Shortest Path with Normalized Single Valued Trapezoidal Neutrosophic Numbers

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ABSTRACT

In this paper we studied the network with Single Valued Trapezoidal Neutrosophic (SVTN) numbers. We propose an algorithm by transforming single valued trapezoidal neutrosophic (SVTN) numbers into normalized single valued trapezoidal neutrosophic (NSVTN) numbers and obtain an optimal value of the short path problem using defuzzification and scoring function. Finally, a numerical example is used to illustrate the efficiency of the proposed approach.

Keywords: Normalized single valued trapezoidal neutrosophic number, Score function, Network, Shortest path problem.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Lotfi A. Zadeh in 1965\cite{1}. Since then fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy set uses one real value $\mu_A(x) \in [0,1]$ to represent the grade of membership of fuzzy set $A$ defined on universe $X$. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy set was proposed \cite{2} to capture the uncertainty of grade of membership. Interval valued fuzzy set uses an interval value $[\mu_A^L(x), \mu_A^U(x)]$ with $0 \leq \mu_A^L(x) \leq \mu_A^U(x) \leq 1$ to represent the grade of membership of fuzzy set $A$. In 1986, Atanassov introduced the intuitionistic fuzzy sets and provably equivalent to interval...
valued fuzzy sets. The intuitionistic fuzzy sets consider both truth-membership \( t_A(x) \) and falsity-membership \( f_A(x) \), with \( t_A(x), f_A(x) \in [0,1] \) and \( 0 \leq t_A(x) + f_A(x) \leq 1 \). Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief system. In intuitionistic fuzzy sets, indeterminacy is \( 1 - t_A(x) - f_A(x) \) by default.

In 1998, the concept of the neutrosophic set (NS for short) and neutrosophic logic were introduced by Smarandache in [3,4] in order to efficiently handle the indeterminate and inconsistent information in real world. Neutrosophic set is a generalization of the theory of fuzzy set, intuitionistic fuzzy sets, interval-valued fuzzy sets and interval-valued intuitionistic fuzzy sets. The concept of the neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) are completely independently, which are within the real standard or nonstandard unit interval \( [0,1] \). However, the neutrosophic theory is difficult to be directly applied in real scientific and engineering areas. To easily use it in science and engineering areas, Wang et al. [5] proposed the concept of SVNS, which is an instance of a neutrosophic set, whose functions of truth, indeterminacy and falsity lie in \([0,1]\). Recent research works on neutrosophic set theory and its applications in various fields are progressing rapidly. Recently, based on the neutrosophic set theory, Subas presented the concept of triangular and trapezoidal neutrosophic number and applied to multiple-attribute decision making problems. Then Biswas et al presented a special case of trapezoidal neutrosophic numbers and applied to multiple-attribute decision making problems by introducing the cosine similarity measure. Deli and Subas presented the single valued trapezoidal neutrosophic numbers (SVN-number) as a generalization of the intuitionistic trapezoidal fuzzy numbers and proposed a methodology for solving multiple-attribute decision making problems with SVN-number. In addition, Thamaraiselvi and Santhi introduced a mathematical representation of a transportation problems in neutrosophic environment based n single valued trapezoidal neutrosophic numbers and also provided the solution method.

The shortest path problem (SPP) is one of the most fundamental and well-known combinatorial problems that appear in various fields of science and engineering, e.g, road
networks application, transportation, routing in communication channels and scheduling problems. The main objective of the shortest path problem is to find a path with minimum length between any pair of vertices. The edge (arc) length of the network may represent the real life quantities such as, time, cost, etc. In a classical shortest path problem, the distances of the edge between different nodes of a network are assumed to be certain. Numerous algorithms have been developed with the weights on edges on network being fuzzy numbers, intuitionistic fuzzy numbers, vague numbers. Recently, Broumi et al. [6] Presented the concept of neutrosophic graphs, interval valued neutrosophic graphs and bipolar single valued neutrosophic graphs. To this day, only a few papers dealing with shortest path problem in neutrosophic environment. The paper proposed by Broumi et al. is one of the first on this subject. The same authors proposed another algorithm for solving shortest path problem in a bipolar neutrosophic environment. Also, in [7] they proposed the shortest path algorithm in a network with its edge lengths as interval valued neutrosophic numbers.

The goal of this work is to propose an approach for solving shortest path problem in a network where edge weights are characterized by a normalized single valued trapezoidal neutrosophic numbers.

II. PRELIMINARIES

DEFINITION 2.1[3]

Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A neutrosophic set $A$ in $X$ is characterized by a truth-membership function $T_A$, an indeterminacy-membership-function $I_A$ and a falsity-membership function $F_A$. The neutrosophic set $A$ ($NSA$) is an object having the form $A = \left\{ x : T_A(x), I_A(x), F_A(x) > x \in X \right\}$. The function $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $[0,1]$.

That is, $T_A : X \rightarrow [0,1], I_A : X \rightarrow [0,1], F_A : X \rightarrow [0,1]$
There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so 
$-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$. That is, its components $T_A(x), I_A(x), F_A(x)$ are 
non-standard subsets included in the unitary nonstandard interval $[-1,1]$ or standard subsets 
included in the unitary standard interval $[0,1]$ as in the intuitionistic fuzzy set. 

**Example 2.1**

When we ask the opinion of an expert about certain statement, he/she may say that the 
possibility in which the statement is true is 0.8 and the statement is false is 0.6 and the degree in 
which he/she is not sure is 0.3. For, neutrosophic notation it can be expressed as $x(0.8,0.3,0.6)$.

**DEFINITION 2.2[5]**

Let $X$ be a space of points (object) with generic elements in $X$ denoted by $x$. A single 
valued neutrosophic set $A$ (SVNS $A$) is characterized by truth-membership function $T_A(x)$, an 
indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each 
point $x$ in $X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$. A SVNS $A$ can be written as 

$$A = \{ <x: T_A(x), I_A(x), F_A(x)>, x \in X \}.$$ 

**DEFINITION 2.3[8]**

A single valued trapezoidal neutrosophic number (SVTN-number) 

$$a = (a_1, b_1, c_1, d_1)$$ 

is a special neutrosophic set on the real number set $R$, whose truth-

membership, indeterminacy-membership, and a falsity-membership are given as follows 

$$T_a(x) = \begin{cases} 
(x - a_1)T_a / (b_1 - a_1) & (a_1 \leq x \leq b_1) \\
0 & \text{otherwise} 
\end{cases}$$ 

$$T_a(b_1 \leq x \leq c_1)$$ 

$$T_a(c_1 \leq x \leq d_1)$$
\[ I_a(x) = \begin{cases} 
\frac{(b_1 - x + I_a(x - a_1))}{(b_1 - a_1)} & (a_1 \leq x \leq b_1) \\
I_a(b_1) & (b_1 \leq x \leq c_1) \\
\frac{(x - c_1 + I_a(d_1 - x))}{(d_1 - c_1)} & (c_1 \leq x \leq d_1) \\
1 & \text{otherwise}
\end{cases} \]

\[ F_a(x) = \begin{cases} 
\frac{(b_1 - x + F_a(x - a_1))}{(b_1 - a_1)} & (a_1 \leq x \leq b_1) \\
F_a(b_1) & (b_1 \leq x \leq c_1) \\
\frac{(x - c_1 + F_a(d_1 - x))}{(d_1 - c_1)} & (c_1 \leq x \leq d_1) \\
1 & \text{otherwise}
\end{cases} \]

Where \( 0 \leq T_a \leq 1; 0 \leq I_a \leq 1; 0 \leq F_a \leq 1 \) and \( 0 \leq T_a + I_a + F_a \leq 3; a_1, b_1, c_1, d_1 \in \mathbb{R} \)

DEFINITION 2.4[9]

Let \( \tilde{A}_1 = (a_1, a_2, a_3, a_4); T_1, I_1, F_1 > \) and \( \tilde{A}_2 = (b_1, b_2, b_3, b_4); T_2, I_2, F_2 > \) be two single valued trapezoidal neutrosophic numbers. Then, the operations for SVTN-numbers are defined as below

i) \( \tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2 > \)

ii) \( \tilde{A}_1 \otimes \tilde{A}_2 = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2 > \)

iii) \( \lambda \tilde{A}_1 = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda > \)

DEFINITION 2.5[8]

Let \( \tilde{A}_1 = (a_1, a_2, a_3, a_4); T_1, I_1, F_1 > \) be a single valued trapezoidal neutrosophic number. Then, the score function \( s(\tilde{A}_1) \) and accuracy function \( a(\tilde{A}_1) \) of a SVTN-numbers are defined as follows:
i. \( s(\tilde{A}_1) = \left(\frac{1}{2}\right) \left[ a_1 + a_2 + a_3 + a_4 \right] \times \left[ 2 + T_i - I_i - F_i \right] \)

ii. \( a(\tilde{A}_1) = \left(\frac{1}{2}\right) \left[ a_1 + a_2 + a_3 + a_4 \right] \times \left[ 2 + T_i - I_i + F_i \right] \)

**DEFINITION 2.6**[8]

Let \( \tilde{A}_1 \) and \( \tilde{A}_2 \) be two SVTN-numbers the ranking of \( \tilde{A}_1 \) and \( \tilde{A}_2 \) by score function is defined as follows:

1) If \( s(\tilde{A}_1) < s(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \)

2) If \( s(\tilde{A}_1) = s(\tilde{A}_2) \) and if

   i. \( a(\tilde{A}_1) < a(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \)

   ii. \( a(\tilde{A}_1) \geq a(\tilde{A}_2) \) then \( \tilde{A}_1 \geq \tilde{A}_2 \)

   iii. \( a(\tilde{A}_1) = a(\tilde{A}_2) \) then \( \tilde{A}_1 = \tilde{A}_2 \)

**DEFINITION 2.7**[10]

Let \( a = [a_i, b_i, c_i, d_i] \) be a trapezoidal fuzzy number and \( a_i \leq b_i \leq c_i \leq d_i \) then the centre of gravity (COG) of \( K \) can be defined as

\[
\text{COG}(K) = \begin{cases} 
\tilde{a} & \text{if } a_i = b_i = c_i = d_i \\
\frac{1}{3} \left[ a_i + b_i + c_i + d_i - \frac{c_i d_i - a_i b_i}{c_i + d_i - a_i - b_i} \right] & \text{otherwise}
\end{cases}
\]

**DEFINITION 2.8**[10]

Let \( \tilde{A}_1 = (a_i, a_2, a_3, a_4); T_i, I_i, F_i \) be aSVTN-numbers, and then the score function,
accuracy function, and certainty functions are defined as follows:

\[
E(\tilde{A}_1) = COG(K) \times \left( \frac{2 + T_1 - I_1 - F_1}{3} \right), \quad A(\tilde{A}_1) = COG(K) \times (T_1 - F_1), \quad C(\tilde{A}_1) = COG(K) \times T_1
\]

**DEFINITION 2.9[10]**

Let \( \tilde{A}_1 = (a_1, a_2, a_3, a_4); T_1, I_1, F_1 > \) and \( \tilde{A}_2 = (b_1, b_2, b_3, b_4); T_2, I_2, F_2 > \) be two SVTN-numbers the ranking of \( \tilde{A}_1 \) and \( \tilde{A}_2 \) by score function is defined as follows:

1. If \( E(\tilde{A}_1) > E(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \)
2. If \( E(\tilde{A}_1) = E(\tilde{A}_2) \) and if \( A(\tilde{A}_1) > A(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \)
3. If \( E(\tilde{A}_1) = E(\tilde{A}_2) \) and if \( A(\tilde{A}_1) < A(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \)
4. If \( E(\tilde{A}_1) = E(\tilde{A}_2) \) and if \( A(\tilde{A}_1) < A(\tilde{A}_2) \)
5. and \( C(\tilde{A}_1) < C(\tilde{A}_2) \) then \( \tilde{A}_1 < \tilde{A}_2 \)
6. If \( E(\tilde{A}_1) = E(\tilde{A}_2) \) and if \( A(\tilde{A}_1) = A(\tilde{A}_2) \) and \( C(\tilde{A}_1) > C(\tilde{A}_2) \) then \( \tilde{A}_1 > \tilde{A}_2 \)
7. If \( E(\tilde{A}_1) = E(\tilde{A}_2) \) and if \( A(\tilde{A}_1) = A(\tilde{A}_2) \) and \( C(\tilde{A}_1) = C(\tilde{A}_2) \) then \( \tilde{A}_1 = \tilde{A}_2 \)

**DEFINITION 2.10[11]**

Let \( \tilde{a} = (a_1, a_2, c_1, d_1); T_a, I_a, F_a > \). Then, a normalized SVTN-number of \( \tilde{a} \) is defined by

\[
\left( \left( \left( \frac{a_1}{a_1 + b_1 + c_1 + d_1}, \frac{b_1}{a_1 + b_1 + c_1 + d_1}, \frac{c_1}{a_1 + b_1 + c_1 + d_1}, \frac{d_1}{a_1 + b_1 + c_1 + d_1} \right); T_a, I_a, F_a \right) \right)
\]

(1)

**Example 2.2** Assume that \( \tilde{a} = (2, 4, 8, 9); 0.5, 0.3, 0.1 > \). Then, a normalized SVTN-number of \( \tilde{a} \) is computed as \( (0.09, 0.17, 0.35, 0.39); 0.5, 0.3, 0.1 > \).
III. NORMALIZED SINGLE VALUED TRAPEZOIDAL NEUTROSOPHIC PATH PROBLEM

3.1 Discussion on shortcoming of some of the existing methods

Broumi et al. first proposed a method to find the shortest path under trapezoidal fuzzy number (TNS) environment. It is very well known and popular paper in the field of neutrosophic set and system. However, the authors used $S(\tilde{r} + \tilde{s}) = S(\tilde{r}) + S(\tilde{s})$ to solve the problem which may be invalid in some cases. This has been discussed in discussed Example 3.1.

Example 3.1 Broumi et al. [12]

Here authors have considered two arbitrary i.e., $\tilde{r}, \tilde{s}$ be the following TNS number:

$$\tilde{r} = \langle (1,2,3,4); 0.4,0.6,0.7 \rangle,$$

$$\tilde{s} = \langle (1,5,7,9); 0.7,0.6,0.8 \rangle.$$

We observe that the authors used an invalid mathematical assumption to solve the problem. i.e., $S(\tilde{r} + \tilde{s}) = S(\tilde{r}) + S(\tilde{s})$

Our objective is to show that above considered assumption is not valid such as $S(\tilde{r} + \tilde{s}) \neq S(\tilde{r}) + S(\tilde{s})$.

Solution: According to the method of Broumi et al. [12] [see; iteration 4, page no 420, ref. Broumi et al.[12]], we have:

$$\tilde{r} + \tilde{s} = \langle (1,2,3,4); 0.4,0.6,0.7 \rangle \oplus \langle (1,5,7,9); 0.7,0.6,0.8 \rangle = \langle (2,7,10,13); 0.82,0.36,0.56 \rangle.$$

Therefore, we get, $S(\tilde{r} + \tilde{s}) = 5.006$. but $S(\tilde{r}) + S(\tilde{s}) = 3.3$. Hence, it is clear that $S(\tilde{r} + \tilde{s}) \neq S(\tilde{r}) + S(\tilde{s})$. Therefore, we can say that the method of Broumi et al. is not valid.
3.2 Transformation of single valued trapezoidal neutrosophic numbers into normalized single valued trapezoidal neutrosophic numbers

In this section, we slightly modified the SVTN-numbers into NSVTN-numbers. Using this equation

\[
\left( \frac{a_1}{a_i + b_i + c_i + d_i}, \frac{b_1}{a_i + b_i + c_i + d_i}, \frac{c_1}{a_i + b_i + c_i + d_i}, \frac{d_1}{a_i + b_i + c_i + d_i} \right); T'_a, I'_a, F'_a
\]

Example 3.2

Let us consider the two single valued trapezoidal neutrosophic numbers

\[
\bar{u} = \langle (2, 4, 8, 9); 0.5, 0.3, 0.1 \rangle, \quad \bar{v} = \langle (7, 8, 9, 10); 0.3, 0.2, 0.6 \rangle,
\]

and we use the Eq (1) then we get the normalized single valued trapezoidal neutrosophic number.

i.e., \( \bar{u} = \langle (0.09, 0.17, 0.35, 0.39); 0.5, 0.3, 0.1 \rangle \), \( \bar{v} = \langle (0.21, 0.24, 0.26, 0.29); 0.3, 0.2, 0.6 \rangle \).

Now we have to show that NSVTN-numbers satisfies the condition \( S(\bar{u} + \bar{v}) \neq S(\bar{u}) + S(\bar{v}) \).

\( \bar{u} + \bar{v} = \langle (0.3, 0.41, 0.61, 0.68); 0.65, 0.06, 0.06 \rangle \). Therefore, we get, \( S(\bar{u} + \bar{v}) = 0.42 \). But \( S(\bar{u}) + S(\bar{v}) = 0.30 \). Hence, it is clear that NSVTN-number satisfies the condition.

3.3. ALGORITHM

In this section, an algorithm is proposed to find the shortest path of the network with single valued neutrosophic numbers from the starting node (f) to the terminating node (g).

The main steps of the algorithm are as follows:

**Step 1:** Convert the single valued trapezoidal neutrosophic numbers into normalized single valued neutrosophic numbers using the Definition 2.10.

**Step 2:** Let us consider the graph with starting node f and terminating node g. Find the score function \( \{ S(N_{ij}) \ for \ i, j \} \) of each arc length for the given network with normalized single valued trapezoidal neutrosophic numbers using the Definition 2.8.
Step 3: Find \( \min\{S(N_{fj}) \text{ for } j = 2,3,\ldots,g\} \)

Step 4: Suppose \( \min\{S(N_{fj}) \text{ for } j = 2,3,\ldots,g\} = S(N_{fk}) \) corresponding to the arc \((f,k)\)

Step 5: Repeat step 3 and step 4 from the node \(k\) until the terminating node \(g\).

Step 6: Calculate the summation of the score function for each arc corresponding to the path which is the shortest path with starting node \(f\) and terminating node \(g\).

IV. NUMERICAL EXAMPLE

Example 4.1: Consider the network shown in figure 1. We want to obtain the shortest path from node 1 to node 6 where edges have a normalized single valued trapezoidal neutrosophic numbers (SVTN-numbers)

![Network Diagram]

In this network each edge has been assigned to a single valued trapezoidal neutrosophic number as follows:

<table>
<thead>
<tr>
<th>Edges</th>
<th>Single valued trapezoidal neutrosophic cost</th>
<th>Edges</th>
<th>Single valued trapezoidal neutrosophic cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>((1,2,3,4);0.4,0.6,0.7)</td>
<td>3-4</td>
<td>((2,4,8,9);0.5,0.3,0.1)</td>
</tr>
<tr>
<td>1-3</td>
<td>((2,5,7,8);0.2,0.3,0.4)</td>
<td>3-5</td>
<td>((3,4,5,10);0.3,0.4,0.7)</td>
</tr>
<tr>
<td>2-3</td>
<td>((3,7,8,9);0.1,0.4,0.6)</td>
<td>4-6</td>
<td>((7,8,9,10);0.3,0.2,0.6)</td>
</tr>
<tr>
<td>2-5</td>
<td>((1,5,7,9);0.7,0.6,0.8)</td>
<td>5-6</td>
<td>((2,4,5,7);0.6,0.5,0.3)</td>
</tr>
</tbody>
</table>

Solution:

Step 1: Convert the single valued trapezoidal neutrosophic numbers into normalized single valued neutrosophic numbers using the definition 2.10, we get the following table
<table>
<thead>
<tr>
<th>Edges</th>
<th>Normalized single valued trapezoidal neutrosophic cost</th>
<th>Edges</th>
<th>Normalized single valued trapezoidal neutrosophic cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>&lt;(0.1,0.2,0.3,0.4);0.4,0.6,0.7&gt;</td>
<td>3-4</td>
<td>&lt;(0.09,0.17,0.35,0.39);0.5,0.3,0.1&gt;</td>
</tr>
<tr>
<td>1-3</td>
<td>&lt;(0.09,0.23,0.32,0.36);0.2,0.3,0.4&gt;</td>
<td>3-5</td>
<td>&lt;(0.14,0.18,0.23,0.45);0.3,0.4,0.7&gt;</td>
</tr>
<tr>
<td>2-3</td>
<td>&lt;(0.11,0.26,0.30,0.33);0.1,0.4,0.6&gt;</td>
<td>4-6</td>
<td>&lt;(0.21,0.24,0.26,0.29);0.3,0.2,0.6&gt;</td>
</tr>
<tr>
<td>2-5</td>
<td>&lt;(0.05,0.23,0.32,0.41);0.7,0.6,0.8&gt;</td>
<td>5-6</td>
<td>&lt;(0.11,0.22,0.28,0.39);0.6,0.5,0.3&gt;</td>
</tr>
</tbody>
</table>

Applying steps 2-5 as in the proposed Algorithm, we get the NSVTN shortest path 1→2→5→6 with the length <(0.26,0.65,0.9,1.2);0.93,0.18,0.17> and the score function value of node 1-2 is 0.09, node 2-5 is 0.11, node 5-6 is 0.15. It is clear that the range of NSVTN shortest path length is 0.26 to 1.2 and we get an optimal solution 0.09+0.11+0.15=0.35 which lies inside the region.

V. CONCLUSION

In this paper, we have introduced an algorithm to get the shortest path of the network with single valued trapezoidal neutrosophic numbers by transforming normalized single valued trapezoidal neutrosophic numbers.

REFERENCE


