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Semi-Compact and Semi-*Lindelöf* Spaces via Neutrosophic Crisp Set Theory

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Abstract

The aim of this paper is devoted to introduce and study the concepts of semi-compact (resp. semi-Lindelöf, locally semi-compact) spaces in a neutrosophic crisp topological space. Several properties, functions properties of neutrosophic crisp semi-compact spaces are studied. In addition to these, we introduce and study the definition of neutrosophic crisp semi-Lindelöf spaces and neutrosophic crisp locally semi-compact spaces. We show that neutrosophic crisp semi-compact spaces is preserved under neutrosophic crisp irresolute function and neutrosophic crisp pre-semi-closed function with neutrosophic crisp semi-compact point inverses.

Keywords: Neutrosophic crisp semi-compact spaces, Neutrosophic crisp semi-Lindelöf spaces, Neutrosophic crisp locally semi-compact spaces**.** Neutrosophic topological spaces

1. Introduction and preliminaries

Neutrosophic Crisp Sets were introduced by Salama & Smarandache in 2015. Neutrosophic topological spaces and many applications have been investigated by Salama et al. [5, 7, 8, 9] and [11-21]. The notions and terminologies not explained in this paper may be found in [9]. Some definitions and results which will be needed in this paper are recalled here. *In this paper, we generalize the crisp semi-compact spaces* [1] *and some notions in [2, 3, 4, 6] to the notion of neutrosophic crisp semi-compact spaces.*

Definition 1.1 [9] For any non-empty fixed set X, a neutrosophic crisp set (NC-set, for short) A is an object having the form $A=\langle A_1, A_2, A_3\rangle$, where A_1, A_2 and A_3 are subsets of Xsatisfying$A_1\cap A_2 =\emptyset$, $A_1\cap A_3 = \emptyset$ and $A_3 \cap A_2 = \emptyset$.

Several relations and operations between NC-sets were defined in [8].

Definition 1.2 [9] A neutrosophic crisp topology (NCT, for short) on a non-empty set X is a family Γ of neutrosophic crisp subsets of X satisfying the following axioms

i) $\emptyset_N, X_N\in\Gamma$.
ii)$A_1\cap A_2\in\Gamma$ for any A_1 and $A_2\in\Gamma$.
iii) $\cup A_j\in\Gamma$ for any $\{A_j: j \in J\} \subseteq\Gamma$.

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In this case the pair (X, Γ) is called a neutrosophic crisp topological space ($NCTS$, for short) in X. The elements in Γ are called neutrosophic crisp open sets (NC-open sets for short) in X. A NC-set F is said to be neutrosophic crisp closed set (NC-closed set, for short) if and only if its complement $F^c$is a NC-open set.

Definition 1.3 [8] Let (X, Γ) be a $NCTS$ and $A=\langle A_1, A_2, A_3\rangle$ be a NC-set in X. Then the neutrosophic crisp closure of A ($NCcl(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by:

(i) $NCcl(A)=\cap \{K: K$ is a NC-closed set in X and $A\subseteq K\}$

(ii) $NCint(A)=\cup \{G: G$ is a NC-open set in X and $G\subseteq A\}$,

It can be also shown that $NCcl(A)$ is a NC-closed set, and $NCint(A)$ is a NC-open set in X.

Definition 1.4 [7] Let (X, Γ) be a $NCTS$ and $A =\langle A_1, A_2, A_3\rangle$ be a NCS in X, then A is called:

i) Neutrosophic crisp α-open set iff $A\subseteq NCint(NCcl(NCint(A))$.

ii) Neutrosophic crisp semi-open set iff $A\subseteq NCcl\ (NCint\ (A))$.

iii)Neutrosophic crisp pre-open set iff $A\subseteq NCint(NCcl(A))$.

Definition 1.5 [3,10] A subset A of space X is called semi-compact relative to X if any semi-open cover of A in X has a finite subcover of A.

Definition 1.6 [10] A subset A of a space X is called semi-Lindelöf in X if any semi-open cover of A in X has a countable subcover of A.

Definition 1.7 [5] Let (X, Γ) be a $NCTS$ and $A =\langle A_1, A_2, A_3\rangle$ be a NCS in X, then $f: X\longrightarrow X$ is NCsemi-continuous if the inverse image of NCsemi-open set is NCsemi- open.

2. Neutrosophic Crisp Semi-compact Spaces.

Definition 2.1 Let (X, Γ) be a $NCTS$.

(i) If a family $\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle: i\in I\}$ of NC-semiopen sets in X satisfies the condition $X_N= \cup\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle: i\in I\}$, then it is called a NC-semiopen cover of X.

(ii) A finite subfamily of a NC-semiopen cover $\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle: i=1,2, 3, \ldots, n\}$ on X, which is also a NC-semiopen cover of X, is called a finite sub cover of NC-semiopen sets.

Definition 2.2A $NCTS$ (X, Γ) is called neutrosophic crisp semi-compact spaces (NC-semi-compact , for short) if any NC-semiopen cover of X has a finite subcover.

Definition 2.3

A family $\{\langle k_{i_1},k_{i_2},k_{i_3}\rangle: i\in I\}$ of NC-semiclosed sets in X satisfies the finite intersection property (FIP for short) iff every finite subfamily $\{\langle k_{i_1},k_{i_2},k_{i_3}\rangle: i=1, 2,3\ldots\ldots,n\}$ of the family satisfies the condition $\bigcap_i \{\langle k_{i_1},k_{i_2},k_{i_3}\rangle: i=1,2,3\ldots\ldots,n\}\neq \Phi_N$.

Theorem 2.4 A $NCTS$ (X, Γ) is NC-semi-compact iff every family $\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle: i\in I\}$ of NC-semiclosed sets in X having the FIP has a nonempty intersection.

Proof. Let X be a NC-semi-compact space and $\mathcal{G}$=$\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle$: i∈I$\}$be a cover of NC-semiopen sets of X having the FIP. Suppose that $\bigcap_{i}\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle$:$i$∈I$\}$= Φ_N, then $\{X\backslash\langle G_{i_1},G_{i_2},G_{i_3}\rangle$:$i$∈I$\}$ is a NC-semiopen cover of X and must contain a finite subcover $\{X\backslash\langle G_{i_1},G_{i_2},G_{i_3}\rangle$: i=1,2,3……,$n\}$ for X. This implies that $\bigcap_{i}\{\langle G_{i_1},G_{i_2}, G_{i_3}\rangle$: i= 1,2, 3……, $n\}$ = Φ_N this contradicts our assumption that $\mathcal{G}$ has a FIP. Conversely, assume that X is not NC-semi-compact. Then there exists a NC-semiopen cover $\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle$:$i$∈I$\}$ for X, which contain a finite subcover for X. Thus, $\{X\backslash\langle G_{i_1},G_{i_2},G_{i_3}\rangle$:$i$∈I$\}$ is a family of NC-semiclosed sets of X having the FIP. Moreover, we have $\bigcap_{i}\{X\backslash\langle G_{i_1},G_{i_2}, G_{i_3}\rangle$: i∈I$\}$= Φ_N. This complete the proof.

Definition 2.5A subset u=$\langle u_1, u_2,u_3\rangle$ of a $NCTS$ (X, Γ) is called NC-semi-compact relative to X if any NC-semiopen cover of u in X has a finite subcover of u. By NC-semi-compact in X, we will mean NC-semi-compact relative to X.

Definition 2.6 A subset u=$\langle u_1, u_2,u_3\rangle$ of a $NCTS$ (X, Γ) is called NC-semi-Lindelöf in X if any NC-semiopen cover of u in X has a countable subcover of u.

Remark 2.7

Since the family of all $NC\alpha$-open subset of a $NCTS$ (X, Γ), denoted by Γ^{α} is NCT on X finer that Γ, then the family of all NC-semiopen subsets of (X,Γ^{α}) is equal to the family of all NC-semiopen subsets of (X,Γ^{α}). Hence, it easily to see that a NC-set u of (X, Γ) is NC-semi-compact (respNC-semi-Lindelöf) in X iff it is NC-semi-compact (resp. NC-semi-Lindelöf) in (X,Γ^{α}).

Theorem 2.8 The finite (resp. countable) union of NC-semi-compact (resp. NC-semi-Lindelöf) sets in a $NCTSX$ is a NC-semi-compact (resp. NC-semi-Lindelöf) in X.

Proof . obvious.

Lemma 2.9 Let u⊆v⊆X, where X is a $NCTS$. Then u is NC-semiopen set in v, if u is NC-semiopen set in X.

Theorem 2.10 Let v be a NC-preopen subset of a $NCTSX$ and u⊆v. If u is NC-semi-compact (resp. NC-semi-Lindelöf) in X, then u is NC-semi-compact (resp. NC-semi-Lindelöf) in v.

Proof. Suppose that $\mathcal{G}$= $\{\langle G_{i_1},G_{i_2},G_{i_3}\rangle$: i∈I$\}$ is a cover of u by NC-semiopen sets in v. Using lemma 2.9, G_{i_j}=S_{i_j}∩ v for each i∈I, j=1,2,3,where S_{i_j} is NC-semiopen set in X for each i∈I, j=1,2,3. Thus ξ=$\{\langle S_{i_1},S_{i_2},S_{i_3}\rangle$: $i\in I$ $\}$ is a cover of u by NC-semiopen set in X, but u is NC-semi-compact in X, so there exists i=1,2,3……,n, j=1,2,3. Such that $u\subseteq\bigcup_{i=1}^{n} S_{i_j}$ and thus $u\subseteq\bigcup_{i=1}^{n}(S_{i_j}\cap v)$ $=\bigcup_{i=1}^{n} G_{i_j}$.Hence u is NC-semi-compact in v.

The other case is similar.

Corollary 2.11 Let v be NC-open ($NC\alpha$-open) set of $NCTS$ and u⊆v, if u is NC-semi-compact (resp. NC-semi-Lindelöf) in X, then u is NC-semi compact (resp. NC-semi-Lindelöf) in v.

Proof. It is obviously, since each NC-open set is $NC\alpha$-open set and also NC-preopen set.

Lemma 2.12 Let $u \subseteq v \subseteq X$, where X is a $NCTS$ and v is a NC-preopen set in X, then u is NC-semiopen (resp. NC-semiclosed) in $v$iff$u= S \cap v$, where S is NC-semiopen (resp. NC-semiclosed) in X.

Proof. Obvious.

Theorem 2.13 Let v be a NC-preopen subset of $NCTSX$ and $u \subseteq v$. Then u is NC-semi compact (resp. NC-semi-Lindelöf) in Xiff$u$ is NC-semi compact (resp. NC-semi-Lindelöf) in v.

Proof. Necessity. It follows from Theorem 2.8 sufficiency. Suppose that $\xi= \{\langle S_{i_1}, S_{i_2}, S_{i_3}\rangle: i \in I\}$ is a cover of u be NC-semiopen sets in X. Then $\mathcal{G}= \{S_{i_j} \cap v: i \in I, j=1,2,3\}$ is a cover of u. Since S_{i_j} is NC-semiopen in X for each $i \in I$ and v is NC-preopen in X, it follows from Lemma 2.12 that $S_{i_j} \cap v$ is NC-semiopen set in v for each $i \in I$, j=1,2,3, but u is NC-semi-compact in v, so there exists $i \in I$, j=1,2,3 that $u \subseteq \bigcup_{i=1}^{n} S_{i_j} \cap v \subseteq \bigcup_{i=1}^{n} S_{i_j}$. Hence, u is NC-semi-compact in X.

The other case is similar.

Corollary 2.14 A NC-preopen subset u of X is NC-semi compact (resp. NC-semi-Lindelöf) iff$u$ is NC-semi compact (resp. NC-semi-Lindelöf) in X.

Corollary 2.15 A NC-open ($NC\alpha$-open) subset u of X is NC-semi compact (resp. NC-semi-Lindelöf) iff$u$ is NC-semi compact (resp. NC-semi-Lindelöf) in X.

Theorem 2.16 Let v be a NC-semi-compact (resp. NC-semi-Lindelöf) set in a $NCTSX$ and v be is NC-semiclosed of X. Then $u \cap v$ is NC-semi-compact (resp. NC-semi-Lindelöf) in X.

Proof. Suppose that $\mathcal{G}= \{G_{i_j}: i \in \mathrm{I}, j=1,2,3\}$ is a cover of $u \cap v$ by NC-semiopen set in X. Then $\mathcal{G}= \{G_{i_j}: i \in \mathrm{I}, j=1,2,3\} \cup \{X \backslash v\}$ is a cover of u by NC-semiopen sets in X, but u is NC-semi-compact in X, so there exists i= 1.2.3……n, j=1.2.3 such that $u \subseteq (\bigcup_{i=1}^{n} S_{i_j}) \cup \{X \backslash v\}$. Thus $u \cap v \subseteq \bigcup_{i=1}^{n} (S_{i_j} \cap v) \subseteq \bigcup_{i=1}^{n} S_{i_j}$. Hence, $u \cap v$ is NC-semi-compact in X.

The other case is similar.

Corollary 2.17A NC-semiclosed subset u of a NC-semi-compact (resp. NC-semi-Lindelöf) space X is NC-semi-compact (resp. NC-semi-Lindelöf) in X.

Remark 2.18 From the Definition 2.1 of NC-semi-compact space, one may deduce that: NC-semi-compact space $\Longrightarrow NC$-compact space, but the inverse direction may not be true in general as show by the following example.

Example 2.19 Let (X, Γ) be a $NCTS$, where X is infinite, and $\Gamma= \{X_N, \Phi_N\} \cup \{P\}$ where $P=\langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$ be a NC-point in X. Then (X, Γ) is NC-compact but not NC-semi-compact, since $\{\langle \{x, p_1\}, \{x, p_2\}, \{x, p_3\} \rangle: x \in X\}$ is NC-semiopen cover of X which has no finite subcover.

3. Functions and Neutrosophic Crisp Semi-compact Spaces

Definition 3.1 A function f from a $NCTSX$ into a $NCTSY$ is called NC-irresolute if the inverse image of each NC-semiopen set in X, is a NC-semiopen set in Y.

Theorem 3.2 Let f:(X,Γ_1)⟶(Y,Γ_2) be a NC-irresolute function. Then

(i) If u is NC-semi-Lindelöf in X, then $f(A)$ is NC-semi-Lindelöf in Y.

(ii) If u is NC-semi-compact in X, then $f(A)$ is NC-semi-compact in Y.

Proof. We will proof (i) and (ii) is similar.

Suppose that $\mathcal{G}$= {⟨G_{i_j}⟩: i∈I, j=1, 2, 3} is a cover of $f(A)$ by NC-semiopen sets in Y. Then $\mathfrak{Z}$= {⟨f^{-1}(G_{i_j})⟩:i∈I, j=1,2,3} is a cover of u, but f is NC-irresolute function, so ⟨f^{-1}(G_{i_j})⟩ is NC-semiopen sets in X for each i∈I, j=1,2,3. Since u is NC-semi-Lindelöf in X, there exists i_1,i_2, i_3,…..,∈I such that $u \subseteq \bigcup_{i=1}^{\infty} \langle f^{-1}(G_{i_j})\rangle$. Thus $f(u) \subseteq \bigcup_{i=1}^{\infty} \langle f(f^{-1}(G_{i_j}))\rangle \subseteq \bigcup_{i=1}^{\infty} \langle G_{i_j}\rangle$. Hence, $f(A)$ is NC-semi-Lindelöf in X.

Corollary 3.3 If a function f:(X,Γ_1)⟶(Y,Γ_2) is a NC-irresolute (resp. NC-semi continuous) surjective and X is NC-semi-compact, then Y is NC-semi-compact (resp. NC-compact).

Definition 3.4 A function f from a $NCTSX$ into a $NCTSY$ is called NC-pre-semiopen (resp. NC-pre-semiclosed) if the image of each NC-semiopen (resp. NC-semiclosed) subsets of X is NC-semiopen (resp. NC-semiclosed) subsets of Y.

Theorem 3.5 Let f:(X,Γ_1)⟶(Y,Γ_2) be a NC-pre-semiclosed surjection. If for each NC-point y= ⟨{y_1},{y_2},{y_3}⟩ in Y, $f^{-1}(y)$=⟨f^{-1}{y_1},f^{-1}{y_2},f^{-1}{y_3}⟩ is NC-semi-compact (resp. NC-semi-Lindelöf) in X, then $f^{-1}(u)$ is NC-semi-compact (resp. NC-semi-Lindelöf) in X, where u is NC-semi-compact (resp. NC-semi-Lindelöf) in Y.

Proof. Will show the case when u is NC-semi-compact in X, the other case is similar. Let $\mathcal{G}$= {⟨f^{-1}($\mathcal{G}_{i_j}$)⟩:i∈I, j=1,2,3} is a cover of $f^{-1}(u)$ by NC-semiopen sets in X. Then it follows by assumption that for each NC-point y=⟨{y_1}, {y_2}, {y_3} ⟩ in Y, there exists a finite subcollection $\mathcal{G}_j^y$ of $\mathcal{G}$ such that $f^{-1}(y) \subseteq \cup \mathcal{G}_j^y$. Let H_{y_j}=$\cup \mathcal{G}_j^y$. Then H_y is NC-semiopen in X where any union of NC-semiopen sets is NC-semiopen. Let F_{y_j}= $Y \backslash f(X \backslash H_{y_j})$. Then $F_{y_j}$is NC-semiopen in Y where f is NC-pre-semiclosed, also y_i∈F_{y_j}; for each y_i∈u, since $f^{-1}(y) \subseteq H_{y_j}$. Thus the family {$H_{y_j}$:$y_j$∈$u$} is a cover of u by NC-semiclosed sets in Y, but u is NC-semi-compact in Y, so there exists y_1, y_2,…..,y_n∈u such that $u \subseteq \bigcup_{i=1}^{n} F_{y_{ij}}$, j=1,2,3. Thus $f^{-1}(u) \subseteq \bigcup_{i=1}^{n} f^{-1}(F_{y_{ij}}) \subseteq F_{y_{ij}}$.

Since $\mathcal{G}_j^{y_i}$ is a finite sub collection of $\mathcal{G}$ for each i=1,2, …..,n, j=1,2,3, it follows that $\bigcup_{i=1}^{n} \mathcal{G}_j^{y_i}$ is a finite sub collection of $\mathcal{G}$. Hence, $f^{-1}(u)$ is NC-semi-compact in X.

Corollary 3.6 Let f:(X,Γ_1)⟶(Y,Γ_2) be a NC-pre-semiclosed surjection. and $f^{-1}(y)$ is NC-semi-compact in X, for each NC-point y=⟨{y_1}, {y_2},{y_3}⟩ in Y. If Y is NC-semi-compact, so is X.

Definition 3.7 A $NCTS$ (X, Γ) is called NC-Hausdorff space if for each distinct NC-points$x$ and y of X, there exists two disjoint NC-open sets u and v of X containing x and y, respectively.

Theorem 3.8 Let f:$(X,\Gamma_1)\longrightarrow(Y,\Gamma_2)$ is a NC-irresolute function from a NC-semi-compact space X into aNC-Hausdorff space Y, then

(i) f is NC-pre-semiclosed.

(ii) f is NC-semi-homomorphism if it is bijective.

Proof. Let u be a NC -semiclosed set of X. Then u is NC-semi-compact in X, (by Corollary 2.17). By Theorem 3.2, $f(u)$ is NC-semi-compact in Y and hence it is NC-semi-compact. Since Y is NC-Hausdorff, then $f(A)$ is NC-closed set in Y and NC-semiclosed. hence f is NC-pre-semiclosed.

(ii) Obvious.

4. Locally Neutrosophic Crisp Semi-compact Spaces

Definition 4.1 A $NCTSX$ is said to be locally neutrosophic crisp semi-compact (LNC-semi-compact, for short) if each NC-point of X has a NC-open neighborhood which is a NC-semi-compact X.

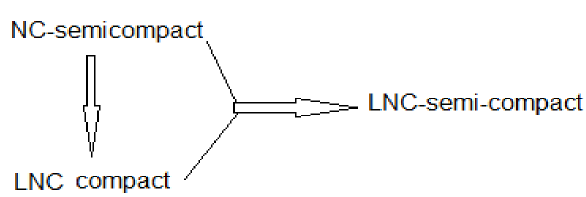
Remark 4.2 It is obvious that every NC-semi-compact space is LNC-semi-compact but the converse may not be true as show by the following example.

Example 4.3 Let (X, Γ) be an infinite discrete $NCTS$. It is obvious that (X, Γ) is LNC-semi-compact but not NC-semi-compact.

Remark 4.4 Every LNC-semi-compact space is LNC-compact, but the converse may not be true as shown by the following example.

Example 4.5 By Example 2.19 shows that a $NCTS$ (X, Γ) is LNC-compact but not LNC-semi-compact.

Remark 4.6 From the above discussion one can draw the following diagram:



Theorem 4.7 A $NCTSX$is LNC-semi-compact iff for each NC-point $x\in X$, there exists a NC-open set$u$ in X which is LNC-semi-compact containing$x$.

Proof. Let u= $\{\langle u_{i_1},u_{i_2},u_{i_3}\rangle: i\in I\}$ be a NC-open set in$X$ containing $x=\langle\{x_1\}, \{x_2\}, \{x_3\}\rangle$which is LNC-semi-compact. Then there exists a NC-open neighbourhood v= $\{\langle v_{i_1},v_{i_2},v_{i_3}\rangle: i\in I\}$ of x in u which is a NC-semi-compact in u. Since u is NC-open in X, so is v and by Corollary 2.11, v is NC-semi-compact in X. This shows that X is LNC-semi-compact.

The proof of the converse is obvious.

Theorem 4.8 A $NCTSX$is LNC-semi-compact iff for each NC-point of X has a NC-open neighbourhood which is LNC-semi-compact in X.

Proof. This follows from Corollary 2.15.

Theorem 4.9 Let f:(X, Γ_1)⟶(Y, Γ_2) be a NC-open, NC-semi continuous surjection. and X is LNC-semi-compact space, then Y is LNC-semi-compact.

Proof. For any NC-point y∈Y, there exists NC-point x∈X such that $f(x)$=y. Since X is LNC-semi-compact, there exists a NC-open neighborhoodU_x of x which is NC-semi-compact in X. Hence $f(U_x)$ is NC-open neighborhood of y which is NC-semi-compact in Y. Therefore, by Theorem 4.8 is LNC-semi-compact.

Theorem 4.10 Let f:(X, Γ_1)⟶(Y, Γ_2) be a NC-pre-semiclosed, NC-continuous surjection. and $f^{-1}(y)$ is NC-semi-compact in X, for each NC-point y∈Y. If Y is LNC-semi-compact, so is X.

Proof. Let x is NC-point of X, by Theorem 4.8, there exists a NC-open neighborhood v of $f(x)$ such that v is NC-semi-compact in X. Then $f^{-1}(v)$ is a NC-open neighborhood of X. By Theorem 3.5, $f^{-1}(v)$ is NC-semi-compact in X. This shows that X is LNC-semi-compact.

5. Conclusion

The paper deals with the concept of semi-compact ness (resp. semi-Lindelöf) in the generalized setting of a neutrosophic crisp topological space. We achieve a number of a neutrosophic crisp semi-compact (resp. neutrosophic crisp semi-Lindelöf) space. Also, we introduce and study the concept of neutrosophic crisp locally semi-compact spaces.

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