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Ordinary Single Valued Neutrosophic Topological Spaces

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Abstract: We define an ordinary single valued neutrosophic topology and obtain some of its basic properties. In addition, we introduce the concept of an ordinary single valued neutrosophic subspace. Next, we define the ordinary single valued neutrosophic neighborhood system and we show that an ordinary single valued neutrosophic neighborhood system has the same properties in a classical neighborhood system. Finally, we introduce the concepts of an ordinary single valued neutrosophic base and an ordinary single valued neutrosophic subbase, and obtain two characterizations of an ordinary single valued neutrosophic base and one characterization of an ordinary single valued neutrosophic subbase.

Keywords: ordinary single valued neutrosophic (co)topology; ordinary single valued neutrosophic subspace; α-level; ordinary single valued neutrosophic neighborhood system; ordinary single valued neutrosophic base; ordinary single valued neutrosophic subbase

1. Introduction

In 1965, Zadeh [1] introduced the concept of fuzzy sets as the generalization of an ordinary set. In 1986, Chang [2] was the first to introduce the notion of a fuzzy topology by using fuzzy sets. After that, many researchers [3–13] have investigated several properties in fuzzy topological spaces.

However, in their definitions of fuzzy topology, fuzziness in the notion of openness of a fuzzy set was absent. In 1992, Samanta et al. [14,15] introduced the concept of gradation of openness (closedness) of fuzzy sets in X in two different ways, and gave definitions of a smooth topology and a smooth co-topology on X satisfying some axioms of gradation of openness and some axioms of gradation of closedness of fuzzy sets in X, respectively. After then, Ramadan [16] defined level sets of a smooth topology and smooth continuity, and studied some of their properties. Demirci [17] defined a smooth neighborhood system and a smooth Q-neighborhood system, and investigated their properties. Chattopadhyay and Samanta [18] introduced a fuzzy closure operator in smooth topological spaces. In addition, they defined smooth compactness in the sense of Lowen [8,9], and obtained its properties. Peters [19] gave the concept of initial smooth fuzzy structures and found its properties. He [20] also introduced a smooth topology in the sense of Lowen [8] and proved that the collection of smooth topologies forms a complete lattice. Al Tahan et al. [21] defined a topology such that the hyperoperation is pseudocontinuous, and showed that there is no relation in general between pseudotopological and strongly pseudotopological hypergroupoids. In addition, Onassanya and Hošková-Mayerová [22] investigated some topological properties of α-level subsets' topology of a fuzzy subset. Moreover, Çoker and Demirci [23], and Samanta and Mondal [24,25] defined intuitionistic gradation of openness (in short IGO) of fuzzy sets in Šostak's sense [26] by using intuitionistic fuzzy sets introduced by Atanassov [27]. They mainly dealt with intuitionistic gradation of openness of fuzzy sets in the sense of Chang. However, in 2010, Lim et al. [28] investigated intuitionistic smooth topological spaces in Lowen's sense. Recently, Kim et al. [29] studied continuities and neighborhood systems in intuitionistic smooth topological spaces. In addition, Choi et al. [30] studied an interval-valued smooth topology by gradation of openness of interval-valued fuzzy sets introduced by Gorzalczany [31] and Zadeh [32], respectively. In particular, Ying [33] introduced the concept of the topology (called a fuzzifying topology) considering the degree of openness of an ordinary subset of a set. In 2012, Lim et al. [34] studied general properties in ordinary smooth topological spaces. In addition, they [35–37] investigated closures, interiors and compactness in ordinary smooth topological spaces.

In 1998, Smarandache [38] defined the concept of a neutrusophic set as the generalization of an intuitionistic fuzzy set. Salama et al. [39] introduced the concept of a neutrosophic crisp set and neutrosophic crisp relation (see [40] for a neutrosophic crisp set theory). After that, Hur et al. [41,42] introduced categories $\mathbf{NSet}(H)$ and $\mathbf{NCSet}$ consisting of neutrosophic sets and neutrosophic crisp sets, respectively, and investigated them in a topological universe view-point. Smarandache [43] defined the notion of neutrosophic topology on the non-standard interval and Lupiáñez proved that Smarandache's definitions of neutrsophic topology are not suitable as extensions of the intuitionistic fuzzy topology (see Proposition 3 in [44,45]). In addition, Salama and Alblowi [46] defined a neutrosophic topology and obtained some of its properties. Salama et al. [47] defined a neutrosophic crisp topology and studied some of its properties. Wang et al. [48] introduced the notion of a single valued neutrosophic set. Recently, Kim et al. [49] studied a single valued neutrosophic relation, a single valued neutrosophic equivalence relation and a single valued neutrosophic partition.

In this paper, we define an ordinary single valued neutrosophic topology and obtain some of its basic properties. In addition, we introduce the concept of an ordinary single valued neutrosophic subspace. Next, we define the ordinary single valued neutrosophic neighborhood system and we show that an ordinary single valued neutrosophic neighborhood system has the same properties in a classical neighborhood system. Finally, we introduce the concepts of an ordinary single valued neutrosophic base and an ordinary single valued neutrosophic subbase, and obtain two characterizations of an ordinary single valued neutrosophic base and one characterization of an ordinary single valued neutrosophic subbase.

2. Preliminaries

In this section, we introduce the concepts of single valued neutrosophic set, the complement of a single valued neutrosophic set, the inclusion between two single valued neutrosophic sets, the union and the intersection of them.

Definition 1 ([43])**.** *Let X be a non-empty set. Then, A is called a neutrosophic set (in sort, NS) in X, if A has the form $A = (T_A, I_A, F_A)$, where*

$$T_A : X \to]^-0, 1^+[, \quad I_A : X \to]^-0, 1^+[, \quad F_A : X \to]^-0, 1^+[.$$

Since there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, for each $x \in X$,

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

Moreover, for each $x \in X$, $T_A(x)$ (resp., $I_A(x)$ and $F_A(x)$) represent the degree of membership (resp., indeterminacy and non-membership) of x to A.

From Example 2.1.1 in [17], we can see that every IFS (intutionistic fuzzy set) A in a non-empty set X is an NS in X having the form

$$A = (T_A, 1 - (T_A + F_A), F_A),$$

where $(1 - (T_A + F_A))(x) = 1 - (T_A(x) + F_A(x))$.

Definition 2 ([43]). *Let A and B be two NSs in X. Then, we say that A is contained in B, denoted by $A \subset B$, if, for each $x \in X$, $inf\ T_A(x) \leq inf\ T_B(x)$, $sup\ T_A(x) \leq sup\ T_B(x)$, $inf\ I_A(x) \geq inf\ I_B(x)$, $sup\ I_A(x) \geq sup\ I_B(x)$, $inf\ F_A(x) \geq inf\ F_B(x)$ and $sup\ F_A(x) \geq sup\ F_B(x)$.*

Definition 3 ([48]). *Let X be a space of points (objects) with a generic element in X denoted by x. Then, A is called a single valued neutrosophic set (in short, SVNS) in X, if A has the form $A = (T_A, I_A, F_A)$, where $T_A,\ I_A,\ F_A : X \to [0, 1]$.*

In this case, $T_A,\ I_A,\ F_A$ are called truth-membership function, indeterminacy-membership function, falsity-membership function, respectively, and we will denote the set of all SVNSs in X as $SVNS(X)$.

Furthermore, we will denote the empty SVNS (resp. the whole SVNS] in X as 0_N (resp. 1_N) and define by $0_N(x) = (0, 1, 1)$ (resp. $1_N = (1, 0, 0)$), for each $x \in X$.

Definition 4 ([48]). *Let $A \in SVNS(X)$. Then, the complement of A, denoted by A^c, is an SVNS in X defined as follows: for each $x \in X$,*

$$T_{A^c}(x) = F_A(x),\ I_{A^c}(x) = 1 - I_A(x) \text{ and } F_{A^c}(x) = T_A(x).$$

Definition 5 ([50]). *Let $A,\ B \in SVNS(X)$. Then,*

(i) A is said to be contained in B, denoted by $A \subset B$, if, for each $x \in X$,

$$T_A(x) \leq T_B(x),\ I_A(x) \geq I_B(x) \text{ and } F_A(x) \geq F_B(x),$$

(ii) A is said to be equal to B, denoted by $A = B$, if $A \subset B$ and $B \subset A$.

Definition 6 ([51]). *Let $A,\ B \in SVNS(X)$. Then,*

(i) the intersection of A and B, denoted by $A \cap B$, is a SVNS in X defined as:

$$A \cap B = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B),$$

where $(T_A \wedge T_B)(x) = T_A(x) \wedge T_B(x)$, $(F_A \vee F_B) = F_A(x) \vee F_B(x)$, for each $x \in X$,

(ii) the union of A and B, denoted by $A \cup B$, is an SVNS in X defined as:

$$A \cup B = (T_A \vee T_B, I_A \wedge I_B, F_A \wedge F_B).$$

Remark 1. *Definitions 5 and 6 are different from the corresponding definitions in [48].*

Result 1 ([51], Proposition 2.1). *Let $A,\ B \in SVNS(X)$. Then,*

(1) *$A \subset A \cup B$ and $B \subset A \cup B$,*

(2) *$A \cap B \subset A$ and $A \cap B \subset B$,*

(3) *$(A^c)^c = A$,*

(4) *$(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$.*

The following are immediate results of Definitions 5 and 6.

Proposition 1. *Let $A,\ B,\ C \in SVNS(X)$. Then,*

(1) (Commutativity) *$A \cup B = B \cup A$, $A \cap B = B \cap A$,*

(2) (Associativity) $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$,
(3) (Distributivity) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,
(4) (Idempotency) $A \cup A = A$, $A \cap A = A$,
(5) (Absorption) $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$,
(5) (DeMorgan's laws) $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$,
(7) $A \cap 0_N = 0_N$, $A \cup 1_N = 1_N$,
(8) $A \cup 0_N = A$, $A \cap 1_N = A$.

Definition 7 (see [46]). *Let $\{A_\alpha\}_{\alpha \in \Gamma} \subset SVNS(X)$. Then,*

(i) the union of $\{A_\alpha\}_{\alpha \in \Gamma}$, denoted by $\bigcup_{\alpha \in \Gamma} A_\alpha$, is a single valued neutrosophic set in X defined as follows: for each $x \in X$,

$$(\bigcup_{\alpha \in \Gamma} A_\alpha)(x) = (\bigvee_{\alpha \in \Gamma} T_{A_\alpha}(x), \bigwedge_{\alpha \in \Gamma} I_{A_\alpha}(x), \bigwedge_{\alpha \in \Gamma} F_{A_\alpha}(x)),$$

(ii) the intersection of $\{A_\alpha\}_{\alpha \in \Gamma}$, denoted by $\bigcap_{\alpha \in \Gamma} A_\alpha$, is a single valued neutrosophic set in X defined as follows: for each $x \in X$,

$$(\bigcap_{\alpha \in \Gamma} A_\alpha)(x) = (\bigwedge_{\alpha \in \Gamma} T_{A_\alpha}(x), \bigvee_{\alpha \in \Gamma} I_{A_\alpha}(x), \bigvee_{\alpha \in \Gamma} F_{A_\alpha}(x)).$$

The following are immediate results of the above definition.

Proposition 2. *Let $A \in SVNS(X)$ and let $\{A_\alpha\}_{\alpha \in \Gamma} \subset SVNS(X)$. Then,*
(1) (Generalized Distributivity)

$$A \cup (\bigcap_{\alpha \in \Gamma} A_\alpha) = \bigcap_{\alpha \in \Gamma} (A \cup A_\alpha), \quad A \cap (\bigcup_{\alpha \in \Gamma} A_\alpha) = \bigcup_{\alpha \in \Gamma} (A \cap A_\alpha),$$

(2) (Generalized DeMorgan's laws)

$$(\bigcup_{\alpha \in \Gamma} A_\alpha)^c = \bigcap_{\alpha \in \Gamma} A_\alpha^c, \quad (\bigcap_{\alpha \in \Gamma} A_\alpha)^c = \bigcup_{\alpha \in \Gamma} A_\alpha^c.$$

3. Ordinary Single Valued Neutrosophic Topology

In this section, we define an ordinary single valued neutrosophic topological space and obtain some of its properties. Throughout this paper, we denote the set of all subsets (resp. fuzzy subsets) of a set X as 2^X (resp. I^X).

For $T_\alpha, I_\alpha, F_\alpha \in I$, $\alpha = (T_\alpha, I_\alpha, F_\alpha) \in I \times I \times I$ is called a single valued neutrosophic value. For two single valued neutrosophic values α and β,
(i) $\alpha \leq \beta$ iff $T_\alpha \leq T_\beta$, $I_\alpha \geq I_\beta$ and $F_\alpha \geq F_\beta$,
(ii) $\alpha < \beta$ iff $T_\alpha < T_\beta$, $I_\alpha > I_\beta$ and $F_\alpha > F_\beta$.
In particular, the form $\alpha^* = (\alpha, 1-\alpha, 1-\alpha)$ is called a single valued neutrosophic constant.

We denote the set of all single valued neutrosophic values (resp. constant) as **SVNV** (resp. **SVNC**) (see [49]).

Definition 8. *Let X be a nonempty set. Then, a mapping $\tau = (T_\tau, I_\tau, F_\tau) : 2^X \to I \times I \times I$ is called an ordinary single valued neutrosophic topology (in short, osvnt) on X if it satisfies the following axioms: for any $A, B \in 2^X$ and each $\{A_\alpha\}_{\alpha \in \Gamma} \subset 2^X$,*

(OSVNT1) $\tau(\phi) = \tau(X) = (1, 0, 0)$,
(OSVNT2) $T_\tau(A \cap B) \geq T_\tau(A) \wedge T_\tau(B)$, $I_\tau(A \cap B) \leq I_\tau(A) \vee I_\tau(B)$, $F_\tau(A \cap B) \leq F_\tau(A) \vee F_\tau(B)$,
(OSVNT3) $T_\tau(\bigcup_{\alpha \in \Gamma} A_\alpha) \geq \bigwedge_{\alpha \in \Gamma} T_\tau(A_\alpha)$, $I_\tau(\bigcup_{\alpha \in \Gamma} A_\alpha) \leq \bigvee_{\alpha \in \Gamma} I_\tau(A_\alpha)$, $F_\tau(\bigcup_{\alpha \in \Gamma} A_\alpha) \leq \bigvee_{\alpha \in \Gamma} F_\tau(A_\alpha)$.

The pair (X,τ) is called an ordinary single valued neutrosophic topological space (in short, osvnts). We denote the set of all ordinary single valued neutrosophic topologies on X as $OSVNT(X)$.

Let $2=\{0,1\}$ and let $\tau : 2^X \to 2\times 2\times 2$ satisfy the axioms in Definition 8. Since we can consider as $(1,0,0)=1$ and $(0,1,1)=0$, $\tau \in T(X)$, where $T(X)$ denotes the set of all classical topologies on X. Thus, we can see that $T(X)\subset OSVNT(X)$.

Example 1. *(1) Let $X=\{a,b,c\}$. Then, $2^X=\{\phi, X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$. We define the mapping $\tau : 2^X \to I\times I\times I$ as follows:*

$\tau(\phi)=\tau(X)=(1,0,0)$,
$\tau(\{a\})=(0.7,0.3,0.4), \tau(\{b\})=(0.6,0.2,0.3), \tau(\{c\})=(0.8,0.1,0.2)$,
$\tau(\{a,b\})=(0.6,0.3,0.4), \tau(\{b,c\})=(0.7,0.1,0.2), \tau(\{a,c\})=(0.8,0.2,0.3)$.

Then, we can easily see that $\tau\in OSVNT(X)$.

(2) Let X be a nonempty set. We define the mapping $\tau_\phi : 2^X \to I\times I\times I$ as follows: for each $A\in 2^X$,

$$\tau_\phi(A)=\begin{cases}(1,0,0) & \textit{if } \text{either } A=\phi \text{ or } A=X,\\ (0,1,1) & \textit{otherwise.}\end{cases}$$

Then, clearly, $\tau_\phi \in OSVT(X)$.

In this case, τ_ϕ (resp. (X,τ_ϕ)) is called the ordinary single valued neutrosophic indiscrete topology on X (resp. the ordinary single valued neutrosophic indiscrete space].

(3) Let X be a nonempty set. We define the mapping $\tau_X : 2^X \to I\times I\times I$ as follows: for each $A\in 2^X$,

$$\tau_X(A)=(1,0,0).$$

Then, clearly, $\tau_X\in OSVNT(X)$.

In this case, τ_X (resp. (X,τ_X)) is called the ordinary single valued neutrosophic discrete topology on X (resp. the ordinary single valued neutrosophic discrete space].

*(4) Let X be a set and let $\alpha=(T_\alpha, I_\alpha, F_\alpha)\in$ **SVNV** be fixed, where $T_\alpha\in I_1$ and $I_\alpha,\ F_\alpha\in I_0$. We define the mapping $\tau : 2^X \to I\times I\times I$ as follows: for each $A\in 2^X$,*

$$\tau(A)=\begin{cases}(1,0,0) & \textit{if } \text{either } A=\phi \text{ or } A^c \text{ is finite},\\ \alpha & \textit{otherwise.}\end{cases}$$

Then, we can easily see that $\tau\in OSVNT(X)$.

In this case, τ is called the α-ordinary single valued neutrosophic finite complement topology on X and will be denoted by $OSVNCof(X)$. $OSVNCof(X)$ is of interest only when X is an infinite set because if X is finite, then $OSVNCof(X)=\tau_\phi$.

*(5) Let X be an infinite set and let $\alpha=(T_\alpha, I_\alpha, F_\alpha)\in$ **SVNV** be fixed, where $T_\alpha\in I_1$ and $I_\alpha,\ F_\alpha\in I_0$. We define the mapping $\tau : 2^X \to I\times I\times I$ as follows: for each $A\in 2^X$,*

$$\tau(A)=\begin{cases}(1,0,0) & \textit{if } \text{either } A=\phi \text{ or } A^c \text{ is countable},\\ \alpha & \textit{otherwise.}\end{cases}$$

Then, clearly, $\tau\in OSVNT(X)$.

In this case, τ is called the α-ordinary single valued neutrosophic countable complement topology on X and is denoted by $OSVNCoc(X)$.

(6) Let T be the topology generated by $\mathcal{S} = \{(a,b] : a,b \in \mathbb{R}, a < b\}$ as a subbase, let T_0 be the family of all open sets of $\mathbb{R}$ with respect to the usual topology on $\mathbb{R}$ and let $\alpha = (T_\alpha, I_\alpha, F_\alpha) \in$ **SVNV** *be fixed, where $T_\alpha \in I_1$ and $I_\alpha,\ F_\alpha \in I_0$. We define the mapping $\tau : 2^{\mathbb{R}} \to I \times I \times I$ as follows: for each $A \in I^{\mathbb{R}}$,*

$$\tau(A) = \begin{cases} (1,0,0) & \text{if } A \in T_0, \\ \alpha & \text{if } A \in T \setminus T_0, \\ (0,1,1) & \text{otherwise.} \end{cases}$$

Then, we can easily see that $\tau \in OSVNT(X)$.

(7) Let $T \in T(X)$. We define the mapping $\tau_T : 2^X \to I \times I \times I$ as follows : for each $A \in 2^X$,

$$\tau_T(A) = \begin{cases} (1,0,0) & \text{if } A \in T, \\ (0,1,1) & \text{otherwise.} \end{cases}$$

Then, it is easily seen that $\tau_T \in OSVNT(X)$. Moreover, we can see that if T is the classical indiscrete topology, then $\tau_T = \tau_\phi$ and if T is the classical discrete topology, then $\tau_T = \tau_X$.

Remark 2. *(1) If $I = 2$, then we can think that Definition 8 also coincides with the known definition of classical topology.*

(2) Let (X, τ) be an osvnsts. We define two mappings $[\ ]\tau,\ <\ >\tau : 2^X \to I \times I \times I$, respectively, as follows : for each $A \in 2^X$,

$$([\ ]\tau)(A) = (T_\tau(A), I_\tau(A), 1 - T_\tau(A)),\ (<\ >\tau)(A) = (1 - F_\tau(A), I_\tau(A), F_\tau(A)).$$

Then, we can easily see that $[\ ]\tau,\ <\ >\tau \in OSVNT(X)$.

Definition 9. *Let X be a nonempty set. Then, a mapping $\mathcal{C} = (\mu_{\mathcal{C}}, \nu_{\mathcal{C}}) : 2^X \to I \times I \times I$ is called an ordinary single valued neutrosophic cotopology (in short, osvnct) on X if it satisfies the following conditions: for any $A, B \in 2^X$ and each $\{A_\alpha\}_{\alpha \in \Gamma} \subset 2^X$,*

(OSVNCT1) $\mathcal{C}(\phi) = \mathcal{C}(X) = (1,0,0)$,

(OSVNCT2) $T_{\mathcal{C}}(A \cup B) \geq T_{\mathcal{C}}(A) \wedge T_{\mathcal{C}}(B),\quad I_{\mathcal{C}}(A \cup B) \leq I_{\mathcal{C}}(A) \vee I_{\mathcal{C}}(B),$
$F_{\mathcal{C}}(A \cup B) \leq F_{\mathcal{C}}(A) \vee F_{\mathcal{C}}(B)$,

(OSVNCT3) $T_{\mathcal{C}}(\bigcap_{\alpha \in \Gamma} A_\alpha) \geq \bigwedge_{\alpha \in \Gamma} T_{\mathcal{C}}(A_\alpha),\quad I_{\mathcal{C}}(\bigcap_{\alpha \in \Gamma} A_\alpha) \leq \bigvee_{\alpha \in \Gamma} I_{\mathcal{C}}(A_\alpha),$
$F_{\mathcal{C}}(\bigcap_{\alpha \in \Gamma} A_\alpha) \leq \bigvee_{\alpha \in \Gamma} F_{\mathcal{C}}(A_\alpha)$.

The pair $(X, \mathcal{C})$ is called an ordinary single valued neutrosophic cotopological space (in short, osvncts).

The following is an immediate result of Definitions 8 and 9.

Proposition 3. *We define two mappings $f : OSVNT(X) \to OSVNCT(X)$ and $g : OSVNCT(X) \to OSVNT(X)$ respectively as follows:*

$$[f(\tau)](A) = \tau(A^c) \quad \text{for any } \tau \in OSVNT(X) \text{ and any } A \in 2^X$$

and

$$[g(\mathcal{C})](A) = \mathcal{C}(A^c) \quad \text{for any } \mathcal{C} \in OSVNCT(X) \text{ and any } A \in 2^X.$$

Then, f and g are well-defined. Moreover, $g \circ f = 1_{OSVNT(X)}$ and $f \circ g = 1_{OSVNCT(X)}$.

Remark 3. *(1) For each $\tau \in OSVNT(X)$ and each $\mathcal{C} \in OSVNCT(X)$, let $f(\tau) = \mathcal{C}_\tau$ and $g(\mathcal{C}) = \tau_{\mathcal{C}}$. Then, from Proposition 3, we can see that $\tau_{\mathcal{C}_\tau} = \tau$ and $\mathcal{C}_{\tau_{\mathcal{C}}} = \mathcal{C}$.*

(2) Let $(X, \mathcal{C})$ be an osvncts. We define two mappings $[\ ]\mathcal{C}$, $<\ \ >\mathcal{C} : 2^X \to I \times I \times I$, respectively, as follows: for each $A \in 2^X$,

$$([\ ]\mathcal{C})(A) = (T_{\mathcal{C}}(A), I_{\mathcal{C}}(A), 1 - T_{\mathcal{C}}(A)),\ (<\ \ >\mathcal{C})(A) = (1 - F_{\mathcal{C}}(A), I_{\mathcal{C}}(A), F_{\mathcal{C}}(A)).$$

Then, we can easily see that $[\ ]\mathcal{C}$, $<\ \ >\mathcal{C} \in OSVNCT(X)$.

Definition 10. *Let $\tau_1, \tau_2 \in OSVNT(X)$ and let $\mathcal{C}_1, \mathcal{C}_2 \in OSVNCT(X)$.*

(i) We say that τ_1 is finer than τ_2 or τ_2 is coarser than τ_1, denoted by $\tau_2 \preceq \tau_1$, if $\tau_2(A) \leq \tau_1(A)$, i.e., for each $A \in 2^X$,

$$T_{\tau_2}(A) \leq T_{\tau_1}(A),\ I_{\tau_2}(A) \geq I_{\tau_1}(A),\ F_{\tau_2}(A) \geq F_{\tau_1}(A).$$

(ii) We say that $\mathcal{C}_1$ is finer than $\mathcal{C}_2$ or $\mathcal{C}_2$ is coarser than $\mathcal{C}_1$, denoted by $\mathcal{C}_2 \preceq \mathcal{C}_1$, if $\mathcal{C}_2(A) \leq \mathcal{C}_1(A)$, i.e., for each $A \in 2^X$,

$$T_{\mathcal{C}_2}(A) \leq T_{\mathcal{C}_1}(A),\ I_{\mathcal{C}_2}(A) \geq I_{\mathcal{C}_1}(A),\ F_{\mathcal{C}_2}(A) \geq F_{\mathcal{C}_1}(A).$$

We can easily see that τ_1 is finer than τ_2 if and only if $\mathcal{C}_{\tau_1}$ is finer than $\mathcal{C}_{\tau_2}$, and $(OSVNT(X), \preceq)$ and $(OSVNCT(X), \preceq)$ are posets, respectively.

From Example 1 (2) and (3), it is obvious that τ_ϕ is the coarsest ordinary single valued neutrosophic topology on X and τ_X is the finest ordinary single valued neutrosophic topology on X.

Proposition 4. *If $\{\tau_\alpha\}_{\alpha \in \Gamma} \subset OSVNT(X)$, then $\bigcap_{\alpha \in \Gamma} \tau_\alpha \in OSVNT(X)$, where $[\bigcap_{\alpha \in \Gamma} \tau_\alpha](A) = (\bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(A), \bigvee_{\alpha \in \Gamma} I_{\tau_\alpha}(A), \bigvee_{\alpha \in \Gamma} F_{\tau_\alpha}(A)),\ \forall\, A \in 2^X$.*

Proof. Let $\tau = \bigcap_{\alpha \in \Gamma} \tau_\alpha$ and let $\alpha \in \Gamma$. Since $\tau_\alpha \in OSVNT(X)$, $\tau_\alpha(X) = \tau_\alpha(\phi) = (1, 0, 0)$, i.e.,

$$T_{\tau_\alpha}(X) = T_{\tau_\alpha}(\phi) = 1, \quad I_{\tau_\alpha}(X) = I_{\tau_\alpha}(\phi) = 0, \quad F_{\tau_\alpha}(X) = F_{\tau_\alpha}(\phi) = 0.$$

Then, $T_\tau(X) = \bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(X) = 1$, $I_\tau(X) = \bigvee_{\alpha \in \Gamma} I_{\tau_\alpha}(X) = 0 = F_\tau(X)$. Similarly, we have $T_\tau(\phi) = 1$, $I_\tau(\phi) = 0 = F_\tau(\phi)$. Thus, the condition (OSVNT1) holds.

Let $A, B \in 2^X$. Then,

$$\begin{aligned} T_\tau(A \cap B) &= \textstyle\bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(A \cap B) && \text{[By the definition of } \tau] \\ &\geq \textstyle\bigwedge_{\alpha \in \Gamma}(T_{\tau_\alpha}(A) \wedge T_{\tau_\alpha}(B)) && \text{[Since } \tau_\alpha \in OSVNT(X)] \\ &= (\textstyle\bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(A)) \wedge (\textstyle\bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(B)) \\ &= T_\tau(A) \wedge T_\tau(B) && \text{[By the definition of } \tau] \end{aligned}$$

and

$$\begin{aligned} I_\tau(A \cap B) &= \textstyle\bigvee_{\alpha \in \Gamma} I_{\tau_\alpha}(A \cap B) && \text{[By the definition of } \tau] \\ &\leq \textstyle\bigvee_{\alpha \in \Gamma}(I_{\tau_\alpha}(A) \vee I_{\tau_\alpha}(B)) && \text{[Since } \tau_\alpha \in OSVNT(X)] \\ &= (\textstyle\bigvee_{\alpha \in \Gamma} I_{\tau_\alpha}(A)) \vee (\textstyle\bigvee_{\alpha \in \Gamma} I_{\tau_\alpha}(B)) \\ &= I_\tau(A) \vee I_\tau(B). && \text{[By the definition of } \tau] \end{aligned}$$

Similarly, we have $F_\tau(A \cap B) \leq F_\tau(A) \vee F_\tau(B)$. Thus, the condition (OSVNT2) holds:

Now, let $\{A_j\}_{j \in J} \subset 2^X$. Then,

$$\begin{aligned} T_\tau(\textstyle\bigcup_{j \in J} A_j) &= \textstyle\bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(\bigcup_{j \in J} A_j) && \text{[By the definition of } \tau] \\ &\geq \textstyle\bigwedge_{\alpha \in \Gamma}(\bigwedge_{j \in J} T_{\tau_\alpha}(A_j)) && \text{[Since } \tau_\alpha \in OSVNT(X)] \\ &= \textstyle\bigwedge_{j \in J}(\bigwedge_{\alpha \in \Gamma} T_{\tau_\alpha}(A_j)) \\ &= \textstyle\bigwedge_{j \in J}[\bigcap_{\alpha \in \Gamma} T_{\tau_\alpha}](A_j) && \text{[By the definition of } \tau] \\ &= \textstyle\bigvee_{j \in J} T_\tau(A_j) \end{aligned}$$

and

$$
\begin{aligned}
I_\tau(\bigcup_{j\in J} A_j) &= \bigvee_{\alpha\in\Gamma} I_{\tau_\alpha}(\bigcup_{j\in J} A_j) && \text{[By the definition of } \tau\text{]}\\
&\le \bigvee_{\alpha\in\Gamma}(\bigvee_{j\in J} I_{\tau_\alpha}(A_j)) && \text{[Since } \tau_\alpha \in OSVNT(X)\text{]}\\
&= \bigvee_{j\in J}(\bigvee_{\alpha\in\Gamma} I_{\tau_\alpha}(A_j))\\
&= \bigvee_{j\in J}[\bigcup_{\alpha\in\Gamma} I_{\tau_\alpha}](A_j) && \text{[By the definition of } \tau\text{]}\\
&= \bigvee_{j\in J} I_\tau(A_j).
\end{aligned}
$$

Similarly, we have $F_\tau(\bigcup_{j\in J} A_j) \le \bigvee_{j\in J} F_\tau(A_j)$. Thus, the condition (OSVNT3) holds. This completes the proof. □

From Definition 10 and Proposition 4, we have the following.

Proposition 5. *$(OSVNT(X), \preceq)$ is a meet complete lattice with the least element τ_ϕ and the greatest element τ_X.*

Definition 11. *Let (X, τ) be an osvnts and let $\alpha \in$ **SVNV**. We define two sets $[\tau]_\alpha$ and $[\tau]^*_\alpha$ as follows, respectively:*

(i) $[\tau]_\alpha = \{A \in 2^X : T_\tau(A) \ge T_\alpha,\ I_\tau(A) \le I_\alpha,\ I_\tau(A) \le F_\alpha\}$,

(ii) $[\tau]^*_\alpha = \{A \in 2^X : T_\tau(A) > T_\alpha,\ I_\tau(A) < I_\alpha,\ F_\tau(A) < F_\alpha\}$.

In this case, $[\tau]_\alpha$ (resp. $[\tau]^*_\alpha$) is called the α-level (resp. strong α-level] of τ. If $\alpha = (0, 1, 1)$, then $[\tau]_{(0,1,1)} = 2^X$, i.e., $[\tau]_{(0,1,1)}$ is the classical discrete topology on X and if $\alpha = (1, 0, 0)$, then $[\tau]^*_{(1,0,0)} = \phi$. Moreover, we can easily see that for any $\alpha \in$ **SVNV**, $[\tau]^*_\alpha \subset [\tau]_\alpha$.

Lemma 1. *Let $\tau \in OSVNT(X)$ and let $\alpha,\ \beta \in$ **SVNV**. Then,*

(1) $[\tau]_\alpha \in T(X)$,

(2) *if* $\alpha \le \beta$, *then* $[\tau]_\beta \subset [\tau]_\alpha$,

(3) $[\tau]_\alpha = \bigcap_{\beta<\alpha} [\tau]_\beta$, *where* $\alpha \in I_0 \times I_1 \times I_1$,

$(1)'$ $[\tau]^*_\alpha \in T(X)$, *where* $\alpha \in I_1 \times I_0 \times I_0$,

$(2)'$ *if* $\alpha \le \beta$, *then* $[\tau]^*_\beta \subset [\tau]^*_\alpha$,

$(3)'$ $[\tau]^*_\alpha = \bigcup_{\beta>(\alpha} [\tau]^*_\beta$, *where* $\alpha \in I_1 \times I_0 \times I_0$.

Proof. The proofs of (1), $(1)'$, (2) and $(2)'$ are obvious from Definitions 8 and 11.

(3) From (2), $\{[\tau]_\alpha\}_{\alpha\in I_0\times I_1\times I_1}$ is a descending family of classical topologies on X. Then, clearly, $[\tau]_\alpha \subset \bigcap_{\beta<\alpha}[\tau]_\beta$, for each $\alpha \in I_0 \times I_1 \times I_1$.

Suppose $A \notin [\tau]_\alpha$. Then, $T_\tau(A) < T_\alpha$ or $I_\tau(A) > I_\alpha$ or $F_\tau(A) > F_\alpha$. Thus,

$$\text{there exists } T_\beta \in I_0 \text{ such that } T_\tau(A) < T_\beta < T_\alpha$$

or

$$\text{there exists } I_\beta \in I_1 \text{ such that } I_\tau(A) > I_\beta > I_\alpha$$

or

$$\text{there exists } F_\beta \in I_1 \text{ such that } F_\tau(A) > F_\beta > F_\alpha.$$

Thus, $A \notin [\tau]_\beta$, for some $\beta \in$ **SVNV** such that $\beta < \alpha$, i.e., $A \notin \bigcap_{\beta<\alpha} [\tau]_\beta$. Hence, $\bigcap_{\beta<\alpha} [\tau]_\beta \subset [\tau]_\alpha$.

Therefore, $[\tau]_\alpha = \bigcap_{\beta<\alpha} [\tau]_\beta$.

$(3)'$ The proof is similar to (3). □

Remark 4. *From (1) and (2) in Lemma 1, we can see that, for each $\tau \in OSVNT(X)$, $\{[\tau]_\alpha\}_{\alpha \in \mathbf{SVNV}}$ is a family of descending classical topologies called the α-level classical topologies on X with respect to τ.*

The following is an immediate result of Lemma 1.

Corollary 1. *Let (X, τ) be an osvnts. Then, $[\tau]_{\alpha^*} = \bigcap_{\beta<\alpha} [\tau]_{\beta^*}$ for each $\alpha^* \in \mathbf{SVNC}$, where $\alpha \in I_0$.*

Lemma 2. (1) *Let $\{\tau_\alpha\}_{\alpha \in \mathbf{SVNV}}$ be a descending family of classical topologies on X such that $\tau_{(0,1,1)}$ is the classical discrete topology on X. We define the mapping $\tau : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,*

$$\tau(A) = (\bigvee_{A \in \tau_\alpha} T_\alpha, \bigwedge_{A \in \tau_\alpha} I_\alpha, \bigwedge_{A \in \tau_\alpha} F_\alpha).$$

Then, $\tau \in OSVNT(X)$.
(2) *If $\tau_\alpha = \bigcap_{\beta<\alpha} \tau_\alpha$, for each $\alpha \in \mathbf{SVNV}$ ($\alpha \in I_0 \times I_1 \times I_1$), then $[\tau]_\alpha = \tau_\alpha$.*
(3) *If $\tau_\alpha = \bigcup_{\beta>\alpha} \tau_\beta$, for each $\alpha \in \mathbf{SVNV}$ ($\alpha \in I_1 \times I_0 \times I_0$), then $[\tau]^*_\alpha = \tau_\alpha$.*

Proof. The proof is similar to Lemma 3.9 in [28]. □

The following is an immediate result of Lemma 2.

Corollary 2. *Let $\{\tau_{\alpha^*}\}_{\alpha \in I_0}$ be a descending family of classical topologies on X such that $\tau_{(0,1,1)}$ is the classical discrete topology on X. We define the mapping $\tau : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,*

$$\tau(A) = (\bigvee_{A \in \tau_{\alpha^*}} \alpha, \bigwedge_{A \in \tau_{\alpha^*}} (1-\alpha), \bigwedge_{A \in \tau_{\alpha^*}} (1-\alpha)).$$

Then, $\tau \in OSVNT(X)$ and $[\tau]_{\alpha^*} = \bigcap_{\beta<\alpha} \tau_{\beta^*} = \tau_{\alpha^*}$ $\forall\, \alpha \in I_0$.

From Lemmas 1 and 2, we have the following result.

Proposition 6. *Let $\tau \in OSVNT(X)$ and let $[\tau]_\alpha$ be the α-level classical topology on X with respect to τ. We define the mapping $\eta : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,*

$$\eta(A) = (\bigvee_{A \in [\tau]_\alpha} T_\alpha, \bigwedge_{A \in [\tau]_\alpha} I_\alpha, \bigwedge_{A \in [\tau]_\alpha} F_\alpha).$$

Then, $\eta = \tau$.

The fact that an ordinary single valued neutrosophic topological space fully determined by its decomposition in classical topologies is restated in the following theorem.

Theorem 1. *Let $\tau_1, \tau_2 \in OSVNT(X)$. Then, $\tau_1 = \tau_2$ if and only if $[\tau_1]_\alpha = [\tau_2]_\alpha$ for each $\alpha \in \mathbf{SVNV}$, or alternatively, if and only if $[\tau_1]^*_\alpha = [\tau_2]^*_\alpha$ for each $\alpha \in \mathbf{SVNV}$.*

Remark 5. *In a similar way, we can construct an ordinary single valued neutrosophic cotopology $\mathcal{C}$ on a set X, by using the α-levels,*

$$[\mathcal{C}]_\alpha = \{A \in I^X : T_{\mathcal{C}}(A) \geq T_\alpha,\ I_{\mathcal{C}}(A) \leq I_\alpha,\ F_{\mathcal{C}}(A) \leq F_\alpha\}$$

and

$$[\mathcal{C}]^*_\alpha = \{A \in I^X : T_{\mathcal{C}}(A) > T_\alpha,\ I_{\mathcal{C}}(A) < I_\alpha,\ F_{\mathcal{C}}(A) < F_\alpha\},$$

for each $\alpha \in \mathbf{SVNV}$.

Definition 12. *Let $T \in T(X)$ and let $\tau \in OSVNT(X)$. Then, τ is said to be compatible with T if $T = S(\tau)$, where $S(\tau) = \{A \in 2^X : T_\tau(A) > 0, I_\tau(A) < 1,\ F_\tau(A) < 1\}$.*

Example 2. *(1) Let τ_ϕ be the ordinary single valued neutrosophic indiscrete topology on a nonempty set X and let T_0 be the classical indiscrete topology on X. Then, clearly,*

$$S(\tau_\phi) = \{A \in 2^X : T_{\tau_\phi}(A) > 0, I_{\tau_\phi}(A) < 1,\ F_{\tau_\phi}(A) < 1\} = \{\phi, X\} = T_0.$$

Thus, τ_ϕ is compatible with T_0.

(2) Let τ_X be the ordinary single valued neutrosophic discrete topology on a nonempty set X and let T_1 be the classical discrete topology on X. Then, clearly,

$$S(\tau_X) = \{A \in 2^X : T_{\tau_X}(A) > 0, I_{\tau_X}(A) < 1,\ F_{\tau_X}(A) < 1\} = 2^X = T_1.$$

Thus, τ_X is compatible with T_1.

*(3) Let X be a nonempty set and let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_0 \times I_1 \times I_1$. We define the mapping $\tau : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,*

$$\tau(A) = \begin{cases} (1,0,0) & \textit{if} \text{ either } A = \phi \text{ or } A = X, \\ \alpha & \text{otherwise.} \end{cases}$$

Then, clearly, $\tau \in OSVNT(X)$ and τ is compatible with T_1.

Furthermore, every classical topology can be considered as an ordinary single valued neutrosophic topology in the sense of the following result.

Proposition 7. *Let (X, τ) be a classical topological space and and let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_0 \times I_1 \times I_1$. Then, there exists $\tau^\alpha \in OSVNT(X)$ such that τ^α is compatible with T. Moreover, $[\tau^\alpha]_\alpha = \tau$.*

In this case, τ^α is called the α-th ordinary single valued neutrosophic topology on X and (X, τ^α) is called the α-th ordinary single valued neutrosophic topological space.

Proof. Let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_0 \times I_1 \times I_1$ and we define the mapping $\tau^\alpha : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,

$$\tau^\alpha(A) = \begin{cases} (1,0,0) & \text{if either } A = \phi \text{ or } A = X, \\ \alpha & \text{if } A \in \tau \setminus \{\phi, X\}, \\ (0,1,1) & \text{otherwise.} \end{cases}$$

Then, we can easily see that $\tau^\alpha \in OSVNT(X)$ and $[\tau^\alpha]_\alpha = \tau$. Moreover, by the definition of τ^α,

$$S(\tau^\alpha) = \{A \in 2^X : T_{\tau^\alpha}(A) > 0,\ I_{\tau^\alpha}(A) < 1,\ F_{\tau^\alpha}(A) < 1\} = \tau.$$

Thus, τ^α is compatible with τ. □

Proposition 8. *Let (X, T) be a classical topological space, let $C(T)$ be the set of all osvnts on X compatible with T, let $\widetilde{T} = T \setminus \{\phi, X\}$ and let $(I \times I \times I)^{\widetilde{T}}_{(0,1,1)}$ be the set of all mappings $f : \widetilde{T} \to I \times I \times I$ satisfying the following conditions: for any $A,\ B \in \widetilde{T}$ and each $(A_j)_{j \in J} \subset \widetilde{T}$,*

(1) $f(A) \neq (0,1,1)$,

(2) $T_f(A \cap B) \geq T_f(A) \wedge T_f(B)$, $\quad I_f(A \cap B) \leq I_f(A) \vee T_f(B)$,
$F_f(A \cap B) \leq F_f(A) \vee F_f(B)$,

(3) $T_f(\bigcup_{j \in J} A_j) \geq \bigwedge_{j \in J} T_f(A_j)$, $\quad I_f(\bigcup_{j \in J} A_j) \leq \bigvee_{j \in J} I_f(A_j)$,
$F_f(\bigcup_{j \in J} A_j) \leq \bigvee_{j \in J} F_f(A_j)$.

Then, there is a one-to-one correspondence between $C(T)$ *and* $(I \times I \times I)^{\widetilde{T}}_{(0,1,1)}$.

Proof. We define the mapping $F : (I \times I \times I)^{\widetilde{T}}_{(0,1,1)} \to C(T)$ as follows: for each $f \in (I \times I \times I)^{\widetilde{T}}_{(0,1,1)}$,

$$F(f) = \tau_f,$$

where $\tau_f : 2^X \to I \times I \times I$ is the mapping defined by: for each $A \in 2^X$,

$$\tau_f(A) = \begin{cases} (1,0,0) & \text{if either } A = \phi \text{ or } A = X, \\ f(A) & \text{if } A \in \widetilde{T}, \\ (0,1,1) & \text{otherwise.} \end{cases}$$

Then, we easily see that $\tau_f \in C(T)$.

Now, we define the mapping $G : C(T) \to (I \times I \times I)^{\widetilde{T}}_{(0,1,1)}$ as follows: for each $\tau \in C(T)$,

$$G(\tau) = f_\tau,$$

where $f_\tau : \widetilde{T} \to I \times I \times I$ is the mapping defined by: for each $A \in \widetilde{T}$,

$$f_\tau(A) = \tau(A).$$

Then, clearly, $f_\tau \in (I \times I \times I)^{\widetilde{T}}_{(0,1,1)}$. Furthermore, we can see that $F \circ G = id_{C(T)}$ and $G \circ F = id_{(I \times I \times I)^{\widetilde{T}}_{(0,1,1)}}$. Thus, $C(T)$ is equipotent to $I \times I \times I)^{\widetilde{T}}_{(0,1,1)}$. This completes the proof. □

Proposition 9. *Let* (X, τ) *be an osvnts and let* $Y \subset X$. *We define the mapping* $\tau_Y : 2^Y \to I \times I \times I$ *as follows: for each* $A \in 2^Y$,

$$\tau_Y(A) = (\bigvee_{B \in 2^X,\ A = B \cap Y} T_\tau(B), \bigwedge_{B \in 2^X,\ A = B \cap Y} I_\tau(B), \bigwedge_{B \in 2^X,\ A = B \cap Y} F_\tau(B)).$$

Then, $\tau_Y \in OSVNT(Y)$ *and for each* $A \in 2^Y$,

$$T_{\tau_Y}(A) \geq T_\tau(A),\ I_{\tau_Y}(A) \leq I_\tau(A),\ F_{\tau_Y}(A) \leq F_\tau(A).$$

In this case, (Y, τ_Y) is called an ordinary single valued neutrosophic subspace of (X, τ) and τ_Y is called the induced ordinary single valued neutrosophic topology on A by τ.

Proof. It is obvious that the condition (OSVNT1) holds, i.e., $\tau_Y(\phi) = \tau_Y(Y) = (1,0,0)$.

Let $A, B \in 2^Y$. Then, by proof of Proposition 5.1 in [34], $T_{\tau_Y}(A \cap B) \geq T_{\tau_Y}(A) \wedge T_{\tau_Y}(B)$.

Let us show that $I_{\tau_Y}(A \cap B) \leq I_{\tau_Y}(A) \vee I_{\tau_Y}(B)$. Then,

$$\begin{aligned} I_{\tau_Y}(A) \vee I_{\tau_Y}(B) &= (\textstyle\bigwedge_{C_1 \in 2^X,\ A = Y \cap C_1} I_\tau(C_1)) \vee (\bigwedge_{C_2 \in 2^X,\ B = Y \cap C_2} I_\tau(C_2)) \\ &= \textstyle\bigwedge_{C_1,\ C_1 \in 2^X,\ A \cap B = Y \cap (C_1 \cap C_2)} [I_\tau(C_1) \vee I_\tau(C_2)] \\ &\geq \textstyle\bigwedge_{C_1,\ C_1 \in 2^X,\ A \cap B = Y \cap (C_1 \cap C_2)} I_\tau(C_1 \cap C_2) \\ &= I_{\tau_Y}(A \cap B). \end{aligned}$$

Similarly, we have $F_{\tau_Y}(A \cap B) \leq F_{\tau_Y}(A) \vee F_{\tau_Y}(B)$. Thus, the condition (OSVNT2) holds.

Now, let $\{A_\alpha\}_{\alpha \in \Gamma} \subset 2^Y$. Then, by the proof of Proposition 5.1 in [34], $T_{\tau_Y}(\bigcup_{\alpha \in \Gamma} A_\alpha) \geq \bigwedge_{\alpha \in \Gamma} T_{\tau_Y}(A_\alpha)$. On the other hand,

$$\begin{aligned} I_{\tau_Y}(\textstyle\bigcup_{\alpha \in \Gamma} A_\alpha) &= \textstyle\bigwedge_{B_\alpha \in 2^X,\ (\bigcup_{\alpha \in \Gamma} B_\alpha) \cap Y = \bigcup_{\alpha \in \Gamma} A_\alpha} I_\tau(\bigcup_{\alpha \in \Gamma} B_\alpha) \\ &\leq \textstyle\bigwedge_{B_\alpha \in 2^X,\ (\bigcup_{\alpha \in \Gamma} B_\alpha) \cap Y = \bigcup_{\alpha \in \Gamma} A_\alpha} [\bigwedge_{\alpha \in \Gamma} I_\tau(B_\alpha)] \\ &= \textstyle\bigwedge_{\alpha \in \Gamma} [\bigwedge_{B_\alpha \in 2^X,\ (\bigcup_{\alpha \in \Gamma} B_\alpha) \cap Y = \bigcup_{\alpha \in \Gamma} A_\alpha} I_\tau(B_\alpha)] \end{aligned}$$

$= \bigwedge_{\alpha\in\Gamma} I_{\tau_Y}(A_\alpha)$.

Similarly, we have $F_{\tau_Y}(\bigcup_{\alpha\in\Gamma} A_\alpha) \leq \bigwedge_{\alpha\in\Gamma} F_{\tau_Y}(A_\alpha)$. Thus, the condition (OSVNT3) holds. Thus, $\tau_Y \in OSVNT(Y)$.

Furthermore, we can easily see that for each $A \in 2^Y$,

$$T_{\tau_Y}(A) \geq T_\tau(A), \quad I_{\tau_Y}(A) \leq I_\tau(A), \quad F_{\tau_Y}(A) \leq F_\tau(A).$$

This completes the proof. □

The following is an immediate result of Proposition 9.

Corollary 3. *Let (Y, τ_Y) be an ordinary single valued neutrosaophic subspace of (X, τ) and let $A \in 2^Y$.*

(1) $\mathcal{C}_Y(A) = (\bigvee_{B\in 2^X, A=B\cap Y} T_{\mathcal{C}}(B), \bigwedge_{B\in 2^X, A=B\cap Y} I_{\mathcal{C}}(B), \bigwedge_{B\in 2^X, A=B\cap Y} F_{\mathcal{C}}(B))$*, where* $\mathcal{C}_Y(A) = \tau_Y(Y - A)$.

(2) *If $Z \subset Y \subset X$, then $\tau_Z = (\tau_Y)_Z$.*

4. Ordinary Single Valued Neutrosophic Neighborhood Structures of a Point

In this section, we define an ordinary single valued neutrosophic neighborhood system of a point, and prove that it has the same properties in a classical neighborhood system.

Definition 13. *Let (X, τ) be an osvnts and let $x \in X$. Then, a mapping $\mathcal{N}_x : 2^X \to I \times I \times I$ is called the ordinary single valued neutrosophic neighborhood system of x if, for each $A \in 2^X$,*

$$A \in \mathcal{N}_x := \exists B(B \in \tau) \wedge (x \in B \subset A)),$$

i.e.,

$$[A \in \mathcal{N}_x] = \mathcal{N}_x(A) = (\bigvee_{x\in B\subset A} T_\tau(B), \bigwedge_{x\in B\subset A} I_\tau(B), \bigwedge_{x\in B\subset A} F_\tau(B)).$$

Lemma 3. *Let (X, τ) be an osvnts and let $A \in 2^X$. Then,*

$$\bigwedge_{x\in A} \bigvee_{x\in B\subset A} T_\tau(B) = T_\tau(A),$$

$$\bigvee_{x\in A} \bigwedge_{x\in B\subset A} I_\tau(B) = I_\tau(A)$$

and

$$\bigvee_{x\in A} \bigwedge_{x\in B\subset A} F_\tau(B) = F_\tau(A).$$

Proof. By Theorem 3.1 in [33], it is obvious that $\bigwedge_{x\in A} \bigvee_{x\in B\subset A} T_\tau(B) = T_\tau(A)$.

On the other hand, it is clear that $\bigvee_{x\in A} \bigwedge_{x\in B\subset A} I_\tau(B) \geq I_\tau(A)$. Now, let $\mathcal{B}_x = \{B \in 2^X : x \in B \subset A\}$ and let $f \in \Pi_{x\in A}\mathcal{B}_x$. Then, clearly, $\bigcup_{x\in A} f(x) = A$. Thus,

$$\bigvee_{x\in A} I_\tau(f(x)) \leq I_\tau(\bigcup_{x\in A} f(x)) = I_\tau(A).$$

Thus,

$$\bigvee_{x\in A} \bigwedge_{x\in B\subset A} I_\tau(B) = \bigwedge_{f\in\Pi_{x\in A}} \bigvee_{x\in A} I_\tau(f(x)) \leq I_\tau(A).$$

Hence, $\bigvee_{x\in A} \bigwedge_{x\in B\subset A} I_\tau(B) = I_\tau(A)$. Similarly, we have

$$\bigvee_{x\in A} \bigwedge_{x\in B\subset A} F_\tau(B) = F_\tau(A).$$

□

Theorem 2. *Let (X, τ) be an osvnts, let $A \in 2^X$ and let $x \in X$. Then,*

$$\models (A \in \tau) \leftrightarrow \forall x(x \in A \to \exists B(B \in \mathcal{N}_x) \wedge (B \subset A)),$$

i.e.,

$$[A \in \tau] = [\forall x(x \in A \to \exists B(B \in \mathcal{N}_x) \wedge (B \subset A))],$$

i.e.,

$$[A \in \tau] = (\bigwedge_{x\in A} \bigvee_{B\subset A} T_{\mathcal{N}_x}(B), \bigvee_{x\in A} \bigwedge_{B\subset A} I_{\mathcal{N}_x}(B), \bigvee_{x\in A} \bigwedge_{B\subset A} F_{\mathcal{N}_x}(B)).$$

Proof. From Theorem 3.1 in [33], it is clear that $T_\tau(A) = \bigwedge_{x\in A} \bigvee_{B\subset A} T_{\mathcal{N}_x}(B)$.

On the other hand,

$$\begin{aligned} I_\tau(A) &= \textstyle\bigvee_{x\in A} \bigwedge_{x\in C\subset A} I_\tau(C) && \text{[By Lemma 3]} \\ &= \textstyle\bigvee_{x\in A} \bigwedge_{B\subset A} \bigwedge_{x\in C\subset B} I_\tau(C) \\ &= \textstyle\bigvee_{x\in A} \bigwedge_{B\subset A} I_{\mathcal{N}_x}(B). && \text{[By Definition 13]} \end{aligned}$$

Similarly, we have $F_\tau(A) = \bigvee_{x\in A} \bigwedge_{B\subset A} F_{\mathcal{N}_x}(B)$. This completes the proof. □

Definition 14. *Let $\mathcal{A}$ be a single valued neutrosophic set in a set 2^X. Then, $\mathcal{A}$ is said to be normal if there is $A_0 \in 2^X$ such that $\mathcal{A}(A_0) = (1, 0, 0)$.*

We will denote the set of all normal single valued neutrosophic sets in 2^X as $(I \times I \times I)_N^{2^X}$.

From the following result, we can see that an ordinary single valued neutrosophic neighborhood system has the same properties in a classical neighborhood system.

Theorem 3. *Let (X, τ) be an osvnts and let $\mathcal{N} : X \to (I \times I \times I)_N^{2^X}$ be the mapping given by $\mathcal{N}(x) = \mathcal{N}_x$, for each $x \in X$. Then, $\mathcal{N}$ has the following properties:*

(1) *for any $x \in X$ and $A \in 2^X$, $\models A \in \mathcal{N}_x \to x \in A$,*

(2) *for any $x \in X$ and $A, B \in 2^X$, $\models (A \in \mathcal{N}_x) \wedge (B \in \mathcal{N}_x) \to A \cap B \in \mathcal{N}_x$,*

(3) *for any $x \in X$ and $A, B \in 2^X$, $\models (A \subset B) \to (A \in \mathcal{N}_x \to B \in \mathcal{N}_x)$,*

(4) *for any $x \in X$, $\models (A \in \mathcal{N}_x) \to \exists C((C \in \mathcal{N}_x) \wedge (C \subset A) \wedge \forall y(y \in C \to C \in \mathcal{N}_y))$.*

Conversely, if a mapping $\mathcal{N} : X \to (I \times I \times I)_N^{2^X}$ satisfies the above properties (2) *and* (3)*, then there is an ordinary single valued neutrosophic topology $\tau : 2^X \to I \times I \times I$ on X defined as follows: for each $A \in 2^X$,*

$$A \in \tau := \forall x(x \in A \to A \in \mathcal{N}_x),$$

i.e.,

$$[A \in \tau] = \tau(A) = (\bigwedge_{x\in A} T_{\mathcal{N}_x}(A), \bigvee_{x\in A} I_{\mathcal{N}_x}(A), \bigvee_{x\in A} F_{\mathcal{N}_x}(A)).$$

In particular, if $\mathcal{N}$ also satisfies the above properties (1) and (4), then, for each $x \in X$, $\mathcal{N}_x$ is an ordinary single valued neutrosophic neighborhood system of x with respect to τ.

Proof. (1) Since $A \in 2^X$, we can consider A as a special single valued neutrosophic set in x represented by $A = (\chi_A, \chi_{A^c}, \chi_{A^c})$. Then,

$$[x \in A] = A(x) = (\chi_A(x), \chi_{A^c}(x), \chi_{A^c}(x)) = (1, 0, 0).$$

On the other hand,

$$[A \in \mathcal{N}_x] = (\bigvee_{x \in C \subset A} T_\tau(C), \bigwedge_{x \in C \subset A} I_\tau(C), \bigwedge_{x \in C \subset A} F_\tau(C)) \leq (1, 0, 0).$$

Thus, $[A \in \mathcal{N}_x] \leq [x \in A]$.
(2) By the definition of $\mathcal{N}_x$,

$$[A \cap B \in \mathcal{N}_x] = (\bigvee_{x \in C \subset A \cap B} T_\tau(C), \bigwedge_{x \in C \subset A \cap B} I_\tau(C)), \bigwedge_{x \in C \subset A \cap B} F_\tau(C)).$$

From the proof of Theorem 3.2 (2) in [33], it is obvious that

$$T_{\mathcal{N}_x}(A \cap B) \geq T_{\mathcal{N}_x}(A) \wedge T_{\mathcal{N}_x}(B).$$

Thus, it is sufficient to show that $I_{\mathcal{N}_x}(A \cap B) \leq I_{\mathcal{N}_x}(A) \vee I_{\mathcal{N}_x}(B)$:

$$\begin{aligned} I_{\mathcal{N}_x}(A \cap B) &= \bigwedge_{x \in C \subset A \cap B} I_\tau(C) = \bigwedge_{x \in C_1 \subset A,\, x \in C_2 \subset B} I_\tau(C_1 \cap C_2) \\ &\leq \bigwedge_{x \in C_1 \subset A,\, x \in C_2 \subset B} (I_\tau(C_1) \vee I_\tau(C_2)) \\ &= \bigwedge_{x \in C_1 \subset A} I_\tau(C_1) \vee \bigwedge_{x \in C_2 \subset B} I_\tau(C_2) \\ &= I_{\mathcal{N}_x}(A) \vee I_{\mathcal{N}_x}(B). \end{aligned}$$

Similarly, we have $F_{\mathcal{N}_x}(A \cap B) \leq F_{\mathcal{N}_x}(A) \vee F_{\mathcal{N}_x}(B)$. On the other hand,

$$[(A \in \mathcal{N}_x) \wedge (B \in \mathcal{N}_x)] = (T_{\mathcal{N}_x}(A) \wedge T_{\mathcal{N}_x}(B), I_{\mathcal{N}_x}(A) \vee I_{\mathcal{N}_x}(B), F_{\mathcal{N}_x}(A) \vee F_{\mathcal{N}_x}(B)).$$

Thus, $[A \cap B \in \mathcal{N}_x] \geq [(A \in \mathcal{N}_x) \wedge (B \in \mathcal{N}_x)]$.
(3) From the definition of $\mathcal{N}_x$, we can easily show that $[A \in \mathcal{N}_x] \leq [B \in \mathcal{N}_x]$.
(4) It is clear that

$$\begin{aligned} &[\exists C((C \in \mathcal{N}_x) \wedge (C \subset A) \wedge \forall y(y \in C \rightarrow C \in \mathcal{N}_y))] \\ &= (\bigvee_{C \subset A}[T_{\mathcal{N}_x}(C) \wedge \bigwedge_{y \in C} T_{\mathcal{N}_y}(C)],\ \bigwedge_{C \subset A}[I_{\mathcal{N}_x}(C) \vee \bigvee_{y \in C} I_{\mathcal{N}_y}(C)], \\ &\qquad \bigwedge_{C \subset A}[F_{\mathcal{N}_x}(C) \vee \bigvee_{y \in C} F_{\mathcal{N}_y}(C)]). \end{aligned}$$

Then, by the proof of Theorem 3.2 (4) in [33], it is obvious that

$$\bigvee_{C \subset A}[T_{\mathcal{N}_x}(C) \wedge \bigwedge_{y \in C} T_{\mathcal{N}_y}(C)] \geq T_{\mathcal{N}_x}(A).$$

From Lemma 3, $\bigvee_{y \in C} I_{\mathcal{N}_y}(C) = \bigvee_{y \in C} \bigwedge_{y \in D \subset C} I_\tau(D) = I_\tau(C)$. Thus,

$$\begin{aligned} \bigwedge_{C \subset A}[I_{\mathcal{N}_x}(C) \vee \bigvee_{y \in C} I_{\mathcal{N}_y}(C)] &= \bigwedge_{C \subset A}[I_{\mathcal{N}_x}(C) \vee I_\tau(C)] = \bigwedge_{C \subset A} I_\tau(C) \\ &\leq \bigwedge_{x \in C \subset A} I_\tau(C) = I_{\mathcal{N}_x}(A). \end{aligned}$$

Similarly, we have $\bigwedge_{C \subset A}[F_{\mathcal{N}_x}(C) \vee \bigvee_{y \in C} F_{\mathcal{N}_y}(C)] \leq \bigwedge_{x \in C \subset A} F_\tau(C) = F_{\mathcal{N}_x}(A)$. Thus,

$$[\exists C((C \in \mathcal{N}_x) \wedge (C \subset A) \wedge \forall y(y \in C \rightarrow C \in \mathcal{N}_y))] \geq [A \in \mathcal{N}_x].$$

Conversely, suppose $\mathcal{N}$ satisfies the above properties (2) and (3) and let $\tau : 2^X \rightarrow I \times I \times I$ be the mapping defined as follows: for each $A \in 2^X$,

$$\tau(A) = (\bigwedge_{x \in A} T_{\mathcal{N}_x}(A), \bigvee_{x \in A} I_{\mathcal{N}_x}(A), \bigvee_{x \in A} F_{\mathcal{N}_x}(A)).$$

Then, clearly, $\tau(\phi) = (1, 0, 0)$. Since $\mathcal{N}_x$ is single valued neutrosophic normal, there is $A_0 \in 2^X$ such that $\mathcal{N}_x(A_0) = (1, 0, 0)$. Thus, $\mathcal{N}_x(X) = (1, 0, 0)$. Thus,

$$\tau(X) = (\bigwedge_{x \in X} T_{\mathcal{N}_x}(X), \bigvee_{x \in X} I_{\mathcal{N}_x}(X), \bigvee_{x \in X} F_{\mathcal{N}_x}(X)) = (1, 0, 0).$$

Hence, τ satisfies the axiom (OSVNT1).
From the proof of Theorem 3.2 in [33], it is clear that $T_\tau(A \cap B) \geq T_\tau(A) \wedge T_\tau(B)$.
On the other hand,

$$\begin{aligned} I_\tau(A \cap B) &= \bigvee_{x \in A \cap B} I_{\mathcal{N}_x}(A \cap B) \leq \bigvee_{x \in A \cap B}(I_{\mathcal{N}_x}(A) \vee I_{\mathcal{N}_x}(B)) \\ &= \bigvee_{x \in A \cap B} I_{\mathcal{N}_x}(A) \vee \bigvee_{x \in A \cap B} I_{\mathcal{N}_x}(B) \\ &\leq \bigvee_{x \in A} I_{\mathcal{N}_x}(A) \vee \bigvee_{x \in B} I_{\mathcal{N}_x}(B) \\ &= I_\tau(A) \vee I_\tau(B). \end{aligned}$$

Similarly, we have $F_\tau(A \cap B) \leq F_\tau(A) \vee F_\tau(B)$. Then, τ satisfies the axiom (OSVNT2). Moreover, we can easily see that τ satisfies the axiom (OSVNT3). Thus, $\tau \in OSVNT(X)$.

Now, suppose $\mathcal{N}$ satisfies additionally the above properties (1) and (4). Then, from the proof of Theorem 3.2 in [33], we have $T_{\mathcal{N}_x}(A) = \bigvee_{x \in B \subset A} T_\tau(B)$ for each $x \in X$ and each $A \in 2^X$.

Let $x \in X$ and let $A \in 2^X$. Then, by property (4),

$$I_{\mathcal{N}_x}(A) \geq \bigwedge_{C \subset A} [I_{\mathcal{N}_x}(C) \vee \bigvee_{y \in C} I_{\mathcal{N}_y}(C)].$$

From the property (1), $I_{\mathcal{N}_x}(C) = 1$ for any $x \notin C$. Thus,

$$\begin{aligned} I_{\mathcal{N}_x}(A) &\geq \bigwedge_{x \in C \subset A}[I_{\mathcal{N}_x}(C) \vee \bigvee_{y \in C} I_{\mathcal{N}_y}(C)] \\ &\geq \bigwedge_{x \in C \subset A} \bigvee_{y \in C} I_{\mathcal{N}_y}(C) \\ &= \bigwedge_{x \in B \subset A} I_\tau(B). \end{aligned}$$

Now, suppose $x \in C \subset A$. Then, clearly, $\bigvee_{y \in C} I_{\mathcal{N}_y}(C) \geq I_{\mathcal{N}_x}(C) \geq I_{\mathcal{N}_x}(A)$.
Thus,

$$\bigwedge_{x \in B \subset A} I_\tau(B) = \bigwedge_{x \in C \subset A} \bigvee_{y \in C} I_{\mathcal{N}_y}(C) \geq I_{\mathcal{N}_x}(A).$$

Thus, $I_{\mathcal{N}_x}(A) = \bigwedge_{x \in B \subset A} I_\tau(B)$. Similarly, we have $F_{\mathcal{N}_x}(A) = \bigwedge_{x \in B \subset A} F_\tau(B)$. This completes the proof. □

5. Ordinary Single Valued Neutrosophic Bases and Subbases

In this section, we define an ordinary single valued neutrosophic base and subbase for an ordinary single valued neutrosophic topological space, and investigated general properties. Moreover, we obtain two characterizations of an ordinary single valued neutrosophic base and one characterization of an ordinary single valued neutrosophic subbase.

Definition 15. *Let (X, τ) be an osvnts and let $\mathcal{B} : 2^X \to I \times I \times I$ be a mapping such that $\mathcal{B} \leq \tau$, i.e., $T_\mathcal{B} \leq T_\tau$, $I_\mathcal{B} \geq I_\tau$, $F_\mathcal{B} \geq F_\tau$. Then, $\mathcal{B}$ is called an ordinary single valued neutrosophic base for τ if, for each $A \in 2^X$,*

$$T_\tau(A) = \bigvee_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\ A = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigwedge_{\alpha \in \Gamma} T_\mathcal{B}(B_\alpha),$$

$$I_\tau(A) = \bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\ A = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} I_\mathcal{B}(B_\alpha),$$

$$F_\tau(A) = \bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\ A = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} F_\mathcal{B}(B_\alpha).$$

Example 3. *(1) Let X be a set and let $\mathcal{B} : 2^X \to I \times I \times I$ be the mapping defined by:*

$$\mathcal{B}(\{x\}) = (1, 0, 0)\ \ \forall x \in X.$$

Then, $\mathcal{B}$ is an ordinary single valued neutrosophic base for τ_X.

*(2) Let $X = \{a, b, c\}$, let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_1 \times I_0 \times I_0$ and let $\mathcal{B} : 2^X \to I \times I \times I$ be the mapping as follows: for each $A \in 2^X$,*

$$\mathcal{B}(A) = \begin{cases} (1, 0, 0) & \textit{if } \text{either } A = \{a, b\} \text{ or } \{b, c\} \text{ or } X, \\ \alpha & \text{otherwise.} \end{cases}$$

Then, $\mathcal{B}$ is not an ordinary single valued neutrosophic base for an osvnt on X.

Suppose that $\mathcal{B}$ is an ordinary single valued neutrosophic base for an osvnt τ on X. Then, clearly, $\mathcal{B} \leq \tau$. Moreover, $\tau(\{a, b\}) = \tau(\{b, c\}) = (1, 0, 0)$. Thus,

$$T_\tau(\{b\}) = T_\tau(\{a, b\} \cap \tau(\{b, c\}) \geq T_\tau(\{a, b\} \wedge T_\tau(\{b, c\} = 1$$

and

$$I_\tau(\{b\}) = I_\tau(\{a, b\} \cap \tau(\{b, c\}) \leq I_\tau(\{a, b\} \wedge I_\tau(\{b, c\} = 0.$$

Similarly, we have $F_\tau(\{b\}) = 0$. Thus, $\tau(\{b\}) = (1, 0, 0)$. On the other hand, by the definition of $\mathcal{B}$,

$$T_\tau(\{b\}) = \bigvee_{\{A_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ \{b\}=\bigcup_{\alpha\in\Gamma} A_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\mathcal{B}}(A_\alpha) = T_\alpha$$

and

$$I_\tau(\{b\}) = \bigwedge_{\{A_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ \{b\}=\bigcup_{\alpha\in\Gamma} A_\alpha} \bigvee_{\alpha\in\Gamma} I_{\mathcal{B}}(A_\alpha) = I_\alpha.$$

Similarly, we have $F_\tau(\{b\}) = F_\alpha$. This is a contradiction. Hence, $\mathcal{B}$ is not an ordinary single valued neutrosophic base for an osvnt on X

Theorem 4. *Let (X, τ) be an osvnts and let $\mathcal{B} : 2^X \to I \times I \times I$ be a mapping such that $\mathcal{B} \leq \tau$. Then, $\mathcal{B}$ is an ordinary single valued neutrosophic base for τ if and only if for each $x \in X$ and each $A \in 2^X$,*

$$T_{\mathcal{N}_x}(A) \leq \bigvee_{x\in B\subset A} T_{\mathcal{B}}(B),$$

$$I_{\mathcal{N}_x}(A) \geq \bigwedge_{x\in B\subset A} I_{\mathcal{B}}(B),$$

$$F_{\mathcal{N}_x}(A) \geq \bigwedge_{x\in B\subset A} F_{\mathcal{B}}(B).$$

Proof. ($\Rightarrow$): Suppose $\mathcal{B}$ is an ordinary single valued neutrosophic base for τ. Let $x \in X$ and let $A \in 2^X$. Then, by Theorem 4.4 in [34], it is obvious that $T_{\mathcal{N}_x}(A) \leq \bigvee_{x\in B\subset A} T_{\mathcal{B}}(B)$. On the other hand,

$$\begin{aligned} I_{\mathcal{N}_x}(A) &= \textstyle\bigwedge_{x\in B\subset A} I_\tau(B) && \text{[By Definition 13]} \\ &= \textstyle\bigwedge_{x\in B\subset A} \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ B=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_{\mathcal{B}}(B_\alpha). && \text{[By Definition 15]} \end{aligned}$$

If $x \in B \subset A$ and $B = \bigcup_{\alpha\in\Gamma} B_\alpha$, then there is $\alpha_0 \in \Gamma$ such that $x \in B_{\alpha_0}$. Thus,

$$\bigvee_{\alpha\in\Gamma} I_{\mathcal{B}}(B_\alpha) \geq I_{\mathcal{B}}(B_{\alpha_0}) \geq \bigwedge_{x\in B\subset A} I_{\mathcal{B}}(B).$$

Thus, $I_{\mathcal{N}_x}(A) \geq \bigwedge_{x\in B\subset A} I_{\mathcal{B}}(B)$. Similarly, we have $F_{\mathcal{N}_x}(A) \geq \bigwedge_{x\in B\subset A} F_{\mathcal{B}}(B)$. Hence, the necessary condition holds.

($\Leftarrow$): Suppose the necessary condition holds. Then, by Theorem 4.4 in [34], it is clear that

$$T_\tau(A) = \bigvee_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\mathcal{B}}(B_\alpha).$$

Let $A \in 2^X$. Suppose $A = \bigcup_{\alpha\in\Gamma} B_\alpha$ and $\{B_\alpha\} \subset 2^X$. Then,

$$\begin{aligned} I_\tau(A) &\leq \textstyle\bigvee_{\alpha\in\Gamma} I_\tau(B_\alpha) && \text{[By the axiom (OSVNT3)]} \\ &\leq \textstyle\bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha). && [\text{Since } \mathcal{B} \leq \tau] \end{aligned}$$

Thus,

$$I_\tau(A) \leq \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha). \tag{1}$$

On the other hand,

$$\begin{aligned} I_\tau(A) &= \textstyle\bigvee_{x\in A} \bigwedge_{x\in B\subset A} I_\tau(B) && \text{[By Lemma 3]} \\ &= \textstyle\bigvee_{x\in A} I_{\mathcal{N}_x}(A) && \text{[By Definition 13]} \\ &= \textstyle\bigvee_{x\in A} \bigwedge_{x\in B\subset A} I_\mathcal{B}(B) && \text{[By the hypothesis]} \\ &= \textstyle\bigwedge_{f\in\Pi_{x\in A}\mathcal{B}_x} \bigvee_{x\in A} I_\mathcal{B}(f(x)), \end{aligned}$$

where $\mathcal{B}_x = \{B \in 2^X : x \in B \subset A\}$. Furthermore, $A = \bigcup_{x\in A} f(x)$ for each $f \in \Pi_{x\in A}\mathcal{B}_x$. Thus,

$$\bigwedge_{f\in\Pi_{x\in A}\mathcal{B}_x} \bigvee_{x\in A} I_\mathcal{B}(f(x)) = \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha).$$

Hence,

$$I_\tau(A) \geq \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha). \tag{2}$$

By (1) and (2), $I_\tau(A) = \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha)$. Similarly, we have $F_\tau(A) = \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_\mathcal{B}(B_\alpha)$. Therefore, $\mathcal{B}$ is an ordinary single valued neutrosophic base for τ. □

Theorem 5. *Let $\mathcal{B} : 2^X \to I \times I \times I$ be a mapping. Then, $\mathcal{B}$ is an ordinary single valued neutrosophic base for some oist τ on X if and only if it has the following conditions:*

(1) $\bigvee_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_\mathcal{B}(B_\alpha) = 1$,
$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha) = 0$,
$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_\mathcal{B}(B_\alpha) = 0$,

(2) *for any $A_1,\ A_2 \in 2^X$ and each $x \in A_1 \cap A_2$,*

$$T_\mathcal{B}(A_1) \wedge T_\mathcal{B}(A_2) \leq \bigvee_{x\in A\subset A_1\cap A_2} T_\mathcal{B}(A),$$

$$I_\mathcal{B}(A_1) \vee I_\mathcal{B}(A_2) \geq \bigwedge_{x\in A\subset A_1\cap A_2} I_\mathcal{B}(A),$$

$$F_\mathcal{B}(A_1) \vee F_\mathcal{B}(A_2) \geq \bigwedge_{x\in A\subset A_1\cap A_2} F_\mathcal{B}(A).$$

In fact, $\tau : 2^X \to I \times I \times I$ is the mapping defined as follows: for each $A \in 2^X$,

$$T_\tau(A) = \begin{cases} 1 & if\ A = \phi \\ \bigvee_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_\mathcal{B}(B_\alpha) & otherwise, \end{cases}$$

$$I_\tau(A) = \begin{cases} 0 & if\ A = \phi \\ \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_\mathcal{B}(B_\alpha) & otherwise, \end{cases}$$

$$F_\tau(A) = \begin{cases} 0 & if\ A = \phi \\ \bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ A=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_\mathcal{B}(B_\alpha) & otherwise. \end{cases}$$

In this case, τ is called an ordinary single valued neutrosophic topology on X induced by $\mathcal{B}$.

Proof. ($\Rightarrow$): Suppose $\mathcal{B}$ is an ordinary single valued neutrosophic base for some *osvnt* τ on X. Then, by Definition 15 and the axiom (OSVNT1),

$$\bigvee_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha}\ \bigwedge_{\alpha\in\Gamma} T_{\mathcal{B}}(B_\alpha) = T_\tau(X) = 1,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha}\ \bigvee_{\alpha\in\Gamma} I_{\mathcal{B}}(B_\alpha)) = I_\tau(X) = 0,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma}\subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha}\ \bigvee_{\alpha\in\Gamma} F_{\mathcal{B}}(B_\alpha)) = F_\tau(X) = 0.$$

Thus, condition (1) holds.

Let $A_1,\ A_2 \in 2^X$ and let $x \in A_1 \cap A_2$. Then, by the proof of Theorem 4.2 in [33], it is obvious that $T_{\mathcal{B}}(A_1) \wedge T_{\mathcal{B}}(A_2) \leq \bigvee_{x\in A\subset A_1\cap A_2} T_{\mathcal{B}}(A)$. On the other hand,

$$I_{\mathcal{B}}(A_1) \vee I_{\mathcal{B}}(A_2) \geq I_\tau(A_1) \vee I_\tau(A_2) \geq I_\tau(A_1 \cap A_2) \geq I_{\mathcal{N}_x}(A_1 \cap A_2) \geq \bigwedge_{x\in A\subset A_1\cap A_2} I_{\mathcal{B}}(A).$$

Thus,

$$I_{\mathcal{B}}(A_1) \vee I_{\mathcal{B}}(A_2) \geq \bigwedge_{x\in A\subset A_1\cap A_2} I_{\mathcal{B}}(A).$$

Similarly, we have

$$F_{\mathcal{B}}(A_1) \vee F_{\mathcal{B}}(A_2) \geq \bigwedge_{x\in A\subset A_1\cap A_2} F_{\mathcal{B}}(A).$$

Thus, condition (2) holds.

($\Leftarrow$): Suppose the necessary conditions (1) and (2) are satisfied. Then, by the proof of Theorem 4.2 in [33], we can see that the following hold:

$T_\tau(X) = T_\tau(\phi) = 1$,
$T_\tau(A \cap B) \geq T_\tau(A) \wedge T_\tau(B)$ for any $A,\ B \in 2^X$

and

$T_\tau(\bigcup_{\alpha\in\Gamma} A_\alpha) \geq \bigwedge_{\alpha\in\Gamma} T_\tau(A_\alpha)$ for each $\{A_\alpha\}_{\alpha\in\Gamma} \subset 2^X$.

From the definition of τ, it is obvious that $I_\tau(X) = I_\tau(\phi) = 0$. Similarly, we have $F_\tau(X) = F_\tau(\phi) = 0$. Thus, τ satisfies the axiom (OSVNT1).

Let $\{A_\alpha\}_{\alpha\in\Gamma} \subset 2^X$ and let $\mathcal{B}_\alpha = \{\{B_{\delta_\alpha} : \delta_\alpha \in \Gamma_\alpha\} : \bigcup_{\delta_\alpha\in\Gamma_\alpha} B_{\delta_\alpha} = A_\alpha\}$. Let $f \in \Pi_{\alpha\in\Gamma}\mathcal{B}_\alpha$. Then, clearly, $\bigcup_{\alpha\in\Gamma}\bigcup_{B_{\delta_\alpha}\in f(\alpha)} B_{\delta_\alpha} = \bigcup_{\alpha\in\Gamma} A_\alpha$. Thus,

$$\begin{aligned}
I_\tau(\textstyle\bigcup_{\alpha\in\Gamma} A_\alpha) &= \textstyle\bigwedge_{\bigcup_{\delta\in\Gamma} B_\delta=\bigcup_{\alpha\in\Gamma} A_\alpha} \bigvee_{\delta\in\Gamma} I_{\mathcal{B}}(B_\delta) \\
&\leq \textstyle\bigwedge_{f\in\Pi_{\alpha\in\Gamma}\mathcal{B}_\alpha} \bigvee_{\alpha\in\Gamma} \bigvee_{B_{\delta_\alpha}\in f(\alpha)} I_{\mathcal{B}}(B_{\delta_\alpha}) \\
&= \textstyle\bigvee_{\alpha\in\Gamma} \bigwedge_{\{B_{\delta_\alpha}:\delta_\alpha\in\Gamma_\alpha\}\in\mathcal{B}_\alpha} \bigvee_{\delta_\alpha\in\Gamma_\alpha} I_{\mathcal{B}}(B_{\delta_\alpha}) \\
&= \textstyle\bigvee_{\alpha\in\Gamma} I_\tau(A_\alpha).
\end{aligned}$$

Similarly, we have $F_\tau(\bigcup_{\alpha\in\Gamma} A_\alpha) \leq \bigvee_{\alpha\in\Gamma} F_\tau(A_\alpha)$. Thus, τ satisfies the axiom (OSVNT3).

Now, let $A,\ B \in 2^X$ and suppose $I_\tau(A) < I_\alpha$ and $I_\tau(B) < I_\alpha$ for $\alpha \in$ **SVNV**. Then, there are $\{A_{\alpha_1} : \alpha_1 \in \Gamma_1\}$ and $\{B_{\alpha_2} : \alpha_2 \in \Gamma_2\}$ such that $\bigcup_{\alpha_1\in\Gamma_1} A_{\alpha_1} = A$, $\bigcup_{\alpha_2\in\Gamma_2} B_{\alpha_2} = B$ and $I_{\mathcal{B}}(A_{\alpha_1}) < I_\alpha$ for each $\alpha_1 \in \Gamma_1$, $I_{\mathcal{B}}(B_{\alpha_2}) < I_\alpha$ for each $\alpha_2 \in \Gamma_2$. Let $x \in A \cap B$. Then, there are $\alpha_{1x} \in \Gamma_1$ and $\alpha_{2x} \in \Gamma_2$ such that $x \in A_{\alpha_{1x}} \cap B_{\alpha_{2x}}$. Thus, from the assumption,

$$I_\alpha > I_{\mathcal{B}}(A_{\alpha_{1x}}) \vee I_{\mathcal{B}}(B_{\alpha_{2x}}) \geq \bigwedge_{x\in C\subset A_{\alpha_{1x}}\cap B_{\alpha_{2x}}} I_{\mathcal{B}}(C).$$

Moreover, there is C_x such that $x \in C_x \subset A_{\alpha_{1x}} \cap B_{\alpha_{2x}} \subset A \cap B$ and $I_{\mathcal{B}}(C_x) < I_\alpha$. Since $\bigcup_{x \in A \cap B} C_x = A \cap B$, we obtain

$$I_\alpha \geq \bigvee_{x \in A \cap B} I_{\mathcal{B}}(C_x) \geq \bigwedge_{\bigcup_{\alpha \in \Gamma} B_\alpha = A \cap B} \bigvee_{\alpha \in \Gamma} I_{\mathcal{B}}(B_\alpha) = I_\tau(A \cap B).$$

Now, let $I_\beta = I_\tau(A) \vee I_\tau(B)$ and let n be any natural number, where $I_\beta \in I$. Then, $I_\tau(A) < I_\beta + 1/n$ and $I_\tau(B) < I_\beta + 1/n$. Thus, $I_\tau(A \cap B) \leq I_\beta + 1/n$. Thus, $I_\tau(A \cap B) \leq I_\beta = I_\tau(A) \vee I_\tau(B)$. Similarly, we have $F_\tau(A \cap B) \leq F_\tau(A) \vee F_\tau(B)$. Hence, τ satisfies the axiom (OSVNT2). This completes the proof. □

Example 4. *(1) Let $X = \{a, b, c\}$ and let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_1 \times I_0 \times I_0$. We define the mapping $\mathcal{B} : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,*

$$T_{\mathcal{B}}(A) = \begin{cases} 1 & \text{if } A = \{b\} \text{ or } \{a,b\} \text{ or } \{b,c\} \\ T_\alpha & \text{otherwise,} \end{cases}$$

$$I_{\mathcal{B}}(A) = \begin{cases} 0 & \text{if } A = \{b\} \text{ or } \{a,b\} \text{ or } \{b,c\} \\ I_\alpha & \textit{otherwise,} \end{cases}$$

$$F_{\mathcal{B}}(A) = \begin{cases} 0 & \text{if } A = \{b\} \text{ or } \{a,b\} \text{ or } \{b,c\} \\ F_\alpha & \textit{otherwise.} \end{cases}$$

Then, we can easily see that $\mathcal{B}$ satisfies conditions (1) and (2) in Theorem 5. Thus, $\mathcal{B}$ is an ordinary single valued neutrosophic base for an osvnt τ on X. In fact, $\tau : 2^X \to I \times I \times I$ is defined as follows: for each $A \in 2^X$,

$$T_\tau(A) = \begin{cases} 1 & \text{if } A \in \{\phi, \{b\}, \{a,b\}, \{b,c\}, X\} \\ T_\alpha & \text{otherwise,} \end{cases}$$

$$I_\tau(A) = \begin{cases} 0 & \text{if } A \in \{\phi, \{b\}, \{a,b\}, \{b,c\}, X\} \\ I_\alpha & \text{otherwise,} \end{cases}$$

$$F_\tau(A) = \begin{cases} 0 & \text{if } A \in \{\phi, \{b\}, \{a,b\}, \{b,c\}, X\} \\ F_\alpha & \text{otherwise.} \end{cases}$$

*(2) Let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_1 \times I_0 \times I_0$. We define the mapping $\mathcal{B} : 2^{\mathbb{R}} \to I \times I \times I$ as follows: for each $A \in 2^{\mathbb{R}}$,*

$$T_{\mathcal{B}}(A) = \begin{cases} 1 & \text{if } A = (a,b) \text{ for } a,\ b \in \mathbb{R} \text{ with } a \leq b \\ T_\alpha & \text{otherwise,} \end{cases}$$

$$I_{\mathcal{B}}(A) = \begin{cases} 0 & \text{if } A = (a,b) \text{ for } a,\ b \in \mathbb{R} \text{ with } a \leq b \\ I_\alpha & \text{otherwise,} \end{cases}$$

$$F_{\mathcal{B}}(A) = \begin{cases} 0 & \text{if } A = (a,b) \text{ for } a,\ b \in \mathbb{R} \text{ with } a \leq b \\ F_\alpha & \text{otherwise.} \end{cases}$$

Then, it can be easily seen that $\mathcal{B}$ satisfies the conditions (1) and (2) in Theorem 5. Thus, $\mathcal{B}$ is an ordinary single valued neutrosophic base for an osvnt τ_α on $\mathbb{R}$.

In this case, τ_α is called the α-ordinary single valued neutrosophic usual topology on $\mathbb{R}$.

*(3) Let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_1 \times I_0 \times I_0$. We define the mapping $\mathcal{B} : 2^{\mathbb{R}} \to I \times I \times I$ as follows: for each $A \in 2^{\mathbb{R}}$,*

$$T_{\mathcal{B}}(A) = \begin{cases} 1 & \text{if } A = [a,b) \text{ for } a,\ b \in \mathbb{R} \text{ with } a \leq b \\ T_\alpha & \text{otherwise,} \end{cases}$$

$$I_{\mathcal{B}}(A) = \begin{cases} 0 & \text{if } A = [a,b) \text{ for } a,\ b \in \mathbb{R} \text{ with } a \leq b \\ I_\alpha & \text{otherwise,} \end{cases}$$

$$F_{\mathcal{B}}(A) = \begin{cases} 0 & \text{if } A = [a,b) \text{ for } a,\ b \in \mathbb{R} \text{ with } a \leq b \\ F_\alpha & \text{otherwise.} \end{cases}$$

Then, we can easily see that $\mathcal{B}$ satisfies the conditions (1) and (2) in Theorem 5. Thus, $\mathcal{B}$ is an ordinary single valued neutrosophic base for an osvnt τ_l on $\mathbb{R}$.

In this case, τ_l is called the α-ordinary single valued neutrosophic lower-limit topology on $\mathbb{R}$.

Definition 16. *Let $\tau_1,\ \tau_2 \in OSVNT(X)$, and let $\mathcal{B}_1$ and $\mathcal{B}_1$ be ordinary single valued neutrosophic bases for τ_1 and τ_2, respectively. Then, $\mathcal{B}_1$ and $\mathcal{B}_1$ are said to be equivalent if $\tau_1 = \tau_2$.*

Theorem 6. *Let $\tau_1,\ \tau_2 \in OSVNT(X)$, and let $\mathcal{B}_1$ and $\mathcal{B}_1$ be ordinary single valued neutrosophic bases for τ_1 and τ_2 respectively. Then, τ_1 is coarser than τ_2, i.e.,*

$$T_{\tau_1} \leq T_{\tau_2},\ I_{\tau_1} \geq I_{\tau_2},\ F_{\tau_1} \geq F_{\tau_2}$$

if and only if for each $A \in 2^X$ and each $x \in A$,

$$T_{\mathcal{B}_1}(A) \leq \bigvee_{x \in B \subset A} T_{\mathcal{B}_2}(B), \quad I_{\mathcal{B}_1}(A) \geq \bigwedge_{x \in B \subset A} I_{\mathcal{B}_2}(B), \quad F_{\mathcal{B}_1}(A) \geq \bigwedge_{x \in B \subset A} F_{\mathcal{B}_2}(B).$$

Proof. ($\Rightarrow$): Suppose τ_1 is coarser than τ_2. For each $x \in X$, let $x \in A \in 2^X$. Then, by Theorem 4.8 in [34], $T_{\mathcal{B}_1}(A) \leq \bigvee_{x \in B \subset A} T_{\mathcal{B}_2}(B)$. On the other hand,

$$\begin{aligned} I_{\mathcal{B}_1}(A) &\geq I_{\tau_1}(A) && \text{[since } \mathcal{B}_1 \text{ is an ordinary single valued neutrosophic base for } \tau_1\text{]} \\ &\geq I_{\tau_2}(A) && \text{[By the hypothesis]} \\ &= \textstyle\bigwedge_{\{A_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\ A = \bigcup_{\alpha \in \Gamma} A_\alpha} \bigvee_{\alpha \in \Gamma} I_{\mathcal{B}_2}(A_\alpha). && \text{[Since } \mathcal{B}_2 \text{ is an ordinary single valued neutrosophic base for } \tau_2\text{]} \end{aligned}$$

Since $x \in A$ and $A = \bigcup_{\alpha \in \Gamma} A_\alpha$, there is $\alpha_0 \in \Gamma$ such that $x \in A_{\alpha_0}$. Thus,

$$\bigwedge_{\{A_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\ A = \bigcup_{\alpha \in \Gamma} A_\alpha} \bigvee_{\alpha \in \Gamma} I_{\mathcal{B}_2}(A_\alpha) \geq I_{\mathcal{B}_2}(A_{\alpha_0}) \geq \bigwedge_{x \in B \subset A} I_{\mathcal{B}_2}(B).$$

Thus, $I_{\mathcal{B}_1}(A) \geq \bigwedge_{x \in B \subset A} I_{\mathcal{B}_2}(B)$. Similarly, we have $F_{\mathcal{B}_1}(A) \geq \bigwedge_{x \in B \subset A} F_{\mathcal{B}_2}(B)$.

($\Leftarrow$): Suppose the necessary condition holds. Then, by Theorem 4.8 in [34], $T_{\tau_1} \leq T_{\tau_2}$. Let $A \in 2^X$. Then,

$$\begin{aligned} I_{\tau_1}(A) &= \textstyle\bigvee_{x \in A} \bigwedge_{x \in B \subset A} I_{\mathcal{B}_1}(B) && \text{[By Lemma 3]} \\ &\geq \textstyle\bigvee_{x \in A} \bigwedge_{x \in B \subset A} \bigwedge_{x \in C \subset B} I_{\mathcal{B}_2}(C) && \text{[By the hypothesis]} \\ &= \textstyle\bigwedge_{x \in C \subset A} \bigvee_{x \in A} I_{\mathcal{B}_2}(C) \\ &= \textstyle\bigwedge_{\{C_x\}_{x \in A} \subset 2^X,\ A = \bigcup_{x \in A} C_x} \bigvee_{x \in A} I_{\mathcal{B}_2}(C_x) \\ &= I_{\tau_2}(A). \end{aligned}$$

Thus, $I_{\tau_1} \geq I_{\tau_2}$. Similarly, we have $F_{\tau_1} \geq F_{\tau_2}$. Thus, τ_1 is coarser than τ_2. This completes the proof. □

The following is an immediate result of Definition 16 and Theorem 6.

Corollary 4. *Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be ordinary single valued neutrosophic bases for two ordinary single valued neutrosophic topologies on a set X, respectively. Then,*

$\mathcal{B}_1$ and $\mathcal{B}_2$ are equivalent if and only if the following two conditions hold:

(1) *for each* $B_1 \in 2^X$ *and each* $x \in B_1$,

$$T_{\mathcal{B}_1}(B_1) \leq \bigvee_{x \in B_2 \subset B_1} T_{\mathcal{B}_2}(B_2),$$

$$I_{\mathcal{B}_1}(B_1) \geq \bigwedge_{x \in B_2 \subset B_1} I_{\mathcal{B}_2}(B_2),$$

$$F_{\mathcal{B}_1}(B_1) \geq \bigwedge_{x \in B_2 \subset B_1} F_{\mathcal{B}_2}(B_2),$$

(2) *for each* $B_2 \in 2^X$ *and each* $x \in B_2$,

$$T_{\mathcal{B}_2}(B_2) \leq \bigvee_{x \in B_1 \subset B_2} T_{\mathcal{B}_1}(B_1),$$

$$I_{\mathcal{B}_2}(B_2) \geq \bigwedge_{x \in B_1 \subset B_2} I_{\mathcal{B}_1}(B_1),$$

$$F_{\mathcal{B}_2}(B_2) \geq \bigwedge_{x \in B_1 \subset B_2} F_{\mathcal{B}_1}(B_1).$$

It is obvious that every ordinary single valued neutrosophic topology itself forms an ordinary single valued neutrosophic base. Then, the following provides a sufficient condition for one to see if a mapping $\mathcal{B} : 2^X \to I \times I \times I$ such that $T_{\mathcal{B}} \leq T_\tau$, $I_{\mathcal{B}} \geq I_\tau$ and $F_{\mathcal{B}} \geq F_\tau$ is an ordinary single valued neutrosophic base for $\tau \in OSVNT(X)$.

Proposition 10. *Let* (X, τ) *be an osvnts and let* $\mathcal{B} : 2^X \to I \times I \times I$ *be a mapping such that* $T_{\mathcal{B}} \leq T_\tau$, $I_{\mathcal{B}} \geq I_\tau$ *and* $F_{\mathcal{B}} \geq F_\tau$. *For each* $A \in 2^X$ *and each* $x \in A$, *suppose* $T_\tau(A) \leq \bigvee_{x \in B \subset A} T_{\mathcal{B}}(B)$, $I_\tau(A) \geq \bigwedge_{x \in B \subset A} I_{\mathcal{B}}(B)$ *and* $F_\tau(A) \geq \bigwedge_{x \in B \subset A} F_{\mathcal{B}}(B)$. *Then,* $\mathcal{B}$ *is an ordinary single valued neutrosophic base for* τ.

Proof. From the proof of Proposition 4.10 in [34], it is clear that the first part of the condition (1) of Theorem 5 holds, i.e., $\bigvee_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigwedge_{\alpha \in \Gamma} T_{\mathcal{B}}(B_\alpha) = 1$. On the other hand,

$$\begin{aligned}
& \textstyle\bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} I_{\mathcal{B}}(B_\alpha) \\
\geq & \textstyle\bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} I_\tau(B_\alpha) && [\text{since } I_{\mathcal{B}} \geq I_\tau] \\
\geq & \textstyle\bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} I_\tau(\bigcup_{\alpha \in \Gamma} B_\alpha) && [\text{by the axiom (OSVNT3)}] \\
= & I_\tau(X) \\
= & \textstyle\bigvee_{x \in X} \bigwedge_{x \in B \subset X} I_\tau(B) && [\text{By Lemma 3}] \\
\geq & \textstyle\bigvee_{x \in X} \bigwedge_{x \in B \subset X} \bigwedge_{x \in C \subset B} I_{\mathcal{B}}(C) && [\text{By the hypothesis}] \\
= & \textstyle\bigwedge_{x \in C \subset X} \bigvee_{x \in X} I_{\mathcal{B}}(C) \\
= & \textstyle\bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} I_{\mathcal{B}}(B_\alpha).
\end{aligned}$$

Since $\tau \in OSVNT(X)$, $I_\tau(X) = 0$. Thus, $\bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} I_{\mathcal{B}}(B_\alpha) = 0$. Similarly, we have $\bigwedge_{\{B_\alpha\}_{\alpha \in \Gamma} \subset 2^X,\, X = \bigcup_{\alpha \in \Gamma} B_\alpha} \bigvee_{\alpha \in \Gamma} F_{\mathcal{B}}(B_\alpha) = 0$. Thus, condition (1) of Theorem 5 holds.

Now, let $A_1,\ A_2 \in 2^X$ and let $x \in A_1 \cap A_2$. Then, by the proof of Proposition 4.10 in [34], it is obvious that $T_{\mathcal{B}}(A_1) \wedge T_{\mathcal{B}}(A_2) \leq \bigvee_{x \in A \subset A_1 \cap A_2} T_{\mathcal{B}}(A)$. On the other hand,

$$\begin{aligned}
I_{\mathcal{B}}(A_1) \vee I_{\mathcal{B}}(A_2) & \geq I_\tau(A_1) \vee I_\tau(A_2) && [\text{Since } I_{\mathcal{B}} \geq I_\tau] \\
& \geq I_\tau(A_1 \cap A_2) && [\text{by the axiom (OSVNT2)}] \\
& \geq \textstyle\bigwedge_{x \in A \subset A_1 \cap A_2} I_{\mathcal{B}}(A). && [\text{by the hypothesis}]
\end{aligned}$$

Similarly, we have $F_{\mathcal{B}}(A_1) \vee F_{\mathcal{B}}(A_2) \geq \bigwedge_{x \in A \subset A_1 \cap A_2} F_{\mathcal{B}}(A)$. Thus, condition (2) of Theorem 5 holds. Thus, by Theorem 5, $\mathcal{B}$ is an ordinary single valued neutrosophic base for τ. This completes the proof. □

Definition 17. *Let (X, τ) be an osvnts and let $\simeq : 2^X \to I \times I \times I$ be a mapping. Then, φ is called an ordinary single valued neutrosophic subbase for τ, if $\varphi^{\sqcap}$ is an ordinary single valued neutrosophic base for τ, where $\varphi^{\sqcap} : 2^X \to I \times I \times I$ is the mapping defined as follows: for each $A \in 2^X$,*

$$T_{\varphi^{\sqcap}}(A) = \bigvee_{\{B_\alpha\} \sqsubset 2^X,\ A=\bigcap_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\simeq}(B_\alpha),$$

$$I_{\varphi^{\sqcap}}(A) = \bigwedge_{\{B_\alpha\} \sqsubset 2^X,\ A=\bigcap_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_{\simeq}(B_\alpha),$$

$$F_{\varphi^{\sqcap}}(A) = \bigwedge_{\{B_\alpha\} \sqsubset 2^X,\ A=\bigcap_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_{\simeq}(B_\alpha),$$

where $\sqsubset$ stands for "a finite subset of".

Example 5. *Let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_1 \times I_0 \times I_0$. We define the mapping $\simeq : 2^{\mathbb{R}} \to I \times I \times I$ as follows: for each $A \in 2^{\mathbb{R}}$,*

$$T_{\simeq}(A) = \begin{cases} 1 & \text{if } A = (a, \infty) \text{ or } (-\infty, b) \text{ or } (a, b) \\ T_\alpha & \text{otherwise,} \end{cases}$$

$$I_{\simeq}(A) = \begin{cases} 0 & \text{if } A = (a, \infty) \text{ or } (-\infty, b) \text{ or } (a, b) \\ I_\alpha & \text{otherwise,} \end{cases}$$

$$F_{\simeq}(A) = \begin{cases} 0 & \text{if } A = (a, \infty) \text{ or } (-\infty, b) \text{ or } (a, b) \\ F_\alpha & \text{otherwise,} \end{cases}$$

where $a,\ b \in \mathbb{R}$ such that $a < b$. Then, we can easily see that $\simeq$ is an ordinary single valued neutrosophic subbase for the α-ordinary single valued neutrosophic usual topology $\mathcal{U}_\alpha$ on $\mathbb{R}$.

Theorem 7. *Let $\simeq : 2^X \to I \times I \times I$ be a mapping. Then, $\simeq$ is an ordinary single valued neutrosophic subbase for some osvnt if and only if*

$$\bigvee_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\simeq}(B_\alpha) = 1,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_{\simeq}(B_\alpha) = 0,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X=\bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_{\simeq}(B_\alpha) = 0.$$

Proof. ($\Rightarrow$): Suppose $\simeq$ is an ordinary single valued neutrosophic subbase for some *osvnt*. Then, by Definition 17, it is clear that the necessary condition holds.

($\Leftarrow$): Suppose the necessary condition holds. We only show that $\varphi^{\sqcap}$ satisfies the condition (2) in Theorem 5. Let $A,\ B \in 2^X$ and $x \in A \cap B$. Then, by the proof of Theorem 4.3 in [33], it is obvious that $T_{\varphi^{\sqcap}}(A) \wedge T_{\varphi^{\sqcap}}(B) \leq \bigvee_{x\in C\subset A\cap B} T_{\varphi^{\sqcap}}(C)$. On the other hand,

$$\begin{aligned}
& I_{\varphi^{\sqcap}}(A) \vee I_{\varphi^{\sqcap}}(B) \\
&= (\textstyle\bigwedge_{\bigcap_{\alpha_1\in\Gamma_1} B_{\alpha_1}=A} \bigvee_{\alpha_1\in\Gamma_1} I_{\simeq}(B_{\alpha_1})) \vee (\bigwedge_{\bigcap_{\alpha_2\in\Gamma_2} B_{\alpha_2}=B} \bigvee_{\alpha_2\in\Gamma_2} I_{\simeq}(B_{\alpha_2})) \\
&= \textstyle\bigwedge_{\bigcap_{\alpha_1\in\Gamma_1} B_{\alpha_1}=A} \bigwedge_{\bigcap_{\alpha_2\in\Gamma_2} B_{\alpha_2}=B} (\bigvee_{\alpha_1\in\Gamma_1} I_{\simeq}(B_{\alpha_1}) \vee \bigvee_{\alpha_2\in\Gamma_2} I_{\simeq}(B_{\alpha_2})) \\
&\geq \textstyle\bigwedge_{\bigcap_{\alpha\in\Gamma} B_\alpha=A\cap B} \bigvee_{\alpha\in\Gamma} I_{\simeq}(B_\alpha) \\
&= I_{\varphi^{\sqcap}}(A \cap B).
\end{aligned}$$

Since $x \in A \cap B$, $I_{\varphi^{\sqcap}}(A) \vee I_{\varphi^{\sqcap}}(B) \geq I_{\varphi^{\sqcap}}(A \cap B) \geq \bigwedge_{x \in C \subset A \cap B} I_{\varphi^{\sqcap}}(C)$. Similarly, we have $F_{\varphi^{\sqcap}}(A) \vee F_{\varphi^{\sqcap}}(B) \geq F_{\varphi^{\sqcap}}(A \cap B) \geq \bigwedge_{x \in C \subset A \cap B} F_{\varphi^{\sqcap}}(C)$. Thus, $\varphi^{\sqcap}$ satisfies the condition (2) in Theorem 5. This completes the proof. □

Example 6. *Let $X = \{a, b, c, d, e\}$ and let $\alpha \in$ **SVNV** be fixed, where $\alpha \in I_1 \times I_0 \times I_0$. We define the mapping $\simeq : 2^X \to I \times I \times I$ as follows: for each $A \in 2^X$,*

$$T_{\simeq}(A) = \begin{cases} 1 & \text{if } A \in \{\{a\}, \{a,b,c\}, \{b,c,d\}, \{c,e\}\} \\ T_\alpha & \text{otherwise,} \end{cases}$$

$$I_{\simeq}(A) = \begin{cases} 0 & \text{if } A \in \{\{a\}, \{a,b,c\}, \{b,c,d\}, \{c,e\}\} \\ I_\alpha & \text{otherwise,} \end{cases}$$

$$F_{\simeq}(A) = \begin{cases} 0 & \text{if } A \in \{\{a\}, \{a,b,c\}, \{b,c,d\}, \{c,e\}\} \\ F_\alpha & \text{otherwise.} \end{cases}$$

Then, $X = \{a\} \cup \{b,c,d\} \cup \{c,e\}$,

$$T_{\varphi^{\sqcap}}(\{a\}) = T_{\varphi^{\sqcap}}(\{b,c,d\}) = T_{\varphi^{\sqcap}}(\{c,e\}) = 1,$$
$$I_{\varphi^{\sqcap}}(\{a\}) = I_{\varphi^{\sqcap}}(\{b,c,d\}) = I_{\varphi^{\sqcap}}(\{c,e\}) = 0.$$
$$F_{\varphi^{\sqcap}}(\{a\}) = F_{\varphi^{\sqcap}}(\{b,c,d\}) = F_{\varphi^{\sqcap}}(\{c,e\}) = 0.$$

Thus,

$$\bigvee_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\simeq}(B_\alpha) = 1,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_{\simeq}(B_\alpha) = 0,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_{\simeq}(B_\alpha) = 0.$$

Thus, by Theorem 7, $\simeq$ is an ordinary single valued neutrosophic subbase for some osvnt.

The following is an immediate result of Corollary 4 and Theorem 7.

Proposition 11. *$\simeq_1$, $\simeq_2 : 2^X \to I \times I \times I$ be two mappings such that*

$$\bigvee_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\simeq_1}(B_\alpha) = 1,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_{\simeq_1}(B_\alpha) = 0,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_{\simeq_1}(B_\alpha) = 0$$

and

$$\bigvee_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigwedge_{\alpha\in\Gamma} T_{\simeq_2}(B_\alpha) = 1,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} I_{\simeq_2}(B_\alpha) = 0,$$

$$\bigwedge_{\{B_\alpha\}_{\alpha\in\Gamma} \subset 2^X,\ X = \bigcup_{\alpha\in\Gamma} B_\alpha} \bigvee_{\alpha\in\Gamma} F_{\simeq_2}(B_\alpha) = 0.$$

Suppose the two conditions hold:

(1) *for each* $S_1 \in 2^X$ *and each* $x \in S_1$,

$$T_{\simeq_1}(S_1) \leq \bigvee_{x \in S_2 \subset S_1} T_{\simeq_2}(S_2),\ I_{\simeq_1}(S_1) \geq \bigwedge_{x \in S_2 \subset S_1} I_{\simeq_2}(S_2),\ F_{\simeq_1}(S_1) \geq \bigwedge_{x \in S_2 \subset S_1} F_{\simeq_2}(S_2),$$

(2) *for each* $S_2 \in 2^X$ *and each* $x \in S_2$,

$$T_{\simeq_2}(S_2) \leq \bigvee_{x \in S_1 \subset S_2} T_{\simeq_1}(S_1),\ I_{\simeq_2}(S_2) \geq \bigwedge_{x \in S_1 \subset S_2} I_{\simeq_1}(S_1),\ f_{\simeq_2}(S_2) \geq \bigwedge_{x \in S_1 \subset S_2} f_{\simeq_1}(S_1).$$

Then, $\simeq_1$ *and* $\simeq_2$ *are ordinary single valued neutrosophic subbases for the same ordinary single valued neutrosophic topology on* X.

6. Conclusions

In this paper, we defined an ordinary single valued neutrosophic topology and level set of an *osvnst* to study some topological characteristics of neutrosophic sets and obtained some their basic properties. In addition, we defined an ordinary single valued neutrosophic subspace. Next, the concepts of an ordinary single valued neutrosophic neighborhood system and an ordinary single valued neutrosophic base (or subbase) were introduced and studied. Their results are summarized as follows:

First, an ordinary single valued neutrosophic neighborhood system has the same properties in a classical neighborhood system (see Theorem 3).

Second, we found two characterizations of an ordinary single valued neutrosophic base (see Theorems 4 and 5).

Third, we obtained one characterization of an ordinary single valued neutrosophic subbase (see Theorem 7).

Finally, we expect that this paper can be a guidance for the research of separation axioms, compactness, connectedness, etc. in ordinary single valued neutrosophic topological spaces. In addition, one can deal with single valued neutrosophic topology from the viewpoint of lattices.

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