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Semi-Idempotents in Neutrosophic Rings

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Abstract: In complex rings or complex fields, the notion of imaginary element i with $i^2 = -1$ or the complex number i is included, while, in the neutrosophic rings, the indeterminate element I where $I^2 = I$ is included. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operations of R. In this paper we obtain a characterization theorem for a semi-idempotent to be in $\langle Z_p \cup I \rangle$, the neutrosophic ring of modulo integers, where p a prime. Here, we discuss only about neutrosophic semi-idempotents in these neutrosophic rings. Several interesting properties about them are also derived and some open problems are suggested.

Keywords: semi-idempotent; neutrosophic rings; modulo neutrosophic rings; neutrosophic semi-idempotent

MSC: 16-XX; 17C27

1. Introduction

According to Gray [1], an element $\alpha \neq 0$ of a ring R is called a semi-idempotent if and only if α is not in the proper two-sided ideal of R generated by $\alpha^2 - \alpha$, that is $\alpha \notin R(\alpha^2 - \alpha)R$ or $R = R(\alpha^2 - \alpha)R$. Here, 0 is a semi-idempotent, which we may term as trivial semi-idempotent. Semi-idempotents have been studied for group rings, semigroup rings and near rings [2–9].

An element I was defined by Smarandache [10] as an indeterminate element. Neutrosophic rings were defined by Vasantha and Smarandache [11]. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and the indeterminate element I ($I^2 = I$) under the operations of R [11]. The concept of neutrosophic rings is further developed and studied in [12–16]. As the newly introduced notions of neutrosophic triplet groups [17,18] and neutrosophic triplet rings [19], neutrosophic triplets in neutrosophic rings [20] and their relations to neutrosophic refined sets [21,22] depend on idempotents, thus the relative study of semi-idempotents will be an innovative research for any researcher interested in these fields. Finding idempotents is discussed in [18,23–25]. One can also characterize and study neutrosophic idempotents in these situations as basically neutrosophic idempotents are trivial neutrosophic semi-idempotents. A new angle to this research can be made by studying quaternion valued functions [26].

We call a semi-idempotents x in $\langle R \cup I \rangle$ as neutrosophic semi-idempotents if $x = a + bI$ and $b \neq 0; a, b \in \langle R \cup I \rangle$. Several interesting results about semi-idempotents are derived for neutrosophic rings in this paper. As the study pivots on idempotents it has much significance for the recent studies on neutrosophic triplets, duplets and refined sets.

Here, the notion of semi-idempotents in the case of neutrosophic rings is introduced and several interesting properties associated with them are analyzed. We discuss only about neutrosophic semi-idempotents in these neutrosophic rings. This paper is organized into three sections. Section 1 is introductory in nature. In Section 2, the notion of semi-idempotents in the case of

$$\langle Z_n \cup I \rangle = \{a + bI | a, b \in Z_n; n < \infty; I^2 = I\}$$

is considered. Section 3 gives conclusions and proposes some conjectures based on our study.

2. Semi-Idempotents in the Modulo Neutrosophic Rings $\langle Z_n \cup I \rangle$

Throughout this paper, $\langle Z_n \cup I \rangle = \{a + bI/a, b \in Z_n, 2 \leq n < \infty; I^2 = I\}$ denotes the neutrosophic ring of modulo integers. We illustrate some semi-idempotents of $\langle Z_n \cup I \rangle$ by examples and derive some interesting results related with them.

Example 1. *Let $S = \langle Z_2 \cup I \rangle = \{a + bI/a, b \in Z_2, I^2 = I\}$ be the neutrosophic ring of modulo integers. Clearly, $I^2 = I$ and $(1 + I)^2 = 1 + I$ are the two non-trivial idempotents of S. Here, 0 and 1 are trivial idempotents of S. Thus, S has no non-trivial semi-idempotents as all idempotents are trivial semi-idempotents of S.*

Example 2. *Let*

$$R = \langle Z_3 \cup I \rangle = \{a + bI | a, b \in Z^3, I^2 = I\} = \{0, 1, 2, I, 2I, 1 + I, 2 + I, 1 + 2I, 2 + 2I\}$$

be the neutrosophic ring of modulo integers. The trivial idempotents of R are 0 and 1. The non-trivial neutrosophic idempotents are I and $1 + 2I$. Thus, the idempotents I and $1 + 2I$ are trivial neutrosophic semi-idempotents of R. Clearly, 2 and $2 + 2I$ are units of R as 2×2 = 1(mod 3) and $2 + 2I \times 2 + 2I$ = 1(mod 3). $1 + I \in R$ is such that

$$(1 + I)^2 - (1 + I) = 1 + 2I + I - (1 + I) = 1 + 2 + 2I = 2I.$$

Thus, $1 + I$ is a semi-idempotent as the ideal generated by $1 + I$ is $\langle (1 + I)^2 - (1 + I) \rangle = \langle 2I \rangle$ is such that $1 + I \notin R$. However, it is important to note that $(1 + I) \in R$ is a unit as $(1 + I)^2 = 1 + 2I + I = 1$, hence $1 + I$ is a unit in R but it is also a non-trivial semi-idempotent of R. $2 + I$ is not a semi-idempotent as

$$(2 + I)^2 - (2 + I) = 1 + 4I + I - (2 + I) = 2 + I;$$

hence the claim. $2 + 2I \in R$ is a unit, now $(2 + 2I)^2 = 4 + 8I + 4I^2 = 1$, thus $2 + 2I$ is a unit. However, $(2 + 2I)^2 - (2 + 2I) = 1 + 1 + I = 2 + I$.

Now, the ideal generated by $\langle 2 + I \rangle$ does not contain $2 + 2I$ as $\langle 2 + I \rangle = \{0, 2 + I, 1 + 2I\}$, thus $2 + 2I$ is also a non-trivial semi-idempotent even though $2 + 2I$ is a unit of R. Thus, it is important to note that units in modulo neutrosophic rings contribute to non-trivial semi-idempotents. Let $P = \{0, 2 + 2I, 2 + I, 1 + 2I, I, 1 + I, 1\}$ be the collection of trivial and non-trivial semi-idempotents. $2I$ is not a semi-idempotent as $(2I)^2 - 2I = I + I = 2I$, hence the claim. Thus, P is not closed under sum or product.

Theorem 1. *Let $S = \{\langle Z_p \cup I \rangle, +, \times\}$ be the ring of neutrosophic modulo integers where p is a prime. x is semi-idempotent if and only if $x \in \langle Z_p \cup I \rangle \setminus \{Z_p I, 0, 1, a + bI$ with $a + b = 0\}$.*

Proof. The elements $x = a + bI \in S$ with $b = 0$ are such that $x^2 - x$ generates the ideal, which is S, thus x is a semi-idempotent. Let $y = a + bI$; if $a = 0$, the ideal generated by y is $Z_p I$, thus $y \in Z_p I \subset S$, hence $y \in Z_p I$, therefore y is not a semi-idempotent.

Consider $z = a + bI \in S$ with $a + b = 0(mod\ p)$; then, $z^2 - z$ generates an ideal M of S such that every element $x = d + cI$ in M is such that $d + c \equiv 0(mod\ p)$, thus z is not a semi-idempotent of S. Let $x = a + bI \in S(a \neq 0, b \neq 0$ and $a + b \neq 0)$.

$$x^2 - x = \begin{cases} m & m \in Z_p \text{ or} \\ nI & n \in Z_p \text{ or} \\ n + mI & m + n \neq 0 \end{cases}$$

If $x^2 - x = m$, then the ideal generated by $x^2 - x$ is S, thus x is a semi-idempotent. If $x^2 - x = nI$, then the ideal generated by nI is Z_pI, thus $x \notin Z_pI$, hence again x is a semi-idempotent. If $x^2 - x = n + mI(m + n \neq 0)$, then the ideal generated by $n + mI$ is S, thus x is a semi-idempotent by using properties of Z_p, p a prime. Hence, the theorem is proved. □

If we take $S = \{\langle Z_n \cup I\rangle, +, \times\}$ as a neutrosophic ring where n is not a prime, it is difficult to find all semi-idempotents.

Example 3. *Let $S = \{\langle Z_{15} \cup I\rangle, +, \times\}$ be the neutrosophic ring. How can the non-trivial semi-idempotents of S be found? Some of the neutrosophic idempotents of S are $\{1 + 9I,\ 6 + 4I,\ 1 + 5I,\ 1 + 14I, 6 + 5I, 6 + 9I, I, 6I, 10I, 10, 6, 6 + 10I, 10 + 11I, 10 + 6I, 10 + 5I\}$.*

The semi-idempotents are $\{1 + I, 1 + 2I, 1 + 3I, 1 + 4I, 1 + 6I, 1 + 7I, 1 + 8I, 1 + 10I, 1 + 11I, 1 + 12I, 1 + 13I, 6 + I, 6 + 2I, 6 + 3I, 6 + 6I, 6 + 7I, 6 + 8I, 6 + 11I, 6 + 12I, 6 + 13I, 6 + 14I, 10 + I, 10 + 2I, 10 + 3I, 10 + 4I, 10 + 7I, 10 + 8I, 10 + 9I, 10 + 10I, 10 + 12I, 10 + 13I, 10 + 14I\}$.

Are there more non-trivial neutrosophic idempotents and semi-idempotents?

However, we are able to find all idempotents and semi-idempotents of S other than the once given. In view of all these, we have the following theorem.

Theorem 2. *Let $S = \{\langle Z_{pq} \cup I\rangle; \times, +\}$ where p and q are two distinct primes:*

1. *There are two idempotents in Z_{pq} say r and s.*
2. *$\{r, s, rI, sI, I, r + tI, s + tI | t \in \{Z_{pq} \setminus 0\}\}$ such that $r + t = s$, 1 or 0 and $s + t = 0, 1$ or r is the partial collection of idempotents and semi-idempotents of S.*

Proof. Given $S = \{\langle Z_{pq} \cup I\rangle, +, \times\}$ is a neutrosophic ring where p and q are primes, we know from [12,17,18,20,23–25] that Z_{pq} has two idempotents r and s to prove $A = \{r, s, rIsI, I, r + tI$ and $s + tI / t \in Z_{pq} \setminus \{0\}\}$ are idempotents or semi-idempotents of $S. \{r, s, rI, sI, I\}$ are non-trivial idempotents of S. Now, $r + tI \in A$ and $(r + tI)^2 - (r + tI) = mI$ as$r^2 = r$, thus the ideal generated by mI does not contain r_tI. Therefore, r_tI is a non-trivial semi-idempotent. Similarly, $s + tI$ is a non-trivial semi-idempotent. Hence, the theorem is proved. □

We in addition to this theorem propose the following problem.

Problem 1. *Let $S = \{\langle Z_{pq} \cup I\rangle, I, \times\}$, where p and q are two distinct primes, be the neutrosophic ring. Can S have non-trivial idempotents and non-trivial semi-idempotents other than the ones mentioned in (b) of the above theorem?*

Problem 2. *Can the collection of all trivial and non-trivial semi-idempotents have any algebraic structure defined on them?*

We give an example of Z_{pqr}, where p, q and r are three distinct primes, for which we find all the neutrosophic idempotents.

Example 4. *Let $S = \{\langle Z_{30} \cup I\rangle, +, \times\}$, be the neutrosophic ring. The idempotents of Z_{30} are 6, 10, 15, 16, 21 and 25. The non-trivial semi-idempotents of S are $\{1 + I, 1 + 2I, 1 + 3I, 1 + 4I, 1 + 6I, 1 + 7I, 1 + 8I, 1 + 10I, 1 + 11I, 1 + 13I, 1 + 12I, 1 + 16I, 1 + 17I, 1 + 18I, 1 + 19I, 1 + 21I, 1 + 22I, 1 + 23I, 1 + 25I, 1 + 26I, 1 + 27I, 1 + 28I\}$.*

$P_1 = \{1+5I, 1+9I, 1+14I, 1+15I, 1+20I, 1+24I, 1+29I\}$ *are non-trivial idempotents of S.* $J_2 = \{6+I, 6+2I, 6+3I, 6+5I, 6+6I, 6+7I, 6+8I, 6+11I, 6+12I, 6+13I, 6+14I, 6+16I, 6+17I,$ $6+18I, 6+20I, 6+21I, 6+22I, 6+23I, 6+26I, 6+27I, 6+28I, 6+29I\}$ *are non-trivial neutrosophic semi-idempotents of S.* $P_2 = \{6+4I, 6+9I, 6+10I, 6+15I, 6+24I, 6+19I, 6+25I\}$ *are non-idempotents of S.*

Now, we list the non-trivial semi-idempotents associated with 10 of Z_{30}. $J_3 = \{10+I, 10+2I,$ $10+3I, 10+4I, 10+7I, 10+8I, 10+9I, 10+10I, 10+11I, 10+12I, 10+13I, 10+14I, 10+16I, 10+17I,$ $10+18I, 10+19I, 10+22I, 10+23I, 10+24I, 10+25I, 10+27I, 10+28I, 10+29I\}$

$P_3 = \{10+5, 10+6I, 10+15I, 10+20I, 10+21I, 10+26I, 10+11I\}$ *are the collection of non-trivial idempotent related with the idempotents. Now, we find the non-trivial idempotents associated with 15:* $J_4 = \{15+2I, 15+3I, 15+4I, 15+7I, 15+8I, 15+9I, 15+11I, 15+12I, 15+13I, 15+14I, 15+17I, 15+18I, 15+19I, 15+20I, 15+22I, 15+23I, 15+24I, 15+25I, 15+26I, 15+27I, 15+28I, 15+29I\}$.

$P_4 = \{15+I, 15+5I, 15+6I, 15+10I, 15+15I, 15+16I, 15+21I\}$ *are the non-trivial idempotents associated with 15. The collection of non-trivial semi-idempotents associated with 16 are:* $J_5 = \{16+I,$ $16+2I, 16+3I, 16+4I, 16+6I, 16+7I, 16+8I, 16+10I, 16+19I, 16+27I, 16+21I, 16+22I, 16+23I, 16+25I, 16+11I, 16+12I, 16+13I, 16+17I, 16+18I, 16+28I.$ $P_5 = \{16+14I, 16+15I, 16+20I, 16+24I, 16+29I, 16+5I, 16+9I\}$ *are the set of non-trivial idempotents related with the idempotent. We find the non-trivial semi-idempotents associated with the idempotent 21:* $J_6 = \{21+I, 21+2I, 21+3I, 21+5I, 21+6I, 21+7I, 21+8I, 21+12I, 21+11I, 21+13I, 21+14I, 21+16I, 21+17I, 21+18I, 21+20I, 21+21I, 21+22I, 21+23I, 21+26I, 21+27I, 21+28I, 21+29I\}$. $P_6 = \{21+4I, 21+9I, 21+10I, 21+15I, 21+19I, 21+24I, 21+25I\}$ *is the collection of non-trivial idempotents related with the real idempotent 21. The collection of all non-trivial semi-idempotents associated with the idempotent 25.* $J_7 = \{25+I, 25+2I, 25+3I, 25+4I, 25+7I, 25+8I, 25+9I, 25+10I, 25+12I, 25+13I, 25+14I, 25+16I, 25+24I, 25+17I, 25+18I, 25+19I, 25+22I, 25+23I, 25+27I, 25+28I, 25+29I\}$ $P_7 = \{25+5I, 25+6I, 25+11I, 25+15I, 25+20I, 25+21I, 25+26I\}$ *are the non-trivial collection of neutrosophic semi-idempotents related with the idempotent 25.*

We tabulate the neutrosophic idempotents associated with the real idempotents in Table 1. Based on that table, we propose some open problems.

Table 1. Idempotents.

S.No	Real	Neutrosophic	Sum	Missing
1	1	$1+5I$ $1+9I$ $1+14I$ $1+15I$ $1+20I$ $1+24I$ $1+29I$	1 + 5 = 6 1 + 9 = 10 1 + 14 = 15 1 + 15 = 16 1 + 20 = 21 1 + 24 = 25 1 + 29 = 0	1
2	6	$6+4I$ $6+9I$ $6+10I$ $6+15I$ $6+24I$ $6+19I$ $6+25I$	6 + 4 = 10 6 + 9 = 15 6 + 10 = 16 6 + 15 = 1 6 + 24 = 0 6 + 19 = 25 6 + 25 ≡ 1	6

Table 1. *Cont.*

S.No	Real	Neutrosophic	Sum	Missing
3	10	$10+5I$ $10+6I$ $10+15I$ $10+20I$ $10+21I$ $10+26I$ $10+11I$	10 + 5 = 15 10 + 6 = 16 10 + 15 = 25 $10 + 20 \equiv 0$ $10 + 21 \equiv 1$ $10 + 26 \equiv 6$ 10 + 11 = 21	10
4	15	$15+I$ $15+5I$ $15+6I$ $15+10I$ $15+15I$ $15+16I$ $15+21I$	15 + 1 = 16 15 + 5 = 20 15 + 6 = 21 15 + 10 = 25 $15 + 15 \equiv 0$ $15 + 16 \equiv 1$ $15 + 21 \equiv 6$	15
5	16	$16+14I$ $16+15I$ $16+20I$ $16+24I$ $16+29I$ $16+5I$ $16+9I$	$16+14 \equiv 0$ $16+15 \equiv 1$ $16+20 \equiv 6$ $16+24 \equiv 10$ $16+29 \equiv 15$ 16 + 5 = 21 16 + 9 = 25	16
6	21	$21+4I$ $21+9I$ $21+10I$ $21+15I$ $21+19I$ $21+24I$ $21+25I$	21 + 4 = 25 $21 + 9 \equiv 0$ $21 + 10 \equiv 1$ $21 + 15 \equiv 6$ $21 + 19 \equiv 10$ $21 + 24 \equiv 15$ $21 + 25 \equiv 16$	21
7	25	$25+I$ $25+5I$ $25+6I$ $25+10I$ $25+16I$ $25+21I$ $25+26I$	$25 + 5 \equiv 0$ $25 + 6 \equiv 1$ $25 + 11 \equiv 6$ $25 + 15 \equiv 10$ $25 + 20 \equiv 15$ $25 + 21 \equiv 16$ $25 + 26 \equiv 21$	25

We see there are eight idempotents including 0 and 1. It is obvious that using 0 we get only idempotents or trivial semi-idempotents.

In view of all these, we conjecture the following.

Conjecture 1. *Let $S = \{\langle Z_n \cup I\rangle, +, \times\}$ be the neutrosophic ring $n = pqr$, where p, q and r are three distinct primes.*

1. *$Z_n = Z_{pqr}$ has only six non-trivial idempotents associated with it.*
2. *If m_1, m_2, m_3, m_4, m_5 and m_6 are the idempotents, then, associated with each real idempotent m_i, we have seven non-trivial neutrosophic idempotents associated with it, i.e. $\{m_i + n_j I, j = 1, 2, \ldots, 7\}$, such that $m_i + n_j \equiv t$, where t_j takes the seven distinct values from the set $\{0, 1, m_k, k \neq i; k = 1, 2, 3, \ldots 6\}$. $i = 1, 2, \ldots, 6$.*

This has been verified for large values of p, q and r, where p, q and r are three distinct primes.

3. Conjectures, Discussion and Conclusions

We have characterized the neutrosophic semi-idempotents in $\langle Z_p \cup I \rangle$, with p a prime. However, it is interesting to find neutrosophic semi-idempotents of $\langle Z_n \cup I \rangle$, with n a non-prime composite number. Here, we propose a few new open conjectures about idempotents in Z_n and semi-idempotents in $\langle Z_n \cup I \rangle$.

Conjecture 2. *Given $\langle Z_n \cup I \rangle$, where $n = p_1, p_2, \ldots p_t; t > 2$ and p_is are all distinct primes, find:*

1. *the number of idempotents in Z_n;*
2. *the number of idempotents in $\langle Z_n \cup I \rangle \setminus Z_n$;*
3. *the number of non-trivial semi-idempotents in Z_n; and*
4. *the number of non-trivial semi-idempotents in $\langle Z_n \cup I \rangle \setminus Z_n$.*

Conjecture 3. *Prove if $\langle Z_n \cup I \rangle$ and $\langle Z_m \cup I \rangle$ are two neutrosophic rings where $n > m$ and $n = p^t q$ ($t > 2$, and p and q two distinct primes) and $m = p_1 p_2 \ldots p_s$ where p_is are distinct primes. $1 \leq i \leq s$, then*

1. *prove Z_n has more number of idempotents than Z_m; and*
2. *prove $\langle Z_m \cup I \rangle$ has more number of idempotents and semi-idempotents than $\langle Z_n \cup I \rangle$.*

Finding idempotents in the case of Z_n has been discussed and problems are proposed in [18,23,24]. Further, the neutrosophic triplets in Z_n are contributed by Z_n. In the case of neutrosophic duplets, we see units in Z_n contribute to them. Both units and idempotents contribute in general to semi-idempotents.

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