Abstract- The aim of this paper is to introduce the new concept of Penta partitioned neutrosophic Pythagorean topological space and discussed some of its properties.

Keywords – Penta partitioned Neutrosophic set, Penta partitioned neutrosophic topological space, Pentapartitioned Neutrosophic Pythagorean topological space

I. INTRODUCTION
The fuzzy set was introduced by Zadeh [13] in 1965. The concept of Neutrosophic set was introduced by F. Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Smarandache is proposed neutrosophic set[ 11]. In neutrosophic sets, the indeterminacy membership function walks along independently of the truth membership or of the falsity membership. Neutrosophic theory has been widely explored by researchers for application purpose in handling real life situations involving uncertainty.

Rama Malik and Surpati Pramanik [7] introduced Pentapartitioned neutrosophic set and its properties. Here indeterminacy is divided into three parts as contradiction, ignorance and unknown membership function.

Also we introduced the concept of Penta partitioned neutrosophic Pythagorean set [4] and establish some of its properties in our previous work. Now we have extended our work in this Pentapartitioned neutrosophic Pythagorean set as a topological space.

II Preliminaries
2.1 Definition
Let X be a non-empty set. A PNS \( A \) over X characterizes each element \( p \) in X by a truth-membership function \( T_A \), a contradiction membership function \( C_A \), an ignorance membership function \( I_A \), unknown membership function \( U_A \) and a falsity membership function \( F_A \), such that for each \( p \in X \),

\[
0 \leq T_A + C_A + U_A + I_A + F_A \leq 5
\]

2.2 Definition
Let X be a universe. A Pentapartitioned neutrosophic pythagorean set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

\[
A = \{< x, T_A, C_A, I_A, U_A, F_A > : x \in X \}
\]

Where \( T_A + F_A \leq 1, C_A + U_A \leq 1 \) and \((T_A)^2 + (C_A)^2 \leq 3\),

\[
(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3
\]
Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

2.3 Definition
A Pentapartitioned neutrosophic pythagorean set $A$ is contained in another Pentapartitioned neutrosophic pythagorean set $B$ (i.e) $A \subseteq B$ if $T_A \leq T_B$, $C_A \leq C_B$, $I_A \geq I_B$, $U_A \geq U_B$ and $F_A \geq F_B$.

2.4 Definition
The complement of a Pentapartitioned neutrosophic pythagorean set $(F, A)$ on $X$ denoted by $(F, A)^c$ and is defined as $F^c(x) = \{ <x, F_A, U_A, 1 - I_A, C_A, T_A \geq x \in X \}$

2.5 Definition
Let $X$ be a non-empty set, $A = <T_A, C_A, I_A, U_A, F_A>$ and $B = <T_B, C_B, I_B, U_B, F_B>$ are two Pentapartitioned neutrosophic pythagorean sets. Then $A \cup B = <x, \max(T_A, T_B), \max(C_A, C_B), \min(I_A, I_B), \max(U_A, U_B), \max(F_A, F_B)>$

2.6 Definition
A Pentapartitioned neutrosophic pythagorean set $(F, A)$ over the universe $X$ is said to be empty Pentapartitioned neutrosophic soft set $0_X$ with respect to the parameter $A$ if $T_{F(e)} = 0, C_{F(e)} = 0, I_{F(e)} = 1, U_{F(e)} = 1, F_{F(e)} = 0, \forall x \in X, \forall e \in A$. It is denoted by $0_X$

2.7 Definition
A Pentapartitioned neutrosophic pythagorean set $(F, A)$ over the universe $X$ is said to be universe Pentapartitioned neutrosophic pythagorean set with respect to the parameter $A$ if $T_{F(e)} = 1, C_{F(e)} = 1, I_{F(e)} = 0, U_{F(e)} = 0, F_{F(e)} = 0, \forall x \in X, \forall e \in A$. It is denoted by $1_X$

III Pentapartitioned Neutrosophic Pythagorean Topological Space

3.1 Definition
A Pentapartitioned Neutrosophic Pythagorean topology on a non-empty set $M$ is a $\tau$ of Pentapartitioned Neutrosophic Pythagorean sets satisfying the following axioms.

i) $0_M, 1_M \in \tau$

ii) The union of the elements of any sub collection of $\tau$ is in $\tau$

iii) The intersection of the elements of any finite sub collection $\tau$ is in $\tau$

The pair $(M, \tau)$ is called an Pentapartitioned Neutrosophic Pythagorean Topological Space over $M$.

3.2 Note
1. Every member of $\tau$ is called a PNP open set in $M$.

2. The set $A_M$ is called a PNP closed set in $M$ if $A_M \in \tau^c$, where $\tau^c = \{ A_M^c : A_M \in \tau \}$.

3.3 Example
Let $M = \{ b_1, b_2 \}$ and Let $A_M, B_M, C_M$ be Pentapartitioned Neutrosophic Pythagorean sets where $A_M = \{ <b_1, 0.5, 0.1, 0.5, 0.7, 0.2 >, <b_2, 0.7, 0.5, 0.6, 0.2, 0.1 > <b_3, 0.6, 0.5, 0.8, 0.4, 0.3 > \}$

$B_M = \{ <b_1, 0.6, 0.7, 0.6, 0.1, 0.2 >, <b_2, 0.2, 0.3, 0.6, 0.4, 0.7 >, <b_3, 0.5, 0.6, 0.7, 0.1, 0.3 > \}$

$C_M = \{ <b_1, 0.6, 0.7, 0.5, 0.1, 0.2 >, <b_2, 0.7, 0.5, 0.6, 0.2, 0.1 >, <b_3, 0.6, 0.6, 0.7, 0.1, 0.3 > \}$

$\tau = \{ A_M, B_M, C_M, 0_M, 1_M \}$ is an Pentapartitioned Neutrosophic Pythagorean topology on $M$. 


3.4 Proposition

Let \((M, \tau_1)\) and \((M, \tau_2)\) be two Penta Partitioned Neutrosophic Pythagorean topological space on M. Then \(\tau_1 \cap \tau_2\) is a Penta Partitioned Neutrosophic Pythagorean topology on M where \(\tau_1 \cap \tau_2 = \{A_M, A_M \in \tau_1 \text{ and } A_M \in \tau_2\}\)

**Proof:**

Obviously \(0_M, 1_M \in \tau\).

Let \(A_M, B_M \in \tau_1 \cap \tau_2\)

Then \(A_M, B_M \in \tau_1 \text{ and } A_M, B_M \in \tau_2\)

We know that \(\tau_1\) and \(\tau_2\) are two Penta Partitioned Neutrosophic Pythagorean topological spaces on M.

\(\tau_1 \cap \tau_2 = \{A_M, A_M \in \tau_1 \text{ and } A_M \in \tau_2\}\)

3.5 Example

Let \(A_M\) and \(B_M\) be two Penta Partitioned Neutrosophic Pythagorean topological space on M.

Define \(\tau_1 = \{0_M, 1_M, A_M\}\)

\(\tau_2 = \{0_M, 1_M, B_M\}\)

Then \(\tau_1 \cap \tau_2 = \{0_M, 1_M\}\) is a Penta Partitioned Neutrosophic Pythagorean topological space on M.

But \(\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M\}\),

\(\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cup B_M\}\) and

\(\tau_1 \cap \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cap B_M\}\) are not Penta Partitioned Neutrosophic Pythagorean topological space on M.

### IV Properties of Pentapartitioned Neutrosophic Pythagorean Topological Spaces

4.1 Definition

Let \((M, \tau)\) be a Penta Partitioned Neutrosophic Pythagorean topological space on M and let \(A_M\) belongs to Penta Partitioned Neutrosophic Pythagorean set on M. Then the interior of \(A_M\) is denoted as \(\text{PNPInt}(A_M)\). It is defined by \(\text{PNPInt}(A_M) = \cup \{B_M \in \tau: B_M \subseteq A_M\}\)

4.2 Definition

Let \((M, \tau)\) be a Penta Partitioned Neutrosophic Pythagorean topological space on M and let \(A_M\) belongs to Penta Partitioned Neutrosophic Pythagorean set M. Then the closure of \(A_M\) is denoted as \(\text{PNPCL}(A_M)\). It is defined by \(\text{PNPCL}(A_M) = \cap \{B_M \in \tau^C: A_M \subseteq B_M\}\)

4.3 Theorem

Let \((M, \tau)\) be a Penta Partitioned Neutrosophic Pythagorean topological space over M. Then the following properties are hold.

i) \(0_M\) and \(1_M\) are Penta Partitioned Neutrosophic Pythagorean closed sets over M

ii) The intersection of any number of Penta Partitioned Neutrosophic Pythagorean closed set is a Penta
Partitioned Neutrosophic Pythagorean closed set over $M$.

iii) The union of any two Penta Partitioned Neutrosophic Pythagorean closed set is an Penta Partitioned Neutrosophic Pythagorean closed set over $M$.

*Proof*
It is obviously true.

4.4 Theorem

Let $(M, \tau)$ be a be a Penta Partitioned Neutrosophic Pythagorean topological space over $M$ and Let $A_M \in \tau$ Penta Partitioned Neutrosophic Pythagorean topological space. Then the following properties hold.

(i) $PNPInt (A_M) \subseteq A_M$

(ii) $A_M \subseteq B_M$ implies $PNPInt (A_M) \subseteq PNPInt (B_M)$.

(iii) $PNPInt (A_M) \in \tau$.

(iv) $A_M$ is a PNP open set implies $PNPInt (A_M) = A_M$.

(v) $PNPInt (PNPInt (A_M)) = PNPInt(A_M)$

(vi) $PNPInt (0_M) = 0_M$, $PNPInt (1_M) = 1_M$.

*Proof:*

(i) and (ii) are obviously true.

(iii) obviously $\cup \{B_M \in \tau : B_M \subseteq A_m\} \in \tau$

Note that $\cup \{B_M \in \tau : B_M \subseteq A_m\} = QNSInt (A_M)$

$\therefore PNPInt (A_M) \in \tau$

(iv) Necessity: Let $A_M$ be a PNP open set. ie., $A_M \in \tau$. By (i) and (ii) $PNPInt (A_M) \subseteq A_m$.

Since $A_M \in \tau$ and $A_M \subseteq A_m$

Then $A_M \subseteq \cup \{B_M \in \tau : B_M \subseteq A_m\} = QNSInt (A_M)$

$A_M \subseteq PNPInt (A_M)$

Thus $PNPInt = A_m$.

Sufficiency: Let $PNPInt (A_m) = A_m$

By (iii) $PNPInt (A_m) \in \tau$, ie., $A_m$ is a PNP open set.

(v) To prove $PNPInt (PNPInt (A_m)) = PNPInt(A_m)$

By (iii) $PNPInt (A_m) \in \tau$.

By (iv) $PNPInt (PNPInt (A_m)) = PNPInt (A_m)$.

(vi) We know that $0_M$ and $1_M$ are in $\tau$

By (iv) $PNPInt (0_M) = 0_M$, $PNPInt (1_M) = 1_M$. Hence the result.
4.5 Theorem

Let $(M, \tau)$ be a a Penta Partitioned Neutrosophic Pythagorean topological space over $M$ and let $A_M$ is in the Penta Partitioned Neutrosophic Pythagorean topological space. Then the following properties hold.

(i) $A_M \subseteq \text{PNPCI}(A_M)$
(ii) $A_M \subseteq B_M$ implies $\text{PNPCI}(A_M) \subseteq \text{QNSCl}(B_M)$.
(iii) $\text{PNPCI}(A_M)^c \in \tau$.
(iv) $A_M$ is a PNP closed set implies $\text{PNPCI}(A_M) = A_M$.
(v) $\text{PNPCI}(\text{PNPCI}(A_M)) = \text{PNPCI}(A_M)$
(vi) $\text{PNPCI}(0_M) = 0_M, \text{PNPCI}(1_M) = 1_M$.

Proof:

(i) and (ii) are obviously true.

(iii) By theorem, $\text{PNPInt}(A_M^c) \in \tau$.

Therefore $\text{PNPCI}(A_M)^c = (\cap \{B_M \in \tau^c : B_M \subseteq A_M\})^c$

$= \cup \{B_M \in \tau : B_M \not\subseteq A_M^c\} = \text{PNPInt}(A_M^c)$

$\therefore [\text{PNPCI}(A_M)]^c \in \tau$

(iv) Necessity:

By theorem, $A_M \subseteq \text{PNPCI}(A_M)$

Let $A_M$ be a PNP closed set i.e. $A_M \in \tau^c$.

Since $A_M \in \tau$ and $A_M \subseteq A_M$

$\text{PNPCI}(A_M) = \cap \{B_M \in \tau^c : A_M \subseteq B_M\} \subseteq \{B_M \in \tau^c : A_M \subseteq A_M\}$

Thus $A_M = \text{PNPCI}(A_M)$

Sufficiency: This is obviously true by (iii)

(v) and (vi) can be proved by (iii) and (iv)

4.6 Theorem

Let $(M, \tau)$ be a a Penta Partitioned Neutrosophic Pythagorean topological space over $M$ and let $A_M, B_M$ are in Penta Partitioned Neutrosophic Pythagorean topological space $M$. Then the following properties hold.

(i) $\text{PNPInt}(A_M) \cap \text{PNPInt}(B_M) = \text{PNPInt}(A_M \cap B_M)$
(ii) $\text{PNPInt}(A_M) \cup \text{QNSInt}(B_M) \subseteq \text{PNPInt}(A_M \cup B_M)$
(iii) $\text{PNPCI}(A_M) \cup \text{QNSCl}(B_M) \subseteq \text{PNPCI}(A_M \cup B_M)$
(iv) $\text{PNPCI}(A_M \cup B_M) \subseteq \text{PNPCI}(A_M) \cap \text{PNPCI}(B_M)$
\[(v) \quad (\text{PNPInt}(A_M))^c = \text{PNPCL}(A_M^c)\]
\[(vi) \quad (\text{PNPCL}(A_M))^c = \text{PNPInt}(A_M^c)\]

**Proof:**
(i) Since \(A_M \cap B_M \subseteq A_m\) for any \(m\) in \(M\)

By theorem, \(\text{PNPInt}(A_M \cap B_M) \subseteq \text{PNPInt}(A_M)\)

Similarly, \(\text{PNPInt}(A_M \cap B_M) \subseteq \text{PNPInt}(B_M)\)

\(\text{PNPInt}(A_M \cap B_M) \subseteq \text{PNPInt}(A_M) \cap \text{PNPInt}(B_M)\)

By theorem, \(\text{PNPInt}(A_M) \subseteq A_M\) and \(\text{PNPInt}(B_M) \subseteq B_M\)

Thus \(\text{PNPInt}(A_M \cap B_M) \subseteq A_M \cap B_M\)

Therefore, \(\text{PNPInt}(A_M) \cap \text{PNPInt}(B_M) = \text{PNPInt}(A_M \cap B_M)\)

Similarly we can prove (ii), (iii) and (iv).

\[v) \quad (\text{PNPInt}(A_M))^c = (\cap \{B_M \in \tau: B_M \subseteq A_m\})^c\]
\[= \cap \{B_M \in \tau^c: A_M^c \subseteq B_m\}\]
\[= \text{PNPCL}(A_M^c)\]

Similarly we can prove (vi)

4.7 Example
Let \(M = \{b_1, b_2\}\) and let \(A_M, B_M, C_M\) be Penta Partitioned Neutrosophic Pythagorean where
\(A_M = \{<b_1, 0.3,0.3,0.2,0,1,0.3>, <b_2, 0.6,0.4,0.2,0,3,0.1>\}\)
\(B_M = \{<b_1, 0.2,0.3,0.5,0,1,0.5>, <b_2, 0.6,0.5,0.2,0,3,0.2>\}\)
\(C_M = \{<b_1, 0.3,0.3,0.2,0,3,0.3>, <b_2, 0.6,0.5,0.2,0,3,0.1>\}\)
\(\tau = \{A_M, B_M, C_M, 0_M, 1_M\}\) is an Penta Partitioned Neutrosophic Pythagorean topology on \(M\).

i) \(\text{PNPInt}(A_M) = 0_M = \text{PNPInt}(B_M)\)

Then \(A_M \cup B_M = C_M\)

\(\text{PNPInt}(A_M) \cup \text{PNPInt}(B_M) = 0_M \cup 0_M = 0_M\)

And \(\text{PNPInt}(A_M \cup B_M) = \text{PNPInt}(C_M) = C_M\)

\(\text{PNPInt}(A_M) \cup \text{PNPInt}(B_M) \neq \text{PNPInt}(A_M \cup B_M)\)

ii) \(\text{PNPCL}(B_M)^c = (\text{PNPCL}(B_M))^c = 0_M = 1_M\)

Similarly, \(\text{PNPCL}(C_M)^c = X_M\)

\(\text{PNPCL}(A_M)^c \cap \text{PNPCL}(B_M)^c = 1_M \cap 1_M = 1_M\)

Similarly, \(\text{PNPCL}(A_M^c \cap B_M^c) = \text{PNPCL}(A_M \cap B_M)^c\)
\[= \text{PNPInt}(A_M \cup B_M)^c\]
\[= C_M^c\]

\(\text{PNPCL}(A_M^c \cap B_M^c) \neq \text{PNPCL}(A_M)^c \cap (\text{PNPCL}(B_M))^c\)

V. CONCLUSION

In this paper, we have studied the properties of Pentapartitioned Neutrosophic Pythagorean topological space and in future I have extended the concept to heptapartitioned neutrosophic topological space.
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