TÜBİTAK

Turkish Journal of Mathematics

http://journals.tubitak.gov.tr/math/

Research Article

Turk J Math
(2019) 43: 498 – 510
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doi:10.3906/mat-1805-110

Separation axioms on neutrosophic soft topological spaces

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Received: 22.05.2018 • **Accepted/Published Online:** 31.12.2018 • **Final Version:** 18.01.2019

Abstract: A neutrosophic set, proposed by Smarandache, considers a truth membership function, an indeterminacy membership function, and a falsity membership function. Neutrosophic soft sets combined by Maji have been utilized successfully to model uncertainty in several areas of application such as control, reasoning, pattern recognition, and computer vision. In the present paper, some basic notions of neutrosophic soft sets have been redefined and the neutrosophic soft point concept has been introduced. Later we give the neutrosophic soft T_i-space and the relationships between them are discussed in detail.

Key words: Neutrosophic soft set, neutrosophic soft point, neutrosophic soft separation axioms

1. Introduction

The concept of a neutrosophic set was introduced by Smarandache [13]. This theory is a generalization of classical sets, fuzzy set theory [14], intuitionistic fuzzy set theory [1], etc. Some works have been done on neutrosophic sets by some researchers in many area of mathematics [4, 11]. Many practical problems in economics, engineering, environment, social science, medical science, etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. These difficulties may be due to the inadequacy of the theories of parametrization tools. Each of these theories has its inherent difficulties, as was pointed out by Molodtsov in [10]. Molodtsov initiated a completely new approach for modeling uncertainties and applied it successfully in directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and so on. Shabir and Naz [12] first introduced the notion of soft topological spaces, which are defined over an initial universe with a fixed set of parameters, and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces were also done by some authors in [2, 3, 6, 8]. Neutrosophic soft sets were first defined by Maji [9] and later this concept was modified by Deli and Broumi [7]. Later neutrosophic soft topological spaces were presented by Bera in [5].

The first aim of this paper is to reintroduce the concept of operations on neutrosophic soft sets and construct a neutrosophic soft topology on neutrosophic soft sets. Later the notions of neutrosophic soft point and neutrosophic soft neighborhood are defined and some of their important properties are studied. Finally, the concept of separation axioms of neutrosophic soft topological spaces is given. Furthermore, properties of neutrosophic soft T_i-spaces $(i = 0, 1, 2, 3, 4)$ and some relations between them are discussed. Characterization

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2010 *AMS Mathematics Subject Classification:* 54A05, 03E75

theorems of them are also proved. We hope that these results will be useful for future study on neutrosophic soft topology to carry out a general framework for practical applications.

2. Preliminaries

In this section we now state certain useful definitions, theorems, and several existing results for neutrosophic soft sets that we require in the next sections.

Definition 2.1 *[13] A neutrosophic set A on the universe set X is defined as:*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T,\ I,\ F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2 *[10] Let X be an initial universe, E be a set of all parameters, and $P(X)$ denote the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of e-elements of the soft set (F, E), or as the set of e-approximate elements of the soft set, i.e.*

$$(F, E) = \{(e, F(e)) : e \in E,\ F : E \to P(X)\}.$$

After the neutrosophic soft set was defined by Maji [9], this concept was modified by Deli and Broumi [7] as given below:

Definition 2.3 *[7] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $\left(\widetilde{F}, E\right)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $\left(\widetilde{F}, E\right)$. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs:*

$$\left(\widetilde{F}, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

where $T_{\widetilde{F}(e)}(x),\ I_{\widetilde{F}(e)}(x),\ F_{\widetilde{F}(e)}(x) \in [0, 1]$ are respectively called the truth-membership, indeterminacy-membership, and falsity-membership function of $\widetilde{F}(e)$. Since the supremum of each $T,\ I,\ F$ is 1, the inequality $0 \leq T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.4 *[5] Let $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set over the universe set X. The complement of $\left(\widetilde{F}, E\right)$ is denoted by $\left(\widetilde{F}, E\right)^c$ and is defined by:*

$$\left(\widetilde{F}, E\right)^c = \left\{\left(e, \left\langle x, F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x), T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$$

It is obvious that $\left(\left(\widetilde{F}, E\right)^c\right)^c = \left(\widetilde{F}, E\right)$.

Definition 2.5 *[9] Let* $\left(\widetilde{F},E\right)$ *and* $\left(\widetilde{G},E\right)$ *be two neutrosophic soft sets over the universe set* X. $\left(\widetilde{F},E\right)$ *is said to be a neutrosophic soft subset of* $\left(\widetilde{G},E\right)$ *if* $T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \leq I_{\widetilde{G}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x)$, $\forall e \in E,\ \forall x \in X$. *It is denoted by* $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)$. $\left(\widetilde{F},E\right)$ *is said to be neutrosophic soft equal to* $\left(\widetilde{G},E\right)$ *if* $\left(\widetilde{F},E\right)$ *is a neutrosophic soft subset of* $\left(\widetilde{G},E\right)$ *and* $\left(\widetilde{G},E\right)$ *is a neutrosophic soft subset of* $\left(\widetilde{F},E\right)$. *It is denoted by* $\left(\widetilde{F},E\right) = \left(\widetilde{G},E\right)$.

3. Neutrosophic soft points and their properties

Definition 3.1 *Let* $\left(\widetilde{F}_1,E\right)$ *and* $\left(\widetilde{F}_2,E\right)$ *be two neutrosophic soft sets over the universe set* X. *Then their union is denoted by* $\left(\widetilde{F}_1,E\right) \cup \left(\widetilde{F}_2,E\right) = \left(\widetilde{F}_3,E\right)$ *and is defined by:*

$$\left(\widetilde{F}_3,E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

where

$$\begin{aligned}
T_{\widetilde{F}_3(e)}(x) &= \max\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\}, \\
I_{\widetilde{F}_3(e)}(x) &= \max\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\}, \\
F_{\widetilde{F}_3(e)}(x) &= \min\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}.
\end{aligned}$$

Definition 3.2 *Let* $\left(\widetilde{F}_1,E\right)$ *and* $\left(\widetilde{F}_2,E\right)$ *be two neutrosophic soft sets over the universe set* X. *Then their intersection is denoted by* $\left(\widetilde{F}_1,E\right) \cap \left(\widetilde{F}_2,E\right) = \left(\widetilde{F}_3,E\right)$ *and is defined by:*

$$\left(\widetilde{F}_3,E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

where

$$\begin{aligned}
T_{\widetilde{F}_3(e)}(x) &= \min\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\}, \\
I_{\widetilde{F}_3(e)}(x) &= \min\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\}, \\
F_{\widetilde{F}_3(e)}(x) &= \max\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}.
\end{aligned}$$

Definition 3.3 *A neutrosophic soft set* $\left(\widetilde{F},E\right)$ *over the universe set* X *is said to be a null neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 0,\ I_{\widetilde{F}(e)}(x) = 0,\ F_{\widetilde{F}(e)}(x) = 1;\ \forall e \in E, \forall x \in X$. *It is denoted by* $0_{(X,E)}$.

Definition 3.4 *A neutrosophic soft set* $\left(\widetilde{F},E\right)$ *over the universe set* X *is said to be an absolute neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 1,\ I_{\widetilde{F}(e)}(x) = 1,\ F_{\widetilde{F}(e)}(x) = 0;\ \forall e \in E, \forall x \in X$. *It is denoted by* $1_{(X,E)}$.

Clearly, $0^c_{(X,E)} = 1_{(X,E)}$ *and* $1^c_{(X,E)} = 0_{(X,E)}$.

Definition 3.5 *Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X and $\tau \subset NSS(X,E)$. Then τ is said to be a neutrosophic soft topology on X if:*

1. $0_{(X,E)}$ *and* $1_{(X,E)}$ *belong to* τ,
2. *the union of any number of neutrosophic soft sets in* τ *belongs to* τ,
3. *the intersection of a finite number of neutrosophic soft sets in* τ *belongs to* τ.

Then (X,τ,E) is said to be a neutrosophic soft topological space over X. Each member of τ is said to be a neutrosophic soft open set.

Definition 3.6 *Let (X,τ,E) be a neutrosophic soft topological space over X and $\left(\widetilde{F},E\right)$ be a neutrosophic soft set over X. Then $\left(\widetilde{F},E\right)$ is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.*

Definition 3.7 *Let NS be the family of all neutrosophic sets over the universe set X and $x \in X$. The neutrosophic set $x_{(\alpha,\beta,\gamma)}$ is called a neutrosophic point, for $0 < \alpha,\beta,\gamma \leq 1$, and is defined as follows:*

$$x_{(\alpha,\beta,\gamma)}\left(y\right) = \begin{cases} \left(\alpha,\beta,\gamma\right), & \text{if } y = x \\ \left(0,0,1\right), & \text{if } y \neq x. \end{cases}$$

It is clear that every neutrosophic set is the union of its neutrosophic points.

Definition 3.8 *Suppose that $X = \{x_1, x_2\}$. Then neutrosophic set*

$$A = \left\{\left\langle x_1, 0.1, 0.3, 0.5\right\rangle, \left\langle x_2, 0.5, 0.4, 0.7\right\rangle\right\}$$

is the union of neutrosophic points $x_{1_{(0.1,0.3,0.5)}}$ and $x_{2_{(0.5,0.4,0.7)}}$.

Now we define the concept of neutrosophic soft points for neutrosophic soft sets.

Definition 3.9 *Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X. Then neutrosophic soft set $x^e_{(\alpha,\beta,\gamma)}$ is called a neutrosophic soft point, for every $x \in X, 0 < \alpha,\beta,\gamma \leq 1$, $e \in E$, and is defined as follows:*

$$x^e_{(\alpha,\beta,\gamma)}\left(e'\right)\left(y\right) = \begin{cases} \left(\alpha,\beta,\gamma\right) & \text{if } e' = e \text{ and } y = x, \\ \left(0,0,1\right) & \text{if } e' \neq e \text{ or } y \neq x. \end{cases}$$

Definition 3.10 *Suppose that the universe set X is given by $X = \{x_1, x_2\}$ and the set of parameters by $E = \{e_1, e_2\}$. Let us consider neutrosophic soft set $\left(\widetilde{F},E\right)$ over the universe X as follows:*

$$\left(\widetilde{F},E\right) = \left\{ \begin{array}{c} e_1 = \left\{\left\langle x_1, 0.3, 0.7, 0.6\right\rangle, \left\langle x_2, 0.4, 0.3, 0.8\right\rangle\right\}, \\ e_2 = \left\{\left\langle x_1, 0.4, 0.6, 0.8\right\rangle, \left\langle x_2, 0.3, 0.7, 0.2\right\rangle\right\} \end{array} \right\}.$$

It is clear that $\left(\widetilde{F},E\right)$ *is the union of its neutrosophic soft points* $x^{e_1}_{1(0.3,0.7,0.6)}, x^{e_2}_{1(0.4,0.6,0.8)}, x^{e_1}_{2(0.4,0.3,0.8)}$, *and* $x^{e_2}_{2(0.3,0.7,0.2)}$. *Here*

$$\begin{aligned}
x^{e_1}_{1(0.3,0.7,0.6)} &= \left\{ \begin{array}{c} e_1 = \{\langle x_1, 0.3, 0.7, 0.6\rangle, \langle x_2, 0, 0, 1\rangle\}, \\ e_2 = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0, 0, 1\rangle\} \end{array} \right\}, \\
x^{e_2}_{1(0.4,0.6,0.8)} &= \left\{ \begin{array}{c} e_1 = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0, 0, 1\rangle\}, \\ e_2 = \{\langle x_1, 0.4, 0.6, 0.8\rangle, \langle x_2, 0, 0, 1\rangle\} \end{array} \right\}, \\
x^{e_1}_{2(0.4,0.3,0.8)} &= \left\{ \begin{array}{c} e_1 = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0.4, 0.3, 0.8\rangle\}, \\ e_2 = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0, 0, 1\rangle\} \end{array} \right\}, \\
x^{e_2}_{2(0.3,0.7,0.2)} &= \left\{ \begin{array}{c} e_1 = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0, 0, 1\rangle\}, \\ e_2 = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0.3, 0.7, 0.2\rangle\} \end{array} \right\}.
\end{aligned}$$

Definition 3.11 *Let* $\left(\widetilde{F},E\right)$ *be a neutrosophic soft set over the universe set* X. *We say that* $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$ *read as belonging to the neutrosophic soft set* $\left(\widetilde{F},E\right)$ *whenever* $\alpha \leq T_{\widetilde{F}(e)}(x), \beta \leq I_{\widetilde{F}(e)}(x)$ *and* $\gamma \geq F_{\widetilde{F}(e)}(x)$.

Definition 3.12 *Let* (X,τ,E) *be a neutrosophic soft topological space over* X. *A neutrosophic soft set* $\left(\widetilde{F},E\right)$ *in* (X,τ,E) *is called a neutrosophic soft neighborhood of the neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$, *if there exists a neutrosophic soft open set* $\left(\widetilde{G},E\right)$ *such that* $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right)$.

Theorem 3.13 *Let* (X,τ,E) *be a neutrosophic soft topological space and* $\left(\widetilde{F},E\right)$ *be a neutrosophic soft set over* X. *Then* $\left(\widetilde{F},E\right)$ *is a neutrosophic soft open set if and only if* $\left(\widetilde{F},E\right)$ *is a neutrosophic soft neighborhood of its neutrosophic soft points.*

Proof Let $\left(\widetilde{F},E\right)$ be a neutrosophic soft open set and $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$. Then $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right) \subset \left(\widetilde{F},E\right)$. Therefore, $\left(\widetilde{F},E\right)$ is a neutrosophic soft neighborhood of $x^e_{(\alpha,\beta,\gamma)}$.

Conversely, let $\left(\widetilde{F},E\right)$ be a neutrosophic soft neighborhood of its neutrosophic soft points. Let $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$. Since $\left(\widetilde{F},E\right)$ is a neutrosophic soft neighborhood of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$, there exists $\left(\widetilde{G},E\right) \in \tau$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right)$. Since $\left(\widetilde{F},E\right) = \cup\left\{x^e_{(\alpha,\beta,\gamma)} : x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)\right\}$, it follows that $\left(\widetilde{F},E\right)$ is a union of neutrosophic soft open sets and hence $\left(\widetilde{F},E\right)$ is a neutrosophic soft open set.

The neighborhood system of a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$, denoted by $Ų\left(x^e_{(\alpha,\beta,\gamma)},E\right)$, is the family of all its neighborhoods. □

Theorem 3.14 *The neighborhood system* $Ų\left(x^e_{(\alpha,\beta,\gamma)},E\right)$ *at* $x^e_{(\alpha,\beta,\gamma)}$ *in a neutrosophic soft topological space* (X,τ,E) *has the following properties:*

1) If $\left(\widetilde{F},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$*, then* $x^{e}_{(\alpha,\beta,\gamma)}\in\left(\widetilde{F},E\right)$*;*

2) If $\left(\widetilde{F},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$ *and* $\left(\widetilde{F},E\right)\subset\left(\widetilde{H},E\right)$*, then* $\left(\widetilde{H},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$*;*

3) If $\left(\widetilde{F},E\right)$*,* $\left(\widetilde{G},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$*, then* $\left(\widetilde{F},E\right)\cap\left(\widetilde{G},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$*;*

4) If $\left(\widetilde{F},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$*, then there exists a* $\left(\widetilde{G},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$ *such that* $\left(\widetilde{G},E\right)\in Ų\left(y^{e'}_{(\alpha',\beta',\gamma')},E\right)$*, for each* $y^{e'}_{(\alpha',\beta',\gamma')}\in\left(\widetilde{G},E\right)$*.*

Proof The proof of 1), 2), and 3) is obvious from Definition 3.12.

4) If $\left(\widetilde{F},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$, then there exists a neutrosophic soft open set $\left(\widetilde{G},E\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)}\in\left(\widetilde{G},E\right)\subset\left(\widetilde{F},E\right)$. From Proposition 3.13, $\left(\widetilde{G},E\right)\in Ų\left(x^{e}_{(\alpha,\beta,\gamma)},E\right)$, so for each $y^{e'}_{(\alpha',\beta',\gamma')}\in\left(\widetilde{G},E\right)$, $\left(\widetilde{G},E\right)\in Ų\left(y^{e'}_{(\alpha',\beta',\gamma')},E\right)$ is obtained. □

Definition 3.15 *Let* $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *be two neutrosophic soft points. For the neutrosophic soft points* $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *over a common universe* X*, we say that the neutrosophic soft points are distinct points if* $x^{e}_{(\alpha,\beta,\gamma)}\cap\ y^{e'}_{(\alpha',\beta',\gamma')}=0_{(X,E)}$*.*

It is clear that $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *are distinct neutrosophic soft points if and only if* $x\neq y$ *or* $e'\neq e$*.*

4. Neutrosophic soft separation axioms

In this section, we consider neutrosophic soft separation axioms and neutrosophic soft topological subspace consisting of distinct neutrosophic soft points of neutrosophic soft topological space over X.

Definition 4.1 *a) Let* (X,τ,E) *be a neutrosophic soft topological space over* X*, and* $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *are distinct neutrosophic soft points. If there exist neutrosophic soft open sets* $\left(\widetilde{F},E\right)$ *and* $\left(\widetilde{G},E\right)$ *such that*

$$\begin{aligned}x^{e}_{(\alpha,\beta,\gamma)} &\in \left(\widetilde{F},E\right) \text{ and } x^{e}_{(\alpha,\beta,\gamma)}\cap\left(\widetilde{G},E\right)=0_{(X,E)} \text{ or}\\ y^{e'}_{(\alpha',\beta',\gamma')} &\in \left(\widetilde{G},E\right) \text{ and } y^{e'}_{(\alpha',\beta',\gamma')}\cap\left(\widetilde{F},E\right)=0_{(X,E)},\end{aligned}$$

then (X,τ,E) *is called a neutrosophic soft* T_0*-space.*

b) Let (X,τ,E) *be a neutrosophic soft topological space over* X *and* $x^{e}_{(\alpha,\beta,\gamma)}$*, and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *are distinct neutrosophic soft points. If there exist neutrosophic soft open sets* $\left(\widetilde{F},E\right)$ *and* $\left(\widetilde{G},E\right)$ *such that*

$$\begin{aligned}x^{e}_{(\alpha,\beta,\gamma)} &\in \left(\widetilde{F},E\right), x^{e}_{(\alpha,\beta,\gamma)}\cap\left(\widetilde{G},E\right)=0_{(X,E)} \text{ and}\\ y^{e'}_{(\alpha',\beta',\gamma')} &\in \left(\widetilde{G},E\right),\ y^{e'}_{(\alpha',\beta',\gamma')}\cap\left(\widetilde{F},E\right)=0_{(X,E)},\end{aligned}$$

then (X,τ,E) *is called a neutrosophic soft* T_1*-space.*

c) Let (X,τ,E) be a neutrosophic soft topological space over X, and $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points. If there exist neutrosophic soft open sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ such that

$$x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right),\ y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right) \text{ and } \left(\widetilde{F},E\right) \cap \left(\widetilde{G},E\right) = 0_{(X,E)},$$

then (X,τ,E) is called a neutrosophic soft T_2-space.

Example 4.2 *Let $X = \{x_1, x_2\}$ be a universe set, $E = \{e_1, e_2\}$ be a parameters set, and $x^{e_1}_{1(0.1,0.4,0.7)}$, $x^{e_2}_{1(0.2,0.5,0.6)}$, $x^{e_1}_{2(0.3,0.3,0.5)}$, and $x^{e_2}_{2(0.4,0.4,0.4)}$ be neutrosophic soft points. Then the family $\tau = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F}_1,E\right), \left(\widetilde{F}_2,E\right), \left(\widetilde{F}_3,E\right), \left(\widetilde{F}_4,E\right), \left(\widetilde{F}_5,E\right), \left(\widetilde{F}_6,E\right), \left(\widetilde{F}_7,E\right), \left(\widetilde{F}_8,E\right)\right\}$, where*

$$\left(\widetilde{F}_1,E\right) = \left\{x^{e_1}_{1(0.1,0.4,0.7)}\right\},$$

$$\left(\widetilde{F}_2,E\right) = \left\{x^{e_2}_{1(0.2,0.5,0.6)}\right\},$$

$$\left(\widetilde{F}_3,E\right) = \left\{x^{e_1}_{2(0.3,0.3,0.5)}\right\},$$

$$\left(\widetilde{F}_4,E\right) = \left(\widetilde{F}_1,E\right) \cup \left(\widetilde{F}_2,E\right),$$

$$\left(\widetilde{F}_5,E\right) = \left(\widetilde{F}_1,E\right) \cup \left(\widetilde{F}_3,E\right),$$

$$\left(\widetilde{F}_6,E\right) = \left(\widetilde{F}_2,E\right) \cup \left(\widetilde{F}_3,E\right),$$

$$\left(\widetilde{F}_7,E\right) = \left(\widetilde{F}_1,E\right) \cup \left(\widetilde{F}_2,E\right) \cup \left(\widetilde{F}_3,E\right),$$

$$\left(\widetilde{F}_8,E\right) = \left\{x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}, x^{e_2}_{2(0.4,0.4,0.4)}\right\},$$

is a neutrosophic soft topology over X. Hence, (X,τ,E) is a neutrosophic soft topological space over X. Also, (X,τ,E) is a neutrosophic soft T_0-space but not a neutrosophic soft T_1-space because for neutrosophic soft points $x^{e_1}_{1(0.1,0.4,0.7)}$ and $x^{e_2}_{2(0.4,0.4,0.4)}$, (X,τ,E) is not a neutrosophic soft T_1-space.

Example 4.3 *Let $X = \mathbb{N}$ be a natural numbers set and $E = \{e\}$ be a parameter set. Here $n^{e}_{(\alpha_n,\beta_n,\gamma_n)}$ are neutrosophic soft points. Here we can give $(\alpha_n,\beta_n,\gamma_n)$ appropriate values and the neutrosophic soft points $n^{e}_{(\alpha_n,\beta_n,\gamma_n)}, m^{e}_{(\alpha_m,\beta_m,\gamma_m)}$ are distinct neutrosophic soft points if and only if $n \neq m$. It is clear that there is one-to-one compatibility between the set of natural numbers and the set of neutrosophic soft points $N^e = \left\{n^{e}_{(\alpha_n,\beta_n,\gamma_n)}\right\}$. Then we give cofinite topology on this set. Then neutrosophic soft set $\left(\widetilde{F},E\right)$ is a neutrosophic soft open set if and only if the finite neutrosophic soft point is discarded from N^e. Hence, (X,τ,E) is a neutrosophic soft T_1-space but not a neutrosophic soft T_2-space.*

Example 4.4 *Let* $X = \{x_1, x_2\}$ *be a universe set,* $E = \{e_1, e_2\}$ *be a parameters set, and* $x^{e_1}_{1(0.1,0.4,0.7)}$, $x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}$, *and* $x^{e_2}_{2(0.4,0.4,0.4)}$ *be neutrosophic soft points. Then the family* $\tau = \left\{0_{(X,E)}, 1_{(X,E)},\right.$ $\left.\left(\widetilde{F}_1, E\right), \left(\widetilde{F}_2, E\right), ..., \left(\widetilde{F}_{15}, E\right)\right\}$, *where*

$$\left(\widetilde{F}_1, E\right) = \left\{x^{e_1}_{1(0.1,0.4,0.7)}\right\},$$

$$\left(\widetilde{F}_2, E\right) = \left\{x^{e_2}_{1(0.2,0.5,0.6)}\right\},$$

$$\left(\widetilde{F}_3, E\right) = \left\{x^{e_1}_{2(0.3,0.3,0.5)}\right\},$$

$$\left(\widetilde{F}_4, E\right) = \left\{x^{e_2}_{2(0.4,0.4,0.4)}\right\},$$

$$\left(\widetilde{F}_5, E\right) = \left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_2, E\right),$$

$$\left(\widetilde{F}_6, E\right) = \left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_3, E\right),$$

$$\left(\widetilde{F}_7, E\right) = \left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_4, E\right),$$

$$\left(\widetilde{F}_8, E\right) = \left(\widetilde{F}_2, E\right) \cup \left(\widetilde{F}_3, E\right),$$

$$\left(\widetilde{F}_9, E\right) = \left(\widetilde{F}_2, E\right) \cup \left(\widetilde{F}_4, E\right),$$

$$\left(\widetilde{F}_{10}, E\right) = \left(\widetilde{F}_3, E\right) \cup \left(\widetilde{F}_4, E\right),$$

$$\left(\widetilde{F}_{11}, E\right) = \left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_2, E\right) \cup \left(\widetilde{F}_3, E\right),$$

$$\left(\widetilde{F}_{12}, E\right) = \left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_2, E\right) \cup \left(\widetilde{F}_4, E\right),$$

$$\left(\widetilde{F}_{13}, E\right) = \left(\widetilde{F}_2, E\right) \cup \left(\widetilde{F}_3, E\right) \cup \left(\widetilde{F}_4, E\right),$$

$$\left(\widetilde{F}_{14}, E\right) = \left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_3, E\right) \cup \left(\widetilde{F}_4, E\right),$$

$$\left(\widetilde{F}_{15}, E\right) = \left\{x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}, x^{e_2}_{2(0.4,0.4,0.4)}\right\},$$

is a neutrosophic soft topology over X. *Hence,* (X, τ, E) *is a neutrosophic soft topological space over* X. *Also,* (X, τ, E) *is a neutrosophic soft* T_2*-space.*

Theorem 4.5 *Let* (X, τ, E) *be a neutrosophic soft topological space over* X. *Then* (X, τ, E) *is a neutrosophic soft* T_1*-space if and only if each neutrosophic soft point is a neutrosophic soft closed set.*

Proof Let (X,τ,E) be a neutrosophic soft T_1-space and $x^{e}_{(\alpha,\beta,\gamma)}$ be an arbitrary neutrosophic soft point. We show that $\left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c}$ is a neutrosophic soft open set. Let $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c}$; then $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points. Hence, $x \neq y$ or $e' \neq e$.

Since (X,τ,E) is a neutrosophic soft T_1-space, there exists a neutrosophic soft open set $\left(\widetilde{G},E\right)$ such that

$$y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right) \text{ and } x^{e}_{(\alpha,\beta,\gamma)} \cap \left(\widetilde{G},E\right) = 0_{(X,E)}.$$

Then, since $x^{e}_{(\alpha,\beta,\gamma)} \cap \left(\widetilde{G},E\right) = 0_{(X,E)}$, we have $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right) \subset \left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c}$. This implies that $\left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c}$ is a neutrosophic soft open set, i.e. $x^{e}_{(\alpha,\beta,\gamma)}$ is a neutrosophic soft closed set.

Suppose that each neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)}$ is a neutrosophic soft closed set. Then $\left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c}$ is a neutrosophic soft open set. Let $x^{e}_{(\alpha,\beta,\gamma)} \cap y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$. Thus, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c}$ and $x^{e}_{(\alpha,\beta,\gamma)} \cap \left(x^{e}_{(\alpha,\beta,\gamma)}\right)^{c} = 0_{(X,E)}$. Therefore, (X,τ,E) is a neutrosophic soft T_1-space over X. □

Theorem 4.6 *Let (X,τ,E) be a neutrosophic soft topological space over X. (X,τ,E) is a neutrosophic soft T_2-space iff for distinct neutrosophic soft points $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft open set $\left(\widetilde{F},E\right)$ containing $x^{e}_{(\alpha,\beta,\gamma)}$ but not $y^{e'}_{(\alpha',\beta',\gamma')}$ such that $y^{e'}_{(\alpha',\beta',\gamma')} \notin \overline{\left(\widetilde{F},E\right)}$.*

Proof Let $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ be two neutrosophic soft points in neutrosophic soft T_2-space (X,τ,E). Then there exist disjoint neutrosophic soft open sets $\left(\widetilde{F},E\right), \left(\widetilde{G},E\right)$ such that

$$x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right),\ y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right).$$

Since $x^{e}_{(\alpha,\beta,\gamma)} \cap y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$ and $\left(\widetilde{F},E\right) \cap \left(\widetilde{G},E\right) = 0_{(X,E)}$, $y^{e'}_{(\alpha',\beta',\gamma')} \notin \left(\widetilde{F},E\right)$. It implies that $y^{e'}_{(\alpha',\beta',\gamma')} \notin \overline{\left(\widetilde{F},E\right)}$.

Next suppose that, for distinct neutrosophic soft points $x^{e}_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft open set $\left(\widetilde{F},E\right)$ containing $x^{e}_{(\alpha,\beta,\gamma)}$ but not $y^{e'}_{(\alpha',\beta',\gamma')}$ such that $y^{e'}_{(\alpha',\beta',\gamma')} \notin \overline{\left(\widetilde{F},E\right)}$. Then $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\overline{\left(\widetilde{F},E\right)}\right)^{c}$, i.e. $\left(\widetilde{F},E\right)$ and $\left(\overline{\left(\widetilde{F},E\right)}\right)^{c}$ are disjoint neutrosophic soft open sets containing $x^{e}_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$, respectively. □

Theorem 4.7 *Let (X,τ,E) be a neutrosophic soft T_1-space for every neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right) \in \tau$. If there exists a neutrosophic soft open set $\left(\widetilde{G},E\right)$ such that*

$$x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right) \subset \overline{\left(\widetilde{G},E\right)} \subset \left(\widetilde{F},E\right),$$

then (X,τ,E) is a neutrosophic soft T_2-space.

Proof Suppose that $x^e_{(\alpha,\beta,\gamma)} \cap y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$. Since (X,τ,E) is a neutrosophic soft T_1-space, $x^e_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ are neutrosophic soft closed sets in τ. Thus, $x^e_{(\alpha,\beta,\gamma)} \in \left(y^{e'}_{(\alpha',\beta',\gamma')}\right)^c \in \tau$. Then there exists a neutrosophic soft open set $\left(\widetilde{G},E\right)$ in τ such that

$$x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right) \subset \overline{\left(\widetilde{G},E\right)} \subset \left(y^{e'}_{(\alpha',\beta',\gamma')}\right)^c.$$

Hence, we have $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\overline{\left(\widetilde{G},E\right)}\right)^c$, $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)$, and $\left(\widetilde{G},E\right) \cap \left(\overline{\left(\widetilde{G},E\right)}\right)^c = 0_{(X,E)}$, i.e. (X,τ,E) is a neutrosophic soft T_2-space. □

Remark 4.8 *Let (X,τ,E) be a neutrosophic soft T_i-space for $i=0,1,2$. For each $x \neq y$, neutrosophic points $x_{(\alpha,\beta,\gamma)}$ and $y_{(\alpha',\beta',\gamma')}$ have neighborhoods satisfying conditions of T_i-space in neutrosophic topological space (X,τ_e) for each $e \in E$ because $x^e_{(\alpha,\beta,\gamma)}$ and $y^e_{(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points.*

Definition 4.9 *Let (X,τ,E) be a neutrosophic soft topological space over X, $\left(\widetilde{F},E\right)$ be a neutrosophic soft closed set, and $x^e_{(\alpha,\beta,\gamma)} \cap \left(\widetilde{F},E\right) = 0_{(X,E)}$. If there exist neutrosophic soft open sets $\left(\widetilde{G_1},E\right)$ and $\left(\widetilde{G_2},E\right)$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G_1},E\right)$, $\left(\widetilde{F},E\right) \subset \left(\widetilde{G_2},E\right)$, and $\left(\widetilde{G_1},E\right) \cap \left(\widetilde{G_2},E\right) = 0_{(X,E)}$, then (X,τ,E) is called a neutrosophic soft regular space. (X,τ,E) is said to be a neutrosophic soft T_3-space if it is both a neutrosophic soft regular and neutrosophic soft T_1-space.*

Theorem 4.10 *Let (X,τ,E) be a neutrosophic soft topological space over X. (X,τ,E) is a neutrosophic soft T_3-space if and only if for every $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right) \in \tau$, there exists $\left(\widetilde{G},E\right) \in \tau$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right) \subset \overline{\left(\widetilde{G},E\right)} \subset \left(\widetilde{F},E\right)$.*

Proof Let (X,τ,E) be a neutrosophic soft T_3-space and $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right) \in \tau$. Since (X,τ,E) is a neutrosophic soft T_3-space for the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ and neutrosophic soft closed set $\left(\widetilde{F},E\right)^c$, there exist $\left(\widetilde{G_1},E\right), \left(\widetilde{G_2},E\right) \in \tau$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G_1},E\right)$, $\left(\widetilde{F},E\right)^c \subset \left(\widetilde{G_2},E\right)$, and $\left(\widetilde{G_1},E\right) \cap \left(\widetilde{G_2},E\right) = 0_{(X,E)}$. Thus, we have $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G_1},E\right) \subset \left(\widetilde{G_2},E\right)^c \subset \left(\widetilde{F},E\right)$. Since $\left(\widetilde{G_2},E\right)^c$ is a neutrosophic soft closed set, $\overline{\left(\widetilde{G_1},E\right)} \subset \left(\widetilde{G_2},E\right)^c$.

Conversely, let $x^e_{(\alpha,\beta,\gamma)} \cap \left(\widetilde{H},E\right) = 0_{(X,E)}$ and $\left(\widetilde{H},E\right)$ be a neutrosophic soft closed set. Thus, $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{H},E\right)^c$ and from the condition of the theorem, we have $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right) \subset \overline{\left(\widetilde{G},E\right)} \subset \left(\widetilde{H},E\right)^c$. Then $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)$, $\left(\widetilde{H},E\right) \subset \left(\overline{\left(\widetilde{G},E\right)}\right)^c$, and $\left(\widetilde{G},E\right) \cap \left(\overline{\left(\widetilde{G},E\right)}\right)^c = 0_{(X,E)}$ are satisfied, i.e. (X,τ,E) is a neutrosophic soft T_3-space. □

Definition 4.11 *A neutrosophic soft topological space* (X,τ,E) *over* X *is called a neutrosophic soft normal space if for every pair of disjoint neutrosophic soft closed sets* $\left(\widetilde{F_1},E\right),\left(\widetilde{F_2},E\right)$*, there exists disjoint neutrosophic soft open sets* $\left(\widetilde{G_1},E\right),\left(\widetilde{G_2},E\right)$ *such that* $\left(\widetilde{F_1},E\right)\subset\left(\widetilde{G_1},E\right)$ *and* $\left(\widetilde{F_2},E\right)\subset\left(\widetilde{G_2},E\right)$.

(X,τ,E) *is said to be a neutrosophic soft* T_4*-space if it is both a neutrosophic soft normal and neutrosophic soft* T_1*-space.*

Theorem 4.12 *Let* (X,τ,E) *be a neutrosophic soft topological space over* X. *Then* (X,τ,E) *is a neutrosophic soft* T_4*-space if and only if, for each neutrosophic soft closed set* $\left(\widetilde{F},E\right)$ *and neutrosophic soft open set* $\left(\widetilde{G},E\right)$ *with* $\left(\widetilde{F},E\right)\subset\left(\widetilde{G},E\right)$*, there exists a neutrosophic soft open set* $\left(\widetilde{D},E\right)$ *such that*

$$\left(\widetilde{F},E\right)\subset\left(\widetilde{D},E\right)\subset\overline{\left(\widetilde{D},E\right)}\subset\left(\widetilde{G},E\right).$$

Proof Let (X,τ,E) be a neutrosophic soft T_4-space, $\left(\widetilde{F},E\right)$ be a neutrosophic soft closed set, and $\left(\widetilde{F},E\right)\subset\left(\widetilde{G},E\right)\in\tau$. Then $\left(\widetilde{G},E\right)^c$ is a neutrosophic soft closed set and $\left(\widetilde{F},E\right)\cap\left(\widetilde{G},E\right)^c=0_{(X,E)}$. Since (X,τ,E) is a neutrosophic soft T_4-space, there exist neutrosophic soft open sets $\left(\widetilde{D_1},E\right)$ and $\left(\widetilde{D_2},E\right)$ such that $\left(\widetilde{F},E\right)\subset\left(\widetilde{D_1},E\right)$, $\left(\widetilde{G},E\right)^c\subset\left(\widetilde{D_2},E\right)$, and $\left(\widetilde{D_1},E\right)\cap\left(\widetilde{D_2},E\right)=0_{(X,E)}$. This implies that

$$\left(\widetilde{F},E\right)\subset\left(\widetilde{D_1},E\right)\subset\left(\widetilde{D_2},E\right)^c\subset\left(\widetilde{G},E\right).$$

$\left(\widetilde{D_2},E\right)^c$ is a neutrosophic soft closed set and $\overline{\left(\widetilde{D_1},E\right)}\subset\left(\widetilde{D_2},E\right)^c$ is satisfied. Thus,

$$\left(\widetilde{F},E\right)\subset\left(\widetilde{D_1},E\right)\subset\overline{\left(\widetilde{D_1},E\right)}\subset\left(\widetilde{G},E\right)$$

is obtained.

Conversely, let $\left(\widetilde{F_1},E\right),\left(\widetilde{F_2},E\right)$ be two disjoint neutrosophic soft closed sets. Then $\left(\widetilde{F_1},E\right)\subset\left(\widetilde{F_2},E\right)^c$. From the condition of theorem, there exists a neutrosophic soft open set $\left(\widetilde{D},E\right)$ such that

$$\left(\widetilde{F_1},E\right)\subset\left(\widetilde{D},E\right)\subset\overline{\left(\widetilde{D},E\right)}\subset\left(\widetilde{F_2},E\right)^c.$$

Thus, $\left(\widetilde{D},E\right),\left(\overline{\left(\widetilde{D},E\right)}\right)^c$ are neutrosophic soft open sets and $\left(\widetilde{F_1},E\right)\subset\left(\widetilde{D},E\right)$, $\left(\widetilde{F_2},E\right)\subset\left(\overline{\left(\widetilde{D},E\right)}\right)^c$, and $\left(\widetilde{D},E\right)\cap\left(\overline{\left(\widetilde{D},E\right)}\right)^c=0_{(X,E)}$ are obtained. Hence, (X,τ,E) is a neutrosophic soft T_4-space. □

Definition 4.13 *Let* (X,τ,E) *be a neutrosophic soft topological space over* X *and* $\left(\widetilde{F},E\right)$ *be an arbitrary neutrosophic soft set. Then* $\tau_{(\widetilde{F},E)}=\left\{\left(\widetilde{F},E\right)\cap\left(\widetilde{H},E\right):\left(\widetilde{H},E\right)\in\tau\right\}$ *is said to be neutrosophic soft topology on* $\left(\widetilde{F},E\right)$ *and* $\left(\left(\widetilde{F},E\right),\tau_{(\widetilde{F},E)},E\right)$ *is called a neutrosophic soft topological subspace of* (X,τ,E).

Theorem 4.14 *Let (X,τ,E) be a neutrosophic soft topological space over X. If (X,τ,E) is a neutrosophic soft T_i-space, then the neutrosophic soft topological subspace $\left(\left(\widetilde{F},E\right),\tau_{\left(\widetilde{F},E\right)},E\right)$ is a neutrosophic soft T_i-space for $i=0,1,2,3$.*

Proof Let $x^{e}_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\left(\widetilde{F},E\right),\tau_{\left(\widetilde{F},E\right)},E\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)} \cap y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$. Thus, there exist neutrosophic soft open sets $\left(\widetilde{F_1},E\right)$ and $\left(\widetilde{F_2},E\right)$ satisfying the conditions of neutrosophic soft T_i-spaces such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F_1},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F_2},E\right)$. Then $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F_1},E\right) \cap \left(\widetilde{F},E\right)$ and $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F_2},E\right) \cap \left(\widetilde{F},E\right)$. Also, the neutrosophic soft open sets $\left(\widetilde{F_1},E\right) \cap \left(\widetilde{F},E\right)$, $\left(\widetilde{F_2},E\right) \cap \left(\widetilde{F},E\right)$ in $\tau_{\left(\widetilde{F},E\right)}$ satisfy the conditions of neutrosophic soft T_i-space for $i=0,1,2,3$. □

Theorem 4.15 *Let (X,τ,E) be a neutrosophic soft topological space over X. If (X,τ,E) is a neutrosophic soft T_4-space and $\left(\widetilde{F},E\right)$ is a neutrosophic soft closed set in (X,τ,E), then $\left(\left(\widetilde{F},E\right),\tau_{\left(\widetilde{F},E\right)},E\right)$ is a neutrosophic soft T_4-space.*

Proof Let (X,τ,E) be a neutrosophic soft T_4--space and $\left(\widetilde{F},E\right)$ be a neutrosophic soft closed set in (X,τ,E). Let $\left(\widetilde{F_1},E\right)$ and $\left(\widetilde{F_2},E\right)$ be two neutrosophic soft closed sets in $\left(\left(\widetilde{F},E\right),\tau_{\left(\widetilde{F},E\right)},E\right)$ such that $\left(\widetilde{F_1},E\right) \cap \left(\widetilde{F_2},E\right) = 0_{(X,E)}$. When $\left(\widetilde{F},E\right)$ is a neutrosophic soft closed set in (X,τ,E), $\left(\widetilde{F_1},E\right)$ and $\left(\widetilde{F_2},E\right)$ are neutrosophic soft closed sets in (X,τ,E). Since (X,τ,E) is a neutrosophic soft T_4-space, there exist neutrosophic soft open sets $\left(\widetilde{G_1},E\right)$ and $\left(\widetilde{G_2},E\right)$ such that $\left(\widetilde{F_1},E\right) \subset \left(\widetilde{G_1},E\right)$, $\left(\widetilde{F_2},E\right) \subset \left(\widetilde{G_2},E\right)$, and $\left(\widetilde{G_1},E\right) \cap \left(\widetilde{G_2},E\right) = 0_{(X,E)}$. Then $\left(\widetilde{F_1},E\right) = \left(\widetilde{G_1},E\right) \cap \left(\widetilde{F},E\right)$, $\left(\widetilde{F_2},E\right) = \left(\widetilde{G_2},E\right) \cap \left(\widetilde{F},E\right)$, and $\left(\left(\widetilde{G_1},E\right) \cap \left(\widetilde{F},E\right)\right) \cap \left(\left(\widetilde{G_2},E\right) \cap \left(\widetilde{F},E\right)\right) = 0_{(X,E)}$. This implies that $\left(\left(\widetilde{F},E\right),\tau_{\left(\widetilde{F},E\right)},E\right)$ is a neutrosophic soft T_4-space. □

5. Conclusion

Neutrosophic soft separation axioms are the most important and interesting concepts via neutrosophic soft topology. We have introduced neutrosophic soft separation axioms in neutrosophic soft topological spaces, which are defined over an initial universe with a fixed set of parameters. We further investigate some essential features of the initiated neutrosophic soft separation axioms. We hope that these results will be useful for future studies on neutrosophic soft topology to carry out a general framework for practical applications. Applications of neutrosophic soft separation axioms in neutrosophic soft topological spaces can be investigated in decision-making problems.

Acknowledgment

The authors express their grateful thanks to the referee for reading the manuscript and making helpful comments.

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