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SELF-CENTERED INTERVAL VALUED NEUTROSOPHIC GRAPH

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Abstract

In this paper, we discuss the concepts of interval valued neutrosophic gragh, single valued neutrosophic signed graph, self centered single valued neutrosophic graph. We present the concept of self-centered interval valued netrosophic graph. We investigate some properties of self-centered interval valued neutrosophic graphs.

Keywords: Neutosophic graph, interval valued graph, self-centered graph.

1. Introduction

Karunambigai M.G and Kalaivani O introduced the concept of self-centered IFG. Smarandache F, Broumi S, Talea M, Bakali A, Dhavaseelan R, Vikram-prasad R, Krishnaraj V introduced the idea of neutrosophic sets by combining the non-standard analysis. Neutrosophic set is a mathematical tool for dealing real life problems having imprecise, indeterminacy and inconsistent data. Neutrosophic set theory, as a generalization of classical set theory, fuzzy set theory and intuitionistic fuzzy set theory, is applied in a variety of fields, including control theory, decision making problems, topology, medicines and inmany more real life problems. Wang et al. Wang H, Samarandache F, ZhangY.Q presented the notion of single-valued neutrosophic sets to apply neutrosophic sets in real life problems more conveniently. A single-valued neutrosophic set has three components: truth membership degree, indeterminacymembership degree and falsity membership degree. These three components of a single-valued neutrosophic set are not dependent and their values arecontained in the standard unit interval [0, 1]. Single-valued neutrosophic sets are the generalization of intuitionistic fuzzy sets. Single-valued neutrosophic sets have been a new hot research topic and many researchers haveaddressed this issue. Akram et al. Akram M and Shahzadi S has discussed several concepts related to single-valued neutrosophic graphs. Majumdarand Samanta Majumdar P and Samanta S.K studied similarity and entropy of single-valued neutrosophic sets. In this paper, we introduce the concepts of length, distance, radius and eccentricity of self-centered of a interval valued neutrosophic graph. We also discuss some interesting properties besides given some examples.

2. Preliminaries

Definition 2.1. *Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A is an object of the form*

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$$

where the functions $T, I, F : X \to]^{-}0, 1^{+}[$ *define respectively the a truth-membership function, indeterminacy-membership function and falsity-membership function of the element* $x \in X$ *to the set A with the condition*

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

The functions $T_A(x), I_A(x)$ *and* $F_A(x)$ *are real standard or non standard subsets of* $]^{-}0, 1^{+}[$.

Definition 2.2. *Let X be a space of points (options) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS) is characterized by truth-membership function* $T_A(x)$*, an indeterminacy-membership function* $I_A(x)$ *and a falsity-membership* $F_A(x)$*. For each point* $x \in X$*,* $T_A(x), I_A(x), F_A(x) \in \{0, 1\}$.
A (SVNS) can be written as

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$$

Definition 2.3. *Let* $A = (T_A, I_A, F_A)$ *and* $B = (T_B, I_B, F_B)$ *be single valued neutrosophic sets on a set on a set X. If* $A = (T_A, I_A, F_A)$ *is a single valued neutrosophic relation on a set X, then*

$$\begin{aligned} T_B(x, y) &\leq min(T_A(x), T_A(y)) \\ I_B(x, y) &\geq max(I_A(x), I_A(x)) \text{ and} \\ F_B(x, y) &\geq max(F_A(x), F_A(x)) \end{aligned}$$

A single valued neutrosophic relation A on X is called symmetric if

$$T_A(x, y) = T_A(y, x) \qquad T_B(x, y) = T_B(y, x)$$
$$I_A(x, y) = I_A(y, x) \qquad I_B(x, y) = I_B(y, x)$$
$$F_A(x, y) = F_A(y, x) \qquad F_B(x, y) = F_B(y, x)$$

$x, y \in X$.

Throughout this paper, we denote $G^* = (V, E)$ a crisp graph, and $G = (A, B)$ a single valued neutrosophic graph.

Definition 2.4. *A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$ where*

1. *The functions $T_A : V \to [0, 1], I_A : V \to [0, 1], F_A : V \to [0, 1]$ denote degree of truth-membership, degree of indeterminacy-membership and degree of falsity-membership of the element $v_i \in V$ respectively and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in V(i = 1, 2, ...n)$*

2. *The functions $T_B : E \leq V \times V \to [0, 1], I_B : E \leq V \times V \to [0, 1]$ and$F_B : E \leq V \times V \to [0, 1]$ are defined by*

$$T_B(v_i, v_j) \leq min[T_A(v_i), T_A(v_j)]$$
$$I_B(v_i, v_j) \geq max[I_A(v_i), I_A(v_j)] and$$
$$F_B(v_i, v_j) \geq max[F_A(v_i), F_A(v_j)]$$

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$$

for all $(v_i, v_j) \in E(i, j = 1, 2, ...n)$

We call A the single valued neutrosophic vertex set of V, B the single valued neutrosophic edge set of E, respectively. Note that B is a symmetric single valued neutrosophic relation on A. We use the notation for an element of E. Thus, $G = (A, B)$ is a single valued neutrosophic graph of $G^* = (V, E)$ if

$T_B : E \leq V \times V \rightarrow [0,1], I_B : E \leq V \times V \rightarrow [0,1]$ *and* $F_B : E \leq V \times V \rightarrow [0,1]$ *are defined by*

$$\begin{aligned} T_B(v_i, v_j) &\leq min[T_A(v_i), T_A(v_j)] \\ I_B(v_i, v_j) &\geq max[I_A(v_i), I_A(v_j)] and \\ F_B(v_i, v_j) &\geq max[F_A(v_i), F_A(v_j)] for\ all\ (v_i, v_j \in E) \end{aligned}$$

Example 2.1. *Consider the graph* G^* *such that* $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. *Let A be a single valued neutrosophic subset of V and let B a single valued neutrosophic subset of E denoted by*

	v_1	v_2	v_3	v_4
T_A	0.4	0.5	0.3	0.2
I_A	0.2	0.2	0.5	0.4
F_A	0.5	0.3	0.6	0.3

	v_1v_2	v_2v_3	v_3v_4	v_4v_5
T_B	0.4	0.3	0.2	0.1
I_B	0.2	0.5	0.5	0.4
F_B	0.5	0.6	0.6	0.5

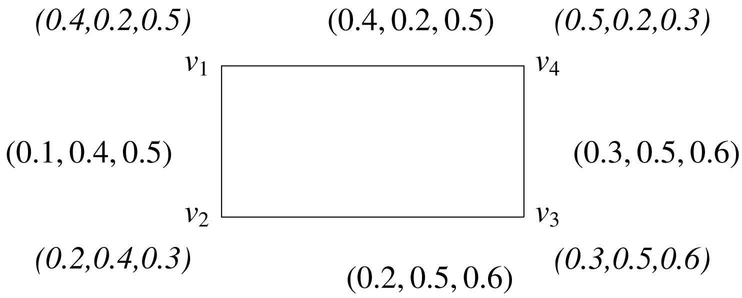


Figure 2.2 Single valued neutrosophic graph

in figure 3,

1. *(v_1,0.4,0.2,0.5) is single valued neutrosophic vertex or SVN-vertex.*

2. *(v_1v_2,0.4,0.2,0.5) is single valued neutrosophic edge or SVN-edge.*

3. *(v_1,0.4,0.2,0.5) and (v_2,0.5,0.2,0.3) are single valued neutrosophic adjacent vertices.*

4. *(v_1v_2,0.4,0.2,0.5) and (v_1v_4,0.1,0.4,0.5) are single valued neutrosophic adjacent edge.*

Definition 2.5. *Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x) = [T_{AL}(x), T_{AU}(x)]$, $I_A(x) = [I_{AL}(x), I_{AU}(x)]$, $F_A(x) = [F_{AL}(x), F_{AU}(x)]$*

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.6. *Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (IVNS)A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x) = [T_{AL}(x), T_{AU}(x)]$, $I_A(x) = [I_{AL}(x), I_{AU}(x)]$, $F_A(x) = [F_{AL}(x), F_{AU}(x)] \subseteq [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.*

Definition 2.7. *An interval valued neutrosophic graph of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$ where*

$$A =< [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}] >$$

is an interval valued neutrosophic set on V and

$$B =< [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}] >$$

is an interval valued neutrosophic set on E satisfies the following condition:
1. $V = \{v_1, v_2...v_n\}$ such that $T_{AL} : V \to [0, 1]$, $T_{AU} : V \to [0, 1]$, $I_{AL} : V \to [0, 1]$, $I_{AU} : V \to [0, 1]$, and $F_{AL} : V \to [0, 1]$, $F_{AU} : V \to [0, 1]$ denote the degree of the truth-membership,the degree of indeterminacy-membership and degree of falsity-membership of the element $y \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$$

for all $v_i \in V(i = 1, 2, ..n)$
2. The functions $T_{BL} : V \times V \to [0, 1]$,$T_{BU} : V \times V \to [0, 1]$, $I_{BL} : V \times V \to [0, 1]$,

$I_{BU}: V \times V \to [0,1]$ *and* $F_{BL}: V \times V \to [0,1]$, $F_{BU}: V \times V \to [0,1]$ *are such that*

$$\begin{aligned}
T_{BL}(\{v_i, v_j\}) &\leq min[T_{AL}(v_i), T_{AL}(v_j)] \\
T_{BU}(\{v_i, v_j\}) &\leq min[T_{AU}(v_i), T_{AU}(v_j)] \\
I_{BL}(\{v_i, v_j\}) &\geq max[I_{AL}(v_i), I_{AL}(v_j)] \\
I_{BU}(\{v_i, v_j\}) &\geq max[I_{AU}(v_i), I_{AU}(v_j)] \text{ and} \\
F_{BL}(\{v_i, v_j\}) &\geq max[F_{AL}(v_i), F_{AL}(v_j)] \\
F_{BU}(\{v_i, v_j\}) &\geq max[F_{AU}(v_i), F_{AU}(v_j)]
\end{aligned}$$

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ *respectively, where*

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$$

for all $\{v_i, v_j\} \in E(i, j = 1, 2, ...n)$ *We call A the IVN vertex set of V, B the IVN edge set of E, respectively. Note that B is a symmetric IVN relation on A. We use the notation* (v_i, v_j) *for an element of E Thus,* $G = (A, B)$ *is an ING of* $G^* = (V, E)$ *if* $T_{BL}(v_i, v_j) \leq min[T_{AL}(v_i), T_{AL}(v_j)]$, $T_{BU}(v_i, v_j) \leq min[T_{AU}(v_i), T_{AU}(v_j)]$, $I_{BL}(v_i, v_j) \geq max[I_{AL}(v_i), I_{AL}(v_j)]$, $I_{BU}(v_i, v_j) \geq max[I_{AU}(v_i), I_{AU}(v_j)]$ *and* $F_{BL}(v_i, v_j) \geq max[F_{AL}(v_i), F_{AL}(v_j)]$, $F_{BU}(v_i, v_j) \geq max[F_{AU}(v_i), F_{AU}(v_j)]$ *for all* $(v_i, v_j) \in E$

Definition 2.8. *A path P in a single valued neutrosophic graph* $G = (A, B)$ *is a sequence of distinct vertices* $v_1, v_2, ...v_n$ *such that either one of the following condition is satisfied (i)* $T_B(v_i, v_j) > 0, I_B(v_i, v_j) > 0$ *and* $F_B(v_i, v_j) = 0$ *for some i and j. (ii)* $T_B(v_i, v_j) = 0$, $I_B(v_i, v_j) = 0$ *and* $F_B(v_i, v_j) > 0$ *for some i and j.*

Definition 2.9. *Let* $G = (A, B)$ *be a connected single valued neutrosophic graph.*

1. *The T-length of a path* $P: v_1, v_2, ...v_n$ *in G,* $l_T(P)$ *is defined as*

$$l_T(P) = \sum_{i=1}^{n-1} \left(\frac{1}{T_B(v_i, v_{i+1})} \right)$$

2. *The I-length of a path* $P: v_1, v_2, ...v_n$ *in G,* $l_I(P)$ *is defined as*

$$l_I(P) = \sum_{i=1}^{n-1} \left(\frac{1}{I_B(v_i, v_{i+1})} \right)$$

3. *The F-length of a path* $P : v_1, v_2, ...v_n$ *in G,* $l_F(P)$ *is defined as*

$$l_F(P) = \sum_{i=1}^{n-1} \left(\frac{1}{F_B(v_i, v_{i+1})} \right)$$

The (T, I, F)*- length of a path* $P : v_1, v_2, ...v_n$ *in G,* $l_{(T,I,F)}(P)$ *is defined as* $l_{(T,I,F)}(P) = (l_T(p), l_I(p), l_F(p))$.

Definition 2.10. *Let* $G = (A, B)$ *be a connected single valued neutrosophic graph.*

1. *The T-distance* $\delta_T(v_i, v_j)$ *is the minimum of the T-length of all the paths joining* v_i *and* v_j *in G, where* $v_i, v_j \in V$. *i.e*

$$\delta_T(v_i, v_j) = min\{l_T(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

2. *The I-distance* $\delta_I(v_i, v_j)$ *is the minimum of the I-length of all the paths joining* v_i *and* v_j *in G, where* $v_i, v_j \in V$. *i.e*

$$\delta_I(v_i, v_j) = min\{l_I(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

3. *The F-distance* $\delta_F(v_i, v_j)$ *is the minimum of the F-length of all the paths joining* v_i *and* v_j *in G, where* $v_i, v_j \in V$. *i.e*

$$\delta_F(v_i, v_j) = min\{l_F(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

The distance $\delta_{(T,I,F)}(v_i, v_j)$ *is defined as* $\delta_{(T,I,F)}(v_i, v_j) = (\delta_T, \delta_I, \delta_F)$

Definition 2.11. *Let* $G = (A, B)$ *be a connected single valued neutrosophic graph.*

1. *For each* $v_i \in V$*, the T-eccentricity of* v_i*, denoted by* $e_T(v_i)$ *and is defined as*

$$e_T(v_i) = max\{\delta_T(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

2. *For each* $v_i \in V$*, the I-eccentricity of* v_i*, denoted by* $e_I(v_i)$ *and is defined as*

$$e_I(v_i) = max\{\delta_I(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

3. *For each* $v_i \in V$*, the F-eccentricity of* v_i*, denoted by* $e_F(v_i)$ *and is defined as*

$$e_F(v_i) = min\{\delta_F(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

For each $v_i \in V$, the eccentricity of v_i denoted by $e(v_i)$ and is defined by $e(v_i) = (e_T(v_i), e_I(v_i), e_F(v_i))$.

Definition 2.12. *Let $G = (A, B)$ be a connected single valued neutrosophic graph.*

1. *The T-radius of G is denoted by $r_T(G)$ and is defined as*

$$r_T(G) = min\{e_T(v_i) : v_i \in V\}.$$

2. *The I-radius of G is denoted by $r_I(G)$ and is defined as*

$$r_I(G) = min\{e_I(v_i) : v_i \in V\}.$$

3. *The F-radius of G is denoted by $r_F(G)$ and is defined as*

$$r_F(G) = min\{e_F(v_i) : v_i \in V\}.$$

The radius of G is denoted by $r(G)$ and is defined as $r(G) = (r_T(G), r_I(G), r_F(G))$.

Definition 2.13. *A connected single valued neutrosophic graph G is a*

1. *T-self-centered single valued neutrosophic graph, if every vertex of G is a T-central vertex. (i.e) $r_T(G) = e_T(v_i), \forall v_i \in V$.*
2. *I-self-centered single valued neutrosophic graph, if every vertex of G is a I-central vertex. (i.e) $r_I(G) = e_I(v_i), \forall v_i \in V$.*
3. *F-self-centered single valued neutrosophic graph, if every vertex of G is a F-central vertex. (i.e) $r_F(G) = e_F(v_i), \forall v_i \in V$.*
4. *single valued neutrosophic self-centered graph, if every vertex of G is a central vertex. (i.e) $r_T(G) = e_T(v_i), r_I(G) = e_I(v_i)$ and $r_F(G) = e_F(v_i), \forall v_i \in V$.*

3. Self-centered Interval Valued Neutrosophhic Graph

Definition 3.1. *A path P in a interval valued neutrosophic graph $G = (A, B)$ is a sequence of distinct vertices $v_1, v_2, ...v_n$ such that either one of the following condition is satisfied*

i $[T_{B_L}(v_i, v_j), T_{B_U}(v_i, v_j)] > 0$, $[I_{B_L}(v_i, v_j), I_{B_U}(v_i, v_j)] > 0$ *and*
$[F_{B_L}(v_i, v_j), F_{B_U}(v_i, v_j)] = 0$ *for some i and j.*

ii $[T_{B_L}(v_i, v_j), T_{B_U}(v_i, v_j)] = 0$, $[I_{B_L}(v_i, v_j), I_{B_U}(v_i, v_j)] = 0$ *and* $[F_{B_L}(v_i, v_j), F_{B_U}(v_i, v_j)] > 0$ *for some i and j.*

Definition 3.2. *Let $G = (A, B)$ be a connected interval valued neutrosophic graph.*

1. *The T-length of a path $P : v_1, v_2, ...v_n$ in G, $l_T(P)$ is defined as*

$$l_T(P) = [l_{T_L}(P), l_{T_U}(P)] \text{ where}$$

$$l_{T_L}(P) = \sum_{i=1}^{n-1} \left(\frac{1}{T_{B_L}(v_i, v_{i+1})} \right)$$

$$l_{T_U}(P) = \sum_{i=1}^{n-1} \left(\frac{1}{T_{B_U}(v_i, v_{i+1})} \right)$$

2. *The I-length of a path $P : v_1, v_2, ...v_n$ in G, $l_I(P)$ is defined as*

$$l_I(P) = [l_{I_L}(P), l_{I_U}(P)] \text{ where}$$

$$l_{I_L}(P) = \sum_{i=1}^{n-1} \left(\frac{1}{I_{B_L}(v_i, v_{i+1})} \right)$$

$$l_{I_U}(P) = \sum_{i=1}^{n-1} \left(\frac{1}{I_{B_U}(v_i, v_{i+1})} \right)$$

3. *The F-length of a path $P : v_1, v_2, ...v_n$ in G, $l_F(P)$ is defined as*

$$l_F(P) = [l_{F_L}(P), l_{F_U}(P)] \text{ where}$$

$$l_{F_L}(P) = \sum_{i=1}^{n-1} \left(\frac{1}{F_{B_L}(v_i, v_{i+1})} \right)$$

$$l_{F_U}(P) = \sum_{i=1}^{n-1} \left(\frac{1}{F_{B_U}(v_i, v_{i+1})} \right)$$

The (T, I, F)- length of a path $P : v_1, v_2, ...v_n$ in G, $l_{(T,I,F)}(P)$ is defined as

$$l_{(T,I,F)}(P) = ([l_{T_L}(P), l_{T_U}(P)], [l_{I_L}(P), l_{I_U}(P)], [l_{F_L}(P), l_{F_U}(P)]).$$

Definition 3.3. *Let $G = (A, B)$ be a connected interval valued neutrosophic graph.*

1. *The T-distance $\delta_T(v_i, v_j)$ is the minimum of the T-length of all the paths joining v_i and v_j in G, where $v_i, v_j \in V$. i.e*

$$\delta_T(v_i, v_j) = [\delta_{T_L}(v_i, v_j), \delta_{T_U}(v_i, v_j)]$$

where

$$\delta_{T_L}(v_i, v_j) = min\{l_{T_L}(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

$$\delta_{T_U}(v_i, v_j) = min\{l_{T_U}(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

2. *The I-distance $\delta_I(v_i, v_j)$ is the minimum of the I-length of all the paths joining v_i and v_j in G, where $v_i, v_j \in V$. i.e*

$$\delta_I(v_i, v_j) = [\delta_{I_L}(v_i, v_j), \delta_{I_U}(v_i, v_j)]$$

where

$$\delta_{I_L}(v_i, v_j) = min\{l_{I_L}(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

$$\delta_{I_U}(v_i, v_j) = min\{l_{I_U}(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

3. *The F-distance $\delta_F(v_i, v_j)$ is the minimum of the F-length of all the paths joining v_i and v_j in G, where $v_i, v_j \in V$. i.e*

$$\delta_F(v_i, v_j) = [\delta_{F_L}(v_i, v_j), \delta_{F_U}(v_i, v_j)]$$

where

$$\delta_{F_L}(v_i, v_j) = min\{l_{F_L}(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

$$\delta_{F_U}(v_i, v_j) = min\{l_{F_U}(P) : P \text{ is a path between } v_i \text{ and } v_j\}$$

The distance $\delta_{(T,I,F)}(v_i, v_j)$ is defined as $\delta_{(T,I,F)}(v_i, v_j) = (\delta_T, \delta_I, \delta_F)$ i.e

$$\delta_{(T,I,F)} = ([\delta_{T_L}, \delta_{I_U}], [\delta_{I_L}, \delta_{T_U}], [\delta_{F_L}, \delta_{F_U}])$$

Definition 3.4. *Let $G = (A, B)$ be a connected interval valued neutrosophic graph.*

1. *For each $v_i \in V$, the T-eccentricity of v_i, denoted by $e_T(v_i)$ and is defined as*

$$e_T(v_i) = [e_{T_L}(v_i), e_{T_U}(v_i)] \text{ where}$$

$$e_{T_L}(v_i) = max\{\delta_{T_L}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

$$e_{T_U}(v_i) = max\{\delta_{T_U}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

2. *For each $v_i \in V$, the I-eccentricity of v_i, denoted by $e_I(v_i)$ and is defined as*

$$e_I(v_i) = [e_{I_L}(v_i), e_{I_U}(v_i)] \text{ where}$$

$$e_{I_L}(v_i) = max\{\delta_{I_L}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

$$e_{I_U}(v_i) = max\{\delta_{I_U}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

3. *For each $v_i \in V$, the F-eccentricity of v_i, denoted by $e_F(v_i)$ and is defined as*

$$e_F(v_i) = [e_{F_L}(v_i), e_{F_U}(v_i)] \text{ where}$$

$$e_{F_L}(v_i) = min\{\delta_{F_L}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

$$e_{F_U}(v_i) = min\{\delta_{F_U}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$$

For each $v_i \in V$, the eccentricity of v_i denoted by $e(v_i)$ and is defined by $e(v_i) = (e_T(v_i), e_I(v_i), e_F(v_i))$. i.e

$$e(v_i) = ([e_{T_L}(v_i), e_{T_U}(v_i)], [e_{I_L}(v_i), e_{I_U}(v_i)], [e_{F_L}(v_i), e_{F_U}(v_i)])$$

Definition 3.5. *Let $G = (A, B)$ be a connected interval valued neutrosophic graph.*

1. *The T-radius of G is denoted by $r_T(G)$ and is defined as*

$$r_T(G) = [r_{T_L}(G), r_{T_U}(G)] \text{ where}$$

$$r_{T_L}(G) = min\{e_{T_L}(v_i) : v_i \in V\}.$$

$$r_{T_U}(G) = min\{e_{T_U}(v_i) : v_i \in V\}.$$

2. *The I-radius of G is denoted by $r_I(G)$ and is defined as*

$$r_I(G) = [r_{I_L}(G), r_{I_U}(G)] \text{ where}$$

$$r_{I_L}(G) = min\{e_{I_L}(v_i) : v_i \in V\}.$$

$$r_{I_U}(G) = min\{e_{I_U}(v_i) : v_i \in V\}.$$

3. *The F-radius of G is denoted by $r_F(G)$ and is defined as*

$$r_F(G) = [r_{F_L}(G), r_{F_U}(G)] \text{ where}$$

$$r_{F_L}(G) = min\{e_{F_L}(v_i) : v_i \in V\}.$$

$$r_{F_U}(G) = min\{e_{F_U}(v_i) : v_i \in V\}.$$

The radius of G is denoted by $r(G)$ and is defined as $r(G) = (r_T(G), r_I(G), r_F(G))$. i.e,

$$r(G) = ([r_{T_L}(G), r_{T_U}(G)], [r_{I_L}(G), r_{I_U}(G)], [r_{F_L}(G), r_{F_U}(G)])$$

Definition 3.6. *A connected interval valued neutrosophic graph G is a*

1. *T-self-centered interval valued neutrosophic graph, if every vertex of G is a T-central vertex. $r_T(G) = e_T(v_i)$ (i.e)*

$$r_{T_L}(G) = e_{T_L}(v_i), \quad r_{T_U}(G) = e_{T_U}(v_i)$$

for all $v_i \in V$

2. *I-self-centered interval valued neutrosophic graph, if every vertex of G is a I-central vertex. $r_I(G) = e_I(v_i)$ (i.e)*

$$r_{I_L}(G) = e_{I_L}(v_i), \quad r_{I_U}(G) = e_{I_U}(v_i).$$

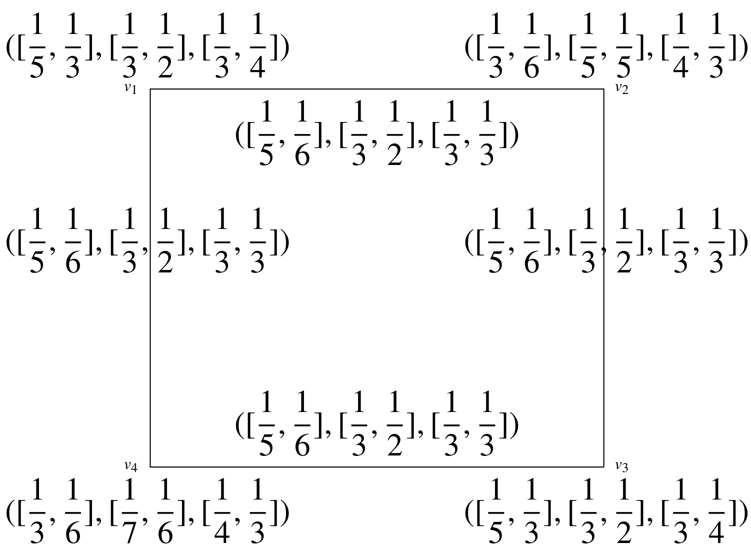
for all $v_i \in V$

3. *F-self-centered interval valued neutrosophic graph, if every vertex of G is a F-central vertex. $r_F(G) = e_F(v_i)$ (i.e)*

$$r_{F_L}(G) = e_{F_L}(v_i), \quad r_{F_U}(G) = e_{F_U}(v_i).$$

for all $v_i \in V$

interval valued neutrosophic self-centered graph, if every vertex of G is a central vertex. (i.e) $r_T(G) = e_T(v_i), r_I(G) = e_I(v_i)$ and $r_F(G) = e_F(v_i), \forall v_i \in V$.



Example 3.1.

Figure 3.1 Self-Centered Interval valued Neutrosophic Graph

Path	(v_1, v_2)	(v_2, v_3)	(v_3, v_4)
Distance $\delta_{(T,I,F)}(v_i, v_j)$	*([5,6],[6,4],[3,3])*	*([5,6],[6,4],[3,3])*	*([5,6],[6,4],[3,3])*

Path	(v_1, v_4)	(v_1, v_3)	(v_2, v_4)
Distance $\delta_{(T,I,F)}(v_i, v_j)$	*([5,6],[6,4],[3,3])*	*([10,12],[6,8],[3,3])*	*([10,12],[6,8],[3,3])*

Then the eccentricity of v_i are $e(v_1) = ([10,12],[6,8],[3,3])$, $e(v_2) = ([10,12],[6,8],[3,3])$, $e(v_3) = ([10,12],[6,8],[3,3])$, $e(v_4) = ([10,12],[6,8],[3,3])$.
Radius of G is $r(G) = ([10,12],[6,8],[3,3])$.
Here $r(G) = e(v_i)$, $\forall v_i \in V$.
Hence G is a self-centered interval valued neutrosophic graph.

Theorem 3.1. *A Interval valued neutrosophic graph $G = (A, B)$ is a self-centered interval valued neutrosophic graph iff $\delta_T(v_i, v_j) \leq r_T(G)$, $\delta_I(v_i, v_j) \leq r_I(G)$, $\delta_F(v_i, v_j) \geq r_F(G)$.*
i.e $\delta_{T_L}(v_i, v_j) \leq r_{T_L}(G)$, $\delta_{T_U}(v_i, v_j) \leq r_{T_U}(G)$, $\delta_{I_L}(v_i, v_j) \leq r_{I_L}(G)$, $\delta_{I_U}(v_i, v_j) \leq r_{I_U}(G)$, $\delta_{F_L}(v_i, v_j) \geq r_{F_L}(G)$, $\delta_{F_U}(v_i, v_j) \geq r_{F_U}(G)$ $\forall (v_i, v_j) \in V$.

Proof. We assume that G is self-centered interval valued neutrosophic graph G. That is $e_{T_L}(v_i) = e_{T_L}(v_j)$, $e_{T_U}(v_i) = e_{T_U}(v_j)$, $e_{I_L}(v_i) = e_{I_L}(v_j)$, $e_{I_U}(v_i) = e_{I_U}(v_j)$, $e_{F_L}(v_i) = e_{F_L}(v_j)$, $e_{F_U}(v_i) = e_{F_U}(v_j)$.
Now we want to prove that

$$\delta_T(v_i, v_j) \leq r_T(G), \text{ i.e}\delta_{T_L}(v_i, v_j) \leq r_{T_L}(G), \delta_{T_U}(v_i, v_j) \leq r_{T_U}(G),$$
$$\delta_I(v_i, v_j) \leq r_I(G), \text{ i.e } \delta_{I_L}(v_i, v_j) \leq r_{I_L}(G), \delta_{I_U}(v_i, v_j) \leq r_{I_U}(G),$$
$$\delta_F(v_i, v_j) \geq r_F(G), \text{ i.e } \delta_{F_L}(v_i, v_j) \geq r_{F_L}(G), \delta_{F_U}(v_i, v_j) \geq r_{F_U}(G).$$

By the definition of eccentricity, we obtain
$\delta_T(v_i, v_j) \leq e_T(v_i)$, $\delta_I(v_i, v_j) \leq e_I(v_i)$, $\delta_F(v_i, v_j) \geq e_F(v_i)$.

$$\text{i.e } \delta_{T_L}(v_i, v_j) \leq e_{T_L}(v_i), \delta_{T_U}(v_i, v_j) \leq e_{T_U}(v_i),$$
$$\delta_{I_L}(v_i, v_j) \leq e_{I_L}(v_i), \delta_{I_U}(v_i, v_j) \leq e_{I_U}(v_i),$$
$$\delta_{F_L}(v_i, v_j) \geq e_{F_L}(v_i), \delta_{F_U}(v_i, v_j) \geq e_{F_U}(v_i) \quad \forall (v_i, v_j) \in V.$$

Since G is self-centered interval valued neutrosophic graph, the above inequality becomes

$$\delta_{T_L}(v_i, v_j) \leq r_{T_L}(G), \delta_{T_U}(v_i, v_j) \leq r_{T_U}(G),$$
$$\delta_{I_L}(v_i, v_j) \leq r_{I_L}(G), \delta_{I_U}(v_i, v_j) \leq r_{I_U}(G),$$
$$\delta_{F_L}(v_i, v_j) \geq r_{F_L}(G), \delta_{F_U}(v_i, v_j) \geq r_{F_U}(G) \quad \forall (v_i, v_j) \in V.$$

$$\text{i.e, } \delta_T(v_i, v_j) \leq r_T(G), \delta_I(v_i, v_j) \leq r_I(G), \delta_F(v_i, v_j) \geq r_F(G) \ .$$

Assume that $\delta_T(v_i, v_j) \leq r_T(G), \delta_I(v_i, v_j) \leq r_I(G), \delta_F(v_i, v_j) \geq r_F(G)$.

$$\text{i.e } \delta_{T_L}(v_i, v_j) \leq r_{T_L}(G), \delta_{T_U}(v_i, v_j) \leq r_{T_U}(G),$$
$$\delta_{I_L}(v_i, v_j) \leq r_{I_L}(G), \delta_{I_U}(v_i, v_j) \leq r_{I_U}(G),$$
$$\delta_{F_L}(v_i, v_j) \geq r_{F_L}(G), \delta_{F_U}(v_i, v_j) \geq r_{F_U}(G) \quad \forall (v_i, v_j) \in V.$$

Then we have to prove that G is self-centered interval valued neutrosophic graph. Suppose that G is not self-centered interval valued neutrosophic graph.
Then $r_T(G) = e_T(v_i)$, $r_I(G) = e_I(v_i)$, $r_F(G) = e_F(v_i)$. i.e $r_{T_L}(G) = e_{T_L}(v_i)$, $r_{T_U}(G) = e_{T_U}(v_i)$, $r_{I_L}(G) = e_{I_L}(v_i)$, $r_{I_U}(G) = e_{I_U}(v_i)$, $r_{F_L}(G) = e_{F_L}(v_i)$, $r_{F_U}(G) = e_{F_U}(v_i)$ for some $v_i \in V$.
Let us assume that $e_{T_L}(v_i)$, $e_{T_U}(v_i)$, $e_{i_L}(v_i)$, $e_{I_U}(v_i)$, $e_{F_L}(v_i)$ and $e_{F_U}(v_i)$ is the least value among all other eccentricity. That is

$$r_{T_L}(G) = e_{T_L}(v_i), r_{T_U}(G) = e_{T_U}(v_i), r_{I_L}(G) = e_{I_L}(v_i), r_{I_U}(G) = e_{I_U}(v_i),$$

$$r_{F_L}(G) = e_{F_L}(v_i) \ \ and \ r_{F_U}(G) = e_{F_U}(v_i). \tag{3.1}$$

where $e_{T_L}(v_i) < e_{T_L}(v_j)$, $e_{T_U}(v_i) < e_{T_U}(v_j)$, $e_{I_L}(v_i) < e_{I_L}(v_j)$, $e_{I_U}(v_i) < e_{I_U}(v_j)$, $e_{F_L}(v_i) < e_{F_L}(v_j)$ and $e_{F_U}(v_i) < e_{F_U}(v_j)$ for some $(v_i, v_j) \in V$ and

$$\delta_{T_L}(v_i, v_j) = e_{T_L}(v_j) > e_{T_L}(v_i), \ \delta_{T_U}(v_i, v_j) = e_{T_U}(v_j) > e_{T_U}(v_i),$$

$$\delta_{I_L}(v_i, v_j) = e_{I_L}(v_j) > e_{I_L}(v_i), \ \delta_{I_U}(v_i, v_j) = e_{I_U}(v_j) > e_{I_U}(v_i),$$

$$\delta_{F_L}(v_i, v_j) = e_{F_L}(v_j) > e_{F_L}(v_i), \ and \ \delta_{F_U}(v_i, v_j) = e_{F_U}(v_j) > e_{F_U}(v_i). \tag{3.2}$$

for some $(v_i, v_j) \in V$.
Hence from equations (1) and (2), we have

$$\delta_{T_L}(v_i, v_j) > r_{T_L}(G), \delta_{T_U}(v_i, v_j) > r_{T_U}(G),$$
$$\delta_{I_L}(v_i, v_j) > r_{I_L}(G), \delta_{I_U}(v_i, v_j) > r_{I_U}(G),$$
$$\delta_{F_L}(v_i, v_j) < r_{F_L}(G), \delta_{F_U}(v_i, v_j) < r_{F_U}(G)$$

for some $(v_i, v_j) \in V$, which is a contradiction to the fact that

$$\delta_{T_L}(v_i, v_j) \leq r_{T_L}(G), \delta_{T_U}(v_i, v_j) \leq r_{T_U}(G),$$
$$\delta_{I_L}(v_i, v_j) \leq r_{I_L}(G), \delta_{I_U}(v_i, v_j) \leq r_{I_U}(G),$$
$$\delta_{F_L}(v_i, v_j) \geq r_{F_L}(G), \delta_{F_U}(v_i, v_j) \geq r_{F_U}(G) \quad \forall (v_i, v_j) \in V.$$

i.e, $\delta_T(v_i, v_j) \leq r_T(G), \delta_I(v_i, v_j) \leq r_I(G), \delta_F(v_i, v_j) \geq r_F(G)$.
Hence G is a self-centered interval valued neutrosophic graph. □

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