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REGULAR SEMICLOSED SETS ON NEUTROSOPHIC CRISP TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce another idea of neutrosophic crisp generalised sets called neutrosophic crisp regular semi closed sets and examined their central properties in neutrosophic crisp topological spaces. We additionally present neutrosophic crisp regular semi closure and neutrosophic crisp regular semi interior and concentrate a portion of their major properties.

1. INTRODUCTION AND PRELIMINARIES

In 1965, Zadeh [10] had introduced a fuzzy set as a degree of membership. In 1986, Atanassove [1] proposed the degree of non-membership to fuzzy sets. In addition to this Smarandache [9] added the degree of indeterminacy in 1998. In [7], Salama and Smarandache introduced the following notions, we select one type alone in each case, as more than two types [3]. Let a $\mathcal{NCS}$ (neutrosophic crisp set) $L = \langle L_1, L_2, L_3\rangle$ of a $X \neq \phi$, where $L_1, L_2, L_3 \subseteq X$, $\phi_N = (\phi, \phi, X)$, $X_N = (X, X, \phi)$]. We will denote the set of all $\mathcal{NCS}$s in X as $\mathcal{NCS}(X)$.

Let $L = (L_1, L_2, L_3), M = (M_1, M_2, M_3) \in NCS(X)$ and $(L_j)_{j\in J} \subset NCS(X)$, where $L_j = (L_{j,1}, L_{j,2}, L_{j,3})$. Then

(i) $L \subset M$, if $L_1 \subset M_1, L_2 \subset M_2, L_3 \supset M_3$.

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(ii) $L = M$, if $L \subset M$ and $M \subset L$.
(iii) $L^C = (L_1^c, L_2^c, L_3^c)$.
(iv) $L \cap M = (L_1 \cap M_1, L_2 \cap M_2, L_3 \cap M_3)$.
(v) $L \cup M = (L_1 \cup M_1, L_2 \cup M_2, L_3 \cup M_3)$.
(vi) $\cap L_j = (\cap L_{j,1}, \cap L_{j,2}, \cap L_{j,3})$.
(vii) $\cup L_j = (\cup L_{j,1}, \cup L_{j,2}, \cup L_{j,3})$.

Let $L, M, C \in NCS(X)$ and $(L_j)_{j \in J} \subset NCS(X)$. Then

(i) $\phi_N \subset L \subset X_N$.
(ii) if $L \subset M$ and $M \subset C$, then $L \subset C$.
(iii) $L \cap M \subset L$ and $L \cap M \subset M$.
(iv) $L \subset L \cup M$ and $M \subset L \cup M$.
(v) $L \subset M$ iff $L \cap M = L$.
(vi) $L \subset M$ iff $L \cup M = M$.
(vii) $L \cup L = L, L \cap L = L$.
(viii) $L \cup M = M \cup L, L \cap M = M \cap L$.
(ix) $L \cup (M \cup C) = (L \cup M) \cup C,\ L \cap (M \cap C) = (L \cap M) \cap C$.
(x) $L \cup (M \cap C) = (L \cup M) \cap (L \cup C)$ and $L \cap (M \cup C) = (L \cap M) \cup (L \cap C)$.
(xi) $L \cup (L \cap M) = L, L \cap (L \cup M) = L$.
(xii) $(L \cup M)^c = L^c \cap M^c, (L \cap M)^c = L^c \cup M^c$.
(xiii) $(L^c)^c = L$.
(xiv) $L \cup \phi_N = L, L \cap \phi_N = \phi_N$.
(xv) $L \cup X_N = X_N, L \cap X_N = L$.
(xvi) $X_N^c = \phi_N, \phi_N^c = X_N$.
(xvii) $L \cup L^c = X_N, L \cap L^c = \phi_N$.
(xviii) $(\cap L_j)^c = \cup L_J^c, (\cup L_j)^c = \cap L_j^c$.
(xix) $L \cap (\cup L_j) = \cup (L \cap L_j), L \cup (\cap L_j) = \cap (L \cup L_j)$. .

Moreover, Salama et al. [5,7,8] applied the concept of neutrosophip crisp sets to concept of $\mathcal{NCT}$ (neutrosophic crisp topology), $\mathcal{NCT}$ (neutrosophic crisp topological space),$\mathcal{NC}cs$ (neutrosophic crisp closed set), $\mathcal{NCT}$ (neutrosophic crisp open set), $\mathcal{NC}cl(L)$ (neutrosophic crisp closure of L) and neutrosophic crisp interior of L. A neutrosophic crisp subset L of a $\mathcal{NCTS}$ (X, Γ) is said to be neutrosophic crisp pre (resp. semi, α *and* β) open set [6] (briefly, $\mathcal{NCP}os$ (resp. $\mathcal{NCS}os$, $\mathcal{NC}\alpha os$ *and* $\mathcal{NC}\beta os$)) if $L \subseteq \mathcal{NC}int(\mathcal{NC}cl(L))$ (resp. $L \subseteq \mathcal{NC}cl(\mathcal{NC}int(L)), L \subseteq \mathcal{NC}int(\mathcal{NC}cl(\mathcal{NC}int(L)))$ *and* $L \subseteq \mathcal{NC}cl(\mathcal{NC}int(\mathcal{NC}cl(L)))$).

The complement of a $\mathcal{NCP}os$ (resp. $\mathcal{NCS}os$, $\mathcal{NC}\alpha os$ and $\mathcal{NC}\beta os$) is called a neutrosophic crisp preclosed (resp. semi, α and β) closed set (briefly, $\mathcal{NCP}cs$ (resp. $\mathcal{NCS}cs$, $\mathcal{NC}\alpha cs$ and $\mathcal{NC}\beta cs$)) in (X, Γ). The family of all $\mathcal{NCP}os$ (resp. $\mathcal{NCP}cs$, $\mathcal{NCS}os$, $\mathcal{NCS}cs$, $\mathcal{NC}\alpha os$, $\mathcal{NC}\alpha cs$, $\mathcal{NC}\beta os$ and $\mathcal{NC}\beta cs$) X is denoted by $\mathcal{NCPOS}(X)$ (resp. $\mathcal{NCPCS}(X)$, $\mathcal{NCSOS}(X)$, $\mathcal{NCSCS}(X)$, $\mathcal{NC}\alpha OS(X)$, $\mathcal{NC}\alpha CS(X)$, $\mathcal{NC}\beta OS(X)$ and $\mathcal{NC}\beta CS(X)$). [6] Let L be a $\mathcal{NCS}$ of $\mathcal{NCTS}$ (X, Γ). Then, the neutrosophic crisp pre (resp. semi, α and β) interior of L is the union of all $\mathcal{NCP}os$ (resp. $\mathcal{NCS}os$, $\mathcal{NC}\alpha os$ and $\mathcal{NC}\beta os$) contained in L and is denoted by $\mathcal{NCP}int(L)$ (respectively $\mathcal{NCS}int(L)$, $\mathcal{NC}\alpha int(L)$ and $\mathcal{NC}\beta int(L)$). the neutrosophic crisp pre (resp. semi, α and β) closure of L is the intersection of all $\mathcal{NCP}cs$ (resp. $\mathcal{NCS}cs$, $\mathcal{NC}\alpha cs$ and $\mathcal{NC}\beta cs$) contains L and is dented by $\mathcal{NCP}cl(L)$ (resp. $\mathcal{NCS}cl(L)$, $\mathcal{NC}\alpha cl(L)$ and $\mathcal{NC}\beta cl(L)$). The undefined notions from [7] and cited therein. In general topology Cameron [2] defined a regular semi open sets and Di Maio and Noiri [4] defined semi regular open sets.

2. Neutrosophic crisp regular semi closed sets

Definition 2.1. *A $\mathcal{NCS}$, L of a $\mathcal{NCTS}$ (X, Γ) is called a neutrosophic crisp (i) regular open (resp. closed) set (briefly, $\mathcal{NC}ros$ (resp. $\mathcal{NC}rcs$)) if $L = \mathcal{NC}int(\mathcal{NC}cl(L))$ (resp. $L = \mathcal{NC}cl(\mathcal{NC}int(L))$). (ii) regular semi closed (resp. open) sets (briefly, $\mathcal{NC}r\mathcal{S}cs$ (resp. $\mathcal{NC}r\mathcal{S}os$)) if $\exists\mathcal{NC}rcs$ (resp. $\mathcal{NC}ros$) H in $X \ni \mathcal{NC}int(H) \subseteq L \subseteq H$ (resp. $H \subseteq L \subseteq \mathcal{NC}cl(H)$).*

$\mathcal{NC}r\mathcal{S}CS(X)$ (resp. $\mathcal{NC}r\mathcal{S}OS(X)$) denotes the family of all $\mathcal{NC}r\mathcal{S}cs$ (resp. $\mathcal{NC}r\mathcal{S}os$) of X

Proposition 2.1. *If a $\mathcal{NCS}$, L is a $\mathcal{NC}ros$ (resp. $\mathcal{NC}rSos$) then L^c is $\mathcal{NC}rcs$ (resp. $\mathcal{NC}rScs$).*

Proposition 2.2. *In a $\mathcal{NCTS}$ (X, Γ), the following hold:*

(i) *Every $\mathcal{NC}ros$ is a $\mathcal{NC}os$ (resp. $\mathcal{NC}r\mathcal{S}os$).*
(ii) *Every $\mathcal{NC}r\mathcal{S}os$ is a $\mathcal{NCS}os$.*
(iii) *Every $\mathcal{NC}os$ is a $\mathcal{NC}rSos$ (resp. $\mathcal{NC}\alpha os$).*
(iv) *Every $\mathcal{NC}\alpha os$ is a $\mathcal{NCS}os$ (resp. $\mathcal{NCP}os$).*
(v) *Every $\mathcal{NCP}os$ is a $\mathcal{NC}\beta os$.*
(vi) *Every $\mathcal{NCS}os$ is a $\mathcal{NC}\beta os$.*

But not conversely.

Definition 2.2. *A neutrosophic crisp subset L of a $\mathcal{NCTS}$ (X, Γ) is called a neutrosophic crisp semi regular open sets (briefly, $\mathcal{NCS}ros$) if it is both $\mathcal{NCS}o$ and $\mathcal{NCS}c$ or equivalently, $L = \mathcal{NCS}int(\mathcal{NCS}cl(L))$. The family of all $\mathcal{NCS}ro$ (resp. $\mathcal{NCSO}$) of X is denoted by $\mathcal{NCS}rO(X)$ (resp. $\mathcal{NCS}O(X)$).*

Theorem 2.1. *For any $\mathcal{NCS}$, L of a $\mathcal{NCTS}$ (X, Γ). (a) (i) $L \in \mathcal{NCS}rO(X)$. (ii) $L = \mathcal{NCS}int(\mathcal{NCS}cl(L))$ (iii) There exist a $\mathcal{NC}ros$ H of $X \ni H \subseteq L \subseteq \mathcal{NC}cl(H)$. are equivalent. (b) (i) $L \in \mathcal{NCS}rC(X)$. (ii) $L = \mathcal{NCS}cl(\mathcal{NCS}int(L))$ (iii) There exist a $\mathcal{NC}rcs$ H of $X \ni \mathcal{NC}int(H) \subseteq L \subseteq H$. are equivalent.*

From this discussion, we have,

$$\begin{tikzcd}
 & \mathcal{NC}os \arrow[r] \arrow[dd] & \mathcal{NC}\alpha os \arrow[r] \arrow[dd] & \mathcal{NCP}os \arrow[dd] \\
\mathcal{NC}ros \arrow[ur] \arrow[dr] & & & \\
 & \mathcal{NC}r\mathcal{S}os \arrow[r] & \mathcal{NCS}os \arrow[r] & \mathcal{NC}\beta os
\end{tikzcd}$$

FIGURE 1

3. NEUTROSOPHIC CRISP REGULAR SEMI CLOSURE (RESP. INTERIOR)

Definition 3.1. *The intersection (resp. union) of all $\mathcal{NC}r\mathcal{S}cs$ (resp. $\mathcal{NC}r\mathcal{S}os$) in a $\mathcal{NCTS}$ (X, Γ) containing (resp. contained in) L is called neutrosophic crisp regular semi closure of L (resp. neutrosophic crisp regular semi interior of L) (briefly, $\mathcal{NC}r\mathcal{S}cl(L)$ (resp. $\mathcal{NC}r\mathcal{S}int(L)$)), $\mathcal{NC}r\mathcal{S}cl(L) = \cap\{M : L \subseteq M, M$ is a $\mathcal{NC}r\mathcal{S}cs\}$ (resp. $\mathcal{NC}r\mathcal{S}int(L) = \cup\{M : B \subseteq L, M$ is a $\mathcal{NC}r\mathcal{S}os\}$).*

Proposition 3.1. *Let L be any neutrsophic crisp set in a $\mathcal{NCTS}$ (X, Γ), the following properties are true:*

(i) $\mathcal{NC}r\mathcal{S}cl(L) = L$ *iff* L *is a* $\mathcal{NC}r\mathcal{S}cs$.
(ii) $\mathcal{NC}r\mathcal{S}int(L) = L$ *iff* L *is a* $\mathcal{NC}r\mathcal{S}os$.
(iii) $\mathcal{NC}r\mathcal{S}cl(L)$ *is the smallest* $\mathcal{NC}r\mathcal{S}cs$ *containing* L .
(iv) $\mathcal{NC}r\mathcal{S}int(L)$ *is the largest* $\mathcal{NC}r\mathcal{S}os$ *contained in* L .

(v) $\mathcal{N}\mathcal{C}r\mathcal{S}int(X_N - L) = X_N - (\mathcal{N}\mathcal{C}r\mathcal{S}cl(L))$.
(vi) $\mathcal{N}\mathcal{C}r\mathcal{S}cl(X_N - L) = X_N - (\mathcal{N}\mathcal{C}r\mathcal{S}int(L))$.

Theorem 3.1. *Let L and M be two neutrsophic crisp set in a $\mathcal{N}CTS$ (X, Γ), the following properties hold:*

(i) $\mathcal{N}\mathcal{C}r\mathcal{S}cl(\phi_N) = \phi_N$, $\mathcal{N}\mathcal{C}r\mathcal{S}cl(X_N) = X_N$.
(ii) $L \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}cl(L)$.
(iii) $L \subseteq M \Rightarrow \mathcal{N}\mathcal{C}r\mathcal{S}cl(L) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}cl(M)$.
(iv) $\mathcal{N}\mathcal{C}r\mathcal{S}cl(L \cap M) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}cl(L) \cap \mathcal{N}\mathcal{C}r\mathcal{S}cl(M)$.
(v) $\mathcal{N}\mathcal{C}r\mathcal{S}cl(L) \cup \mathcal{N}\mathcal{C}r\mathcal{S}cl(M) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}cl(L \cup M)$.
(vi) $\mathcal{N}\mathcal{C}r\mathcal{S}cl(\mathcal{N}\mathcal{C}r\mathcal{S}cl(L)) = \mathcal{N}\mathcal{C}r\mathcal{S}cl(L)$.
(vii) $\mathcal{N}\mathcal{C}r\mathcal{S}int(\phi_N) = \phi_N$, $\mathcal{N}\mathcal{C}r\mathcal{S}int(X_N) = X_N$.
(viii) $\mathcal{N}\mathcal{C}r\mathcal{S}int(L) \subseteq L$.
(ix) $L \subseteq M \Rightarrow \mathcal{N}\mathcal{C}r\mathcal{S}int(L) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}int(M)$.
(x) $\mathcal{N}\mathcal{C}r\mathcal{S}int(L \cap M) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}int(L) \cap \mathcal{N}\mathcal{C}r\mathcal{S}int(M)$.
(xi) $\mathcal{N}\mathcal{C}r\mathcal{S}int(L) \cup \mathcal{N}\mathcal{C}r\mathcal{S}int(M) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}int(L \cup M)$.
(xii) $\mathcal{N}\mathcal{C}r\mathcal{S}int(\mathcal{N}\mathcal{C}r\mathcal{S}int(L)) = \mathcal{N}\mathcal{C}r\mathcal{S}int(L)$.

Proposition 3.2. *For any $\mathcal{N}CS$, L of a $\mathcal{N}CTS$ (X, Γ), then:*

(i) $\mathcal{N}\mathcal{C}rint(L) \subseteq \mathcal{N}\mathcal{C}int(L) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}int(L) \subseteq \mathcal{N}\mathcal{C}\mathcal{S}int(L) \subseteq \mathcal{N}\mathcal{C}\beta int(L) \subseteq L \subseteq \mathcal{N}\mathcal{C}\beta cl(L) \subseteq \mathcal{N}\mathcal{C}\mathcal{S}cl(L) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}cl(L) \subseteq \mathcal{N}\mathcal{C}cl(L) \subseteq \mathcal{N}\mathcal{C}rcl(L)$.
(ii) $\mathcal{N}\mathcal{C}int(L) \subseteq \mathcal{N}\mathcal{C}\alpha int(L) \subseteq \mathcal{N}\mathcal{C}\mathcal{S}int(L) \subseteq \mathcal{N}\mathcal{C}\mathcal{S}cl(L) \subseteq \mathcal{N}\mathcal{C}\alpha cl(L) \subseteq \mathcal{N}\mathcal{C}cl(L)$.
(iii) $\mathcal{N}\mathcal{C}\alpha int(L) \subseteq \mathcal{N}\mathcal{C}\mathcal{P}int(L) \subseteq \mathcal{N}\mathcal{C}\beta int(L) \subseteq \mathcal{N}\mathcal{C}\beta cl(L) \subseteq \mathcal{N}\mathcal{C}\mathcal{P}cl(L) \subseteq \mathcal{N}\mathcal{C}\alpha cl(L)$.

Theorem 3.2. *If a $\mathcal{N}\mathcal{C}r\mathcal{S}os$ L is such that $L \subseteq M \subseteq \mathcal{N}\mathcal{C}cl(L)$, then M is also a $\mathcal{N}\mathcal{C}r\mathcal{S}os$.*

Corollary 3.1. *If a $\mathcal{N}\mathcal{C}r\mathcal{S}cs$ L is such that $\mathcal{N}\mathcal{C}int(L) \subseteq M \subseteq L$, then M is also a $\mathcal{N}\mathcal{C}r\mathcal{S}cs$.*

Theorem 3.3. *A $\mathcal{N}CS$ $L \in \mathcal{N}\mathcal{C}r\mathcal{S}O(X)$ iff for every neutrosophic crisp point $p \in L$, $\exists$ a $\mathcal{N}CS$ $M \in \mathcal{N}\mathcal{C}r\mathcal{S}O(X)$ such that $p \in M \subseteq L$.*

Proposition 3.3. *If $L \in \mathcal{N}\mathcal{C}r\mathcal{S}O(X)$, then $\mathcal{N}\mathcal{C}r\mathcal{S}cl(L) \subseteq \mathcal{N}\mathcal{C}r\mathcal{S}O(X)$.*

Proposition 3.4. *If L is $\mathcal{N}\mathcal{C}r\mathcal{S}os$ in X, then L^c is $\mathcal{N}\mathcal{C}r\mathcal{S}cs$.*

Proposition 3.5. *In a $\mathcal{N}CTS$ (X, Γ), the $\mathcal{N}Crcs$, $\mathcal{N}Cros$ and $\mathcal{N}Crclos$ are $\mathcal{N}Cr\mathcal{S}os$.*

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