008

• For the intersection defined by the default (min) N-norm, we have also that $x(1/2, 0, 0) \in j(1_{\sim}) \cap j(A)$, and this NS is not in the family τ^* .

References

- Atanassov, K.T. (1983), "Intuitionistic fuzzy sets", paper presented at the VII ITKR's Session, Sofia, June.
- Atanassov, K.T. (1988), "Review and new results on intuitionistic fuzzy sets", preprint IM-MFAIS-1-88, Sofia.
- Bayhan, S. and Çoker, D. (2003), "On T₁ and T₂ separation axioms in intuitionistic fuzy topological spaces", J. Fuzzy Math., Vol. 11, pp. 581-92.
- Çoker, D. (1996), "An introduction to fuzzy subspaces in intuitionistic fuzzy topological spaces", J. Fuzzy Math., Vol. 4, pp. 749-64.
- Çoker, D. (1997), "An introduction to intuitionistic fuzzy topological spaces", Fuzzy Sets and Systems, Vol. 88, pp. 81-9.
- Çoker, D. and Eş, A.H. (1995), "On fuzzy compactness in intuitionistic fuzzy topological spaces", J. Fuzzy Math., Vol. 3, pp. 899-909.
- Eş, A.H. and Çoker, D. (1996), "More on fuzzy compactness in intuitionistic fuzzy topological spaces", *Notes IFS*, Vol. 2 No. 1, pp. 4-10.
- Gürçay, H., Çoker, D. and Eş, A.H. (1997), "On fuzzy continuity in intuitionistic fuzzy topological spaces", J. Fuzzy Math., Vol. 5, pp. 365-78.
- Hanafy, J.H. (2003), "Completely continuous functions in intuitionistic fuzzy topological spaces", *Czech, Math. J.*, Vol. 53 No. 128, pp. 793-803.
- Hur, K., Kim, J.H. and Ryou, J.H. (2004), "Intuitionistic fuzzy topologial spaces", J. Korea Soc. Math. Educ., Ser B, Vol. 11, pp. 243-65.
- Lee, S.J. and Lee, E.P. (2000), "The category of intuitionistic fuzzy topological spaces", Bull. Korean Math. Soc., Vol. 37, pp. 63-76.
- Lupiañez, F.G. (2004a), "Hausdorffness in intuitionistic fuzzy topological spaces", J. Fuzzy Math., Vol. 12, pp. 521-5.
- Lupiañez, F.G. (2004b), "Separation in intuitionistic fuzzy topological spaces", Intern. J. Pure Appl. Math., Vol. 17, pp. 29-34.
- Lupiañez, F.G. (2006a), "Nets and filters in intuitionistic fuzzy topological spaces", *Inform. Sci.*, Vol. 176, pp. 2396-404.
- Lupiañez, F.G. (2006b), "On intuitionistic fuzzy topological spaces", Kybernetes, Vol. 35, pp. 743-7.
- Lupiañez, F.G. (2007), "Covering properties in intuitionistic fuzzy topological spaces", *Kybernetes*, Vol. 36, pp. 749-53.
- Lupiañez, F.G. (2008), "On neutrosophic topology", Kybernetes, Vol. 37, pp. 797-800.
- Smarandache, F. (1998), Neutrosophy. Neutrosophic Probability, Set and Logic. Analytic Synthesis & Synthetic Analysis, American Research Press, Rehoboth, NM.
- Smarandache, F. (2002), "A unifying field in logics: neutrosophic logic", Multiple-Valued Logic, Vol. 8, pp. 385-438.
- Smarandache, F. (2003), "Definition of neutrosophic logic. A generalization of the intuitionistic fuzzy logic", Proc. 3rd Conf. Eur. Soc. Fuzzy Logic Tech. (EUSFLAT, 2003), Zittau, Germany, pp. 141-6.

Smarandache, F. (2005a), "Neutrosophic set. A generalization of the intuitionistic fuzzy set", Intern. J. Pure Appl. Math., Vol. 24, pp. 287-97.	On various neutrosophic
Smarandache, F. (2005b), "N-norm and N-conorm in neutrosophic logic and set, and the neutrosophic topologies", A Unfying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, 4th ed., American Research Press, Rehoboth, NM.	topologies
Turanh, N. and Çoker, D. (2000), "Fuzzy connectedness in intuitionistic fuzzy topological spaces", <i>Fuzzy Sets and Systems</i> , Vol. 116, pp. 369-75.	1009

Further reading

Atanassov, K.T. (1986), "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, Vol. 20, pp. 87-96. Robinson, A. (1996), *Non-standard Analysis*, Princeton University Press, Princeton, NJ.

Corresponding author

Francisco Gallego Lupiáñez can be contacted at: fg_lupianez@mat.ucm.es

To purchase reprints of this article please e-mail: **reprints@emeraldinsight.com**