Optimal synthesis of multi-product energy systems under neutrosophic environment

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Abstract

Integration of multiple production technologies to produce energy in various vectors develops more efficient and sustainable multi-product energy systems—this integration results in reduced waste generation, increased efficiency, and higher economic benefits. Synthesis of energy systems requires planning under uncertainties that can result in risky technological investments. Management of these risks can result to a robust and flexible energy system. This study develops a novel neutrosophic optimization model to address these uncertainties. It involves treating product demands, waste targets, and economic benefits as interval-valued neutrosophic numbers. Three characteristic functions under the neutrosophic environment are considered: membership, non-membership, and indeterminacy. Two case studies are used to illustrate the model: one involves a polygeneration plant, and another involves an integrated biorefinery. Sensitivity analyses were performed for each case, adjusting the levels of risk tolerance in the neutrosophic environment. The model generates relevant process design insights such as technology selection and optimal output levels. A design that balances environmental impacts and economic benefits is also generated. The insights that can be generated by the model allows policymakers and plant developers to manage risks with multi-product energy systems.

1. Introduction

Providing a sustainable future is posed by the problem of climate change and overpopulation. This challenge paves the way for the development of sustainable technologies to satisfy the growing energy demand. The integration of different production technologies produces multiple vectors of energy, simultaneously reducing harmful environmental impacts and increasing energy efficiency [1]. The design of multi-product energy systems is possible through process integration. It is a valuable method for the design, synthesis, and operation of energy systems sustainably [2]. It encompasses the techniques such as mathematical programming [3], pinch analysis [4], value chain modeling [5], among others, in examining interactions between processes rather than analyzing individual components. It also allows the design of energy systems sustainably. However, several challenges are present in process integration in energy systems. Such challenges are mainly the uncertainty with new and emerging technologies and the representation of the interactions between technologies in integrated energy systems [6].

Several studies have contributed to the design of energy systems that address the challenges mentioned. The trend of progress in energy systems research was examined using a Scopus database search. Fig. 1 shows the trend of published papers with “energy system” in title, abstract, or keywords over the past 20 years. Only at most 6% of the total publication about energy systems is related to mathematical programming. Then, 30% of mathematical programming approaches are related to risks. This trend implies that only a small part of research on energy systems uses mathematical programming approaches, but a significant number of these approaches incorporate risks in planning it. A state-of-the-art review by Andiappan [7] discusses three stages of developing energy systems: synthesis, design, and operation stages. These stages should be considered in a synergistic manner [8]. Thus, the energy system synthesis must consider the uncertainty of designing a more robust energy system and operating it flexibly. Recent studies on energy systems focus on power-to-gas technology [9], distributed energy systems [10], and electricity-hydrogen energy systems [11]. A P-graph methodology has also been applied to design sustainable energy systems [12]. The method allows for generating optimal or near-optimal design of sustainable energy systems.
Integration of biogas wastewater treatment plant and hybrid renewable energy was performed by Lim et al. [13] using P-graph methodology. It involves optimizing the economic benefits generated under steady-state conditions. A multi-period P-graph approach was developed by Aviso et al. [14], which generated energy systems that are flexible with varying availabilities with raw materials. A review by Prabatha et al. [15] evaluates different methods for considering uncertainties in energy system, which provides energy planners information on which model to use.

Approaches to handling uncertainties in the energy system are summarized by Zeng et al. [16]. These approaches include interval programming [17], stochastic mathematical programming [18], and fuzzy optimization [19]. The interval programming approach treats uncertainties as interval data in which the optimization uses to generate an interval-valued objective function. Huang et al. [20] developed an interval-valued chance-constrained mathematical programming model considering uncertainties in energy supply. This technique was later extended by Li et al. [21] to incorporate dual-interval numbers as uncertainty representation. In interval programming, the uncertainty’s nature is often neglected as it only considers the upper and lower bounds of the uncertainty. Stochastic mathematical programming aims to achieve the highest expected benefit based on the probability distribution of uncertainties. In energy systems, one of the early applications of stochastic programming to multi-product energy systems is by Gamou et al. [22]. Stochastic programming is also used to consider the human psychological perception of the risks in integrated energy systems [23]. The challenge in this approach is the lack of historical or statistical data, especially, when integrating emerging technologies is involved [24,25]. A recent paper about a model for distributed integrated energy system was developed by Mei et al. [26] with power output as the random variable. In energy systems,
fuzzy optimization provides a fair compromised solution between conflicting objectives such as economic benefits and environmental impacts. Martinsen and Krey [27] developed a fuzzy optimization model balancing contradictory targets of environmental impacts, financial benefits, and energy security in energy system. Sadeghi et al. [28] developed a multi-objective fuzzy mathematical programming model to incorporate conflicting objectives and uncertainties simultaneously in energy supply. Fuzzy set and fuzzy programming represents uncertainty as possibilistic degrees where the most possible values in a distribution can be arbitrary. This representation may be a challenge since uncertainties of given information are evaluated based on a membership function that generates it degree of acceptability [29]. This study can address this gap by extending the fuzzy set concept to maximize the energy product satisfaction and consider its consequences when the demand satisfied is under- or overestimated. Applying the concept of neutrosophic sets will represent these consequences in energy product demand satisfaction. Other approaches for handling uncertainty includes planning of hybrid power systems under variable renewable energy output [30], life cycle optimization under uncertainty [31] and application of type-2 fuzzy sets for energy systems management [32].

A neutrosophic linear program (NeLP) is developed to synthesize multi-product energy systems under uncertainty optimally. The concept of neutrosophic sets was developed by Smarandache [33] to generalize fuzzy sets [34] and intuitionistic fuzzy sets [35]. This approach is designed to extend the concept of set belongingness as a function of membership, non-membership, and indeterminacy. The membership function is defined as the degree of belongingness to a set while the non-membership function is defined as the degree of non-belongingness. These two components are decoupled to incorporate the incompleteness of information which is represented by the indeterminacy function. In neutrosophic sets, these three components are treated independently to each other, thus, giving a more general approach towards uncertainty representation. It was applied to represent uncertain data, and improve multi-criteria decision tools to handle neutrosophic data. This concept was used in Analytic Hierarchy Process (AHP) [36], Decision-Making Trial Evaluation Laboratory (DEMATEL) [37], Multi-Objective Optimization on the basis of Ratio Analysis (MUL-TIMOORA) [38], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [39] and Data Envelopment Analysis (DEA) [40]. Recent developments in the application of neutrosophic sets in engineering and sustainable development are as follows. Siksnelytė et al. [41] used a neutrosophic MULTIMOORA developed by Brauers and Zavadskas [42] to handle data uncertainty of sustainability energy indicators of countries in the Baltic Sea Region. A compromise solution approach similar to fuzzy optimization was developed by Rizk-Allah et al. [43] to handle multi-objective optimization under a neutrosophic environment. A neutrosophic optimization approach was used by Ahmad et al. [44] for water gas-shale management under uncertainty. This study involves economic uncertainties when developing pathways that originates from fresh water sources to the disposal site. These studies only associate neutrosophic sets to uncertainty—no explicit representation of how the study applied membership, non-membership, and indeterminacy functions to the problem.

The recent development of decision tools applied with the concept of neutrosophic sets is the ranking and evaluation of negative emissions technologies using neutrosophic data envelopment analysis [46]. This method involves representing input and output data as neutrosophic sets with degrees of performance satisfaction (membership) with consequential degrees of performance dissatisfaction (non-membership) and attainability (indeterminacy). To date, no practical application of neutrosophic sets is provided for synthesis and risk management of multi-product energy systems under a neutrosophic decision environment. Uncertainty in energy system is caused by multiple interlinked and complex human factors, especially on the demand side [45]. Using neutrosophic sets, these uncertainties can be modeled using the available information to energy planners. The application of neutrosophic sets allows to represent the consequences brought by increasing and decreasing of the estimates of the energy demand. This paper also provides the first neutrosophic-based optimization model to generate risk profiles for planning multi-product energy systems, as previous works on fuzzy optimization for energy systems only considers the degree of satisfaction of the energy demand. The optimization approach aims to manage planning risks by considering the trade-offs between economic benefits and environmental impacts associated with synthesis of these systems. Future energy consumption is one of the most important uncertainty in future energy systems [46]. Managing risks with satisfying energy demand can be handled by the NeLP model for energy systems. This technique allows not only manages uncertainties in planning but also how managing the risks associated with these uncertainties affecting the synthesis of the whole system. The rest of the paper is organized as follows. Section 2 discusses the problem statement while Section 3 discusses the mathematical background and optimization model development. Section 4 provides case studies and finally, Section 5 provides conclusions and future work.

2. Problem statement

The formal problem statement to be addressed is discussed as follows:

- The system consists of n streams and m technological options.
- Each technological option \( j \in J \) requires a defined ratio between material and energy flows, denoted by the conversion factor equal to \( A_{ij} \). The scale at which technology \( j \) is designed is represented by a continuous decision variable \( x_i \). The technologies are assumed to operate in a steady-state manner.
- Each stream \( i \in I \) is characterized with a final input or output flow rate of \( y_j \) bounded to a lower limit of \( Y_{ij}^{-} \) and an upper limit of \( Y_{ij}^{+} \).
- The system’s total profit consists of technology costs (i.e., capital costs, and operation and maintenance costs), raw material costs and product revenue. Equipment costs are represented with a linear cost function, having a fixed cost denoted by \( FC \) and variable cost \( VC \) per unit capacity. On the other hand, the raw material costs and product revenues are based on material and energy prices indicated by \( S_i \).
- In a “risk-free” scenario, the objective is to maximize the total profit subject to the assumption that different demand levels are equally satisfactory. This means that as long as the profit is maximized, the final energy output can take any value between the given lower limit and the upper limit. Hence, the risk arising from over- and underproduction of products should be balanced by accounting for the product limits’ neutrosophic nature.
- The demand levels bounded by \( [y_i^{L}, y_i^{U}] \) for valuable product and raw material stream \( s \in S \) is represented by three characteristic functions of neutrosophic sets, i.e., membership, nonmembership and indeterminacy functions. These functions generate the degree of satisfaction from attaining a certain level of production, the degree of dissatisfaction resulting from underproduction, and the degree of indeterminacy as a consequence of overproduction. For raw material streams, it
represents the degree of satisfaction for resource conservation subject to consequences of under- and overconsumption.

- The bounds \([Y^L_i, Y^U_i]\) for waste streams \(w \in W \subseteq I\), is also represented with characteristics function of neutrosophic sets. The membership function gives the degree of satisfaction for waste minimization, while the non-membership denotes unnecessary waste treatment dissatisfaction. The indeterminacy function denotes the degree of uncertainty of the attainability of lower minimization levels due to external factors.

- Optimizing the energy system under a neutrosophic environment also requires setting the bounds for profit to an interval of \([P_{\min}, P_{\max}]\). The bounds can be obtained by generating an optimal design with maximum profit under “risk-free” scenarios and with minimum profit subject to acceptable levels of demands. However, the minimum satisfactory profit is recommended to be higher than the minimum profit obtained.

3. Optimization model

3.1. Deterministic model

The objective is to maximize the total profit generated by the system:

\[
\max P = \sum_i S_i y_i - \sum_j \left( C_j x_j + V_C x_j \right) \tag{1}
\]

where \(S_i\) is the price of the stream \(i\) entering or leaving the system, \(C_j\) is the fixed cost \(j\), and \(V_C\) is the variable cost for investing technology \(j\). The total capacity of technology \(j\), denoted as \(x_j\), determines the process’s optimal preliminary design. The amount of raw materials needed and products produced in the system are indicated as \(y_i\). The scaling factor \(C_j\) should also be greater than or equal to 0.

The material and energy balances in the system can be expressed as an input-output constraint:

\[
\sum_j A_{ij} x_j = y_i \quad \forall i \tag{2}
\]

where \(A_{ij}\) denotes the conversion factor of the stream \(i\) using technology \(j\). A positive value of \(A_{ij}\) means the production of the stream \(i\) while a negative value signifies its consumption.

The streams are bounded between a given lower bound and upper bound:

\[
Y^L_i \leq y_i \leq Y^U_i \quad \forall i \tag{3}
\]

For product or waste streams, the upper bound, \(Y^U_i\) and lower bound \(Y^L_i\) is represented by the demand or emission limit, respectively. The upper and lower bounds for raw material streams are based on resource availability or target resource reduction. Intermediate streams take a value of zero for both upper and lower bound, indicating that it is consumed within the system. The model user can treat a given stream as either a product or an intermediate to check whether it is possible for the given stream to be consumed completely or not. Raw material streams have negative \(y_i\) value.

The big \(M\) constraint restricts candidate technology \(j\) identified for the system design:

\[
x_j \leq M b_j \quad \forall j \tag{4}
\]

where \(b_j\) represents the decision whether to invest in technology \(j\) or not. This upper bound of technological investment will have a value of 0 when the decision-maker does not invest in technology \(j\). Thus, its capacity becomes 0 since it should be positive. Options for technology are based on the topological constraint for each process within the system:

\[
\sum_j T_{jk} b_j \leq 1 \quad \forall k \tag{5}
\]

where \(T_{jk}\) is a binary parameter that denotes whether technology \(j\) is an option for conversion process \(k\). It is assumed that for each process, only one technology is chosen. In an energy system, multiple technological options may serve the same purpose; thus, it can be grouped into a single conversion process. In this way, the decision-maker can select a single technology for a given process.

The deterministic model (Eqn. (1) subject to Eqns. (2)–(5)) assumed that the values within the bounds of the each stream (Eq (3)) are treated as equally acceptable. Thus, the model solves for an optimal design by identifying a certain value within the bounds. In reality, demand satisfaction within an acceptable bound differs from one value to another. It entails different risk levels in demand satisfaction: economic opportunity losses are incurred for highly conservative demand estimates and unnecessary product excess for highly optimistic demand estimate. These are considered and managed by the neutrosophic linear program.

3.2. Neutrosophic model

The development of the NeLP model requires generating the functions for uncertainties under a neutrosophic environment. Preliminaries about neutrosophic sets are explained in the Appendix. The deterministic to NeLP model’s extension involves the construction of degrees of satisfaction, dissatisfaction and indeterminacy for product demand, waste generation and profit attainment. Fig. 2 illustrates these characteristic functions. For the product demand, the degree of satisfaction, \(s_{\text{as}}\), increases along the interval \([Y^L_i, Y^U_i]\), signifying the increasing possibility to satisfy customer needs. Consequently, the possibility of overproduction also increases. It is represented by the degree of indeterminacy, \(\tau_{\text{as}}\) with an expert-defined indeterminacy risk tolerance, \(T_{\text{as}}\). Underproduction can also incur undesirable outcomes such as opportunity losses and penalties; this is represented by the degree of dissatisfaction, \(s_{\text{aw}}\), with an expert-defined dissatisfaction risk tolerance, \(T_{\text{aw}}\). An opposite approach is done for the waste generation with neutrosophic degrees \(s_{\text{aw}}, \beta_{\text{aw}}\) and \(\gamma_{\text{aw}}\) as shown in Fig. 2b. The degree of satisfaction, \(s_{\text{aw}}\), decreases through the interval, which signifies the preference towards waste reduction. However, the consequences lie with the indeterminacy, \(\gamma_{\text{aw}}\) of designing the process to minimize waste generation and the dissatisfaction, \(\beta_{\text{aw}}\) towards higher waste generations. The range of tolerance levels are from 0 to 1. A value of 0 for the \(T_i\) means that the expert considers the maximum variation between the different levels of product demand while a value of 1 means that the expert equally treats them in terms of overproduction risks. A similar analogy as that of \(T_{\text{as}}\) is represented here, however, this is in terms of underproduction risks instead. Note that the tolerance levels are associated with expert’s risk perception towards the uncertainty of the system as a whole. Since there are no frameworks developed yet for determining these tolerance levels, only one set is used for the model.

The profit can also be expressed as an interval that is neutrosophic in nature. The interval is obtained from a minimum acceptable profit, \(P_{\text{min}}\) to the maximum profit, \(P_{\text{max}}\). The upper bound can be obtained from solving the deterministic model while a certain percentage of \(P_{\text{max}}\) can be used as the lower bound. The
The degree of desirability arising from economic returns, \( a \), is measured by the membership function. The possibility of attaining higher economic benefits decreases due to the external condition surrounding the system (e.g., variability in product demand, customer choice, competition with similar industries). This concept can be modeled by the degree of indeterminacy, \( g \), that increases as the profit level increases. However, designing the process to achieve an acceptable level of profit may incur opportunity losses. This effect can be represented by the degree of dissatisfaction, \( b \), which increases towards the lower bound (i.e., minimum acceptable profit).

Expert-defined levels of tolerance, \( T_\beta \) and \( T_\gamma \), are also incorporated for the profit. These factors represent the decision maker’s attitude towards risks arising from the consequences of increasing and decreasing levels, respectively, of the generation of products, wastes, and economic benefits. A higher risk tolerance value means that the expert assumes that the consequences of attaining satisfactory levels of economic returns, product demand and waste minimization are manageable. On the other hand, a lower risk tolerance value means treating the consequences with the utmost importance. These values are important since it allows the transition to different decision environments, which will be explained at the end of this section.

The goal of the NeLP model is to simultaneously maximize the overall degree of satisfaction, minimize the overall degree of dissatisfaction, and minimize the overall degree of indeterminacy:

\[
\text{max } a \quad \text{(6)}
\]

\[
\text{min } b \quad \text{(7)}
\]

\[
\text{min } g \quad \text{(8)}
\]

wherein:

\[
\alpha_s \leq \alpha_w \quad \forall s \subseteq \mathcal{I} \quad \forall w \subseteq \mathcal{I} \quad \alpha_s \leq \alpha_w \quad \forall w \subseteq \mathcal{I} \quad \text{(9)}
\]

\[
\beta_s \leq \beta_w \quad \forall s \subseteq \mathcal{I} \quad \beta_s \leq \beta_w \quad \forall w \subseteq \mathcal{I} \quad \text{(10)}
\]

\[
\gamma_s \leq \gamma_w \quad \forall s \subseteq \mathcal{I} \quad \gamma_s \leq \gamma_w \quad \forall w \subseteq \mathcal{I} \quad \text{(11)}
\]

\[
\alpha \leq \alpha_p \quad \text{(12)}
\]

\[
\beta \geq \beta_p \quad \text{(13)}
\]

\[
\gamma \geq \gamma_p \quad \text{(14)}
\]

To reduce the goal into a single objective and to ensure Pareto optimality, the objective function is expressed as:

\[
\text{max } a - b - \gamma + \frac{1}{M} \left[ \sum_s \alpha_s - \beta_s - \gamma_s + \sum_w \alpha_w - \beta_w - \gamma_w \right] + \alpha_p - \beta_p - \gamma_p + P \quad \text{(15)}
\]

The negative signs in the second and third terms of Eqn. (15) signifies the minimization of overall degrees of dissatisfaction and indeterminacy. The fourth term ensures Pareto optimality, optimizing individual degrees of satisfaction, dissatisfaction, and indeterminacy of product and waste streams and maximizes profit. The weight influence of this term on the overall neutrosophic degrees should be kept negligible. Thus, an infinitesimally small weight factor, \( 1/M \), is introduced. The membership, nonmembership, and indeterminacy functions are expressed as linear functions.
\[ y_s = \alpha_s \left( y_{s_1}^U - y_{s_1}^L \right) + y_{s_1}^L \quad \forall s \in S \subset \mathbb{I} \tag{16} \]

\[ y_s(1 - T_\beta) = \beta_s \left( y_{s_1}^U - y_{s_1}^L \right) + y_{s_1}^L(1 - T_\beta) \quad \forall s \in S \subset \mathbb{I} \tag{17} \]

\[ y_s(1 - T_\gamma) = \gamma_s \left( y_{s_1}^U - y_{s_1}^L \right) + y_{s_1}^L(1 - T_\gamma) \quad \forall s \in S \subset \mathbb{I} \tag{18} \]

The membership function for the product is an increasing function with a maximum value of one at the highest acceptable value (i.e., upper bound) of the demand and a minimum value of zero at the lowest value. The trend is the opposite of nonmembership function. However, the maximum value is lower than one depending on the risk tolerance for nonmembership, \( T_\beta \). An increasing indeterminacy function is defined for neutrosophic products with a maximum value based on the risk tolerance for indeterminacy, \( T_\gamma \). Neutrosophic functions for waste streams have an opposite trend to that of the product. It shares with the same risk tolerances of nonmembership and indeterminacy.

\[ y_w = \alpha_w \left( y_{w_1}^U - y_{w_1}^L \right) + y_{w_1}^L \quad \forall w \in W \subset \mathbb{I} \tag{19} \]

\[ y_w(1 - T_\beta) = \beta_w \left( y_{w_1}^U - y_{w_1}^L \right) + y_{w_1}^L(1 - T_\beta) \quad \forall w \in W \subset \mathbb{I} \tag{20} \]

\[ y_w(1 - T_\gamma) = \gamma_w \left( y_{w_1}^U - y_{w_1}^L \right) + y_{w_1}^L(1 - T_\gamma) \quad \forall w \in W \subset \mathbb{I} \tag{21} \]

The model user or expert defines the tolerance parameters, \( T_\beta \) and \( T_\gamma \) to represent his attitude towards the consequences in the uncertainty's fuzzy nature. It applies to the profit, which we define as an interval-valued neutrosophic number with linear membership, nonmembership, and indeterminacy functions.

\[ P = \alpha_P(P_{\min} - P_{\max}) + P_{\min} \tag{22} \]

\[ P(1 - T_\beta) = \beta_P(P_{\min} - P_{\max}) + P_{\max}(1 - T_\beta) \tag{23} \]

\[ P(1 - T_\gamma) = \gamma_P(P_{\min} - P_{\max}) + P_{\min}(1 - T_\gamma) \tag{24} \]

The NeLP model also incorporates the constraints for material and energy balance (Eqn. (2)), stream bounds (Eqn. (3)), process scaling (Eqn. (4)) and technology selection (Eqn. (5)). This model is solved by having the objective function in Eqn (15) subject to the constraints from Eqn. (16) to Eqn. (24), from Eqn. (9) to Eqn. (14), and from Eqn. (2) to Eqn. (5). The degrees of satisfaction for product generation, waste minimization, and profit attainment are bounded in the interval [0,1] to represent the total performance satisfaction range. The degrees of dissatisfaction and indeterminacy, on the other hand, are bounded on the intervals [0, 1 - \( T_\beta \)] and [0, 1 - \( T_\gamma \)] respectively. The upper bounds represent the highest degrees at which the decision-maker is tolerant of the risks identified here. Note that the model is also flexible in different decision environments. A fuzzy optimization model can be achieved when \( T_\beta = T_\gamma = 1 \), thus, only the degrees of satisfaction are considered. An intuitionistic fuzzy decision environment is established when \( T_{\beta} = 1 \). Here, the degrees of satisfaction and dissatisfaction are both considered. A completely neurotrophic decision environment is achieved when \( T_\beta = T_\gamma = 0 \). All neutrosophic decision-making elements (i.e., membership, nonmembership, and indeterminacy) are fully considered by using this approach. An “indeterminate” fuzzy decision environment is used when \( T_\beta = 1 \) and \( T_\gamma = 0 \) is considered. In this case, the decision maker assumes that opportunity losses are manageable. Two case studies illustrate the optimization model: one applied to the design of polygeneration systems and an integrated biorefinery design. The results provide insights on technology selection neutrosophic risk environment and sustainable multi-product energy systems planning with better environmental performance under acceptable economic benefits.

4. Case studies

4.1. Polygeneration plant

The case study was adopted from Aviso and Tan [47] involving a polygeneration system with four main energy products. Table 1 shows the conversion factors of a polygeneration system with four main products, namely, hot water, chilled water for cooling, steam and electricity. Six technological options produce these products. These options are combined heat and power (CHP), boiler (BL), hot water generator (HWG), heat exchanger (HE), absorption chiller (AC), and electric chiller (EC). For this case, the option between AC and EC is restricted to one cooling technology for the design. The flow rate limits are given in Table 2, representing the polygeneration system’s design bounds under a neutrosophic decision environment. The flow rate for natural gas is set to a negative value for the bounds since it is an input to the system. The consumption of natural gas is decreasing from the given lower bound to the upper bound. The cost for developing the polygeneration system is based on Table 3, showing the fixed cost per equipment unit and the variable cost per flow rate of the main product. The prices for each stream are also given in Table 3.

The optimal design considering the maximum profit targeting any demand levels is shown in Fig. 4. The technologies selected are CHP, HE, and EC. The synthesized polygeneration system involves steam, hot water, and chilled water production at 18.00, 15.00, and 6.00 MW, respectively, while the electricity production is 26% more than the minimum acceptable demand level. The system gives a total profit of $5.676 million/y, in which the total cost for the equipment is $1.533 million. The solution leads to an optimistic design in which steam, hot water, and chilled water demands are assumed to be at the maximum level. However, taking into account the risk for over- and underproduction, a solution using the NeLP can be achieved. For this, the profit is set at a minimum acceptable level of $2.230 million/y, 1.3 times the minimum profit achieved at the given range of demand. The minimum profit is obtained by changing the objective function to minimization, subject to the constraints that the demands for the final products are still satisfied.

The solution in which the decision maker is not tolerant to the risk in both overproduction (i.e. uncertainty tolerance, \( T_\beta = 0 \)) and underproduction (i.e. dissatisfaction tolerance, \( T_\gamma = 0 \)) can be achieved and is shown in Fig. 5. In this case, an additional HWG unit is installed and EC is replaced with AC instead. In this case, an additional HWG unit is installed, and EC is replaced with AC instead. In this case, the electricity demand satisfaction is greater than in the maximum profit scenario. The total profit achieved is $3.708 million/y, with a total equipment cost of $1.497 million/y. The optimal solution presents an acceptable economic benefit level.
while attaining a balance between risks due to over- and under-production. This balance is achieved by compromising 20% of the hot water production and the chilled water generation. The design also generates a 27% reduction in steam generation, while 11% more electricity is produced. Balancing the demand satisfaction and consequential risks results in the system generating more electricity than other energy products in this polygeneration system. CHP, HE, and HWG allow more robust operation in the system.

Table 2
Bounds for net stream flow rate in polygeneration case study.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Gas (MW)</td>
<td>-70</td>
<td>-40</td>
</tr>
<tr>
<td>Hot Water (MW)</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Electricity (MW)</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Chilled Water (MW)</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Steam (MW)</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 3
Economic data for polygeneration case study.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Fixed Cost (€)</th>
<th>Variable Cost (€/MW)</th>
<th>Stream Price (€/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHP</td>
<td>382,500</td>
<td>948,347</td>
<td>Natural Gas 30</td>
</tr>
<tr>
<td>BL</td>
<td>45,500</td>
<td>175,000</td>
<td>Hot Water 30</td>
</tr>
<tr>
<td>HWG</td>
<td>7500</td>
<td>39,474</td>
<td>Electricity 90</td>
</tr>
<tr>
<td>HE</td>
<td>625</td>
<td>4688</td>
<td>Chilled Water 50</td>
</tr>
<tr>
<td>AC</td>
<td>92,500</td>
<td>220,238</td>
<td>Steam 40</td>
</tr>
<tr>
<td>EC</td>
<td>43,750</td>
<td>267,857</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. Optimal design for polygeneration system under maximum profit scenario.

Fig. 5. Optimal solution under risk-averse scenario ($T_d = 0, T_g = 0$).
subject to varying hot water demand, since there are more combinations of part-load operation with more technologies. They also give an insight into an advisable nominal capacity level to invest in for the polygeneration system. In the given solution in Fig. 5, it is easier to adjust to different scenarios since higher flexibility can be done if there are three units (CHP, HWG, and HE) offering the same product than with only two units (CHP and HE). This allows to operate even when the hot water demand is lower than expected; the polygeneration system can be adjusted without having the HE unit to operate at unstable part-load capacity. Such minimum part-load capacity provides the safe and operable level for an equipment, estimated from 55% to 110% by Aguilar et al. [48] for a 7-MW power generation unit.

The tolerance levels can also be adjusted to 0.6 and 0.2 for $T_3$ and $T_r$, respectively. This adjustment gives an optimal solution shown in Fig. 6. The tolerance level chosen indicates a higher level of tolerance to underproduction than in overproduction, representing a conservative approach to polygeneration design. In this case, the optimal design chooses to invest in a small capacity boiler rather than HWG. The demand for chilled water and hot water are satisfied at the minimum level while the electricity demand is satisfied at the maximum level. The solution suggests to install an additional boiler to allow flexibility to different levels of steam demand. This allows the operation of CHP at the stable part-load capacity, typically from 55% to 110% as discussed by Aguilar et al. [48], to satisfy electricity demand. However, the design requires to compromise the total profit from $5.676$ million/y to $3.640$ million/y, 63% more than the minimum acceptable profit level. This solution gives a degree of profit satisfaction of 0.409 to balance the risk associated with demand uncertainties. It results from satisfying certain product demands to a minimum and investing in additional equipment. On the other hand, this solution allows more flexibility in operations by recommending necessary baseline capacities to balance the risks associated with demand uncertainties. A sensitivity analysis is performed in order to determine the effect of different risk perception to the design. Fig. 7 shows the resulting design at different levels of tolerance to both dissatisfaction and uncertainty. Three optimal designs are obtained from the analysis, in which these configurations are optimal at a specific range of tolerance levels. A large region is covered by the first design, indicating that optimizing the polygeneration system for maximum profit can be suggested when the tolerance for uncertainty is more than 0.60 but not less than 0.20 at any levels of dissatisfaction tolerance. Additional equipment such as a boiler, as shown in design (b), and an HWG, as shown in design (c) in Fig. 7, will be needed at low tolerance levels. These are suggested when setting a tolerance at the regions shown. The design also involves the preference for AC rather than EC, which can be suggested when electricity demand is realized at the maximum levels.

A summary of the results is presented in Table 4 with profit reduction based on the maximum possible. Here, the different outputs of each configuration presented are shown as well as their total profit in each scenario. Considering the neutrosophic elements in the model, the design will require additional process units to balance the risks involved. Based on this case study, the following insights can be drawn as to how the polygeneration system can be synthesized:

- The choice between equipment in a polygeneration system will depend on the expert's risk tolerance levels. It balances the risk and economic benefits of a polygeneration system.
- The NeLP model generates different design configurations for the system, where one or more processes are considered. Each design is optimal at a particular risk tolerance levels.
- Energy planners will evaluate how a polygeneration system behaves at different risk perceptions, allowing flexible operation and a robust system for further detailed design.

4.2. Integrated biorefinery

For the second case study, an integrated biorefinery complex case is adopted from Sy et al. [49]. A slight modification was made with the demand data to have a more representative scale of demands in a regional setting. Table 7 shows the conversion factors data, while Table 5 shows the process economics data. In this case, the environmental impacts of the system are considered for the design. Three environmental impacts are included, namely, CO₂ emissions, water consumption and land footprint. In Table 6, the product streams' prices are shown, and the bounds for the
Fig. 7. Sensitivity analysis (a) of the design configurations in (b) to (d) at different tolerance levels of dissatisfaction and indeterminacy.

Table 4
Economic data for polygeneration case study.

<table>
<thead>
<tr>
<th></th>
<th>Risk-free, Maximum Profit Scenario</th>
<th>Risk-Averse Scenario (completely neutrosophic, $T_g - T_r = 0$)</th>
<th>Conservative Scenario ($T_g = 0.6, T_r = 0.2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity (MW)</td>
<td>12.60</td>
<td>14.01</td>
<td>14.00</td>
</tr>
<tr>
<td>Steam (MW)</td>
<td>18.00</td>
<td>13.21</td>
<td>13.20</td>
</tr>
<tr>
<td>Hot Water (MW)</td>
<td>15.00</td>
<td>12.01</td>
<td>12.00</td>
</tr>
<tr>
<td>Chilled Water (MW)</td>
<td>6.00</td>
<td>4.80</td>
<td>4.80</td>
</tr>
<tr>
<td>Profit (million €/y)</td>
<td>5.67</td>
<td>3.71</td>
<td>3.68</td>
</tr>
<tr>
<td>% Profit reduction</td>
<td>–</td>
<td>34.7%</td>
<td>35.8%</td>
</tr>
</tbody>
</table>

Table 5
Equipment costs of plants in case 2.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Capacity Basis</th>
<th>Fixed Cost (USD)</th>
<th>Variable Cost (USD/unit main product)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated Microalgae to Biodiesel Plant</td>
<td>1 t/h Biodiesel</td>
<td>0.00</td>
<td>67,000,000</td>
</tr>
<tr>
<td>Anaerobic Digestion Unit</td>
<td>1 t/h Methane</td>
<td>0.00</td>
<td>4,000,000</td>
</tr>
<tr>
<td>Combined Heat and Power Plant</td>
<td>1 MW Power</td>
<td>459,000</td>
<td>1,138,000</td>
</tr>
<tr>
<td>Methanol Production Plant</td>
<td>1 t/h Methanol</td>
<td>0.00</td>
<td>18,300,000</td>
</tr>
<tr>
<td>Biochar Plant</td>
<td>1 t/h Biochar</td>
<td>0.00</td>
<td>8,670,000</td>
</tr>
<tr>
<td>Purchased Methane</td>
<td>1 t/h Methane</td>
<td>0.00</td>
<td>71,000,000</td>
</tr>
<tr>
<td>Purchased Methanol</td>
<td>1 t/h Methanol</td>
<td>0.00</td>
<td>52,200,000</td>
</tr>
</tbody>
</table>

Table 6
Stream prices and demand bounds for case 2.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Stream Price (USD/t)</th>
<th>Flow rate unit</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biodiesel</td>
<td>12,500</td>
<td>t/h</td>
<td>5.00</td>
<td>15.00</td>
</tr>
<tr>
<td>Glycerol</td>
<td>780</td>
<td>t/h</td>
<td>0.10</td>
<td>5.00</td>
</tr>
<tr>
<td>Power (USD/MWh)</td>
<td>108</td>
<td>MW</td>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>Heat (USD/MWh)</td>
<td>108</td>
<td>MW</td>
<td>500</td>
<td>2000</td>
</tr>
<tr>
<td>Methane (USD/t)</td>
<td>710</td>
<td>t/h</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Methanol (USD/t)</td>
<td>520</td>
<td>t/h</td>
<td>0.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Biochar (USD/t)</td>
<td>500</td>
<td>t/h</td>
<td>0.00</td>
<td>3.00</td>
</tr>
<tr>
<td>CO2 (USD/t)</td>
<td>–</td>
<td>t/h</td>
<td>–4000.00</td>
<td>1200.00</td>
</tr>
<tr>
<td>Water (USD/t)</td>
<td>–</td>
<td>t/h</td>
<td>–4000.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Land (USD/ha)</td>
<td>–</td>
<td>t/h</td>
<td>–20.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Nitrogen (USD/t)</td>
<td>–</td>
<td>t/h</td>
<td>–1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>–</td>
<td>t/h</td>
<td>–0.02</td>
<td>0.00</td>
</tr>
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</table>
Table 7
Process matrix for case 2.

<table>
<thead>
<tr>
<th></th>
<th>Integrated Microalgae to Biodiesel Plant</th>
<th>Anaerobic Digestion Unit</th>
<th>Combined Heat and Power Plant</th>
<th>Methanol Production Plant</th>
<th>Biochar Plant</th>
<th>Purchased Methane</th>
<th>Purchased Methanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biodiesel (t/h)</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Glycerol (t/h)</td>
<td>0.11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Power (MW)</td>
<td>–11.33</td>
<td>–0.783</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Heat (MW)</td>
<td>–24.94</td>
<td>–5.958</td>
<td>1.90</td>
<td>–0.886</td>
<td>–0.297</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Methane (t/h)</td>
<td>0.00</td>
<td>1.00</td>
<td>–0.324</td>
<td>–0.43</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Methanol (t/h)</td>
<td>–0.11</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Biochar (t/h)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Solid Residue (t/h)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>–0.50</td>
<td>–6.67</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Liquid Residue (t/h)</td>
<td>2.40</td>
<td>–8.76</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CO₂ (t/h)</td>
<td>14.65</td>
<td>2.48</td>
<td>0.891</td>
<td>0.00</td>
<td>–2.68</td>
<td>0.07</td>
<td>0.54</td>
</tr>
<tr>
<td>Water (t/h)</td>
<td>–44.46</td>
<td>–320.30</td>
<td>–0.252</td>
<td>–0.40</td>
<td>0.00</td>
<td>–0.13</td>
<td>–1.68</td>
</tr>
<tr>
<td>Nitrogen (t/h)</td>
<td>–0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Phosphorus (t/h)</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Fig. 8. Process flow diagram of the optimal solution for Case Study 2 under completely neutrosophic decision environment.
products, emission, and environmental inputs are indicated. All materials streams are in units of tonnes per hour while the two energy streams, heat in the form of steam and electricity (power), are in units of megawatts (MW). The integrated biorefinery case is set-up to produce biodiesel from 5 t/h to 15 t/h, along with energy outputs of heat from 500 to 2000 MW and electricity from 500 to 1000 MW. Five process units are involved in this case: biodiesel plant, anaerobic digestion (AD) unit, combined heat and power (CHP) plant, methanol plant, and biochar plant. Biodiesel is produced from microalgae, represented by its required nutrients, namely, nitrogen and phosphorus, and land area. Energy requirements for all process units are fulfilled by the CHP plant, whose raw material is methane. Methane requirements can be satisfied by purchasing methane with associated environmental footprints or produced by the AD unit using biodiesel plant residues. Methanol requirement by the biodiesel plant can also be purchased or produced by the methanol plant. The decision whether to purchase these inputs is not mutually exclusive to allow the residues from the biodiesel plant to be processed completely within the system. Methane is also consumed completely in the system in addition to these residues. Note that in Table 5, the calculated variable cost for purchasing methane and methanol is based on its purchase prices given in Table 6.

Solving for the system’s minimum and maximum profit within the bounds of the product streams yields an annual profit of 634 million USD/year to 1567 million USD/year, respectively. Suppose we set the satisfactory profit levels to 800 million USD/year to 1567 million USD/year. The neutrosophic decision environment is set-up such that heat, power, biodiesel, and glycerol are treated as neutrosophic streams. Environmental impacts such as CO2 emissions, water consumption, land area are neutrosophic, being CO2 as a waste product. Nitrogen and phosphorus are also considered neutrosophic input streams.

The neutrosophic model generates 136 constraints and 78 variables (14 are integer variables). The optimal solution under completely neutrosophic environment ($T_b = T_g = 0$) is shown in Fig. 8 while the solution under fuzzy decision environment ($T_b = T_g = 1$) is shown in Fig. 9. These two solutions have similar configuration where there are only small difference between capacities of the process technologies. The process diagram shows that the purchase of methanol to supply the biorefinery plant’s needs is preferred over its production. Thus the solid residue from the biorefinery plant is used as an input to the biochar plant. The

![Process flow diagram of the optimal solution for Case Study 2 under fuzzy decision environment.](image-url)
The overall degree of satisfaction obtained from the model is equal to 0.237, while the overall degree of dissatisfaction and indeterminacy are equal to 0.763 and 0.864, respectively. Considering the risk factors that these neutrosophic components represent, the compromised solution generates an annual profit of 1.31 billion USD. This value is only 16% lower than the system when the profit is maximized, under the condition that the input and output streams are within the bounds established. The solution provided under a completely neutrosophic environment allows a 24% reduction in CO2 emissions, 11% reduction in water consumption, and 9% reduction in land use change the reduction of biodiesel output by 9%. Using the NeLP model, the risks associated with satisfying uncertain demand and its environmental impacts can be minimized. The model can also be used to determine the system’s risk profile and extent to which the system can be synthesized subject to conflicting economic benefits and environmental impacts.

The parameters, $T_b$ and $T_g$, can be adjusted based on the experts’ perception or tolerance towards the risk. The sensitivity analysis performed by adjusting these parameters is shown in Fig. 10. The economic benefit is shown in Fig. 10a and the environmental impacts are shown in Fig. 10b to d. Here, three possible configurations of the system are generated: one under fuzzy decision environment ($T_b = T_g = 0$), under completely neutrosophic environment ($T_b = T_g = 0$), and under “indeterminate” fuzzy decision environment ($T_b = 1, T_g = 0$). The least profit with the lowest environmental impact is generated where the profit is 21% less than the maximum profit possible, in which the CO2 emissions is reduced by 26%, the

![Fig. 10. Risk profile of the system under different tolerance to risks.](image-url)
land footprint is reduced by 17.5%, and the water consumption is reduced by 23%. This insight is generated from the indeterminate fuzzy decision environment, where the opportunity loss by not satisfying higher demand levels is not considered. It allows the model to determine the most plausible reduction to energy products’ production to minimize the environmental impacts while attaining a satisfactory output. However, this solution is only applicable when falsity tolerance values are greater than 0.9, and the value of indeterminacy tolerance is less than 0.10. Another scenario is that if the process synthesis’ goal is to meet the demands at satisfactory levels, it is recommended to optimize under a fuzzy decision environment. In this setting, the biodiesel production, as shown in Fig. 10e is at the maximum while the biochar production is maximized to 1.350 t/h as shown in Fig. 10f. This implies that, in this case, biochar production is maximized to decrease the CO2 footprint of the plant to achieve a satisfactory level. In comparison, when the indeterminacy tolerance ranges from 0.30 to 0.60 depending on the falsity tolerance.

Based from the results of the case study, the following insights can be drawn:

- The optimal balance between environmental and economic impacts of an integrated biorefinery is attained by adjusting the risk tolerance parameters in a neutrosophic decision environment.
- The adjustment of the risk tolerance parameters allows the policy-makers to determine how environmental risk impacts can be minimized subject to adequate economic benefits.
- The investment risks due to opportunity loss for high product demands and due to surplus production for low product demands can be managed in the synthesis of integrated biorefineries.

5. Conclusions and future works

A neutrosophic linear program is developed to synthesize multiproduct energy systems optimally and to manage risks associated with the uncertainty in it. The model incorporates product demands as interval-valued neutrosophic numbers, wherein the membership function represents demand satisfaction. Risks associated with the uncertain product demand are considered for both optimistic and conservative estimates using the non-membership and indeterminacy functions, respectively. The model also incorporates the user’s risk perception towards these estimates wherein two adjustable parameters are included. This approach allows to synthesize alternative preliminary designs of energy systems under varying risk tolerances; thus, creating a risk profile for the energy system. Two case studies are presented. Application of the model in the first case study reveals the optimal choice of technologies for synthesizing a polygeneration system in different decision environments. The risk management using the model in the second case balances the economic benefits and environmental impacts of the system. Through these insights, managing risk involved in integrating different energy production technologies aids plant developers in the design and operation stages. The limitation of the model is that there is no systematic way for determining the expert’s risk tolerance level to be used in the model. Thus, future work includes extending the model to supply chain optimization and network design, and to develop a framework for determining the tolerance values to be used.
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