



Plithogenic n- Super Hypergraph in Novel Multi -Attribute Decision Making

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Abstract

An optimal decision-making environment demands feasible Multi-Attribute Decision-Making methods. Plithogenic n – Super Hypergraph introduced by Smarandache is a novel concept and it involves many attributes. This article aims to bridge the concept of Plithogenic n-Super Hypergraph in the vicinity of optimal decision making. This research work introduces the novel concepts of enveloping vertex, super enveloping vertex, dominant enveloping vertex, classification of the dominant enveloping vertex (input, intervene, output dominant enveloping vertices), plithogenic connectors. An application of Plithogenic n-super hypergraph in making optimum decisions is discussed under various decision-making scenarios. Several insights are drawn from this research work and will certainly benefit the decision-makers to overcome the challenges in building decisions.

Keywords: Plithogenic n-super hypergraph, decision making, attributes, dominant enveloping vertex.

1. Introduction

It is quite inevitable for each one is taking up the role of decision-maker in their instances of life. Decision making isn't an activity, but a process comprising of many tasks. The desired outcomes of decisions are a success, if it fails then the process has to be revived. The cognitive contribution in choosing the best alternative with the consideration of criteria and criteria weights is not a simple task; it demands sequential steps and scientific approach. The managerial of either a start-up company or a multinational organization must possess the skills of making optimal decisions to make their companies march in the path of victory. The decision-making environment is not deterministic always and it is characterized mostly by uncertainty and impreciseness, to tackle these challenges the decision-makers are moving towards Multi-Criteria Decision Making methods (MCDM) to design optimal solutions.

MCDM has been explored for the past seventy years and it has been broadly divided into MADM (Multi-Attribute Decision Making) and MODM (Multi-Objective Decision Making) [1]. The former helps in the selection of the alternatives based on attribute description and the latter is based on optimization of decision maker's multi objectives. MADM methods are gaining impetus in the decision-making environment as they are highly developed with robust mathematical principles and also these methods prevent small and medium-sized companies in purchasing expensive software or executing erudite systems of the decision process. MADM methods are more operative and the most widely used methods are Analytic Hierarchy Process (AHP) and Analytic Network Process (ANP) introduced by Satty [2]; Decision Making Trial and Evaluation Laboratory (DEMATEL) developed by Tzeng and Huang [3]; The Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS) method was proposed by Hwang and Yoon; Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method was developed by Tzeng and Huang.

In the above described MADM methods, the major steps involved are (i) formulation of initial decision-making matrix (IDMM) comprising of values representing the degree of fulfilling the criteria by the alternatives. (ii) Normalization of the values in IDMM (iii) Determination of criterion weight (iv) Ranking of alternatives. In these MADM methods, the alternatives are ranked based only on the extent of criteria satisfaction, but the consistency of ranking is not checked as these methods do not provide space for it. The selection of alternatives is based only on attribute satisfaction and it does not consider any other input such as previous data related to the impacts or the effects of these kinds of chosen alternatives. These other inputs are not brought into the decision-making environment and the previous feedback review is also not incorporated into the decision-making environment.

Let us consider the possible situations of exercising decision making in a company, for example in the selection of personnel, methods of production, the extension of product features, the above instances of decision-making situations are not new to companies, as these processes are routine. In making decisions, certainly the managerial will be aware of the desired target to be achieved and will employ his previous experience or the feedback received by him from various sources as inputs in the selection of alternatives. The above-said MADM does not provide space for such kinds of feedback inputs. A comprehensive decision-making environment must comprise of alternative selection based on several inputs such as attribute satisfaction, feedback, and impact of attributes towards the desired output. To overcome such shortcomings, a novel MADM method is introduced in this research work with the integration of Plithogenic – super Hypergraphs introduced by Smarandache [4]. Plithogenic sets introduced by Smarandache [5] are the extension of neutrosophic sets that are characterized by truth, indeterminacy, and false functions. The robust nature of neutrosophic sets inspired several researchers to employ it in diverse fields. Gayathri et al [6] developed multiple attribute group decision making neutrosophic environments with the utilization of Jaccard index measures. Muhammad Naveed Jafar et al [7] used neutrosophic soft matrices with score function to evaluate new technology in Agriculture. Ajay et al [8] developed a single-valued triangular neutrosophic number approach of multi-objective optimization based on simple ratio analysis based on the MCDM method. Luis Andrés Crespo Berti [9] applied a neutrosophic system to tax havens with a criminal approach. Abdel-Basset [10] developed three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem, also developed a hybrid neutrosophic group ANP-TOPSIS framework for supplier selection

problem [11]. Plithogenic sets that deal with attributes, degree of appurtenance, and degree of contradiction have been extensively used in decision making with quality function deployment for selecting supply chain sustainability metrics and for evaluating hospital medical care systems by Abdel-Basset et al [12,13]. In these decision making approaches plithogenic aggregation operations are used to make decisions based on the best and worst criteria with decision-makers' opinions as inputs. These methods of decision making focus primarily on evaluation and selection of alternatives based on combining plithogenic aggregation operators and do not provide space for any graphical representation of the relational impacts between the alternatives.

In the proposed MADM each alternative is considered as an object encompassing several attributes. The decision-making environment consists of three kinds of objects namely input, intervention, and output. The alternatives are taken as inputs, desired target as output, and intervene (intermediate) objects are the objects that combine with the input objects. A company always works on target based. Personnel design project and work on it tirelessly to achieve various sets of goals. The project never gets accomplished with the attainment of a single goal but a series of goals. The success of a project is defined in various dimensions. In the proposed MADAM, the selection of alternatives is based on the degree of association between the attributes of inputs and the attributes of outputs independent or dependent on intervening objects. This decision-making approach is more comprehensive than the conventional MADM methods as it incorporates attributes and feedback into the input system. Also, many times the company prefers collaborative works and the effects of combined initiatives are high. Conventional MADM does not provide space for it, but the proposed MADM is designed exclusively for measuring the optimal combination. Also in MADM methods, graphical representations are not made so far to represent alternatives, criteria, and their relationship. In this novel MADM, plithogenic n -super hypergraphs are used to represent the objects as enveloping vertices and the association between the vertices by plithogenic connectors.

The article is structured as follows: Section 2 introduces new concepts used in novel MADM; section 3 presents the application of novel MADM in optimal decision making; section 4 discusses the results and the last section concludes the work.

2. Preliminaries

2.1 Enveloping vertex

A vertex representing an object comprising of attributes and sub-attributes in the graphical representation of a multi attribute decision-making environment.

For instance

Let us consider Personnel (V) as an input object, this input has a vital role in target achievement, the output object.

These attributes are like databases.

The attributes like Qualification (V1), Age (V2), Experience (V3) are taken into consideration

Attribute sets = {Qualification, Age, Experience}

Qualification = {Graduation, Graduation with additional degree}

Age = {25-35, 36-45}

Experience = { Local, National, International }

Local {0-5,6-10}, National{0-3,4-6}, International {0-2,2-5}

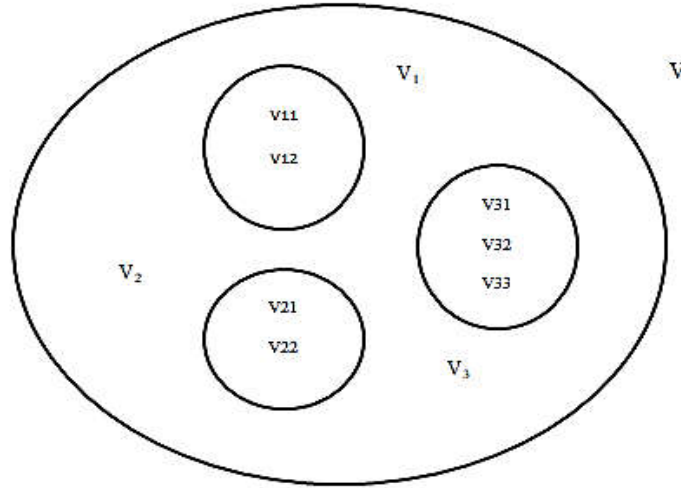


Fig.2.1 Enveloping vertex

Thus an enveloping vertex comprises hyperedges, where each hyperedge represents values of the attributes.

2.2 Super Enveloping vertex

An enveloping vertex comprises of Super hyper edges

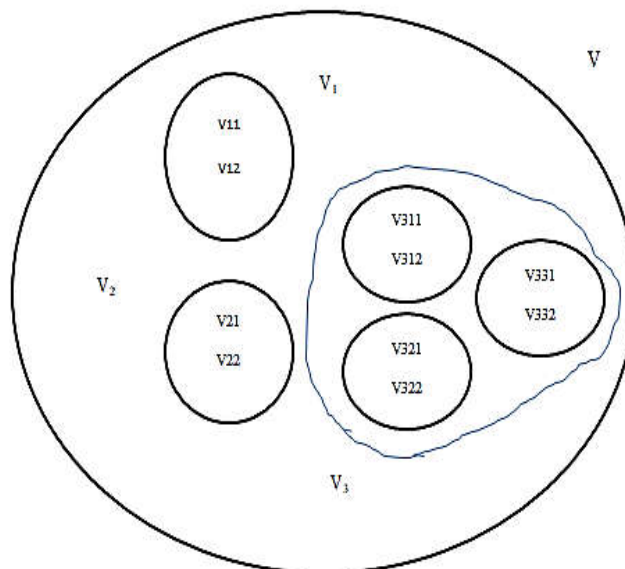


Fig.2.2 Super Enveloping vertex

2.3 Dominant Enveloping Vertex

An enveloping vertex is with dominant attribute values

Attribute sets = {Qualification, Age, Experience}

The dominant attribute values

Qualification = {Graduation, **Graduation with additional degree**}

Age = {25-35, **36-45**}

Experience = { Local, National, **International** }



Fig.2.3 Dominant Enveloping Vertex

2.4 Dominant Super Enveloping Vertex

A super enveloping vertex with dominant attribute values

Attribute sets = {Qualification, Age, Experience}

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Qualification = {Graduation, **Graduation with additional degree**}

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Local {0-5,6-10}, National{0-3,4-6}, International {**0-2,2-5**}

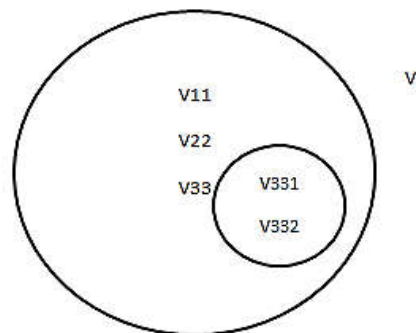


Fig.2.4 Dominant Super Enveloping Vertex

2.5 Classification of Dominant Enveloping Vertex

The dominant enveloping vertex set are classified as input, intervene and output based on the nature of object’s representation.

2.6 Plithogenic Connectors

The connectors associate the input enveloping vertex with output enveloping vertex. These connectors associate the effects of input attributes to output attributes and these connectors are weighted by plithogenic weights.

Let us consider the MADM environment with the product as input object, advertising as intervene object and product success as the output object

Product is the input enveloping vertex, advertising as intervening enveloping vertex and product success as the output enveloping vertex.

Input attributes = {Design, Price}

Design = {creative, conventional}

Price = {High, moderate, low}

Intervene attributes = { Target group, Medium of advertising}

Target group = {female, children}

Medium of advertising = { social networks, media}

Output Attributes = { Profit, Customer Acquisition, Product Reach}

Profit = { Expected, Beyond the target}

Customer Acquisition = { High, Extremely High}

Product Reach = { National, International}

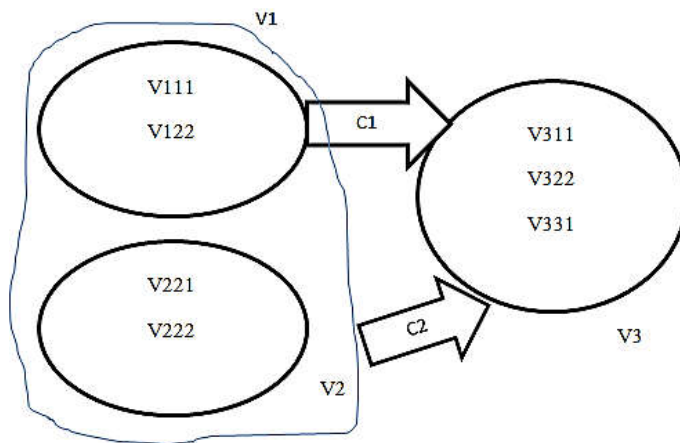


Fig.2.5 Plithogenic Connectors

C1 is the simple plithogenic connector representing the relation between the dominant input attributes to dominant output attributes.

C2 is the combined plithogenic connector representing the relation between the combined dominant input and intervene attributes to dominant output attributes.

Dominant Attribute Relational Matrix Representation

	V311	V322	V331
V111	0.5	0.2	0.3
V122	0.6	0.7	0.8
V111,V221	0.5	0.6	0.4
V111,V222	0.6	0.3	0.8
V122,V221	0.4	0.6	0.8
V122,V222	0.4	0.6	0.7

3. Application of Novel MADM method

3.1 Description of Decision-making Environment

COVID 19 has locked the academic activities to a great extent; the stratagem of Work from Home is employed by the teaching fraternity to engage the learners. One of the biggest challenges to teaching community lies in handling online learning forums and they are badly in need of exposure to the E-learning system of education. To make academicians surpass this task, educational institutions are offering various online courses and organize E-programmes to enhance the professional competency of faculty in partnership with several industries. In this period of the lockdown, the linkage between industries and institutions is getting enhanced in developing countries especially in India. The companies enter institutions as academic partners in establishing virtual laboratories and entertain many online programs in the form of webinars, online courses, and software training programs to handle online classes. The conduct of such programs will certainly contribute to the professional efficiency of faculty. Suppose if an institution decides to conduct any one of the forms of the online program, then it has to decide whether to conduct the program in partnership with industry or independently and also the decision of selecting the kind of online program is based on the feedback acquired from other institutions on the previous organization of such programs. The institution before organizing such programs should decide the component of professional efficiency to be enhanced and determine the contributing factors of the online program towards the same. An optimal solution to this decision-making situation is determined by using the representation of Plithogenic –n Super hypergraph and novel MADM method based on attributes. This decision-making method involves not only the selection process of alternatives based on criteria alike other multi-attribute decision-making methods but it provides space for the selection of alternatives independent or dependent on other alternatives based on their attributes. The outcome of decision making is also considered in the decision-making process. The selection of the alternatives is based on attributes of input objects, intervene objects and output objects.

In this decision-making environment there exist five objects [3 input objects, 1 intervene object and 1 output object] that are represented by enveloping vertices. The input enveloping vertices are Webinars, online

courses, training programs on computer languages, intervene enveloping vertex is Industrial partnership and the output enveloping vertex is Professional Efficiency. The description of the attributes of the objects are presented in Table 3.1

Table 3.1 Description of Attributes

Vertex	Representation	Vertex Attributes		Vertex Sub Attributes			
V1	Webinars	V11	Focus	V111	General	V1111	Education
						V1112	Health
						V1113	Psychology
				V112	Specific	V1121	Physics
						V1122	Chemistry
						V1123	Mathematics
		V1124	Engineering				
		V12	Resource persons	V121	Local	V1211	within the college
						V1212	neighboring colleges
				V122	National	V1221	AICTE affiliated
						V1222	Non-AICTE affiliated
				V123	International	V1231	Affiliation with the host college
						V1232	Non-affiliation with the host college
		V13	Duration	V131	Day	V1311	One day
V1312	Two days						
V1313	Three days						
V132	Week			V1321	One		
				V1322	Two		
				V14	Target Group	V141	Students
V1412	Non-Engineering						

				V ₁₄₂	Research Scholars	V ₁₄₂₁	Engineering		
						V ₁₄₂₂	Non-Engineering		
				V ₁₄₃	Academicians	V ₁₄₃₁	Engineering		
						V ₁₄₃₂	Non-Engineering		
V2	Online courses	V ₂₁	Course nature	V ₂₁₁	Basic	V ₂₁₁₁	remembrance		
						V ₂₁₁₂	understanding		
		V ₂₂	Course Delivery	V ₂₂₁	Synchronous	V ₂₂₁₁	Zoom		
						V ₂₂₁₂	Zoho		
						V ₂₂₁₃	Examineer		
						V ₂₂₁₄	Google meet		
						V ₂₂₂	Asynchronous	V ₂₂₂₁	Google Classroom
								V ₂₂₂₂	Youtube upload
		V ₂₃	Duration	V ₂₃₁	Day	V ₂₃₁₁	One day		
						V ₂₃₁₂	Two days		
						V ₂₃₁₃	Three days		
				V ₂₃₂	Week	V ₂₃₂₁	One		
						V ₂₃₂₂	Two		
						V ₂₄	Target Group	V ₂₄₁	Students
		V ₂₄₁₂	Non-Engineering						
		V ₂₄₂	Research Scholars	V ₂₄₂₁	Engineering				
				V ₂₄₂₂	Non-Engineering				
		V ₂₄₃	Academicians	V ₂₄₃₁	Engineering				
				V ₂₄₃₂	Non-Engineering				
		V3	Training programme on Computer languages	V ₃₁	Course nature	V ₃₁₁	Basic	V ₃₁₁₁	remembrance
V ₃₁₁₂	understanding								
				V ₃₁₂	Moderate	V ₃₁₂₁	understanding		

				V ₃₁₃	Advanced	V ₃₁₂₂	application
						V ₃₁₃₁	Analysis
						V ₃₁₃₂	Evaluation
		V ₃₂	Course Delivery	V ₃₂₁	Synchronous	V ₃₂₁₁	Zoom
						V ₃₂₁₂	Zoho
						V ₃₂₁₃	Examineer
				V ₃₂₂	Asynchronous	V ₃₂₁₄	Google meet
						V ₃₂₂₁	Google Classroom
						V ₃₂₂₂	Youtube upload
		V ₃₃	Duration	V ₃₃₁	Day	V ₃₃₁₁	One day
						V ₃₃₁₂	Two days
						V ₃₃₁₃	Three days
				V ₃₃₂	Week	V ₃₃₂₁	One
						V ₃₃₂₂	Two
		V ₃₄	Target Group	V ₃₄₁	Students	V ₃₄₁₁	Engineering
						V ₃₄₁₂	Non-Engineering
				V ₃₄₂	Research Scholars	V ₃₄₂₁	Engineering
						V ₃₄₂₂	Non-Engineering
				V ₃₄₃	Academicians	V ₃₄₃₁	Engineering
						V ₃₄₃₂	Non-Engineering
V ₄	Industrial Partnership	V ₄₁	MOU	V ₄₁₁	Internship	V ₄₁₁₁	Merit-based
						V ₄₁₁₂	All students
		V ₄₂	Financial Support	V ₄₂₁	Equipment purchase	V ₄₂₁₁	Partial
						V ₄₂₁₂	Complete
				V ₄₂₂	Program organization	V ₄₂₂₁	Partial
		V ₄₂₂₂	Complete				
				V ₄₃₁	Knowledge sharing	V ₄₃₁₁	Periodic

V5	Professional Efficiency	V43	Technical Support			V4312	Regular
				V432	Experts Visit	V4321	Periodic
						V4322	Regular
		V51	Publications	V511	National	V5111	Scopus
						V5112	ICI
				V512	International	V5121	Scopus
						V5122	ICI
		V52	Pedagogy	V521	Teacher-Centered	V5211	lecture
						V5212	chalk & talk
				V522	Learner-Centered	V5221	Blended
						V5222	ICT
		V53	Content preparation	V531	Own	V5311	original
						V5312	modified
				V532	Experts Visit	V5321	Web sources
						V5322	Youtube
		V54	Course Delivery	V541	OER	V5411	Zoom
V5412	Zoho						
V5413	Google meet						
V5414	Examineer						
V542	Asynchronous			V5421	Google Classroom		
				V5422	Youtube upload		

In the above table, the input objects such as webinars, online courses, training programs are represented as the input enveloping vertices V1, V2, and V3 in Fig 3.1,3.2 and 3.3 respectively. The intervening object Industrial Partnership is represented as V4 in Fig 3.4. The output object Professional Efficiency is represented as V5 in Fig 3.5.

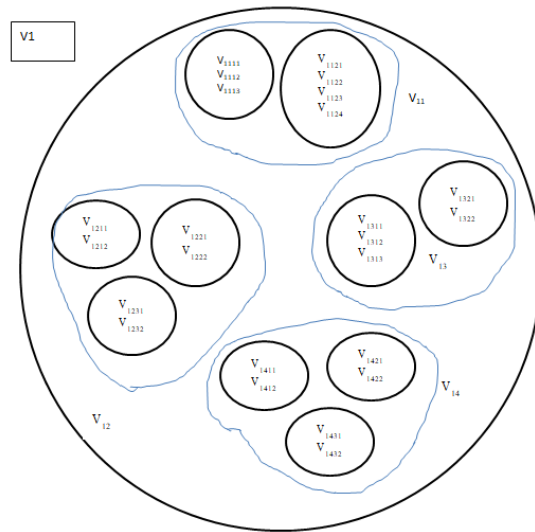


Fig.3.1 Representation of Input Object V1

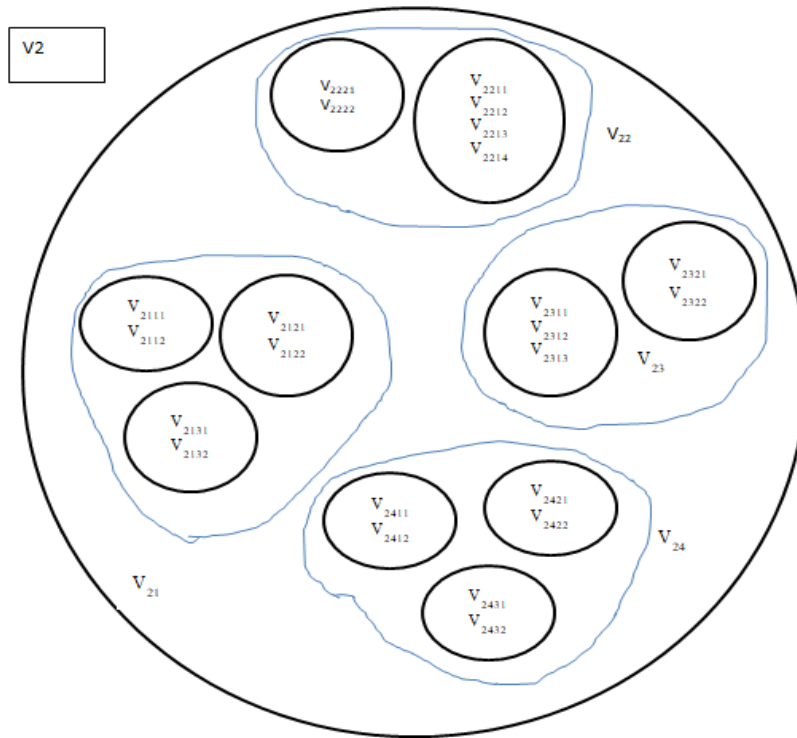


Fig.3.2 Representation of Input Object V2

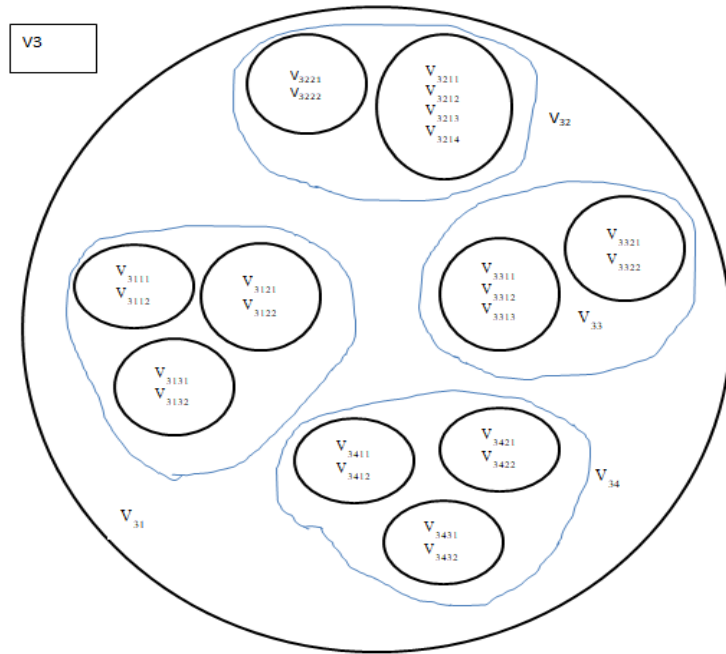


Fig.3.3 Representation of Input Object V3

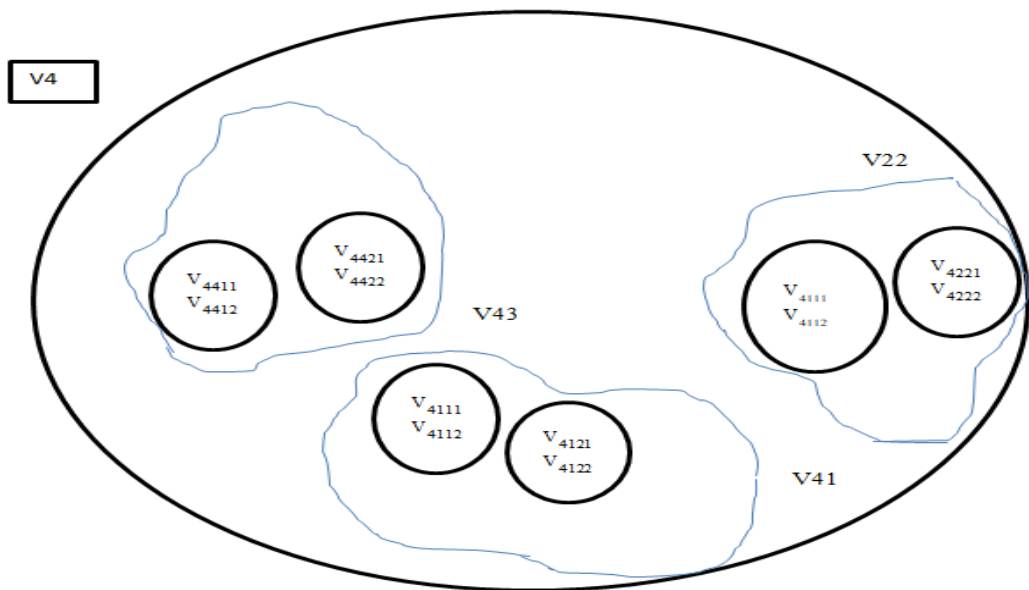


Fig.3.4 Representation of Intervening Object V4

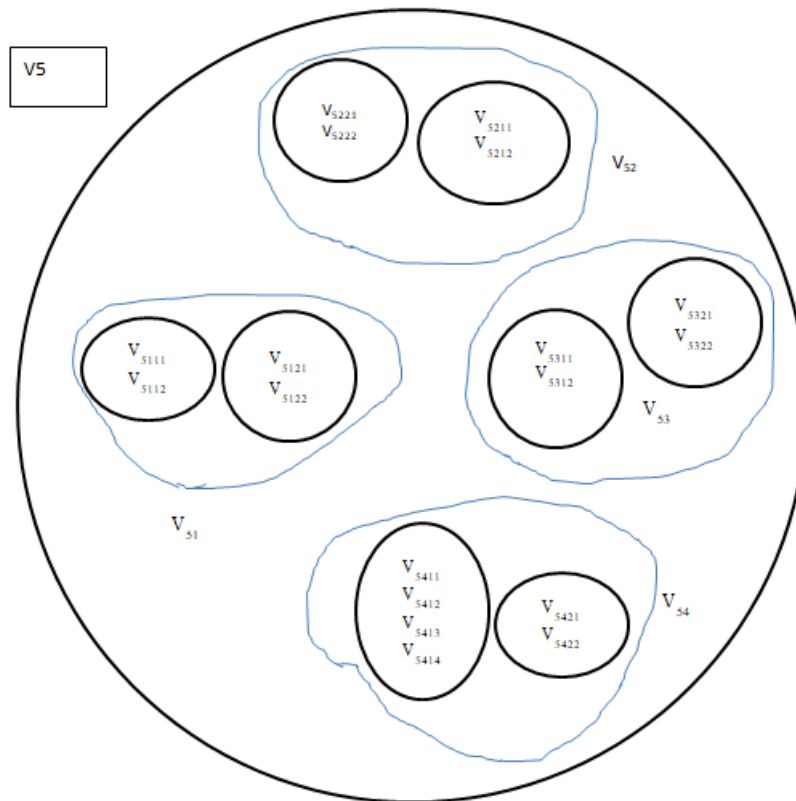


Fig.3.5 Representation of Output Object V5

Each enveloping vertices comprises of many attribute and sub-attribute values. To determine the desired output with and without the combination of input and intervene objects, the dominant attributes are chosen by the decision-makers. The dominant attribute values of the objects are represented in Table 3.2

Table 3.2 Representation of Dominant Attributes

Vertex	Representation	Vertex Attributes		Vertex Sub Attributes			
V1	Webinars	V11	Focus	V111	General	V1111	Education
		V12	Resource persons	V122	National	V1221	AICTE affiliated
		V13	Duration	V131	Day	V1312	Two days
		V14	Target Group	V143	Academicians	V1431	Engineering

V2	Online courses Course Nature,	V ₂₁	Course nature	V ₂₁₂	Moderate	V ₂₁₂₂	application
		V ₂₂	Course Delivery	V ₂₂₂	Asynchronous	V ₂₂₂₁	Google Classroom
		V ₂₃	Duration	V ₂₃₁	Day	V ₂₃₁₂	Two days
		V ₂₄	Target Group	V ₂₄₃	Academicians	V ₂₄₃₁	Engineering
V3	Training program on Computer languages	V ₃₁	Course nature	V ₃₁₁	Moderate	V ₃₁₂₂	application
		V ₃₂	Course Delivery	V ₃₂₁	Synchronous	V ₃₂₁₄	Google meet
		V ₃₃	Duration	V ₃₃₂	Week	V ₃₃₂₁	One
		V ₃₄	Target Group	V ₃₄₃	Academicians	V ₃₄₃₁	Engineering
V4	Industrial Partnership	V ₄₁	MOU	V ₄₁₂	Placement	V ₄₁₂₁	Merit-based
						V ₄₁₂₂	All students
		V ₄₂	Financial Support	V ₄₂₂	Program organization	V ₄₂₂₂	Complete
		V ₄₃	Technical Support	V ₄₃₂	Experts Visit	V ₄₃₂₂	Regular
	V ₄₃₁₂					Regular	
V5	Professional Efficiency	V ₅₁	Publications	V ₅₁₁	National	V ₅₁₁₁	Scopus
		V ₅₂	Pedagogy	V ₅₂₂	Learner-Centered	V ₅₂₂₁	Blended
		V ₅₃	Content preparation	V ₅₃₁	Own	V ₅₃₁₁	original
		V ₅₄	Course Delivery	V ₅₄₂	Asynchronous	V ₅₄₂₁	Google Classroom

The Dominant Enveloping vertices are presented in Fig 3.6

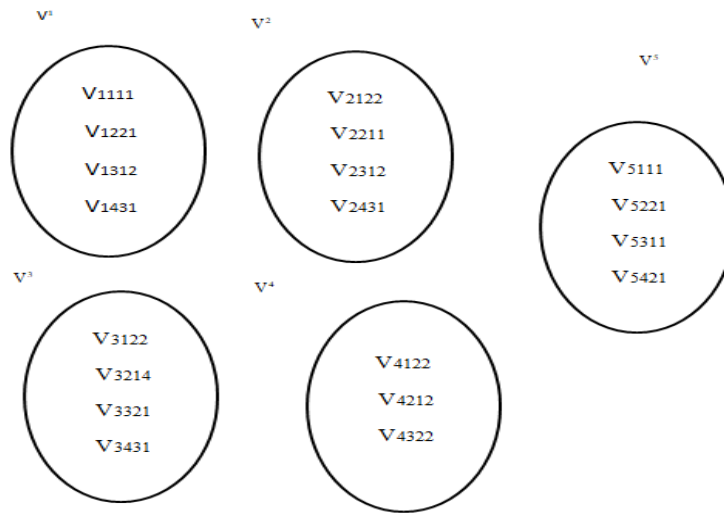


Fig. 3.6 Dominant Enveloping vertices

3.1 Decision-Making Scenario I

The institution is certain of the dominant sub-attributes and makes decisions based on dominant attributes of the input objects. The graphical representation of attribute relation between input dominant enveloping vertices and the output dominating attribute vertex with simple plithogenic fuzzy connectors is presented in Fig 3.7

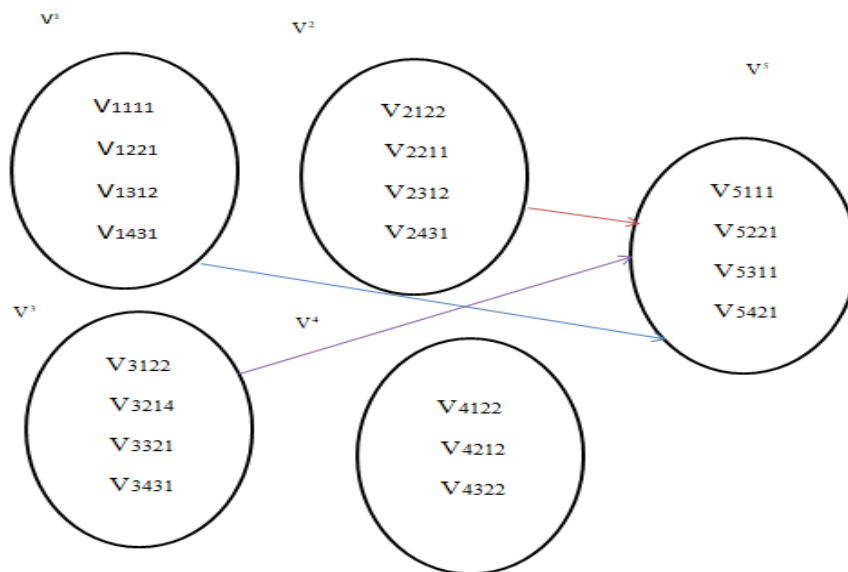


Fig.3.7 Representation of Decision-Making Scenario I

The dominant attribute relational matrix representation between the input objects on the output objects is presented as follows

	V ₅₁₁₁	V ₅₂₂₁	V ₅₃₁₁	V ₅₄₂₁
V ₁₁₁₁	0.55	0.2	0.5	0.6
V ₁₂₂₁	0.8	0.5	0.5	0.8
V ₁₃₁₂	0.75	0.6	0.6	0.9
V ₁₄₃₁	0.65	0.8	0.1	0.4
V ₂₁₂₂	0.5	0.9	0.3	0.6
V ₂₂₁₁	0.3	0.5	0.6	0.8
V ₂₃₁₂	0.45	0.4	0.4	0.9
V ₂₄₃₁	0.6	0.8	0.2	0.4
V ₃₁₂₂	0.85	0.2	0.8	0.8
V ₃₂₁₄	0.9	0.3	0.9	0.9
V ₃₃₂₁	0.5	0.55	0.5	0.4
V ₃₄₃₁	0.6	0.8	0.2	0.4

The frequency matrix as discussed by [14] shall be constructed to rank the dominant attributes of input objects contributing to the dominant attribute of the output object. This is a simple decision-making environment as it does not involve the role of an intervening object.

3.2 Decision Making Scenario II

The institution is certain of the dominant sub-attributes and makes decisions based on dominant attributes of the input and intervene objects. The graphical representation of attribute relation between input and intervene dominant enveloping vertices and the output dominating attribute vertex with combined plithogenic fuzzy connectors is presented in Fig.3.8

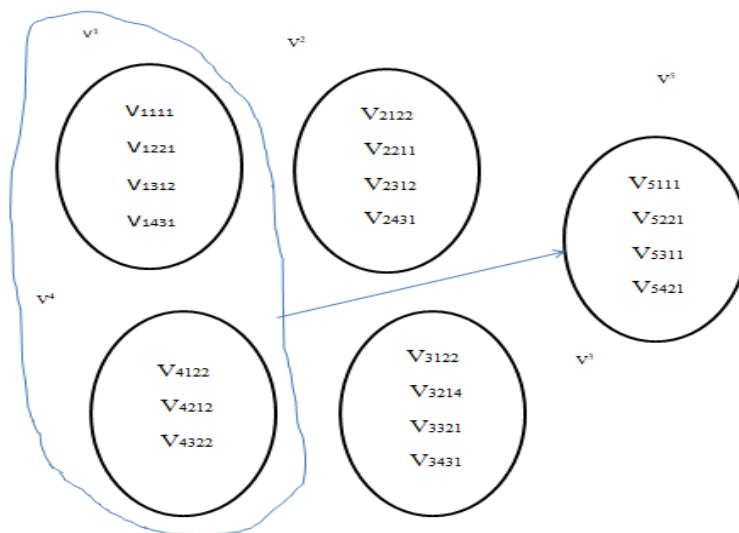


Fig.3.8 Representation of Decision-Making Scenario II

The dominant attribute relational matrix representation between the input and intervene objects on the output objects is presented as follows

	V ₅₁₁₁	V ₅₂₂₁	V ₅₃₁₁	V ₅₄₂₁
V ₁₁₁₁ ,V ₄₁₂₂	0.65	0.6	0.5	0.6
V ₁₁₁₁ ,V ₄₂₁₂	0.8	0.65	0.7	0.8
V ₁₁₁₁ ,V ₄₃₂₂	0.56	0.7	0.9	0.6
V ₁₂₂₁ ,V ₄₁₂₂	0.75	0.8	0.85	0.6
V ₁₂₂₁ ,V ₄₂₁₂	0.9	0.6	0.95	0.8
V ₁₂₂₁ ,V ₄₃₂₂	0.6	0.8	0.45	0.9
V ₁₃₁₂ ,V ₄₁₂₂	0.53	0.7	0.75	0.7
V ₁₃₁₂ ,V ₄₂₁₂	0.43	0.5	0.6	0.7
V ₁₃₁₂ ,V ₄₃₂₂	0.5	0.6	0.78	0.7
V ₁₄₃₁ ,V ₄₁₂₂	0.62	0.85	0.8	0.69
V ₁₄₃₁ ,V ₄₂₁₂	0.67	0.78	0.7	0.63
V ₁₄₃₁ ,V ₄₃₂₂	0.6	0.89	0.58	0.7

The frequency matrix shall be constructed to rank the combined dominant attributes of input and intervene objects contributing to the dominant attribute of the output object. This is a little complex decision-making environment as it involves the role of an intervening object. Fig 3.9 presents the graphical representation of it.

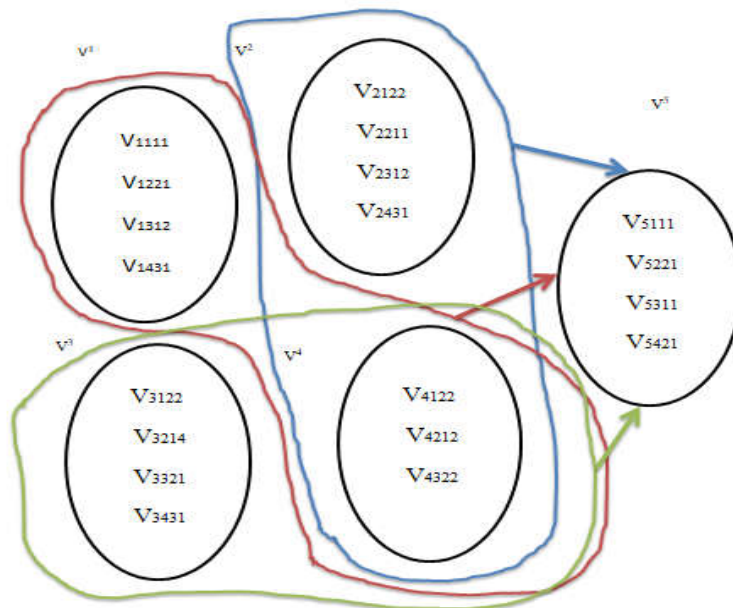


Fig.3.9 Representation of Decision-Making Scenario II with Intervening object

3.3 Decision Making Scenario III

The institution is certain of the dominant sub-attributes. Let us consider a situation, suppose if the institution decides to conduct a webinar with a focus on general, but not able to decide whether to give priority to Education, Health or Psychology, then the decision-making environment becomes more complex. The graphical representation

of all sub-attribute relation between input and the output dominating attribute vertex with simple plithogenic fuzzy connectors is presented in Fig. 3.10

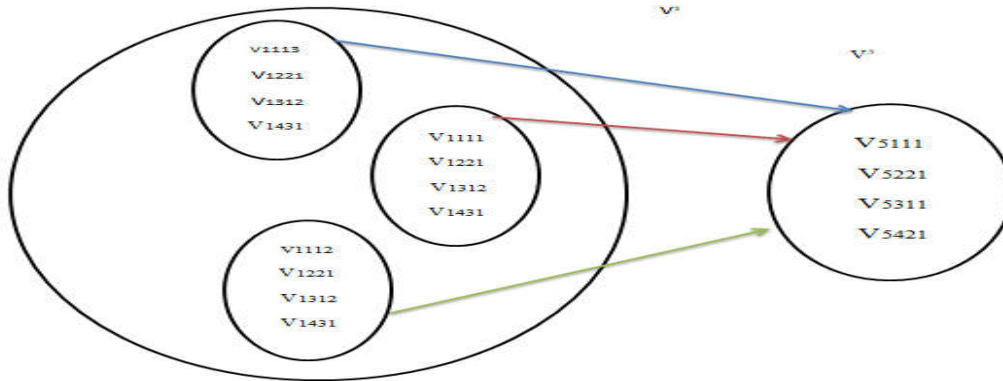


Fig. 3.10 Representation of Decision-Making Scenario III

The dominant attribute relational matrix representation is as follows

	V ₅₁₁₁	V ₅₂₂₁	V ₅₃₁₁	V ₅₄₂₁
V ₁₁₁₁	0.55	0.2	0.5	0.6
V ₁₁₁₂	0.6	0.55	0.7	0.85
V ₁₁₁₃	0.5	0.3	0.6	0.6
V ₁₂₂₁	0.8	0.4	0.4	0.8
V ₁₃₁₂	0.7	0.64	0.6	0.9
V ₁₄₃₁	0.6	0.89	0.62	0.4
V ₂₁₂₂	0.54	0.9	0.73	0.6
V ₂₂₁₁	0.83	0.5	0.6	0.8
V ₂₃₁₂	0.4	0.4	0.45	0.9
V ₂₄₃₁	0.6	0.8	0.72	0.49
V ₃₁₂₂	0.9	0.52	0.8	0.8
V ₃₂₁₄	0.95	0.63	0.9	0.9
V ₃₃₂₁	0.5	0.8	0.5	0.43
V ₃₄₃₁	0.6	0.8	0.52	0.4

3.4 Decision-Making Scenario IV

This decision-making situation is characterized when the institution is uncertain of the dominant sub-attribute values of the input object. Suppose if the institution decides to conduct a webinar with focus on general, but not able to decide whether to give priority to Education, Health or Psychology, In this case, the dominant sub-attribute value is not certain and suppose it wishes to collaborate with the industry then the decision-making environment becomes highly complex. Fig 3.11 presents this graphically

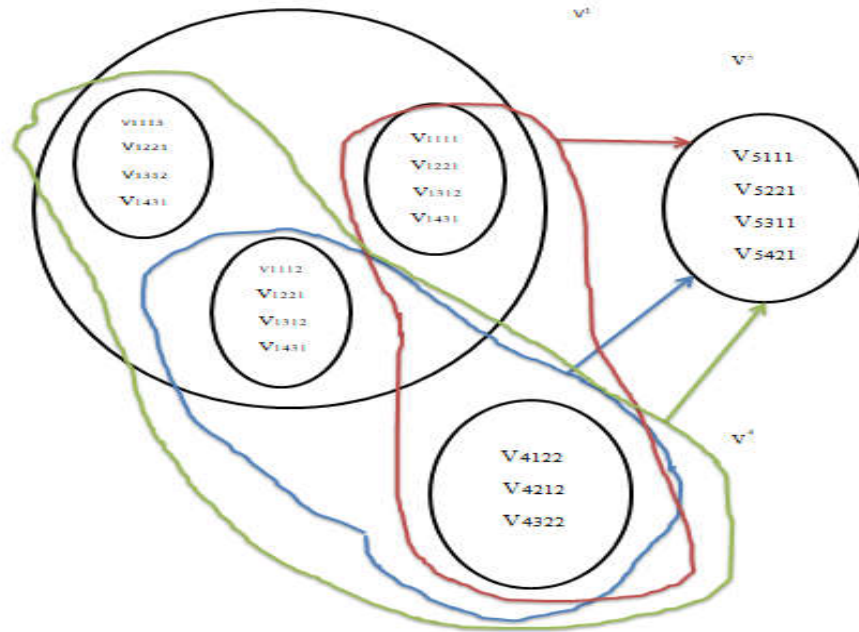


Fig.3.11 Representation of Decision-Making Scenario IV

The dominant attribute relational matrix representation is as follows

	V ₅₁₁₁	V ₅₂₂₁	V ₅₃₁₁	V ₅₄₂₁
V ₁₁₁₁ ,V ₄₁₂₂	0.65	0.6	0.5	0.6
V ₁₁₁₁ ,V ₄₂₁₂	0.8	0.65	0.7	0.8
V ₁₁₁₁ ,V ₄₃₂₂	0.56	0.7	0.9	0.6
V ₁₁₁₂ ,V ₄₁₂₂	0.52	0.68	0.57	0.66
V ₁₁₁₂ ,V ₄₂₁₂	0.45	0.67	0.72	0.56
V ₁₁₁₂ ,V ₄₃₂₂	0.56	0.68	0.74	0.69
V ₁₁₁₃ ,V ₄₁₂₂	0.67	0.79	0.83	0.79
V ₁₁₁₃ ,V ₄₂₁₂	0.57	0.82	0.74	0.68
V ₁₁₁₃ ,V ₄₃₂₂	0.5	0.7	0.7	0.6
V ₁₂₂₁ ,V ₄₁₂₂	0.75	0.8	0.85	0.6
V ₁₂₂₁ ,V ₄₂₁₂	0.9	0.6	0.95	0.8
V ₁₂₂₁ ,V ₄₃₂₂	0.6	0.8	0.45	0.9
V ₁₃₁₂ ,V ₄₁₂₂	0.53	0.7	0.75	0.7
V ₁₃₁₂ ,V ₄₂₁₂	0.43	0.5	0.6	0.7
V ₁₃₁₂ ,V ₄₃₂₂	0.5	0.6	0.78	0.7
V ₁₄₃₁ ,V ₄₁₂₂	0.62	0.85	0.8	0.69
V ₁₄₃₁ ,V ₄₂₁₂	0.67	0.78	0.7	0.63
V ₁₄₃₁ ,V ₄₃₂₂	0.6	0.89	0.58	0.7

The ranking of the attributes contributing to Professional efficiency corresponding to each decision-making environment is presented in Table 3.3

Table 3.3 Ranking of the attributes

Decision Making Environment	Ranking of the Attributes contributing to Professional Efficiency
Decision Making Scenario I	$V_{1312} > V_{1221} > V_{3122} > V_{1221} > V_{2122} > V_{2211} > V_{2431} > V_{1111} > V_{2312} > V_{1431} > V_{3431} > V_{3321}$
Decision Making Scenario II	$V_{1221}, V_{4212} > V_{1221}, V_{4322} > V_{1431}, V_{4122} > V_{1111}, V_{4212}, V_{1221}, V_{4122} > V_{1431}, V_{4322} > V_{1312}, V_{4122} > V_{1312}, V_{4322} > V_{1111}, V_{4122} > V_{1312}, V_{4212}$
Decision Making Scenario III	$V_{3214} > V_{2431} > V_{1312} > V_{2122} > V_{3122} > V_{1111} > V_{2211} > V_{1431} > V_{1221} > V_{3431} > V_{3321} > V_{2312} > V_{1112} > V_{1113}$
Decision Making Scenario IV	$V_{1221}, V_{4212} > V_{1113}, V_{4122} > V_{1221}, V_{4122} > V_{1431}, V_{4122} > V_{1111}, V_{4212} > V_{1113}, V_{4212} > V_{1431}, V_{4212} > V_{1431}, V_{4322} > V_{1111}, V_{4322} > V_{1221}, V_{4322} > V_{1312}, V_{4122} > V_{1112}, V_{4322} > V_{1312}, V_{4322} > V_{1113}, V_{4322} > V_{1112}, V_{4122} > V_{1112}, V_{4212} > V_{1111}, V_{4122} > V_{1312}, V_{4212}$

4. Discussion

The ranking of the input attribute values of the dominant attributes contributing to the output dominant attribute values shows the significance of the individual contribution of each input attribute value. In the first decision-making scenario, the attribute values of the input object are ranked. In the second decision making a scenario the combined attribute values of input and intervene objects are ranked. This helps in finding the combined effect towards the attainment of the output attribute values. In the third decision-making scenario, the ranking of sub-attribute values are made, in this case, there was a choice to choose between Education (V_{1111}), Health (V_{1112}) or Psychology (V_{1113}), but the preference s should be given to Education based on the ranking. In the fourth decision-making scenario, the combined effects of sub-attribute values along with intervening attribute values are ranked and here also the combined effect of the sub-attribute value, Education is gaining more significance. The above decision-making scenarios were focusing on the effects of one input object and the same can be applied to other input objects and the respective results can be determined. The same method of decision making can be applied to production sectors in strategy selection which considers many attribute values and sub-attribute values and this proposed plithogenic –n superhypergraph MADM can be applied in such decision-making scenario.

5. Conclusion

This article presents the application of plithogenic n-super hypergraph in the context of optimal decision making. This research work introduces many new concepts such as enveloping vertex, dominant enveloping vertex, super enveloping vertex, and plithogenic connectors. This research work creates a new avenue in MADM by providing space for comprehensive decision making. A new approach to ranking the attribute values based on the frequency matrix is initiated. The theoretical description of plithogenic n-super hypergraph is translated into practical application in this research work and this will certainly open new vistas of research. This work can be further extended with various plithogenic sets.

References

[1] Ishizaka, A., Labib, A.. "Review of the main developments in the analytic hierarchy process", Expert Systems with Applications, Vol 38, pp.14336-14345, 2011.
 [2] Saaty, T. L., "A scaling method for priorities in hierarchical structures", Journal of Mathematical Psychology. Elsevier: Amsterdam, Vol 15, pp.234-281, 1977.

- [3] Tzeng, G.H.; Huang, J.J. "Multiple Attribute Decision Making: Methods and Applications"; CRC Press: Boca Raton, FL, USA, 2011.
- [4] Smarandache, Florentin., "Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic –n SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-)HyperAlgebra" *Neutrosophic Sets and Systems*, Vol 33, pp.290-296, 2020 .
- [5] Smarandache, F. "Extension of Soft set to Hypersoft Set, and then to Plithogenic Hypersoft Set", *Neutrosophic set and Systems*, 22, pp.68-70, 2018.
- [6] Gayathri.N., Helen.M., Mounika.P., "Utilization of Jaccard Index Measures on Multiple Attribute Group Decision Making under Neutrosophic Environment", *International Journal of Neutrosophic Science*, Vol. 3, pp.67-77, 2020.
- [7] Muhammad Naveed Jafar., Muhammad Saqlain., Ahmad Raza Shafiq., Muhammad Khalid., Hamza Akbar., Aamir Naveed., " New Technology in Agriculture Using Neutrosophic Soft Matrices with the Help of Score Function ", *International Journal of Neutrosophic Science*, Vol. 3, pp.78-88, 2020.
- [8] Ajay.D., Aldring.J., Abirami.S., Jeni Seles Martina.D., " A SVTrN-number approach of multi-objective optimization on the basis of simple ratio analysis based on MCDM method ", *International Journal of Neutrosophic Science* , Vol. 5, pp.16-28, 2020.
- [9] Luis Andrés Crespo Berti., "Application of the neutrosophic system to tax havens with a criminal approach", *International Journal of Neutrosophic Science* , Vol. 5, pp.91-106, 2020.
- [10] Abdel-Basset, M., Manogaran, G., Mohamed, M., Chilamkurti, N., "Three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem". *Future Gener. Comput. Syst.* Vol 89, pp.19–30, 2018.
- [11] Abdel-Basset, M., Mohamed, M., Smarandache, F., "A hybrid neutrosophic group ANP-TOPSIS framework for supplier selection problems". *Symmetry* Vol 10, pp.226, 2018.
- [12] Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F., "A novel model for evaluation Hospital medical care systems based on plithogenic sets". *Artificial intelligence in medicine*, 100, 101710, 2019
- [13] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F., "A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics". *Symmetry*, Vol 11, 903, 2019.
- [14] Shazia Rana ., Madiha Qayyum., Muhammad Saeed., Florentin Smarandache, Bakhtawar Ali Khan., "Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique", *Neutrosophic Sets and Systems*, Vol 28, pp.34-50, 2019.