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Possibility neutrosophic soft sets and PNS-decision making method

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ABSTRACT

In this paper, we introduce concept of possibility neutrosophic soft set and define some related concepts such as possibility neutrosophic soft subset, possibility neutrosophic soft null set, and possibility neutrosophic soft universal set. Then, based on definitions of n-norm and n-conorm, we define set theoretical operations of possibility neutrosophic soft sets such as union, intersection and complement, and investigate some properties of these operations. We also introduce AND-product and OR-product operations between two possibility neutrosophic soft sets. We propose a decision making method called possibility neutrosophic soft decision making method (PNS-decision making method) which can be applied to the decision making problems involving uncertainty based on AND-product operation. We finally give a numerical example to display application of the method that can be successfully applied to the problems.

1. Introduction

Many problems in engineering, medical sciences, economics and social sciences involve uncertainty. Researchers have proposed some theories such as the theory of fuzzy set [34], the theory of intuitionistic fuzzy set [5], the theory of rough set [26], the theory of vague set [19] in order to resolve these problems. However, all of these theories have their own difficulties which are pointed out by Molodtsov [21]. Therefore, he proposed a completely new approach for modeling uncertainty. Molodtsov displayed soft sets in wide range of field including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. After Molodtsov's study [21], the operations of soft sets and some of their properties were given by Maji et al. [24]. Some researchers such as Ali et al. [1], Çağman and Enginoğlu [9], Sezgin and Atagün [30], Zhu and Wen [36], Çağman [12] made some modifications and contributions to the operations of soft sets.

Application of soft set theory in decision making problems was first studied by Maji et al. [23]. Subsequent works were published by many researchers. Çağman and Enginoğlu [9] proposed the uni-int decision making method to reduce the alternatives. Feng et al. [17] generalized the uni-int decision making based on choice value soft sets. Qin et al. [27] improved some algorithms which require relatively fewer calculations compared with the existing decision making algorithms. Zhi et al. [35] presented an efficient decision making approach in incomplete soft set.

In 1986, intuitionistic fuzzy sets were introduced by Atanassov [3] as an extension of Zadeh's [34] notion of fuzzy set. Maji [22] combined intuitionistic fuzzy with soft sets. Çağman and Karataş [11] redefined intuitionistic fuzzy sets and proposed a decision making method using the intuitionistic fuzzy soft sets. Agarwal et al. [2] defined generalized intuitionistic fuzzy soft sets (GIFSS), and investigated properties of GIFSS. They also put forward similarity measure between two GIFSSs. Das and Kar [13] proposed an algorithmic approach based on intuitionistic fuzzy soft set (IFSS) which explores a particular disease reflecting the agreement of all experts. They also demonstrated effectiveness of the proposed approach using a suitable case study. Deli and Çağman [16] defined intuitionistic fuzzy parameterized soft sets as an extension of fuzzy parameterized soft sets [10], and suggested a decision making method based on intuitionistic fuzzy parameterized soft sets. Recently, many researchers published interesting results on intuitionistic fuzzy soft set theory.

Neutrosophic logic theory and neutrosophic sets were proposed by Smarandache [31,32], as a new mathematical tool for dealing with problems involving incomplete, indeterminant, and inconsistent knowledge. Neutrosophy is a new branch of philosophy that generalizes fuzzy logic, intuitionistic fuzzy logic, and paraconsistent logic. Fuzzy sets are characterized by membership functions, and intuitionistic fuzzy sets by membership functions and non-membership functions. In some real life problems for proper description of an object in

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uncertain and ambiguous environment, we need to discuss the indeterminate and incomplete information. However, fuzzy sets and intuitionistic fuzzy sets do not discuss the indeterminant and inconsistent information.

Maji [25] introduced concept of neutrosophic soft set and some operations of neutrosophic soft sets. Then Karaaslan [20] redefined concepts and operations on neutrosophic soft sets. He also gave a decision making method and group decision making method. Recently, the properties of neutrosophic soft sets and applications in decision making problems have been studied increasingly. For example, Broumi [7] defined concept of generalized neutrosophic soft set. Broumi et al. [8] suggested a decision making method on the neutrosophic parameterized soft sets. Şahin and Küçük [29] introduced concept of generalized neutrosophic soft sets and their operations. They also proposed decision making method and similarity measure method under generalized soft neutrosophic environment. Deli [14] defined concept of interval-valued neutrosophic soft set and its operations. Deli and Broumi [15] introduced neutrosophic soft matrices and suggested a decision making method based on neutrosophic soft matrices.

Alkhazaleh et al. [4] first introduced concept of the possibility fuzzy soft sets and their operations, and gave applications of this theory in a decision making problem. They also introduced a similarity measure between two possibility fuzzy soft sets, and gave an application of proposed similarity measure method in a medical diagnosis problem. In 2012, Bashir et al. [6] introduced concept of possibility intuitionistic fuzzy soft set and their operations, and discussed similarity measure between two possibility intuitionistic fuzzy sets. They also gave an application of this similarity measure.

Possibility neutrosophic soft sets are generalization of possibility fuzzy soft sets and possibility intuitionistic fuzzy soft sets. Fuzzy sets and intuitionistic fuzzy sets are very useful in order to model some decision making problems containing uncertainty and incomplete information. However, they sometimes may not suffice to model indeterminate and inconsistent information encountered in real world. This problem also exists in fuzzy soft sets and intuitionistic fuzzy soft sets. Therefore, concepts of neutrosophic set and neutrosophic soft set have a very important role in modeling of some problems which contain indeterminate and inconsistent information. In the definition of neutrosophic soft set [20,25], parameter set is a classical set, and possibility of each element of initial universe related to each parameter are considered as 1. This poses a limitation in modeling of some problems. In a possibility neutrosophic soft set, possibility of each element of initial universe related to each parameter may be different from 1. Therefore, possibility neutrosophic soft sets present a more general perspective than neutrosophic soft sets. If we are to explain the idea of possibility neutrosophic soft set, let us give an example, consider the last ten days of April and the parameter "rainy". First day of last ten days, the possibility of action of rainfall can be 0.8. However, amount of water can be (0.5, 0.3, 0.6). Based on the parameter "rainy", other nine days can be expressed with some possibility neutrosophic values. For last ten days and for parameters more than one, we need possibility neutrosophic soft sets to show all of the corresponding possibility neutrosophic values.

From this point of view, we introduce concept of possibility neutrosophic soft sets based on idea that each of elements of initial universe has got a possibility degree related to each element of parameter set. Furthermore, we define set theoretical operations between two possibility neutrosophic soft sets, and investigate some of their properties. We also propose a new decision making method using AND-product of possibility neutrosophic soft sets called possibility neutrosophic soft(PNS) decision making method. We finally give a numerical example to show the method that can be successfully applied to the problems studied.

2. Preliminary

We first present basic definitions and propositions required in next sections. The concepts given in this section can be found in Refs. [4,6,12,20,21,25].

Throughout this paper U is an initial universe, E is a set of parameters, $P(U)$ is power set of U and Λ is an index set.

Definition 1 *([21])*. Let U be a set of objects, E be a set of parameters and $\emptyset \neq A \subseteq E$. Mapping given by $f: A \to P(U)$ is called a soft set over U and denoted by (f, A).

Set theoretical operations of soft sets are given in [12] as follows:

Let f and g be two soft sets. Then,

1. If $f(e)=\emptyset$ for all $e \in E$, then f is called null soft set and denoted by Φ.
2. If $f(e)=U$ for all $e \in E$, then f is called absolute soft set and denoted by $\hat{U}$.
3. If $f(e) \subseteq g(e)$ for all $e \in E$, f is said to be a soft subset of g and denoted by $f \tilde{\subseteq} g$.
4. If $f \tilde{\subseteq} g$ and $g \tilde{\subseteq} f$, $f=g$.
5. Soft union of f and g, denoted by $f \tilde{\cup} g$, is a soft set over U and defined by $f \tilde{\cup} g : E \to P(U)$ such that $(f \tilde{\cup} g)(e) = f(e) \cup g(e)$ for all $e \in E$.
6. Soft intersection of f and g, denoted by $f \tilde{\cap} g$, is a soft set over U and defined by $f \tilde{\cap} g : E \to P(U)$ such that $(f \tilde{\cap} g)(e) = f(e) \cap g(e)$ for all $e \in E$.
7. Soft complement of f is denoted by $f^{\tilde{c}}$ and defined by $f^{\tilde{c}} : E \to P(U)$ such that $f^{\tilde{c}}(e) = U \setminus f(e)$ for all $e \in E$.

Definition 2 *([20,25])*. A neutrosophic soft set (or namely ns-set) f over U is a neutrosophic set valued function from E to $\mathcal{N}(U)$. It can be written as

$$f = \{(e, \{\langle u, t_{f(e)}(u), i_{f(e)}(u), f_{f(e)}(u)\rangle : u \in U\}) : e \in E\}$$

where, $\mathcal{N}(U)$ denotes set of all neutrosophic sets over U. Here, each of $t_{f(e)}$, $i_{f(e)}$ and $f_{f(e)}$ is a function from U to interval [0, 1] and they denote truth, indeterminacy and falsity membership degrees of element of initial universe U related to parameter $e \in E$, respectively. Note that if $f(e) = \{\langle u, 0, 1, 1\rangle : u \in U\}$, the element $(e, f(e))$ is not appeared in the neutrosophic soft set f. Set of all ns-sets over U is denoted by $\mathcal{NS}$.

Definition 3 *([20])*. Let $f, g \in \mathcal{NS}$. f is said to be neutrosophic soft subset of g, if $t_{f(e)}(u) \leq t_{g(e)}(u)$, $i_{f(e)}(u) \geq i_{g(e)}(u) f_{f(e)}(u) \geq f_{g(e)}(u)$, $\forall e \in E$, $\forall u \in U$. We denote it by $f \sqsubseteq g$. f is said to be neutrosophic soft super set of g if g is a neutrosophic soft subset of f. We denote it by $f \sqsupseteq g$.

If f is neutrosophic soft subset of g and g is neutrosophic soft subset of f. We denote it $f=g$. Here, if we take $i_{f(e)}(u) \leq i_{g(e)}(u)$, it can be shown that the definition is identical to definition of neutrosophic soft subset given by Maji [25].

Definition 4 *([20])*. Let $f \in \mathcal{NS}$. If $t_{f(e)}(u) = 0$ and $i_{f(e)}(u) = f_{f(e)}(u) = 1$ for all $e \in E$ and for all $u \in U$, then f is called null ns-set and denoted by $\tilde{\Phi}$.

Definition 5 *([20])*. Let $f \in \mathcal{NS}$. If $t_{f(e)}(u) = 1$ and $i_{f(e)}(u) = f_{f(e)}(u) = 0$ for all $e \in E$ and for all $u \in U$, then f is called universal ns-set and denoted by $\tilde{U}$.

Definition 6 *([20])*. Let $f, g \in \mathcal{NS}$. Then union and intersection of ns-sets f and g denoted by $f \sqcup g$ and $f \sqcap g$ respectively, are defined by as follows:

$$f \sqcup g = \{(e, \{\langle u, t_{f(e)}(u) \vee t_{g(e)}(u), i_{f(e)}(u) \wedge i_{g(e)}(u), f_{f(e)}(u) \wedge f_{g(e)}(u)\rangle : u \in U\}) : e \in E\}.$$

and ns-intersection of f and g is defined as

$$f \sqcap g = \{(e, \{\langle u, t_{f(e)}(u) \wedge t_{g(e)}(u), i_{f(e)}(u) \vee i_{g(e)}(u), f_{f(e)}(u) \vee f_{g(e)}(u)\rangle : u \in U\}) : e \in E\}.$$

Definition 7 *([20])*. Let $f, g \in \mathcal{NS}$. Then complement of ns-set f, denoted by $f^{\tilde{c}}$, is defined as follows:

$$f^{\tilde{c}} = \{(e, \{\langle u, f_{f(e)}(u), 1 - i_{f(e)}(u), t_{f(e)}(u)\rangle : u \in U\}) : e \in E\}.$$

Proposition 1 ([20]). *Let $f, g, h \in \mathcal{NS}$. Then,*

(1) $\tilde{\Phi} \sqsubseteq f$
(2) $f \sqsubseteq \tilde{U}$
(3) $f \sqsubseteq f$
(4) $f \sqsubseteq g$ *and* $g \sqsubseteq h \rightarrow f \sqsubseteq h$

Proposition 2 ([20]). *Let $f \in \mathcal{NS}$. Then,*

(1) $\tilde{\Phi}^{\tilde{c}} = \tilde{U}$
(2) $\tilde{U}^{\tilde{c}} = \tilde{\Phi}$
(3) $(f^{\tilde{c}})^{\tilde{c}} = f$.

Proposition 3 ([20]). *Let $f, g, h \in \mathcal{NS}$. Then,*

(1) $f \sqcap f = f$ *and* $f \sqcup f = f$
(2) $f \sqcap g = g \sqcap f$ *and* $f \sqcup g = g \sqcup f$
(3) $f \sqcap \tilde{\Phi} = \tilde{\Phi}$ *and* $f \sqcap \tilde{U} = f$
(4) $f \sqcup \tilde{\Phi} = f$ *and* $f \sqcup \tilde{U} = \tilde{U}$
(5) $f \sqcap (g \sqcap h) = (f \sqcap g) \sqcap h$ *and* $f \sqcup (g \sqcup h) = (f \sqcup g) \sqcup h$
(6) $f \sqcap (g \sqcup h) = (f \sqcap g) \sqcup (f \sqcap h)$ *and* $f \sqcup (g \sqcap h) = (f \sqcup g) \sqcap (f \sqcup h)$.

Theorem 1 ([20]). *Let $f, g \in \mathcal{NS}$. Then, De Morgan's law is valid.*

(1) $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$
(2) $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$

Definition 8 *([20])*. Let $f, g \in \mathcal{NS}$. Then 'OR' product of ns-sets f and g denoted by $f \vee g$, is defined as follows:

$$f \vee g = \{((e, e'), \{\langle u, t_{f(e)}(u) \vee t_{g(e)}(u), i_{f(e)}(u) \wedge i_{g(e)}(u), f_{f(e)}(u) \wedge f_{g(e)}(u)\rangle : u \in U\}) : (e, e') \in E \times E\}.$$

Definition 9 *([20])*. Let $f, g \in \mathcal{NS}$. Then 'AND' product of ns-sets f and g denoted by $f \wedge g$, is defined as follows:

$$f \wedge g = \{((e, e'), \{\langle u, t_{f(e)}(x) \wedge t_{g(e)}(u), i_{f(e)}(u) \vee i_{g(e)}(u), f_{f(e)}(u) \vee f_{g(e)}(u)\rangle : u \in U\}) : (e, e') \in E \times E\}.$$

Proposition 4 ([20]). *Let $f, g \in \mathcal{NS}$. Then,*

(1) $(f \vee g)^{\tilde{c}} = f^{\tilde{c}} \wedge g^{\tilde{c}}$
(2) $(f \wedge g)^{\tilde{c}} = f^{\tilde{c}} \vee g^{\tilde{c}}$

Definition 10 *([4])*. Let $U = \{u_1, u_2, \ldots, u_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \rightarrow I^U$ and μ be a fuzzy subset of E, that is $\mu : E \rightarrow I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \rightarrow I^U \times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(u), \mu(e)(u)), \quad \forall u \in U.$$

Then F_μ is called a possibility fuzzy soft set (PFSS in short) over the soft universe (U, E). For each parameter e_i, $F_\mu(e_i) = (F(e_i)(u), \mu(e_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $\mu(e_i)$.

Definition 11 *([6])*. Let $U = \{u_1, u_2, \ldots, u_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to (I \times I)^U \times I^U$ where $(I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all fuzzy subsets of U. Let p be a fuzzy subset of E, that is, $p : E \to I^U$ and let $F_p : E \to (I \times I)^U \times I^U$ be a function defined as follows:

$$F_p(e) = (F(e)(u), p(e)(u)), F(e)(u) = (\mu(u), \nu(u)), \quad \forall u \in U.$$

Then F_p is called a possibility intuitionistic fuzzy soft set (PIFSS in short) over the soft universe (U, E). For each parameter e_i, $F_p(e_i) = (F(e_i)(u), p(e_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $p(e_i)$.

3. Possibility neutrosophic soft sets

In this section, we introduce the concepts of possibility neutrosophic soft set, possibility neutrosophic soft subset, possibility neutrosophic soft null set, possibility neutrosophic soft universal set, and possibility neutrosophic soft set operations.

Definition 12. Let U be an initial universe, E be a parameter set, $\mathcal{N}(\mathcal{U})$ be the collection of all neutrosophic sets of U and I^U is collection of all fuzzy subset of U. A possibility neutrosophic soft set $(PNS-set) f_\mu$ over U is a set of ordered pairs defined by

$$f_\mu = \left\{ \left(e_k, \left\{ \left(\frac{u_j}{f(e_k)(u_j)}, \mu(e_k)(u_j) \right) : u_j \in U \right\} \right) : e_k \in E \right\}$$

or a mapping defined by

$$f_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$$

where, $i \in \Lambda_1$ and $j \in \Lambda_2$, f is a mapping given by $f : E \to \mathcal{N}(\mathcal{U})$ and $\mu(e_k)$ is a fuzzy set such that $\mu : E \to I^U$.

For each parameter $e_k \in E, f(e_k) = \{\langle u_j, t_{f(e_k)}(u_j), i_{f(e_k)}(u_j), f_{f(e_k)}(u_j)\rangle : u_j \in U\}$ indicates neutrosophic value set of parameter e_k and where $t, i, f : U \to [0, 1]$ are the membership functions of truth, indeterminacy and falsity respectively of the element $u_j \in U$. For each $u_j \in U$ and $e_k \in E$, $0 \le t_{f(e_k)}(u_j) + i_{f(e_k)}(u_j) + f_{f(e_k)}(u_j) \le 3$. Also $\mu(e_k)$, degrees of possibility of belongingness of elements of U in $f(e_k)$. So we can write

$$f_\mu(e_k) = \left\{ \left(\frac{u_1}{f(e_k)(u_1)}, \mu(e_k)(u_1) \right), \left(\frac{u_2}{f(e_k)(u_2)}, \mu(e_k)(u_2) \right), \ldots, \left(\frac{u_n}{f(e_k)(u_n)}, \mu(e_k)(u_n) \right) \right\}$$

From now on, we will show set of all possibility neutrosophic soft sets over U with $\mathcal{PN}(U, E)$ such that E is parameter set.

Example 1. Let $U = \{u_1, u_2, u_3\}$ be a set of three cars. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 = cheap, e_2 = equipment, e_3 = fuel consumption and let $\mu : E \to I^U$. We can define a function $f_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ as follows:

$$f_\mu = \begin{Bmatrix} f_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.5, 0.2, 0.6)}, 0.8 \right), \left(\frac{u_2}{(0.7, 0.3, 0.5)}, 0.4 \right), \left(\frac{u_3}{(0.4, 0.5, 0.8)}, 0.7 \right) \right\} \\ f_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.8, 0.4, 0.5)}, 0.6 \right), \left(\frac{u_2}{(0.5, 0.7, 0.2)}, 0.8 \right), \left(\frac{u_3}{(0.7, 0.3, 0.9)}, 0.4 \right) \right\} \\ f_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.6, 0.7, 0.5)}, 0.2 \right), \left(\frac{u_2}{(0.5, 0.3, 0.7)}, 0.6 \right), \left(\frac{u_3}{(0.6, 0.5, 0.4)}, 0.5 \right) \right\} \end{Bmatrix}$$

also we can define a function $g_\nu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ as follows:

$$g_\nu = \begin{Bmatrix} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.6, 0.3, 0.8)}, 0.4 \right), \left(\frac{u_2}{(0.6, 0.5, 0.5)}, 0.7 \right), \left(\frac{u_3}{(0.2, 0.6, 0.4)}, 0.8 \right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.5, 0.4, 0.3)}, 0.3 \right), \left(\frac{u_2}{(0.4, 0.6, 0.5)}, 0.6 \right), \left(\frac{u_3}{(0.7, 0.2, 0.5)}, 0.8 \right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.7, 0.5, 0.3)}, 0.8 \right), \left(\frac{u_2}{(0.4, 0.4, 0.6)}, 0.5 \right), \left(\frac{u_3}{(0.8, 0.5, 0.3)}, 0.6 \right) \right\} \end{Bmatrix}$$

For the purpose of storing a possibility neutrosophic soft set in a computer, we can use matrix notation of possibility neutrosophic soft set f_μ. For example, matrix notation of possibility neutrosophic soft set f_μ can be written as follows: for $m, n \in \Lambda$,

$$f_\mu = \begin{pmatrix} (\langle 0.5, 0.2, 0.6\rangle, 0.8) & (\langle 0.7, 0.3, 0.5\rangle, 0.4) & (\langle 0.4, 0.5, 0.8\rangle, 0.7) \\ (\langle 0.8, 0.4, 0.5\rangle, 0.6) & (\langle 0.5, 0.7, 0.2\rangle, 0.8) & (\langle 0.7, 0.3, 0.9\rangle, 0.4) \\ (\langle 0.6, 0.7, 0.5\rangle, 0.2) & (\langle 0.5, 0.3, 0.7\rangle, 0.6) & (\langle 0.6, 0.5, 0.4\rangle, 0.5) \end{pmatrix}$$

where the m-th row vector shows $f(e_m)$ and n-th column vector shows u_n.

Definition 13. Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. Then, f_μ is said to be a possibility neutrosophic soft subset $(PNS-subset)$ of g_ν, and denoted by $f_\mu \subseteq g_\nu$, if

(1) $\mu(e)$ is a fuzzy subset of $\nu(e)$, for all $e \in E$
(2) f is a neutrosophic subset of g,

Example 2. Let $U=\{u_1, u_2, u_3\}$ be a set of tree houses, and let $E=\{e_1, e_2, e_3\}$ be a set of parameters where e_1 = modern, e_2 = big and e_3 = cheap. Let f_μ be a *PNS*-set defined as follows:

$$f_\mu = \begin{Bmatrix} f_\mu(e_1)=\left\{\left(\frac{u_1}{(0.5,0.2,0.6)},0.8\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.4\right),\left(\frac{u_3}{(0.4,0.5,0.9)},0.7\right)\right\} \\ f_\mu(e_2)=\left\{\left(\frac{u_1}{(0.8,0.4,0.5)},0.6\right),\left(\frac{u_2}{(0.5,0.7,0.2)},0.8\right),\left(\frac{u_3}{(0.7,0.3,0.9)},0.4\right)\right\} \\ f_\mu(e_3)=\left\{\left(\frac{u_1}{(0.6,0.7,0.5)},0.2\right),\left(\frac{u_2}{(0.5,0.3,0.8)},0.6\right),\left(\frac{u_3}{(0.6,0.5,0.4)},0.5\right)\right\} \end{Bmatrix}$$

$g_\nu : E \to \mathcal{N}(U) \times I^U$ be another *PNS*-set defined as follows:

$$g_\nu = \begin{Bmatrix} g_\nu(e_1)=\left\{\left(\frac{u_1}{(0.6,0.1,0.5)},0.9\right),\left(\frac{u_2}{(0.8,0.2,0.3)},0.6\right),\left(\frac{u_3}{(0.7,0.5,0.8)},0.8\right)\right\} \\ g_\nu(e_2)=\left\{\left(\frac{u_1}{(0.9,0.2,0.4)},0.7\right),\left(\frac{u_2}{(0.9,0.5,0.1)},0.9\right),\left(\frac{u_3}{(0.8,0.1,0.9)},0.5\right)\right\} \\ g_\nu(e_3)=\left\{\left(\frac{u_1}{(0.6,0.5,0.4)},0.4\right),\left(\frac{u_2}{(0.7,0.1,0.7)},0.9\right),\left(\frac{u_3}{(0.8,0.2,0.4)},0.7\right)\right\} \end{Bmatrix}$$

it is clear that f_μ is $PNS-subset$ of g_ν.

Definition 14. Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. Then, f_μ and g_ν are called possibility neutrosophic soft equal set and denoted by $f_\mu = g_\nu$, if $f_\mu \subseteq g_\nu$ and $f_\mu \supseteq g_\nu$.

Definition 15. Let $f_\mu \in \mathcal{PN}(U, E)$. Then, f_μ is said to be possibility neutrosophic soft null set, denoted by ϕ_μ, if $\forall e \in E$, $\phi_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ such that $\phi_\mu(e) = \{(\frac{u}{\phi(e)(u)}, \mu(e)(u)) : u \in U\}$, where $\phi(e)=\{\langle u, 0, 1, 1\rangle : u \in U\}$ and $\mu(e)=\{(u, 0) : u \in U\}$.

Definition 16. Let $f_\mu \in \mathcal{PN}(U, E)$. Then, f_μ is said to be possibility neutrosophic soft universal set denoted by U_μ, if $\forall e \in E$, $U_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ such that $U_\mu(e) = \{(\frac{u}{U(e)(u)}, \mu(e)(u)) : u \in U\}$, where $U(e)=\{\langle u, 1, 0, 0\rangle : u \in U\}$ and $\mu(e)=\{(u, 1) : u \in U\}$.

Proposition 5. *Let f_μ, g_ν and $h_\delta \in \mathcal{PN}(U, E)$. Then,*

(1) $\phi_\mu \subseteq f_\mu$
(2) $f_\mu \subseteq U_\mu$
(3) $f_\mu \subseteq f_\mu$
(4) $f_\mu \subseteq g_\nu$ *and* $g_\nu \subseteq h_\delta \to f_\mu \subseteq h_\delta$

Proof. It is clear from Definitions 14–16. □

Definition 17. Let $f_\mu \in \mathcal{PN}(U, E)$, where $f_\mu(e_k)=\{(f(e_k)(u_j), \mu(e_k)(u_j)) : e_k \in E, u_j \in U\}$ and $f(e_k) = \{\langle u, t_{f(e_k)}(u_j), i_{f(e_k)}(u_j), f_{f(e_k)}(u_j)\rangle\}$ for all $e_k \in E$, $u \in U$. Then for $e_k \in E$ and $u_j \in U$,

(1) f^t_μ is said to be truth-membership part of f_μ,

$$f^t_\mu = \{(f^t_{kj}(e_k), \mu_{kj}(e_k))\}$$

and

$$f^t_{kj}(e_k) = \{(u_j, t_{f(e_k)}(u_j))\},\ \mu_{kj}(e_k) = \{(u_j, \mu(e_k)(u_j))\}$$

(2) f^i_μ is said to be indeterminacy-membership part of f_μ,

$$f^i_\mu = \{(f^i_{kj}(e_k), \mu_{kj}(e_k))\}$$

and

$$f^i_{kj}(e_k) = \{(u_j, i_{f(e_k)}(u_j))\},\ \mu_{kj}(e_k) = \{(u_j, \mu(e_k)(u_j))\}$$

(3) f^f_μ is said to be falsity-membership part of f_μ,

$$f^f_\mu = \{(f^f_{kj}(e_k), \mu_{kj}(e_k))\}$$

and

$$f^f_{kj}(e_k) = \{(u_j, f_{f(e_k)}(u_j))\},\ \mu_{kj}(e_k) = \{(u_j, \mu(e_k)(u_j))\}$$

We can write a possibility neutrosophic soft set in form $f_\mu = (f^t_\mu, f^i_\mu, f^f_\mu)$.

A possibility neutrosophic soft set can be expressed in matrix form.

Let us consider possibility neutrosophic soft f_μ given in Example 1. Then, possibility neutrosophic soft f_μ can be expressed in matrix form as follows:

$$f^t_\mu = \begin{pmatrix} (0.5, 0.8) & (0.7, 0.4) & (0.4, 0.7) \\ (0.8, 0.6) & (0.5, 0.8) & (0.7, 0.4) \\ (0.6, 0.2) & (0.5, 0.6) & (0.6, 0.5) \end{pmatrix}$$

$$f^i_\mu = \begin{pmatrix} (0.2, 0.8) & (0.3, 0.4) & (0.5, 0.7) \\ (0.4, 0.6) & (0.7, 0.8) & (0.3, 0.4) \\ (0.7, 0.2) & (0.3, 0.6) & (0.5, 0.5) \end{pmatrix}$$

$$f^f_\mu = \begin{pmatrix} (0.6, 0.8) & (0.5, 0.4) & (0.8, 0.7) \\ (0.5, 0.6) & (0.2, 0.8) & (0.9, 0.4) \\ (0.5, 0.2) & (0.7, 0.6) & (0.4, 0.5) \end{pmatrix}$$

Definition 18 *([28])*. A binary operation $\otimes : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $\otimes$ satisfies the following conditions

(1) $\otimes$ is commutative and associative,
(2) $\otimes$ is continuous,
(3) $a \otimes 1 = a, \forall a \in [0, 1]$,
(4) $a \otimes b \leq c \otimes d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0, 1]$.

Definition 19 *([28])*. A binary operation $\oplus : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm (s-norm) if $\oplus$ satisfies the following conditions

(1) $\oplus$ is commutative and associative,
(2) $\oplus$ is continuous,
(3) $a \oplus 0 = a, \forall a \in [0, 1]$,
(4) $a \oplus b \leq c \oplus d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0, 1]$.

Definition 20. Let $I^3 = [0, 1] \times [0, 1] \times [0, 1]$ and $N(I^3) = \{(a, b, c) : a, b, c \in [0, 1]\}$. Then $(N(I^3), \oplus, \otimes)$ be a lattices together with partial ordered relation $\preceq$, where order relation $\preceq$on $N(I^3)$ can be defined by for $(a, b, c), (d, e, f) \in N(I^3)$

$$(a, b, c) \preceq (e, f, g) \Leftrightarrow a \leq e, b \geq f, c \geq g$$

Smarandache [31] defined n-norm and n-conorm for non-standard unit interval $]^-0, 1^+[$, and referred that definitions of n-norm and n-conorm can be adapted for normal standard real unit interval [0, 1]. Therefore, we can give definitions of n-norm and n-conorm for normal standard real unit interval [0, 1] as follows:

Definition 21. A binary operation

$$\tilde{\otimes} : ([0, 1] \times [0, 1] \times [0, 1])^2 \to [0, 1] \times [0, 1] \times [0, 1]$$

is continuous n-norm if $\tilde{\otimes}$ satisfies the following conditions

(1) $\tilde{\otimes}$ is commutative and associative,
(2) $\tilde{\otimes}$ is continuous,
(3) $a\tilde{\otimes}\hat{0} = \hat{0}, a\tilde{\otimes}\hat{1} = a, \forall a \in [0, 1] \times [0, 1] \times [0, 1], (\hat{1} = (1, 0, 0))$ and $(\hat{0} = (0, 1, 1))$
(4) $a\tilde{\otimes}b \leq c\tilde{\otimes}d$ whenever $a \preceq c$, $b \preceq d$ and $a, b, c, d \in [0, 1] \times [0, 1] \times [0, 1]$.

Here, $a\tilde{\otimes}b = \tilde{\otimes}(\langle t(a), i(a), f(a)\rangle, \langle t(b), i(b), f(b)\rangle) = \langle t(a) \otimes t(b), i(a) \oplus i(b), f(a) \oplus f(b)$

Definition 22. A binary operation

$$\tilde{\oplus} : ([0, 1] \times [0, 1] \times [0, 1])^2 \to [0, 1] \times [0, 1] \times [0, 1]$$

is continuous n-conorm if $\tilde{\oplus}$ satisfies the following conditions

(1) $\tilde{\oplus}$ is commutative and associative,
(2) $\tilde{\oplus}$ is continuous,
(3) $a\tilde{\oplus}\hat{0} = a, a\tilde{\oplus}\hat{1} = \hat{1}, \forall a \in [0, 1] \times [0, 1] \times [0, 1], (\hat{1} = (1, 0, 0))$ and $(\hat{0} = (0, 1, 1))$
(4) $a\tilde{\oplus}b \leq c\tilde{\oplus}d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0, 1] \times [0, 1] \times [0, 1]$.

Here, $a\tilde{\oplus}b = \tilde{\oplus}(\langle t(a), i(a), f(a)\rangle, \langle t(b), i(b), f(b)\rangle) = \langle t(a) \oplus t(b), i(a) \otimes i(b), f(a) \otimes f(b)$

Definition 23. Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. The union of two possibility neutrosophic soft sets f_μ and g_ν over U, denoted by $f_\mu \cup g_\nu$, is defined by

$$f_\mu \cup g_\nu = \{(e_k, \{(\alpha, \mu_{kj}(e_k) \oplus \nu_{kj}(e_k)) : u_j \in U\}) : e_k \in E\}$$

where

$$\alpha = \frac{u_j}{(f^t_{kj}(e_k) \oplus g^t_{kj}(e_k), f^i_{kj}(e_k) \otimes g^i_{kj}(e_k), f^f_{kj}(e_k) \otimes g^f_{kj}(e_k))}$$

Definition 24. Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. The intersection of two possibility neutrosophic soft sets f_μ and g_ν over U, denoted by $f_\mu \cap g_\nu$ is defined by

$$f_\mu \cap g_\nu = \{(e_k, \{(\theta, \mu_{kj}(e_k) \otimes \nu_{kj}(e_k)) : u_j \in U\}) : e_k \in E\}$$

where

$$\theta = \frac{u_j}{(f^t_{kj}(e_k) \otimes g^t_{kj}(e_k), f^i_{kj}(e_k) \oplus g^i_{kj}(e_k), f^f_{kj}(e_k) \oplus g^f_{kj}(e_k))}$$

Example 3. Let us consider the possibility neutrosophic soft sets f_μ and g_ν defined as in Example 1. Let us suppose that t-norm is defined by $a \otimes b = min\{a, b\}$ and the t-conorm is defined by $a \oplus b = max\{a, b\}$ for $a, b \in [0, 1]$. Then,

$$f_\mu \cup g_\nu = \left\{ \begin{array}{l} (f_\mu \cup g_\nu)(e_1) = \left\{ \left(\frac{u_1}{(0.6, 0.2, 0.6)}, 0.8 \right), \left(\frac{u_2}{(0.7, 0.3, 0.5)}, 0.7 \right), \left(\frac{u_3}{(0.4, 0.5, 0.4)}, 0.8 \right) \right\} \\ (f_\mu \cup g_\nu)(e_2) = \left\{ \left(\frac{u_1}{(0.8, 0.4, 0.3)}, 0.6 \right), \left(\frac{u_2}{(0.5, 0.6, 0.2)}, 0.8 \right), \left(\frac{u_3}{(0.7, 0.2, 0.5)}, 0.8 \right) \right\} \\ (f_\mu \cup g_\nu)(e_3) = \left\{ \left(\frac{u_1}{(0.7, 0.3, 0.3)}, 0.8 \right), \left(\frac{u_2}{(0.5, 0.3, 0.6)}, 0.6 \right), \left(\frac{u_3}{(0.8, 0.5, 0.3)}, 0.6 \right) \right\} \end{array} \right\}$$

and

$$f_\mu \cap g_\nu = \left\{ \begin{array}{l} (f_\mu \cap g_\nu)(e_1) = \left\{ \left(\frac{u_1}{(0.5, 0.3, 0.8)}, 0.4 \right), \left(\frac{u_2}{(0.6, 0.5, 0.5)}, 0.4 \right), \left(\frac{u_3}{(0.2, 0.6, 0.8)}, 0.7 \right) \right\} \\ (f_\mu \cap g_\nu)(e_2) = \left\{ \left(\frac{u_1}{(0.5, 0.4, 0.5)}, 0.3 \right), \left(\frac{u_2}{(0.4, 0.7, 0.5)}, 0.6 \right), \left(\frac{u_3}{(0.7, 0.3, 0.9)}, 0.4 \right) \right\} \\ (f_\mu \cap g_\nu)(e_3) = \left\{ \left(\frac{u_1}{(0.6, 0.7, 0.5)}, 0.2 \right), \left(\frac{u_2}{(0.4, 0.5, 0.7)}, 0.5 \right), \left(\frac{u_3}{(0.6, 0.5, 0.4)}, 0.5 \right) \right\} \end{array} \right\}$$

Proposition 6. *Let $f_\mu, g_\nu, h_\delta \in \mathcal{PN}(U, E)$. Then,*

(1) $f_\mu \cap f_\mu = f_\mu$ *and* $f_\mu \cup f_\mu = f_\mu$
(2) $f_\mu \cap g_\nu = g_\nu \cap f_\mu$ *and* $f_\mu \cup g_\nu = g_\nu \cup f_\mu$
(3) $f_\mu \cap \phi_\mu = \phi_\mu$ *and* $f_\mu \cap U_\mu = f_\mu$
(4) $f_\mu \cup \phi = f_\mu$ *and* $f_\mu \cup U_\mu = U_\mu$
(5) $f_\mu \cap (g_\nu \cap h_\delta) = (f_\mu \cap g_\nu) \cap h_\delta$ *and* $f_\mu \cup (g_\nu \cup h_\delta) = (f_\mu \cup g_\nu) \cup h_\delta$
(6) $f_\mu \cap (g_\nu \cup h_\delta) = (f_\mu \cap g_\nu) \cup (f_\mu \cap h_\delta)$ *and* $f_\mu \cup (g_\nu \cap h_\delta) = (f_\mu \cup g_\nu) \cap (f_\mu \cup h_\delta)$.

Proof. The proof can be obtained from Definitions 23 and 24. □

Definition 25 *([18,33])*. A function $N : [0, 1] \to [0, 1]$ is called a negation if $N(0) = 1$, $N(1) = 0$ and N is non-increasing ($x \le y \to N(x) \ge N(y)$). A negation is called a strict negation if it is strictly decreasing ($x < y \to N(x) > N(y)$) and continuous. A strict negation is said to be a strong negation if it is also involutive, i.e. $N(N(x)) = x$.

Smarandache [31] defined unary neutrosophic negation operator by interchanging truth t and falsity f vector components.
Now we define negation and strict negation operators with similar way to Definition 25 as follows:

Definition 26. A function $n_N : [0, 1] \times [0, 1] \times [0, 1] \to [0, 1] \times [0, 1] \times [0, 1]$ is called a negation if $n_N(\hat{0}) = \hat{1}$, $n_N(\hat{1}) = \hat{0}$ and n_N is non-increasing ($x \preceq y \to n_N(x) \succeq n_N(y)$). A negation is called a strict negation if it is strictly decreasing ($x \prec y \to N(x) \succ N(y)$) and continuous.

Definition 27. Let $f_\mu \in \mathcal{PN}(U, E)$. Complement of possibility neutrosophic soft set f_μ, denoted by f^c_μ, is defined by

$$f^c_\mu = \left\{ \left(e, \left\{ \left(\frac{u_j}{n_N(f(e_k))}, N(\mu_{kj}(e_k)(u_j)) \right) : u_j \in U \right\} \right) : e \in E \right\}$$

where

$$(n_N(f_{kj}(e_k)) = (N(f^t_{kj}(e_k)), N(f^i_{kj}(e_k)), N(f^f_{kj}(e_k)), \quad \text{for all } k \in \Lambda_1, j \in \Lambda_2.$$

Example 4. Let us consider the possibility neutrosophic soft set f_μ define in Example 1. Suppose that the negation is defined by $N(f^t_{kj}(e_k)) = f^f_{kj}(e_k)$, $N(f^f_{kj}(e_k)) = f^t_{kj}(e_k)$, $N(f^i_{kj}(e_k)) = 1 - f^i_{kj}(e_k)$ and $N(\mu_{kj}(e_k)) = 1 - \mu_{kj}(e_k)$, respectively. Then, f^c_μ is defined as follow:

$$f^c_\mu = \left\{ \begin{array}{l} f^c_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.6, 0.8, 0.5)}, 0.2 \right), \left(\frac{u_2}{(0.5, 0.7, 0.7)}, 0.6 \right), \left(\frac{u_3}{(0.8, 0.5, 0.4)}, 0.3 \right) \right\} \\ f^c_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.5, 0.6, 0.8)}, 0.4 \right), \left(\frac{u_2}{(0.2, 0.3, 0.5)}, 0.2 \right), \left(\frac{u_3}{(0.9, 0.7, 0.7)}, 0.6 \right) \right\} \\ f^c_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.5, 0.3, 0.6)}, 0.8 \right), \left(\frac{u_2}{(0.7, 0.7, 0.5)}, 0.4 \right), \left(\frac{u_3}{(0.4, 0.5, 0.6)}, 0.5 \right) \right\} \end{array} \right\}$$

Proposition 7. *Let $f_\mu \in \mathcal{PN}(U, E)$. Then,*

(1) $\phi^c_\mu = U_\mu$
(2) $U^c_\mu = \phi_\mu$
(3) $(f^c_\mu)^c = f_\mu$.

Proof. It is clear from Definition 27. □

Proposition 8. *Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. Then, De Morgan's law is valid.*

(1) $(f_\mu \cup g_\nu)^c = f^c_\mu \cap g^c_\nu$
(2) $(f_\mu \cap g_\nu)^c = f^c_\mu \cup g^c_\nu$

Proof.

(1) Let $i, j \in \Lambda$

$$\begin{aligned} (f_\mu \cup g_\nu)^c &= \left\{ \left(e_k, \left\{ \left(\frac{u_j}{(f^t_{kj}(e_k) \oplus g^t_{kj}(e_k), f^i_{kj}(e_k) \otimes g^i_{kj}(e_k), f^f_{kj}(e_k) \otimes g^f_{kj}(e_k))}, \mu_{kj}(e_k) \oplus \nu_{kj}(e_k) \right) : u_j \in U \right\} \right) : e_k \in E \right\}^c \\ &= \left\{ \left(e_k, \left\{ \left(\frac{u_j}{(f^f_{kj}(e_k) \otimes g^f_{kj}(e_k), N(f^i_{kj}(e_k) \otimes g^i_{kj}(e_k)), f^t_{kj}(e_k) \oplus g^t_{kj}(e_k))}, N(\mu_{kj}(e_k) \oplus \nu_{kj}(e_k)) \right) : u_j \in U \right\} \right) : e_k \in E \right\} \\ &= \left\{ \left(e_k, \left\{ \left(\frac{u_j}{(f^f_{kj}(e_k) \otimes g^f_{kj}(e_k), N(f^i_{kj}(e_k)) \oplus N(g^i_{kj}(e_k))), f^t_{kj}(e_k) \oplus g^t_{kj}(e_k))}, N(\mu_{kj}(e_k)) \otimes N(\nu_{kj}(e_k)) \right) : u_j \in U \right\} \right) : e_k \in E \right\} \\ &= \left\{ \left(e_k, \{(\frac{u_j}{(f^f_{kj}(e_k), N(f^i_{kj}(e_k)), f^t_{kj}(e_k))}, N(\mu_{kj}(e_k))) : u_j \in U\} \right) : e_k \in E \right\} \\ &\quad \cap \left\{ \left(e_k, \{(\frac{u_j}{(g^f_{kj}(e_k), N(g^i_{kj}(e_k)), g^t_{kj}(e_k))}, N(\nu_{kj}(e_k))) : u_j \in U\} \right) : e_k \in E \right\} \\ &= \left\{ \left(e_k, \{(\frac{u_j}{(f^t_{kj}(e_k), N(f^i_{kj}(e_k)), f^f_{kj}(e_k))}, \mu_{kj}(e_k)) : u_j \in U\} \right) : e_k \in E \right\}^c \\ &\quad \cap \left\{ \left(e_k, \{(\frac{u_j}{g^t_{kj}(e_k), g^i_{kj}(e_k), g^f_{kj}(e_k)}, \nu_{kj}(e_k)) : u_j \in U\} \right) : e_k \in E \right\}^c = f^c_\mu \cap g^c_\nu. \end{aligned}$$

(2) The techniques used to prove (2) are similar to those used for (1), therefore we skip the proof. □

Definition 28. Let f_μ and $g_\nu \in \mathcal{PN}(U, E)$. Then 'AND' product of PNS-set f_μ and g_ν denoted by $f_\mu \wedge g_\nu$, is defined as follows:

$$f_\mu \wedge g_\nu = \left\{ \left((e_k, e_l), (f^t_{kj}(e_k) \wedge g^t_{lj}(e_l), f^i_{kj}(e_k) \vee g^i_{lj}(e_l), f^f_{kj}(e_k) \vee g^f_{lj}(e_l)), \mu_{kj}(e_k) \wedge \nu_{lj}(e_l) \right) : (e_k, e_l) \in E \times E, j, k, l \in \Lambda \right\}.$$

Definition 29. Let f_μ and $g_\nu \in \mathcal{PN}(U, E)$. Then 'OR' product of PNS-set f_μ and g_ν denoted by $f_\mu \vee g_\nu$, is defined as follows:

$$f_\mu \vee g_\nu = \left\{ \left((e_k, e_l), (f^t_{kj}(e_k) \vee g^t_{lj}(e_l), f^i_{kj}(e_k) \wedge g^i_{lj}(e_l), f^f_{kj}(e_k) \wedge g^f_{lj}(e_l)), \mu_{kj}(e_k) \vee \nu_{lj}(e_l) \right) : (e_k, e_l) \in E \times E, j, k, l \in \Lambda \right\}.$$

4. PNS-decision making method

In this section, we construct a decision making method over the possibility neutrosophic soft set that is called possibility neutrosophic soft decision making method (PNS-decision making method).

Definition 30. Let $g_\nu, h_\rho \in \mathcal{PN}(U, E)$, $f_\mu = g_\nu \wedge h_\rho$, and f_μ^t, f_μ^i and f_μ^f be the truth, indeterminacy and falsity matrices of $\wedge$-product matrix, respectively. Then, weighted matrices of f_μ^t, f_μ^i and f_μ^f, denoted by $\wedge^t$, $\wedge^i$ and $\wedge^f$, are defined as follows:

$\wedge^t(e_{kj}, u_r) = t_{(g_\nu \wedge h_\rho)(e_{kj})}(u_r) + (\nu_{kr}(e_k) \wedge \rho_{jr}(e_j)) - t_{(g_\nu \wedge h_\rho)(e_{kj})}(u_r) \times (\mu_{kr}(e_k) \wedge \nu_{jr}(e_j))$

$\wedge^i(e_{kj}, u_r) = i_{(g_\nu \wedge h_\rho)(e_{kj})}(u_r) \times (\nu_{kr}(e_k) \wedge \rho_{jr}(e_j))$

$\wedge^f(e_{kj}, u_r) = f_{(g_\nu \wedge h_\rho)(e_{kj})}(u_r) \times (\nu_{kr}(e_k) \wedge \rho_{jr}(e_j))$

for $j, k, r \in \Lambda$.

Definition 31. Let $g_\nu, h_\rho \in \mathcal{PN}(U, E)$, $f_\mu = g_\nu \wedge h_\rho$, and let $\wedge^t$, $\wedge^i$ and $\wedge^f$ be the weighed matrices of f_μ^t, f_μ^i and f_μ^f, respectively. Then, in the weighted matrices $\wedge^t$, $\wedge^i$ and $\wedge^f$ scores of $u_n \in U$, denoted by $s^t(u_n)$, $s^i(u_n)$ and $s^f(u_n)$, are defined as follows:

$$s^t(u_n) = \sum_{k,j \in \Lambda} \delta_{kj}^t(u_n)$$

$$s^i(u_n) = \sum_{k,j \in \Lambda} \delta_{kj}^i(u_n)$$

$$s^f(u_n) = \sum_{k,j \in \Lambda} \delta_{kj}^f(u_n)$$

where

$$\delta_{kj}^t(u_n) = \begin{cases} \wedge^t(e_{kj}, u_n), & \wedge^t(e_{kj}, u_n) = \max\{\wedge^t(e_{kj}, u_m) : u_m \in U\} \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{kj}^i(u_n) = \begin{cases} \wedge^i(e_{kj}, u_n), & \wedge^i(e_{kj}, u_n) = \max\{\wedge^i(e_{kj}, u_m) : u_m \in U\} \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{kj}^f(u_n) = \begin{cases} \wedge^f(e_{kj}, u_n), & \wedge^f(e_{kj}, u_n) = \max\{\wedge^f(e_{kj}, u_m) : u_m \in U\} \\ 0, & \text{otherwise} \end{cases}$$

Definition 32. Let $s^t(u_n)$, $s^i(u_n)$ and $s^f(u_n)$ be scores of $u_n \in U$ in the weighted matrices $\wedge^t$, $\wedge^i$ and $\wedge^f$. Then, decision score of $u_n \in U$, denoted by $ds(u_n)$, is defined by

$$ds(u_n) = s^t(u_n) - s^i(u_n) - s^f(u_n)$$

Now, we construct a PNS-decision making method by the following algorithm:

Algorithm.

Step 1: Input the possibility neutrosophic soft sets,
Step 2: Construct the $\wedge$-product matrix,
Step 3: Construct the truth, indeterminacy and falsity matrices of the $\wedge$-product matrix,
Step 4: Construct the weighted matrices $\wedge^t$, $\wedge^i$ and $\wedge^f$,
Step 5: Compute score of $u_t \in U$, for each of the weighted matrices,
Step 6: Compute decision score, for all $u_t \in U$,
Step 7: The optimal decision is to select $u_t = maxds(u_i)$.

Example 5. Assume that $U = \{u_1, u_2, u_3\}$ is a set of houses and $E = \{e_1, e_2, e_3\} = \{cheap, large, moderate\}$ is a set of parameters which is attractiveness of houses. Suppose that Mr.X wants to buy the most suitable house.

Step 1: Based on the choice parameters of *Mr. X*, possibility neutrosophic soft sets g_ν and h_ρ constructed by two experts are as follows:

$$g_\nu = \left\{ \begin{array}{l} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.5, 0.3, 0.7)}, 0.6 \right), \left(\frac{u_2}{(0.6, 0.2, 0.5)}, 0.2 \right), \left(\frac{u_3}{(0.7, 0.6, 0.5)}, 0.4 \right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.35, 0.2, 0.6)}, 0.4 \right), \left(\frac{u_2}{(0.7, 0.8, 0.3)}, 0.5 \right), \left(\frac{u_3}{(0.2, 0.4, 0.4)}, 0.6 \right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.7, 0.2, 0.5)}, 0.5 \right), \left(\frac{u_2}{(0.4, 0.5, 0.2)}, 0.3 \right), \left(\frac{u_3}{(0.5, 0.3, 0.6)}, 0.2 \right) \right\} \end{array} \right\}$$

$$h_\rho = \left\{ \begin{array}{l} h_\rho(e_1) = \left\{ \left(\frac{u_1}{(0.3, 0.4, 0.5)}, 0.2 \right), \left(\frac{u_2}{(0.7, 0.3, 0.4)}, 0.5 \right), \left(\frac{u_3}{(0.4, 0.5, 0.2)}, 0.3 \right) \right\} \\ h_\rho(e_2) = \left\{ \left(\frac{u_1}{(0.4, 0.6, 0.2)}, 0.3 \right), \left(\frac{u_2}{(0.2, 0.5, 0.3)}, 0.7 \right), \left(\frac{u_3}{(0.4, 0.6, 0.2)}, 0.8 \right) \right\} \\ h_\rho(e_3) = \left\{ \left(\frac{u_1}{(0.2, 0.1, 0.6)}, 0.7 \right), \left(\frac{u_2}{(0.8, 0.4, 0.5)}, 0.4 \right), \left(\frac{u_3}{(0.6, 0.4, 0.3)}, 0.4 \right) \right\} \end{array} \right\}$$

Step 2: Let us consider possibility neutrosophic soft set $\wedge$-product $f_\mu = g_\nu \wedge h_\rho$ which is the mapping $\wedge : E \times E \to \mathcal{N}(\mathcal{U}) \times I^U$ given as follows:

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(\langle 0.3, 0.4, 0.7 \rangle, 0.2)$	$(\langle 0.6, 0.3, 0.5 \rangle, 0.2)$	$(\langle 0.4, 0.6, 0.5 \rangle, 0.3)$
e_{12}	$(\langle 0.4, 0.6, 0.7 \rangle, 0.3)$	$(\langle 0.2, 0.5, 0.5 \rangle, 0.2)$	$(\langle 0.4, 0.6, 0.5 \rangle, 0.4)$
e_{13}	$(\langle 0.2, 0.3, 0.7 \rangle, 0.6)$	$(\langle 0.6, 0.4, 0.5 \rangle, 0.2)$	$(\langle 0.6, 0.6, 0.5 \rangle, 0.4)$
e_{21}	$(\langle 0.3, 0.4, 0.6 \rangle, 0.2)$	$(\langle 0.7, 0.8, 0.4 \rangle, 0.5)$	$(\langle 0.2, 0.5, 0.5 \rangle, 0.3)$
e_{22}	$(\langle 0.35, 0.6, 0.6 \rangle, 0.3)$	$(\langle 0.2, 0.8, 0.3 \rangle, 0.5)$	$(\langle 0.2, 0.6, 0.5 \rangle, 0.6)$
e_{23}	$(\langle 0.2, 0.2, 0.6 \rangle, 0.4)$	$(\langle 0.7, 0.8, 0.5 \rangle, 0.4)$	$(\langle 0.2, 0.4, 0.5 \rangle, 0.4)$
e_{31}	$(\langle 0.3, 0.4, 0.5 \rangle, 0.2)$	$(\langle 0.4, 0.5, 0.4 \rangle, 0.3)$	$(\langle 0.4, 0.5, 0.6 \rangle, 0.2)$
e_{32}	$(\langle 0.4, 0.6, 0.5 \rangle, 0.3)$	$(\langle 0.2, 0.5, 0.3 \rangle, 0.3)$	$(\langle 0.4, 0.6, 0.6 \rangle, 0.2)$
e_{33}	$(\langle 0.2, 0.2, 0.6 \rangle, 0.5)$	$(\langle 0.4, 0.5, 0.5 \rangle, 0.3)$	$(\langle 0.5, 0.4, 0.6 \rangle, 0.2)$

Matrix representation of $\wedge$-product

Step 3: We construct matrices f^t_μ, f^i_μ and f^f_μ as follows:

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(0.3, 0.2)$	$(0.6, 0.2)$	$(0.4, 0.3)$
e_{12}	$(0.4, 0.3)$	$(0.2, 0.2)$	$(0.4, 0.4)$
e_{13}	$(0.2, 0.6)$	$(0.6, 0.2)$	$(0.6, 0.4)$
e_{21}	$(0.3, 0.2)$	$(0.7, 0.5)$	$(0.2, 0.3)$
e_{22}	$(0.35, 0.3)$	$(0.2, 0.5)$	$(0.2, 0.6)$
e_{23}	$(0.2, 0.4)$	$(0.7, 0.4)$	$(0.2, 0.4)$
e_{31}	$(0.3, 0.2)$	$(0.4, 0.3)$	$(0.4, 0.2)$
e_{32}	$(0.4, 0.3)$	$(0.2, 0.3)$	$(0.4, 0.2)$
e_{33}	$(0.2, 0.5)$	$(0.4, 0.3)$	$(0.5, 0.2)$

Matrix f^t_μ of $\wedge$-product

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(0.4, 0.2)$	$(0.3, 0.2)$	$(0.6, 0.3)$
e_{12}	$(0.6, 0.3)$	$(0.5, 0.2)$	$(0.6, 0.4)$
e_{13}	$(0.3, 0.6)$	$(0.4, 0.2)$	$(0.6, 0.4)$
e_{21}	$(0.4, 0.2)$	$(0.8, 0.5)$	$(0.5, 0.3)$
e_{22}	$(0.6, 0.3)$	$(0.8, 0.5)$	$(0.6, 0.6)$
e_{23}	$(0.2, 0.4)$	$(0.8, 0.4)$	$(0.4, 0.4)$
e_{31}	$(0.4, 0.2)$	$(0.5, 0.3)$	$(0.5, 0.2)$
e_{32}	$(0.6, 0.3)$	$(0.5, 0.3)$	$(0.6, 0.2)$
e_{33}	$(0.2, 0.5)$	$(0.5, 0.3)$	$(0.4, 0.2)$

Matrix f^i_μ of $\wedge$-product

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(0.7, 0.2)$	$(0.5, 0.2)$	$(0.5, 0.3)$
e_{12}	$(0.7, 0.3)$	$(0.5, 0.2)$	$(0.5, 0.4)$
e_{13}	$(0.7, 0.6)$	$(0.5, 0.2)$	$(0.5, 0.4)$
e_{21}	$(0.6, 0.2)$	$(0.4, 0.5)$	$(0.5, 0.3)$
e_{22}	$(0.6, 0.3)$	$(0.3, 0.5)$	$(0.5, 0.6)$
e_{23}	$(0.6, 0.4)$	$(0.5, 0.4)$	$(0.5, 0.4)$
e_{31}	$(0.5, 0.2)$	$(0.4, 0.3)$	$(0.6, 0.2)$
e_{32}	$(0.5, 0.3)$	$(0.3, 0.3)$	$(0.6, 0.2)$
e_{33}	$(0.6, 0.5)$	$(0.5, 0.3)$	$(0.6, 0.2)$

Matrix f_μ^f of $\wedge$-product

Step 4: We obtain weighted matrices $\wedge^t$, $\wedge^i$ and $\wedge^f$ by using Definition 30 as follows:

$\wedge^t$	u_1	u_2	u_3
e_{11}	0.44	$\underline{0.64}$	0.58
e_{12}	0.58	0.36	$\underline{0.64}$
e_{13}	0.68	0.68	$\underline{0.76}$
e_{21}	0.44	$\underline{0.85}$	0.44
e_{22}	0.55	0.60	$\underline{0.68}$
e_{23}	0.52	$\underline{0.82}$	0.48
e_{31}	0.44	$\underline{0.58}$	0.52
e_{32}	$\underline{0.58}$	0.44	0.52
e_{33}	$\underline{0.60}$	0.58	$\underline{0.60}$

$\wedge^i$	u_1	u_2	u_3
e_{11}	0.08	0.16	$\underline{0.18}$
e_{12}	0.18	0.10	$\underline{0.24}$
e_{13}	0.18	0.08	$\underline{0.24}$
e_{21}	0.08	$\underline{0.40}$	0.15
e_{22}	0.18	$\underline{0.40}$	0.36
e_{23}	0.08	$\underline{0.32}$	0.16
e_{31}	0.08	$\underline{0.15}$	0.10
e_{32}	$\underline{0.18}$	0.15	0.12
e_{33}	0.10	$\underline{0.15}$	0.08

$\wedge^f$	u_1	u_2	u_3
e_{11}	0.14	0.10	$\underline{0.15}$
e_{12}	$\underline{0.21}$	0.10	0.20
e_{13}	$\underline{0.42}$	0.10	0.20
e_{21}	0.12	$\underline{0.20}$	0.15
e_{22}	0.18	0.15	$\underline{0.30}$
e_{23}	$\underline{0.24}$	0.20	$\underline{0.20}$
e_{31}	0.10	$\underline{0.12}$	$\underline{0.12}$
e_{32}	$\underline{0.15}$	0.09	0.12
e_{33}	$\underline{0.30}$	0.15	0.12

Weighed matrices of f_μ^t, f_μ^i and f_μ^f from left to right, respectively.

Step 5: For all $u \in U$, we find scores by using Definition 31 as follows:

$$s^t(u_1) = 1,18, \quad s^t(u_2) = 2,89, \quad s^t(u_3) = 2,68$$

$$s^i(u_1) = 0,18, \quad s^i(u_2) = 1,42, \quad s^i(u_3) = 0,66$$

$$s^f(u_1) = 1,32, \quad s^f(u_2) = 0,32, \quad s^f(u_3) = 0,57$$

Step 6: For all $u \in U$, we find scores by using Definition 31 as follows:

$$ds(u_1) = 1,18 - 0,18 - 1,32 = -0,32$$
$$ds(u_2) = 2,89 - 1,42 - 0,32 = 0,90$$
$$ds(u_3) = 2,68 - 0,66 - 0,57 = 1,45$$

Step 7: Then the optimal selection for *Mr. X* is u_3.

5. Conclusion

In this paper, we introduced the concept of possibility neutrosophic soft set and possibility neutrosophic soft set operations, and studied some properties related with operations defined. Also, we presented a decision making method based on possibility neutrosophic soft set, and gave an application of this method to solve a decision making problem. The proposed method may be used in many different applications to solve the related problems. In future, researchers may study on set theoretical operations of possibility neutrosophic soft sets and integrate decision making methods to the TOPSIS, ELECTRE and other some decision making methods.

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