



SINGLE VALUED NEUTROSOPHIC NUMBER VALUED GENERALIZED NEUTROSOPHIC TRIPLET GROUPS AND ITS APPLICATIONS FOR DECISION MAKING APPLICATIONS

ORHAN ECEMIŞ¹, MEMET ŞAHİN² AND ABDULLAH KARGIN^{2*}

¹Department of Computer Technology, Gaziantep University, Gaziantep 27310, Turkey.

²Department of Mathematics, Gaziantep University, Gaziantep 27310, Turkey.

AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. Author OE designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors MS and AK managed the analyses of the study. Author AK managed the literature searches. All authors read and approved the final manuscript.

ARTICLE INFORMATION

Editor(s):

(1) Dariusz Jacek Jakóbczak, Technical University of Koszalin, Poland.

Reviewers:

(1) L. Sujatha, Auxilium College (Autonomous), India.

(2) Reehana parveen, JBAS College, India.

Received: 27th March 2018

Accepted: 4th June 2018

Published: 11th June 2018

Original Research Article

ABSTRACT

Neutrosophy is a branch of philosophy. Neutrosophy is based on neutrosophic logic, neutrosophic probability and neutrosophic set. Neutrosophic triplet theory is new structure in neutrosophic set theory. In this study; we introduced notion of generalized neutrosophic triplet group. We defined contrary element for generalized neutrosophic triplet group. The contrary element is different from anti element. Contrary element is useful in decision making applications. Then we defined a neutral similarity measure and a contrary similarity measure for single valued neutrosophic number. We studied properties of neutral similarity measure and contrary similarity measure. Also, we introduced single valued neutrosophic number valued generalized neutrosophic triplet set and single valued neutrosophic number valued generalized neutrosophic triplet group by using neutral similarity measure and contrary similarity measure. So we obtained a new structure by using neutrosophic set theory and neutrosophic triplet set theory for the first time. Furthermore, we introduced a new method for decision making application.

Keywords: Neutrosophic triplet group; single valued neutrosophic number; single valued neutrosophic number valued generalized neutrosophic triplet group; application for decision making.

*Corresponding author: Email: abdullahkargin27@gmail.com;

1 Introduction

Neutrosophy is a branch of philosophy, firstly introduced by Smarandache in 1980. Neutrosophy is based on neutrosophic logic, probability and set [1]. Neutrosophic logic is a generalized of many logics such as fuzzy logic which is introduced by Zadeh [2] and intuitionistic fuzzy logic which is introduced by Atanassov [3]. Fuzzy set have function of membership, intuitionistic fuzzy set has function of membership and function of non-membership. Thus; they do not explain the indeterminacy states. But neutrosophic set has function of membership, function of indeterminacy and function of non-membership. Also, many researchers have studied concept of neutrosophic theory [4-22]. Recently Olgun and Bal [23] studied the neutrosophic module; Şahin, Olgun and Kılıçman [24] introduced neutrosophic soft lattices; Şahin, Alkhazaleh and Uluçay [25] studied neutrosophic soft expert set; Şahin, Olgun, Uluçay, Kargin and Smarandache [26] introduced centroid single valued neutrosophic number and its applications; Ji, Zhang and Wang [27] studied multi – valued neutrosophic environments and its application. Also, Smarandache and Ali studied neutrosophic triplet (NT) theory [28] and NT groups [29,30]. A NT has a form $\langle x, \text{neut}(x), \text{anti}(x) \rangle$. Where; $\text{neut}(x)$ is neutral of “x”, $\text{anti}(x)$ is opposite of “x”. Furthermore, $\text{neut}(x)$ is different from the classical unitary element. Also, The NT group is different from the classical group. Recently, Ali, Smarandache and Khan introduced [31] the NT field and NT ring; Smarandache, Şahin and Kargin [32] studied NT G – module; Şahin and Kargin introduced [33] NT metric space, NT vector space and NT normed space; Şahin and Kargin [34] introduced NT inner product.

Many uncertainties and complex situations arise in decision-making applications. It is impossible to come up with these uncertainties and complexities, especially with known numbers. For example, in multi-attribute decision making (MADM), multiple objects are evaluated according to more than one property and there is a choice of the most suitable one. Particularly in multi-attribute group decision making (MAGDM), the most appropriate object selection is made according to the data received from more than one decision maker. Multi - attribute decision making group and multi-attribute decision making problems have been found by many researchers using various methods using intuitionistic fuzzy numbers. For example, Wan and Dong [35] studied trapezoidal intuitionistic fuzzy numbers and application to multi attribute group decision making. Wan, Wang, Li and Dong [36] studied triangular intuitionistic fuzzy numbers and application to multi attribute group decision making. Also, Wang, Smarandache, Zhang and Sunderaman [37] introduced single valued neutrosophic number. Many researchers used various methods using single valued neutrosophic number intuitionistic fuzzy numbers.

In this paper; we introduced generalized neutrosophic triplet group and single valued neutrosophic number valued generalized neutrosophic triplet group. Also, we gave a new similarity measure for single valued neutrosophic number valued generalized neutrosophic triplet groups. Furthermore; we gave a new method for decision making application. In section 2; we give some preliminary results and definition for neutrosophic triplets and similarity measure for single valued neutrosophic numbers. In section 3; generalized neutrosophic triplet set and generalized neutrosophic triplet group are defined. It is show that generalized neutrosophic triplet set and groups are different from the neutrosophic triplet set and groups. In section 4; new similarity measures are given for single valued neutrosophic number and some properties of a these similarity measure are given. In section 5; we defined single valued neutrosophic number valued generalized neutrosophic triplet group and gave a new method for decision making applications. In section 5; conclusions are given.

2 Preliminaries

Definition 2.1: [30] Let N be a set together with a binary operation $*$. Then, N is called a neutrosophic triplet set if for any $a \in N$, there exists a neutral of “a” called $\text{neut}(a)$, different from the classical algebraic unitary element, and an opposite of “a” called $\text{anti}(a)$, with $\text{neut}(a)$ and $\text{anti}(a)$ belonging to N , such that:

$$\begin{aligned} a * \text{neut}(a) &= \text{neut}(a) * a = a, \\ \text{and} \\ a * \text{anti}(a) &= \text{anti}(a) * a = \text{neut}(a). \end{aligned}$$

Definition 2.2: [30] Let $(N, *)$ be a neutrosophic triplet set. Then, N is called a neutrosophic triplet group, if the following conditions are satisfied.

- 1) If $(N, *)$ is well-defined, i.e. for any $a, b \in N$, one has $a*b \in N$.
- 2) If $(N, *)$ is associative, i.e. $(a*b)*c = a*(b*c)$ for all $a, b, c \in N$.

Definition 2.3: [37] Let U be an universe of discourse then the single valued neutrosophic set A is on object having the form $A = \{ (x: T_A(x), I_A(x), F_A(x)) \mid x \in U \}$ where the functions $T, I, F: U \rightarrow [0, 1]$ respectively the degree of membership, the degree of indeterminacy and degree of non-membership of the element $x \in U$ to the set A with the condition.

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

For convenience, we can simply use $x = (T, I, F)$ to represent an element x in single valued neutrosophic numbers and the element x can be called a single valued neutrosophic number.

3 Generalized Neutrosophic Triplet Groups

Definition 3.1: Let N be a neutrosophic triplet set together with a binary operation $*$ and $a, b \in N, N_a \subset N, A_a \subset N$. Then;

- a) N_a is called neutrals set of “ a ” such that $N_a = \{b : b = \text{neut}(a)\}$.
- b) A_a is called anties set of “ a ” such that $A_a = \{b : b = \text{anti}(a)\}$.

Example 3.2: Let $X = \{1, 2, 3\}$ be a set and $P(X)$ is set of all subset of X . Then, $(P(X), \cup)$ is a neutrosophic triplet set. Where, $A = \text{neut}(A)$ and $B = \text{anti}(A)$ for $A \supseteq B$; $A, B \in P(X)$. Also, from definition 3.1; neutrals set of A is $N_A = \{A\}$ and anties of A is $A_A = \{B : A \supseteq B\}$.

Definition 3.3: Let N be a neutrosophic triplet set together with a binary operation $*$, N_a be neutrals set of a , A_a be anties set of a and $a \in N$. If there exists an element $b \in N$ such that $a*b = b*a \in N \setminus (N_a \cup A_a)$, then b is called contrary element of “ a ” and it is shown that $b = \text{con}(a)$.

Example 3.4: From example 3.2; $(P(X), \cup)$ is a neutrosophic triplet set such that $A = \text{neut}(A)$ and $B = \text{anti}(A)$ for $A \supseteq B$; $A, B \in P(X)$. Also, neutrals set of A is $N_A = \{A\}$ and anties of A is $A_A = \{B : A \supseteq B\}$. For $A \in P(X)$, $\text{con}(A) \in P(X) \setminus (A \cup \{B\})$, where $A \supseteq B$. Thus; $\text{con}(A)$ must be element of $P(X) \setminus (A \cup \{B\}) = \{C : A' \supseteq C, A' \text{ is complement of } A\}$. Therefore; there exists a $\text{con}(A)$ for $A \in P(X)$. Because $\{C : A' \supseteq C, A' \text{ is complement of } A\} \neq \emptyset$ and $A \cup \text{con}(A) = \text{con}(A) \cup A$. For example; for $X \in P(X)$, $\text{con}(X) = \{\emptyset\}$.

Definition 3.5: Let N be a neutrosophic triplet set with a binary operation $\#$ and $a, b \in N, C_a \subset N$. Then; N_a is called contraries set of “ a ” such that $C_a = \{b : b = \text{con}(a)\}$.

Example 3.6: From example 3.4; for $A \in P(X)$, $C_A = \{C : A' \supseteq C, A' \text{ is complement of } A\}$.

Definition 3.7: Let N be a neutrosophic triplet set with binary operation $\#$. Then, $(N, \#)$ is called generalized neutrosophic triplet group if following conditions are satisfied.

- i) $a\#b \in N$, for $a, b \in N$,
- ii) $a\#a = a$, $a \in N$.
- iii) If there exists $\text{con}(a)$ for element “ a ” such that $\text{con}(a) = b$, then there exist least “ a ” element $b \in C_a$ such that $a = \text{con}(b)$.

If there exists any $\text{con}(a)$ for $a \in N$, then generalized neutrosophic triplet element is denoted by $(a, (N_a, A_a), C_a)$. If there does not exist $\text{con}(a)$ for $a \in N$, then generalized neutrosophic triplet element is denoted by $(a, (N_a, A_a), \theta)$.

Example 3.8: From example 3.2; $(P(X), \cup)$ is a neutrosophic triplet set such that $A = \text{neut}(A)$ and $B = \text{anti}(A)$ for $A \supseteq B$; $A, B \in P(X)$.

- i) It is clear that $A \cup B \in P(X)$ for $A, B \in P(X)$
- ii) It is clear that $A \cup A = A \in P(X)$.
- ii) From example 3.4; there exists a $\text{con}(A)$ for $A \in P(X)$ and let $\text{con}(A) = C$. From example 3.6; $C_A = \{B: A' \supseteq B, A' \text{ is complement of } A\}$ and $C_C = \{D: C' \supseteq D, C' \text{ is complement of } C\}$. Thus, $C \in A' \supseteq B$ and $A \in C' \supseteq D$ and for $C \in C_A$, there exists $A \in C_C$ such that $A = \text{con}(C)$. Therefore, $(P(X), \cup)$ is a generalized neutrosophic triplet group.

Corollary 3.9: Let N be a generalized neutrosophic triplet group with binary operation $\#$.

- i) There exists $\text{neut}(a)$ for all $a \in N$ such that $a = \text{neut}(a)$.
- ii) There exists $\text{anti}(a)$ for all $a \in N$ such that $a = \text{anti}(a)$.
- iii) There don't have to be $\text{con}(a)$ for $a \in N$.
- vi) If there exists $\text{con}(a)$, then $a \neq \text{con}(a)$ for $a \in N$.
- v) $\text{neut}(a)$ is different from the classical algebraic unitary element.

Proof.

- i) It is clear that there exists $\text{neut}(a)$ for all $a \in N$ such that $a = \text{neut}(a)$ since $a \# a = a$.
- ii) It is clear that there exists $\text{anti}(a)$ for all $a \in N$ such that $a = \text{anti}(a)$ since $a \# a = a$.
- iii) It is clear that there don't have to be $\text{con}(a)$ for $a \in N$ from definition of $\text{con}(a)$.
- iv) If there exists $\text{con}(a)$, then $a \neq \text{con}(a)$ for $a \in N$ since $a = \text{neut}(a) = \text{anti}(a)$ and from definition of $\text{con}(a)$ $a \# a \notin N \setminus (N_a \cup A_a)$.
- v) It is clear from definition of generalized neutrosophic triplet group.

Corollary 3.10: Generalized neutrosophic triplet group is generally different from neutrosophic triplet group. Actually generalized neutrosophic triplet group is a specific form of neutrosophic triplet group.

Example 3.11: Let $N = \{X, Y, Z, T, K\}$ be set of drugs that a pharmaceutical company prepares for the treatment of various diseases and we show the results obtained by using these drugs together in Table 1.

Table 1. Generalized neutrosophic triplet group with binary operation $\#$

#	X	Y	Z	T	K
X	X	X	Y	K	Z
Y	X	Y	Y	Z	X
Z	Y	Y	Z	T	T
T	K	Z	T	T	Y
K	Z	X	T	Y	K

Now, we show that $(N, \#)$ is a generalized neutrosophic triplet group. Where, all $A, B, C \in N$; N_A is set of $\text{neut}(A)$ s; C_A is set of $\text{anti}(A)$ s and C_A is set of $\text{con}(A)$ s.

- i) From Table 1, $A \# B = C \in N$.
- ii) From Table 1, $A \# A = A$.
- iii) For element X ; $N_X = \{X, Y\}$, $A_X = \{X, Z\}$ and $C_X = \{T\}$

For element Y; $N_Y = \{Y, Z\}$, $A_Y = \{Y, T\}$ and $C_Y = \{X, K\}$
 For element Z; $N_Z = \{Z\}$, $A_Z = \{Z\}$ and $C_Z = \{X, Y, T, K\}$
 For element T; $N_T = \{Z, T\}$, $A_T = \{Y, T\}$ and $C_T = \{X\}$
 For element K; $N_K = \{K\}$, $A_K = \{K\}$ and $C_K = \{X, Y, Z, T\}$.

Where, $\text{con}(X) = T$ and $\text{con}(T) = X$
 $\text{con}(Y) = K$ and $\text{con}(K) = Y$
 $\text{con}(Z) = K$ and $\text{con}(K) = Z$
 $\text{con}(T) = X$ and $\text{con}(X) = T$
 $\text{con}(K) = Z$ and $\text{con}(Z) = K$. Thus, $(N, \#)$ is a generalized neutrosophic triplet group.

As a result,

for drug X;

Y, Z can be used together with X and T cannot be used together with X.

For drug Y;

Z, T can be used together and X, K cannot be used together.

For drug Z;

nothing can be used together with Z.

For drug T;

Z, Y can be used together and X cannot be used together.

For drug K;

nothing can be used together with K.

Where, we show that groups can be used in decision making problems. However, it is not always possible to give a definite answer in every problem. In other words, groups may not be useful in a problem where there are uncertainties that this is not the definitive result. So we need to define single valued neutrosophic numbers valued group in order to find solutions to the uncertainties. We can also use the similarity measures for single valued neutrosophic numbers to make a more comprehensive and applicable definition. Now we define a new similarity measure for single valued neutrosophic number. Then, we define single valued neutrosophic number valued generalized neutrosophic triplet group.

4 Neutral Similarity Measure and Contrary Similarity Measure

Definition 4.1: Let $A_1 = \langle T_1, I_1, F_1 \rangle$ and $A_2 = \langle T_2, I_2, F_2 \rangle$ be two single valued neutrosophic numbers. The neutral similarity measure between A_1 and A_2 is

$$S_N(A_1, A_2) = 1 - 2/3 \cdot \left\{ \frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} + \frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} + \frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} \right\}$$

Proposition 4.2: Let S_N be a neutral similarity measure for A_1 and A_2 single valued neutrosophic numbers. Then; S_N is satisfies the following properties.

- i) $0 \leq S_N(A_1, A_2) \leq 1$
- ii) $S_N(A_1, A_2) = S_N(A_2, A_1)$
- iii) $S_N(A_1, A_2) = 1$ if and only if $A_1 = A_2$
- iv) If $A_1 \leq A_2 \leq A_3$ then, $S_N(A_1, A_3) \leq S_N(A_2, A_3)$

Proof:

i) As A_1 and A_2 are single valued neutrosophic number,

$$\max \left\{ 1 - \frac{2}{3} \cdot \left\{ \frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} + \frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} + \frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} \right\} \right\} = 1 \quad \text{and}$$

$$\min \left\{ 1 - \frac{2}{3} \cdot \left\{ \frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} + \frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} + \frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} \right\} \right\} = 0. \text{ Thus,}$$

$$0 \leq S_N(A_1, A_2) \leq 1$$

$$\text{ii) } S_N(A_1, A_2) = 1 - \frac{2}{3} \cdot \left\{ \frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} + \frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} + \frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} \right\}$$

$$= 1 - \frac{2}{3} \cdot \left\{ \frac{\min\{|T_2 - T_1|, |F_2 - F_1|\}}{\max\{|T_2 - T_1|, |F_2 - F_1|\} + 1} + \frac{\min\{|T_2 - T_1|, |I_2 - I_1|\}}{\max\{|T_2 - T_1|, |I_2 - I_1|\} + 1} + \frac{\min\{|I_2 - I_1|, |F_2 - F_1|\}}{\max\{|I_2 - I_1|, |F_2 - F_1|\} + 1} \right\} = S_N(A_2, A_1)$$

$$\text{iii) We suppose that } S_N(A_1, A_2) = 1 - \frac{2}{3} \cdot \left\{ \frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} + \frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} + \frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} \right\} = 1. \text{ Thus,}$$

$$\frac{2}{3} \cdot \left\{ \frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} + \frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} + \frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} \right\} = 0. \text{ Where,}$$

$$\frac{\min\{|T_1 - T_2|, |F_1 - F_2|\}}{\max\{|T_1 - T_2|, |F_1 - F_2|\} + 1} = 0,$$

$$\frac{\min\{|T_1 - T_2|, |I_1 - I_2|\}}{\max\{|T_1 - T_2|, |I_1 - I_2|\} + 1} = 0,$$

$$\frac{\min\{|I_1 - I_2|, |F_1 - F_2|\}}{\max\{|I_1 - I_2|, |F_1 - F_2|\} + 1} = 0. \text{ Thus,}$$

$$\min\{|T_1 - T_2|, |F_1 - F_2|\} = 0$$

$$\min\{|T_1 - T_2|, |I_1 - I_2|\} = 0$$

$$\min\{|I_1 - I_2|, |F_1 - F_2|\} = 0. \text{ Where,}$$

$$|T_1 - T_2| = 0 \quad |F_1 - F_2| = 0 \quad |I_1 - I_2| = 0 \quad \text{and} \quad T_1 = T_2, \quad F_1 = F_2, \quad I_1 = I_2. \text{ Therefore, } A_1 = A_2.$$

Now we suppose that $A_1 = A_2$. It is clear that $S_N(A_1, A_2) = 1$.

iv) We suppose that $A_1 \leq A_2 \leq A_3$. Thus,

$$T_1 \leq T_2 \leq T_3, \quad I_1 \geq I_2 \geq I_3, \quad F_1 \geq F_2 \geq F_3. \text{ Where,}$$

$$\frac{\min\{|T_1 - T_3|, |F_1 - F_3|\}}{\max\{|T_1 - T_3|, |F_1 - F_3|\} + 1} \geq \frac{\min\{|T_2 - T_3|, |F_2 - F_3|\}}{\max\{|T_2 - T_3|, |F_2 - F_3|\} + 1}$$

$$\frac{\min\{|T_1 - T_3|, |I_1 - I_3|\}}{\max\{|T_1 - T_3|, |I_1 - I_3|\} + 1} \geq \frac{\min\{|T_2 - T_3|, |I_2 - I_3|\}}{\max\{|T_2 - T_3|, |I_2 - I_3|\} + 1}$$

$$\frac{\min\{|F_1-F_3|, |I_1-I_3|\}}{\max\{|F_1-F_3|, |I_1-I_3|\}+1} \geq \frac{\min\{|F_2-F_3|, |I_2-I_3|\}}{\max\{|F_2-F_3|, |I_2-I_3|\}+1}. \text{ Thus,}$$

$$1-2/3. \left\{ \frac{\min\{|T_1-T_3|, |F_1-F_3|\}}{\max\{|T_1-T_3|, |F_1-F_3|\}+1} + \frac{\min\{|T_1-T_3|, |I_1-I_3|\}}{\max\{|T_1-T_3|, |I_1-I_3|\}+1} + \frac{\min\{|I_1-I_3|, |F_1-F_3|\}}{\max\{|I_1-I_3|, |F_1-F_3|\}+1} \right\} \leq$$

$$1-2/3. \left\{ \frac{\min\{|T_2-T_3|, |F_2-F_3|\}}{\max\{|T_2-T_3|, |F_2-F_3|\}+1} + \frac{\min\{|T_2-T_3|, |I_2-I_3|\}}{\max\{|T_2-T_3|, |I_2-I_3|\}+1} + \frac{\min\{|I_2-I_3|, |F_2-F_3|\}}{\max\{|I_2-I_3|, |F_2-F_3|\}+1} \right\}. \text{ Therefore;}$$

$$S_N(A_1, A_3) \leq S_N(A_2, A_3).$$

Definition 4.3: Let $A_1 = \langle T_1, I_1, F_1 \rangle$ and $A_2 = \langle T_2, I_2, F_2 \rangle$ be two single valued neutrosophic numbers. The contrary similarity measure between A_1 and A_2 is

$$S_C = 2/3. \left\{ \frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} + \frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} + \frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} \right\}$$

Proposition 4.4: Let S_C be a contrary similarity measure for A_1 and A_2 single valued neutrosophic numbers. Then; S_C is satisfies the following properties.

- i) $0 \leq S_C(A_1, A_2) \leq 1$
- ii) $S_C(A_1, A_2) = S_C(A_2, A_1)$
- iii) $S_C(A_1, A_2) = 0$ if and only if $A_1 = A_2$
- iv) If $A_1 \leq A_2 \leq A_3$ then, $S_N(A_1, A_2) \geq S_N(A_2, A_3)$

Proof:

i) As A_1 and A_2 are single valued neutrosophic number,

$$\max \left\{ 2/3. \left\{ \frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} + \frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} + \frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} \right\} = 1 \text{ and} \right.$$

$$\left. \min \left\{ 2/3. \left\{ \frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} + \frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} + \frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} \right\} \right\} = 0. \text{ Thus; } 0 \leq S_C(A_1, A_2) \leq 1$$

$$\text{ii) } S_C(A_1, A_2) = 2/3. \left\{ \frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} + \frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} + \frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} \right\}$$

$$= 2/3. \left\{ \frac{\min\{|T_2-T_1|, |F_2-F_1|\}}{\max\{|T_2-T_1|, |F_2-F_1|\}+1} + \frac{\min\{|T_2-T_1|, |I_2-I_1|\}}{\max\{|T_2-T_1|, |I_2-I_1|\}+1} + \frac{\min\{|I_2-I_1|, |F_2-F_1|\}}{\max\{|I_2-I_1|, |F_2-F_1|\}+1} \right\} = S_C(A_2, A_1)$$

iii) We suppose that $S_C(A_1, A_2) =$

$$2/3. \left\{ \frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} + \frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} + \frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} \right\} = 0. \text{ Thus,}$$

$$\left\{ \frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} + \frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} + \frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} \right\} = 0. \text{ Where,}$$

$$\frac{\min\{|T_1-T_2|, |F_1-F_2|\}}{\max\{|T_1-T_2|, |F_1-F_2|\}+1} = 0,$$

$$\frac{\min\{|T_1-T_2|, |I_1-I_2|\}}{\max\{|T_1-T_2|, |I_1-I_2|\}+1} = 0,$$

$$\frac{\min\{|I_1-I_2|, |F_1-F_2|\}}{\max\{|I_1-I_2|, |F_1-F_2|\}+1} = 0. \text{ Thus,}$$

$$\min\{|T_1 - T_2|, |F_1 - F_2|\} = 0$$

$$\min\{|T_1 - T_2|, |I_1 - I_2|\} = 0$$

$$\min\{|I_1 - I_2|, |F_1 - F_2|\} = 0. \text{ Where,}$$

$$|T_1 - T_2| = 0 \quad |F_1 - F_2| = 0 \quad |I_1 - I_2| = 0 \text{ and } T_1 = T_2, F_1 = F_2, I_1 = I_2. \text{ Therefore, } A_1 = A_2.$$

Now we suppose that $A_1 = A_2$. It is clear that $S_C(A_1, A_2) = 0$.

iv) We suppose that $A_1 \leq A_2 \leq A_3$. Thus,

$$T_1 \leq T_2 \leq T_3, \quad I_1 \geq I_2 \geq I_3, \quad F_1 \geq F_2 \geq F_3. \text{ Where,}$$

$$\frac{\min\{|T_1 - T_3|, |F_1 - F_3|\}}{\max\{|T_1 - T_3|, |F_1 - F_3|\} + 1} \geq \frac{\min\{|T_2 - T_3|, |F_2 - F_3|\}}{\max\{|T_2 - T_3|, |F_2 - F_3|\} + 1}$$

$$\frac{\min\{|T_1 - T_3|, |I_1 - I_3|\}}{\max\{|T_1 - T_3|, |I_1 - I_3|\} + 1} \geq \frac{\min\{|T_2 - T_3|, |I_2 - I_3|\}}{\max\{|T_2 - T_3|, |I_2 - I_3|\} + 1}$$

$$\frac{\min\{|F_1 - F_3|, |I_1 - I_3|\}}{\max\{|F_1 - F_3|, |I_1 - I_3|\} + 1} \geq \frac{\min\{|F_2 - F_3|, |I_2 - I_3|\}}{\max\{|F_2 - F_3|, |I_2 - I_3|\} + 1}. \text{ Thus,}$$

$$2/3. \left\{ \frac{\min\{|T_1 - T_3|, |F_1 - F_3|\}}{\max\{|T_1 - T_3|, |F_1 - F_3|\} + 1} + \frac{\min\{|T_1 - T_3|, |I_1 - I_3|\}}{\max\{|T_1 - T_3|, |I_1 - I_3|\} + 1} + \frac{\min\{|I_1 - I_3|, |F_1 - F_3|\}}{\max\{|I_1 - I_3|, |F_1 - F_3|\} + 1} \right\} \geq$$

$$2/3. \left\{ \frac{\min\{|T_2 - T_3|, |F_2 - F_3|\}}{\max\{|T_2 - T_3|, |F_2 - F_3|\} + 1} + \frac{\min\{|T_2 - T_3|, |I_2 - I_3|\}}{\max\{|T_2 - T_3|, |I_2 - I_3|\} + 1} + \frac{\min\{|I_2 - I_3|, |F_2 - F_3|\}}{\max\{|I_2 - I_3|, |F_2 - F_3|\} + 1} \right\}. \text{ Therefore;}$$

$$S_C(A_1, A_3) \geq S_C(A_2, A_3).$$

5 Single Valued Neutrosophic Number Valued Generalized Neutrosophic Triplet Group and Its Application

Now we define the single valued neutrosophic number valued generalized neutrosophic triplet set and single valued neutrosophic number valued generalized neutrosophic triplet group using the neural similarity measure and contrary similarity measure in section 4.

Definition 5.1: Let N be a set of single valued neutrosophic numbers, $*$ be a binary operation, S_N be a neutral similarity measure and S_C be a contrary similarity measure. Then, $(N, \alpha^\beta, *)$ is called single valued neutrosophic number valued generalized neutrosophic triplet set if following conditions are satisfied. Where, $\alpha, \beta \in [0, 1]$.

i) $x * x = x$, for $x \in N$.

ii) there exists neut(x) for $x \in N$ such that

$$\text{neut}(x) = \begin{cases} y, & x * y = y * x = x \\ z, & x * z = z * x = t \neq x \text{ and } S_N(x, t) \geq \alpha \end{cases}$$

iii) there exists anti(x) for $x \in N$ such that

$$\text{anti}(x) = \begin{cases} k, & x * k = k * x = y \\ m, & x * k = k * x = m \neq y \text{ and } S_N(x, m) \geq \beta \end{cases}$$

Where, $y = \text{neut}(x)$ from ii).

Definition 5.2: Let $(N, \alpha^\beta, *)$ be a single valued neutrosophic number valued generalized neutrosophic triplet set and S_c is contrary similarity measure. For $x \in N$, if there exists a element $n \in N$ such that

$$n = \begin{cases} p, & x * p = p * x \in N \setminus (N_x \cup A_x) \\ s, & x * p = p * x = s \notin N \setminus (N_x \cup A_x) \text{ and } S_c(x, s) \geq 1 - \beta \end{cases}$$

then n is called contrary of x and it is shown that $\text{con}(x) = n$. Where, N_x is set of neut(x) and A_x is set of anti(x).

Definition 5.3: Let $(N, \alpha^\beta, *)$ be a single valued neutrosophic number valued generalized neutrosophic triplet set. Then $(N, \alpha^\beta, *)$ is called single valued neutrosophic number valued generalized neutrosophic triplet group if following conditions are satisfied.

- i) $x*y$ is a single valued neutrosophic number for $x, y \in N$.
- ii) If there exists $\text{con}(x)$ for element “a” such that $\text{con}(x) = y$, then there exist least “a” element such that $x = \text{con}(y)$.

Corollary 5.4: It is clear that single valued neutrosophic number valued generalized neutrosophic triplet group is generally different from neutrosophic triplet group.

Example 5.5: Let $N = \{X, Y, Z, T, K\}$ be set of drugs that a pharmaceutical company prepares for the treatment of various diseases. We show the effect of drugs on diseases with single valued neutrosophic number such that

$$\begin{aligned} X &= \langle 0.8, 0.3, 0.2 \rangle \\ Y &= \langle 0.6, 0.2, 0.4 \rangle \\ Z &= \langle 0.7, 0.4, 0.1 \rangle \\ T &= \langle 0.9, 0.2, 0.1 \rangle \\ K &= \langle 0.7, 0.1, 0.3 \rangle \end{aligned}$$

Also, we show the results obtained by using these drugs together in Table 2.

Table 2. Results obtained by using drugs together (AA= A#A, A ∈ N)

#	X = <0.8, 0.3, 0.2>	Y = <0.6, 0.2, 0.4>	Z = <0.7, 0.4, 0.1>	T = <0.9, 0.2, 0.1>	K = <0.7, 0.1, 0.3>
X = <0.8, 0.3, 0.2>	XX = <0.8, 0.3, 0.2>	XY = <0.9, 0.1, 0.1>	XZ = <0.5, 0.5, 0.3>	XT = <0.7, 0.4, 0.3>	XK = <0.8, 0.1, 0.2>
Y = <0.6, 0.2, 0.4>	YX = <0.9, 0.1, 0.1>	YY = <0.6, 0.2, 0.4>	YZ = <0.3, 0.6, 0.2>	YT = <0.7, 0.3, 0.3>	YK = <0.9, 0.2, 0.2>
Z = <0.7, 0.4, 0.1>	ZX = <0.5, 0.5, 0.3>	ZY = <0.3, 0.6, 0.2>	ZZ = <0.7, 0.4, 0.1>	ZT = <0.3, 0.8, 0.2>	ZK = <0.9, 0.1, 0.0>
T = <0.9, 0.2, 0.1>	TX = <0.7, 0.4, 0.3>	TY = <0.7, 0.3, 0.3>	TZ = <0.3, 0.8, 0.2>	TT = <0.9, 0.2, 0.1>	TK = <0.8, 0.1, 0.1>
K = <0.7, 0.1, 0.3>	KX = <0.8, 0.1, 0.2>	KY = <0.9, 0.2, 0.2>	KZ = <0.9, 0.1, 0.0>	KT = <0.8, 0.1, 0.1>	KK = <0.7, 0.1, 0.3>

We show that $(\{X, Y, Z, T, K\}, 0.82^{0.8}, \#)$ is a single valued neutrosophic number valued generalized neutrosophic triplet group.

- i) From Table 2, A#B is a single valued neutrosophic number for A, B ∈ N.
- ii) Firstly, we show the results obtained by using S_N neutral similarity measure and S_c contrary similarity measure for drugs.

Table 3. Results obtained by using S_N for drug X.

S_N	XX	XY	XZ	XT	XK
X	1	0.828	0.790	0.818	1

Table 4. Results obtained by using S_C for drug X.

S_C	XX	XY	XZ	XT	XK
X	0	0.171	0.209	0.181	0

From Table 3, set of neutral for X $\rightarrow N_X = \{X, Y, K\}$

From Table 4, set of contrary for X $\rightarrow C_X = \{Z, T\}$

Table 5. Results obtained by using S_N for drug Y.

S_N	YX	YY	YZ	YT	YK
Y	0.743	1	0.659	0.818	0.897

Table 6. Results obtained by using S_C for drug Y

S_C	YX	YY	YZ	YT	YK
Y	0.256	0	0.340	0.181	0.102

From Table 5, set of neutral for Y $\rightarrow N_Y = \{Y, K\}$

From Table 6, set of contrary for Y $\rightarrow C_Y = \{X, Z, T\}$

Table 7. Results obtained by using S_N for drug Z.

S_N	ZX	ZY	ZZ	ZT	ZK
Z	0.777	0.801	1	0.714	0.738

Table 8. Results obtained by using S_C for drug Z.

S_C	ZX	ZY	ZZ	ZT	ZK
Z	0.222	0.198	0	0.285	0.261

From Table 7, set of neutral for Z $\rightarrow N_Z = \{Z\}$

From Table 8, set of contrary for Z $\rightarrow C_Z = \{X, Y, T, K\}$

Table 9. Results obtained by using S_N for drug T.

S_N	TX	TY	TZ	TT	TK
T	0.666	0.7777	0.666	1	0.939

Table 10. Results obtained by using S_C for drug T.

S_C	TX	TY	TZ	TT	TK
T	0.333	0.222	0.333	0	0.06

From Table 9, set of neutral for T $\rightarrow N_T = \{K, T\}$

From Table 10, set of contrary for T $\rightarrow C_T = \{X, Y, Z\}$

Table 11. Results obtained by using S_N for drug K

S_N	KX	KY	KZ	KT	KK
K	0.939	0.828	0.897	0.944	1

Table 12. Results obtained by using S_C for drug K

S_C	KX	KY	KZ	KT	KK
K	0.06	0.171	0.102	0.055	0

From Table 11, set of neutral for K $\rightarrow N_K = \{X, Y, Z, T, K\}$

From Table 12, set of contrary for K $\rightarrow C_K = \{\}$

Where, $\text{con}(X) = T$ and $\text{con}(T) = X$
 $\text{con}(X) = Z$ and $\text{con}(Z) = X$
 $\text{con}(Y) = T$ and $\text{con}(T) = Y$
 $\text{con}(Y) = Z$ and $\text{con}(Z) = Y$. Thus, $(\{X, Y, Z, T, K\}, 0.82^{0.8} \#)$ is a single valued neutrosophic number valued generalized neutrosophic triplet group.

As a result,

- For drug X;
 Y can be used together with X
 T, Z cannot be used together with X.
- For drug Y;
 K can be used together with Y
 X, Z, T cannot be used together with Y.
- For drug Z;
 nothing can be used together with Z.
- For drug T;
 K can be used together with T
 X, Y, Z cannot be used together with T.
- For drug K;
 X, Y, Z, T can be used together with K.

6 Conclusion

In this study we have generalized the neutrosophic triplet groups. Thus, we have added a new structure to the neutrosophic triplet theory, a new and special algebraic structure. We also defined new similarity measures for single valued neutrosophic numbers. Using these measures we have come up with a new solution to decision-making applications by using an algebraic structure, the neutrosophic triplet group together with single valued neutrosophic numbers. Therefore, we obtained a new structure by using neutrosophic set theory and neutrosophic triplet set theory for the first time. In addition, new solutions can be presented to many decision making applications using different similarity measures and using different neutrosophic numbers as in this study.

Competing Interests

Authors have declared that no competing interests exist.

References

- [1] Smarandache F. Neutrosophy: Neutrosophic Probability, Set and Logic. ProQuest Information & Learning. Ann Arbor, MI, USA. 1998;105.
- [2] Zadeh LA. Fuzzy sets. Information and Control. 1965;8(3):338-353.
- [3] Atanassov TK. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 1986;20:87–96.
- [4] Kandasamy WBV, Smarandache F. Basic neutrosophic algebraic structures and their applications to fuzzy and neutrosophic models. Hexis, Frontigan, French. 2004;149.
- [5] Kandasamy WBV, Smarandache F. Some neutrosophic algebraic structures and neutrosophic n-algebraic structures. Hexis, Phoenix, AZ, USA. 2006;209.
- [6] Broumi S, Bakali A, Talea M, Smarandache F, and Vladareanu L. Computation of shortest path problem in a network with SV-Trapezoidal neutrosophic numbers. Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia. 2016;417-422.
- [7] Broumi S, Bakali A, Talea M, Smarandache F, Vladareanu L. Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia. 2016;412-416.
- [8] Liu P, Shi L. The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. Neural Computing and Applications. 2015;26(2):457-471.
- [9] Liu P, Shi L. Some Neutrosophic Uncertain Linguistic Number Heronian Mean Operators and Their Application to Multi-attribute Group Decision making. Neural Computing and Applications; 2015. Available:<https://doi:10.1007/s00521-015-2122-6>
- [10] Liu P, Tang G. Some power generalized aggregation operators based on the interval neutrosophic numbers and their application to decision making. Journal of Intelligent & Fuzzy Systems. 2016; 30:2517-2528.
- [11] Liu P, Tang G. Multi-criteria group decision-making based on interval neutrosophic uncertain linguistic variables and Choquet integral. Cognitive Computation. 2016;8(6):1036-1056.
- [12] Liu P, Wang Y. Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making. Journal of Systems Science & Complexity. 2016;29(3):681-697.
- [13] Liu P, Teng F. Multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator. Internal Journal of Machine Learning and Cybernetics; 2015. Available:<https://doi:10.1007/s13042-015-0385-y>
- [14] Liu P, Zhang L, Liu X, Wang P. Multi-valued Neutrosophic Number Bonferroni mean Operators and Their Application in Multiple Attribute Group Decision Making. Internal Journal of Information Technology & Decision Making. 2016;15(5):1181-1210.
- [15] Sahin M, Kargın A. Neutrosophic triplet metric space and neutrosophic triplet normed space. ICMME -2017, Şanlıurfa, Turkey; 2017.

- [16] Sahin M, Deli I, Ulucay V. Jaccard vector similarity measure of bipolar neutrosophic set based on multi-criteria decision making. In: International Conference on Natural Science and Engineering (ICNASE'16), Kilis, Turkey; 2016.
- [17] Sahin M, Deli I, Ulucay V. Similarity measure of bipolar neutrosophic sets and their application to multiple criteria decision making. *Neural Comput & Applic*; 2016.
Available:[https://doi:10.1007/S00521](https://doi.org/10.1007/S00521)
- [18] Liu C, Luo Y. Power aggregation operators of simplifield neutrosophic sets and their use in multi-attribute group decision making. *IEE/CAA Journal of Automatica Sinica*; 2017.
Available:[https://doi: 10.1109/JAS.2017.7510424](https://doi.org/10.1109/JAS.2017.7510424)
- [19] Sahin R, Liu P. Some approaches to multi criteria decision making based on exponential operations of simplified neutrosophic numbers. *Journal of Intelligent & Fuzzy Systems*. 2017;32(3):2083-2099.
- [20] Liu P, Li H. Multi attribute decision-making method based on some normal neutrosophic bonferroni mean operators. *Neural Computing and Applications*. 2017;28(1):179-194.
- [21] Broumi S, Bakali A, Talea M, Smarandache F. Decision-making method based on the interval valued neutrosophic graph. *Future Technologie, IEEE*. 2016;44-50.
- [22] Liu P. The aggregation operators based on Archimedean t-conorm and t-norm for the single valued neutrosophic numbers and their application to Decision Making. *International Journal of Fuzzy Systems*. 2016;18(5):849-863.
- [23] Olgun N, Bal M. Neutrosophic modules. *Neutrosophic Operational Research*. 2017;2:181-192.
- [24] Şahin MV, Olgun N, Kilicman A. On neutrosophic soft lattices. *Afrika Matematika*. 2017;28(3):379-388.
- [25] Şahin M, Alkhazaleh Ş, Ulucay V. Neutrosophic soft expert sets. *Applied Mathematics*. 2015; 6:116-127.
- [26] Şahin M, Olgun N, Uluçay V, Kargın A, Smarandache F. A new similarity measure based on falsity value between single valued neutrosophic sets based on the centroid points of transformed single valued neutrosophic numbers with applications to pattern recognition. *Neutrosophic Sets and Systems*. 2017;15:31-48.
- [27] Ji P, Zang H, Wang J. A projection – based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. *Neural Computing and Applications*. 2018; 29:221-234.
- [28] Smarandache F, Ali M. Neutrosophic triplet as extension of matter plasma, unmatter plasma and antimatter plasma. *APS Gaseous Electronics Conference*; 2016.
Available:[https://doi: 10.1103/BAPS.2016.GEC.HT6.110](https://doi.org/10.1103/BAPS.2016.GEC.HT6.110)
- [29] Smarandache F, Ali M. The neutrosophic triplet group and its application to physics. Presented by F. S. to Universidad Nacional de Quilmes, Department of Science and Technology, Bernal, Buenos Aires, Argentina; 2014.
- [30] Smarandache F, Ali M. Neutrosophic triplet group. *Neural Computing and Applications*. 2016;29(7): 595-601.

- [31] Ali M, Smarandache F, Khan M. Study on development of neutrosophic triplet ring and neutrosophic triplet field. *Mathematics – MDPI*; 2018.
Available: <https://doi:10.3390/math6040046>
- [32] Smarandache F, Şahin M, Kargin A. Neutrosophic triplet G – module. *Mathematics – MDPI*; 2018.
Available: [https://doi: 10.3390/math6040053](https://doi:10.3390/math6040053)
- [33] Şahin M, Kargin A. Neutrosophic triplet normed space. *Open Physics*. 2017;15:697-704.
- [34] Şahin M, Kargin A. Neutrosophic triplet inner product space. *Neutrosophic Operational Research*. 2017;2:193-215.
- [35] Wan SP, Dong JY. Power geometric operators of trapezoidal intuitionistic fuzzy numbers and application to multi attribute group decision making. *Applied Soft Computing*. 2015;29:153-168.
- [36] Wan SP, Wang F, Li L, Dong JY. Some generalized aggregation operators for triangular intuitionistic fuzzy numbers and application to multi attribute group decision making. *Computer & Industrial Engineering*. 2016;93:286-301.
- [37] Wang H, Smarandache F, Zhang YQ, Sunderraman R. Single valued neutrosophic sets. *Multispace Multistructure*. 2010;4:410–413.