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# SINGLE VALUED NEUTROSOPHIC SUBRINGS AND IDEALS

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ABSTRACT. Single valued neutrosophic set also known as the subclass of neutrosophic set is one of the mighty tool to deal with inexact, indefinite, unsure information, vagueness etc. In this study we have prolonged the theory of neutrosophic rings to SVNS and formed single valued neutrosophic rings . We have also prolonged neutrosophic ideal theory for neutrosophic ideal over a ring to single valued neutrosophic ideal of a single valued neutrosophic ring. Examples are given in order to illustrate the theory and part of single valued neutrosophic notation used.

## 1. INTRODUCTION

The extension of classical set theory which is fuzzy set dealing with vagueness was given by Zadeh [23] allows the gradual assessment of membership function valued in the interval $[0, 1]$ whereas classical set theory allows the membership of elements in the set in binary terms . In many real world problems classical set theory fails with unreliable and incomplete information so it is necessary for the system to be designed in such a way to cope with such data.Fuzzy logic inherently handle imprecise data but sometimes with membership function it is uncertain and hard to be defined by crisp values. Intuitionistic fuzzy set was introduced as a generalization of fuzzy sets by K.

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Atanassov(1983) [7] extends both the concepts by assessment of the elements by two functions namely membership and non membership function valued in the interval $[0, 1]$. Neutrosophic etymologically from neutrosophy commonly known as knowledge of neutral thought represents the distinguish of neutrosophic from fuzzy , intuitionistic fuzzy set was introduced by Samarandache in 1995. It is compressed with the basic indeterminacy membership, truth membership and falsity membership which plays a vital role in most situation.In many real life problems in order to pave solution the system are designed with neutrosophic logic. The subclass of neutrosophic set also known as an extension of intuitionistic fuzzy set, single valued neutrosophic set was given by Wang [19] which gives additional possibility to represent indefinite, unsure and vagueness.

Some known algebraic structure in the literature include neutrosophic groups, semigroups, rings , fields, neutrosophic modules etc. Rosenfield [16] formulated the concept of fuzzy subgroup of a group in 1971 which was redefined by Anthony and Sherwood [3] in 1979. Chakroborthy and Khare [8] gave fuzzy homomorphism between groups and studied its effect on fuzzy subgroup. The concept of fuzzy subring and ideal was introduced by Wang Jin Liu(1982) [19] who fuzzified certain standard results based on rings and ideal . Mukherjee and Sen [13] discussed fuzzy prime ideals. Jun Kim and Yon [9, 21] applied the concept of intuitionistic fuzzy sets to ideals of a near ring. Intuitionistic fuzzy subrings of intutionistic fuzzy ideals of a ring is defined in [4, 6]. Intuitionistic fuzzy ring and its homomorphism has been investigated by Yan [10]. After the introduction of neutrosophic sets by Smarandache [18] several authors [1, 2, 14, 22] have applied the notion of neutrosophic sets to group theory. Pabitrakumar Maji [15] had combined the neutrosophic set with soft sets and introduced a new mathematical model neutrosophic soft set. I.Arockiarani et.al.defined the notion of fuzzy neutrosophic sets [4]. I.Arockiarani and I.R. Sumathi introduced fuzzy neutrosophic groups [6]. J.MartinaJency,I.Arockiarani introduced fuzzy neutrosophic subgroupiods [11]. In this paper we introduce the concept of single valued neutrosophic subrings we also extend neutrosophic ideal theory to form neutrosophic ideal over a ring and single valued neutrosophic ideal of a single valued neutrosophic ring.

## 2. PRELIMINARIES

**Definition 2.1.** *[20] Let X be a space of points(objects),with a generic element in X denoted by x. A single valued neutrosophic set (SVNS)A in X is characterized by truth-membership function $T_A$, indeterminacy-membership function $I_A$ and falsity -membership function $F_A$. For each point x in X $T_A(x), I_A(x), F_A(x) \in [0, 1]$. When X is continuous , a SVNS A can be written as:*

$$A = \int_X \langle T(x), I(x), F(x)\rangle / x, x \in X\,.$$

*When X is discrete, a SVNS A can be written as:*

$$A = \sum_{i=1}^{n} \langle T(xi), I(xi), F(xi)\rangle / xi, xi \in X\,.$$

**Definition 2.2.** *[5] A Fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3\,.$*

**Definition 2.3.** *[5] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$,$x \in X$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle, x \in X\,.$*

(1) $A \subseteq B$ *for all x if $T_A(x) \leq T_B(x)$ ,$I_A(x) \leq I_B(x)$,$F_A(x) \geq F_B(x)$.*
(2) $A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle.$
(3) $A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle.$
(4) $A \backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle.$

**Definition 2.4.** *[5] A fuzzy neutrosophic set A over the universe X is said to be a null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by $0_N$.*

**Definition 2.5.** *[5] A fuzzy neutrosophic set A over the universe X is said to be absolute(universe)fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by $1_N$.*

**Definition 2.6.** *[5] The complement of a fuzzy neutrosophic set A is denoted by $A^c$ and is defined as $A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x)\rangle$ where $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x)$, $F_{A^c}(x) = T_A(x)$. The complement of a fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.*

**Definition 2.7.** *[6] Let $(X, .)$ be a group and let A be a fuzzy neutrosophic set in X. Then A is called a fuzzy neutrosophic group(in short, FNG)in X if it satisfies the following condition:*

(i) $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y), F_A(xy) \leq F_A(x) \vee F_A(y)$.
(ii) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x), F_A(x^{-1}) \leq F_A(x)$ .

**Definition 2.8.** *[17] Let $(X, .)$ be a groupoid and let A and B be two fuzzy neutrosophic sets in X. Then the fuzzy neutrosophic product of A and B, $A \circ B$, is defined as follows, for any $x \in X$ :*

$$T_{A \circ B}(x) = \begin{cases} \vee_{yz=x}[T_A(y) \wedge T_B(z)] \text{for each} (y, z) \in X \times X \text{with, } yz = x \\ 0 \qquad otherwise \end{cases}$$

$$I_{A \circ B}(x) = \begin{cases} \vee_{yz=x}[I_A(y) \wedge I_B(z)] \text{for each} (y, z) \in X \times X \text{with, } yz = x \\ 0 \qquad otherwise \end{cases}$$

$$F_{A \circ B}(x) = \begin{cases} \wedge_{yz=x}[F_A(y) \vee F_B(z)] \text{for each} (y, z) \in X \times X \text{with, } yz = x \\ 0 \qquad otherwise \end{cases}$$

**Definition 2.9.** *[11] Let $(G, .)$ be a groupoid and let $0_N \neq A \in FNS(G)$. Then A is called a fuzzy neutrosophic subgroupoid in G (in short, FNSGP in G) if $A \circ A \subset A$.*

**Definition 2.10.** *[11] Let $(G, .)$ be a groupoid and let $A \in FNS$. Then A is called a fuzzy neutrosophic subgroupoid in G (in short, FNSGP in G) if for any $x, y \in G$,$T_A(xy) \geq T_A(x) \wedge T_A(y)$, $I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$.*

It is clear that $0_N$ and $1_N$ are both $FNSGPs$ of G.

**Definition 2.11.** *[12] Let G be a group and let $A \in FNSGP(G)$. Then A is called a fuzzy neutrosophic subgroup(in short, FNSG) of G if $A(x^{-1}) \geq A(x)$, (i.e.,) $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$ for each $x \in G$ .*

## 3. Single valued neutrosophic subrings and ideals

**Definition 3.1.** *Let $(X, .)$ be a groupoid and let A and B be two single valued neutrosophic sets in X. Then the single valued neutrosophic product of A and B, $A \circ B$, is defined as follows, for any $x \in X$ :*

$$T_{A\circ B}(x)=\begin{cases}\vee_{yz=x}[T_A(y)\wedge T_B(z)]\textit{for each}(y,z)\in X\times X\textit{with,}yz=x\\ 0 \qquad otherwise\end{cases}$$

$$I_{A\circ B}(x)=\begin{cases}\vee_{yz=x}[I_A(y)\wedge I_B(z)]\textit{for each}(y,z)\in X\times X\textit{with,}yz=x\\ 0 \qquad otherwise\end{cases}$$

$$F_{A\circ B}(x)=\begin{cases}\wedge_{yz=x}[F_A(y)\vee F_B(z)]\textit{for each}(y,z)\in X\times X\textit{with,}yz=x\\ 0 \qquad otherwise\end{cases}$$

**Definition 3.2.** *Let $(G, .)$ be a groupoid and let $A \in SVNS$. Then A is called a single valued neutrosophic subgroupoid in G (in short, SVNSGP in G) if for any $x, y \in G$,$T_A(xy) \geq T_A(x) \wedge T_A(y)$, $I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$.*

It is clear that $0_N$ and $1_N$ are both $SVNSGPs$ of G.

**Definition 3.3.** *Let $(R, +, .)$ be a ring and let $0_N \neq A \in SVNS(R)$. Then A is called a single valued neutrosophic subring in R (in short SVNSR in R) if it satisfies the following condition:*

1 *If A is a SVNS(G) with respect to the operation $'+'$ (in sense of Def. 3.2)*
2 *A is a SVNSGP with respect to the operation $'.'$ (in the sense of Def. 3.1)*

**Definition 3.4.** *Let R be a ring and $A \in SVNS(R)$ then A is called a single valued neutrosophic subring of R if it satisfies the following condition. For all $x, y \in R$*

(1) $T_A(x-y) \geq T_A(x) \wedge T_A(y)$
$I_A(x-y) \geq I_A(x) \wedge I_A(y)$
$F_A(x-y) \leq F_A(x) \vee F_A(y)$.
(2) $T_A(xy) \geq T_A(x) \wedge T_A(y)$
$I_A(xy) \geq I_A(x) \wedge I_A(y)$
$F_A(xy) \leq F_A(x) \vee F_A(y)$.

**Example 1.** *Let $R = \{a, b, c, d\}$ be a set with two binary operation as follows*

| + | b | c | d | e |
|---|---|---|---|---|
| b | b | c | d | e |
| c | c | b | e | d |
| d | d | e | c | b |
| e | e | d | b | c |

| + | b | b | b | b |
|---|---|---|---|---|
| b | b | b | b | b |
| c | b | b | b | b |
| d | b | b | b | b |
| e | b | b | c | c |

*Then(R, +, .) is a ring. Let SVNS* $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ *in R defined by* $T_A(b) = 0.7$, $T_A(c) = 0.6$, $T_A(d) = 0.5$, $T_A(e) = 0.2$, $I_A(a) = 0.5$, $I_A(b) = 0.4$ $I_A(c) = I_A(d) = 0.3$ *and* $F_A(a) = 0.2$, $F_A(b) = 0.4$, $F_A(c) = 0.6$ , $F_A(d) = 0.7$ *then* $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ *is an single valued neutrosophic subring of R.*

**Definition 3.5.** *Let R be a ring and let* $0_N \neq A \in SVNS(R)$ *be a* $SVNR$ *in R. Then A is called a single valued neutrosophic left ideal (in short SVNLI ) in R if* $A(xy) \geq A(y)$ *(i.e)*

(1) $T_A(xy) \geq T_A(y)$
$I_A(xy) \geq I_A(y)$
$F_A(xy) \leq F_A(y)$ *for any* $x, y \in R$.

(2) *single valued neutrosophic right ideal (in short SVNRI) in R if* $A(xy) \geq A(x)$ *(i.e)*
$T_A(xy) \geq T_A(x)$
$I_A(xy) \geq I_A(x)$
$F_A(xy) \leq F_A(x)$ *for any* $x, y \in R$.

(3) *single valued neutrosophic ideal (in short SVNI)in R if it is both an* $SVNLI$ *and* $SVNRI$ *in R.*

**Proposition 3.1.** *Let* $A = \langle T_A, I_A, F_A\rangle \in SVNI(R)$. *Then* $T_A(0) \geq T_A(x)$, $I_A(0) \geq I_A(x)$ *and* $F_A(0) \leq F_A(x)$ *for all* $x \in R$.

*Proof.* For all $x \in R$

$$T_A(0) = T_A(x - x) \geq T_A(x) \wedge T_A(x) = T_A(x)$$
$$I_A(0) = I_A(x - x) \geq I_A(x) \wedge I_A(x) = I_A(x)$$
$$F_A(0) = F_A(x - x) \leq F_A(x) \vee F_A(x) = F_A(x)$$

□

**Proposition 3.2.** *Let* $A = \langle T_A, I_A, F_A\rangle \in SVNI(R)$. *Then* $T_A(x) = T_A(-x)$, $I_A(x) = I_A(-x)$ *and* $F_A(x) = F_A(-x)$ *for all* $x \in R$.

*Proof.* For $x \in R$,
$T_A(x) = T_A(-(-x)) \geq T_A(-x) \geq T_A(x)$,
$I_A(x) = I_A(-(-x)) \geq I_A(-x) \geq I_A(x)$
$F_A(x) = F_A(-(-x)) \leq F_A(-x) \geq F_A(x)$ □

**Proposition 3.3.** *If a SVNS* $A = \langle T_A, I_A, F_A\rangle$ *in R satisfies Proposition(3.4) then*

$$T_A(x - y) = T_A(0) \Rightarrow T_A(x) = T_A(y)$$

$$I_A(x-y) = I_A(0) \Rightarrow I_A(x) = I_A(y)$$
$$F_A(x-y) = F_A(0) \Rightarrow F_A(x) = F_A(y)$$

*for all* $x, y \in R$

*Proof.* Let $x, y \in R$ such that $T_A(x-y) = T_A(0)$.Then

i) $T_A(x) = T_A(x-y+y) \geq T_A(x-y) \wedge T_A(y)$
$= T_A(0) \wedge T_A(y) = T_A(y)$

ii) $I_A(x) = I_A(x-y+y) \geq I_A(x-y) \wedge I_A(y)$
$= I_A(0) \wedge I_A(y) = I_A(y)$

iii) $F_A(x) = F_A(x-y+y) \leq F_A(x-y) \vee F_A(y)$
$= F_A(0) \vee F_A(y) = F_A(y)$ .

Similarly

i) $T_A(y) = T_A(x-x+y) = T_A(x-(x-y))$
$\geq T_A(x) \wedge T_A(x-y) = T_A(x)$

ii) $I_A(y) = I_A(x-x+y) = I_A(x-(x-y))$
$\geq I_A(x) \wedge I_A(x-y) = I_A(x)$

iii) $F_A(y) = F_A(x-x+y) = F_A(x-(x-y))$
$\leq T_A(x) \vee F_A(x-y) = F_A(x)$ .

□

**Proposition 3.4.** *If* $\{A_\alpha\}_{\alpha\in\Gamma}$ *be any family of SVNI of R. Then* $\bigcap\limits_{\alpha\in\Gamma} A_\alpha$ *or* $\bigcup\limits_{\alpha\in\Gamma} A_\alpha$ *is a SVNI(SVNLI, SVNRI).*

*Proof.* Suppose$\{A_\alpha\}_{\alpha\in\Gamma}$ be a family of SVNLI of R then

1 (i.) $(\cap T_{A_i})(x-y) = \wedge T_{A_i}(x-y)$
$\geq \wedge(T_{A_i}(x) \wedge T_{A_i}(y))$
$= (\wedge T_{A_i})(x) \wedge (\wedge T_{A_i}(y))$
$= (\cap T_{A_i})(x) \wedge (\cap T_{A_i})(y))$

(ii.) $(\cap I_{A_i})(x-y) = \wedge I_{A_i}(x-y)$
$\geq \wedge(I_{A_i}(x) \wedge I_{A_i}(y))$
$= (\wedge I_{A_i})(x) \wedge (\wedge I_{A_i}(y))$
$= (\cap I_{A_i})(x) \wedge (\cap I_{A_i})(y))$

(iii.) $(\cup F_{A_i})(x-y) = \vee F_{A_i}(x-y)$
$\leq \vee(F_{A_i}(x) \vee F_{A_i}(y))$
$= (\vee F_{A_i})(x) \vee (\vee T_{A_i}(y))$
$= (\cup T_{A_i})(x) \vee (\cup T_{A_i})(y))$

2 (i.) $(\cap T_{A_i})(xy) = \wedge T_{A_i}(xy)$
$\geq \wedge(T_{A_i}(x) \vee T_{A_i}(y))$
$= (\cap T_{A_i})(x) \wedge (\cap T_{A_i})(y))$

(ii.) $(\cap I_{A_i})(xy) = \wedge I_{A_i}(xy)$
$\geq \wedge(I_{A_i}(x) \vee I_{A_i}(y))$
$= (\cap I_{A_i})(x) \wedge (\cap I_{A_i})(y))$

(iii.) $(\cup F_{A_i})(xy) = \vee F_{A_i}(xy)$
$\leq \vee(F_{A_i}(x) \wedge F_{A_i}(y))$
$= (\cup F_{A_i})(x) \wedge (\cup F_{A_i})(y))$

By similar argument we can show that $\bigcup_{\alpha\in\Gamma} A_\alpha$ is a SVNI(SVNLI , SVNRI)

Hence $\bigcap_{\alpha\in\Gamma} A_\alpha$ is a SVNI of R. □

**Lemma 3.1.** *Let R be a ring and let* $A, B \in SVNS(R)$ .

(1) *If* $A, B \in SVNLI(R)$*(resp SVNRI(R) and SVNI(R)), then* $A\cap B \in SVNLI(R)$*(resp SVNRI(R) and SVNI(R)).*
(2) *If* $A \in SVNRI(R)$ *and* $B \in SVNLI(R)$ *then* $A \circ B \subset A \cap B$ .

*Proof.* (1) Suppose $A, B \in SVNLI(R)$ and let $x, y \in R$. Then

i. $T_{A\cap B}(x-y) = T_A(x-y) \wedge T_B(x-y)$
$\geq [T_A(x) \wedge T_A(y)] \wedge [T_B(x) \wedge T_B(y)]$
$= T_{A\cap B}(x) \wedge T_{A\cap B}(y)$ ,

ii. $I_{A\cap B}(x-y) = I_A(x-y) \wedge I_B(x-y)$
$\geq [I_A(x) \wedge I_A(y)] \wedge [I_B(x) \wedge I_B(y)]$
$= I_{A\cap B}(x) \wedge I_{A\cap B}(y)$ ,

iii. $F_{A\cap B}(x-y) = F_A(x-y) \vee F_B(x-y)$
$\leq [F_A(x) \vee F_A(y)] \vee [F_B(x) \vee F_B(y)]$
$= F_{A\cap B}(x) \vee F_{A\cap B}(y)$ .

Also

i. $T_{A\cap B}(xy) = T_A(xy) \wedge T_B(xy)$
$\geq [T_A(y) \wedge T_B(y)]$
$= T_{A\cap B}(y)$ ,

ii. $I_{A\cap B}(xy) = I_A(xy) \wedge I_B(xy)$
$\geq [I_A(y) \wedge I_B(y)]$
$= I_{A\cap B}(y)$ ,

iii. $F_{A\cap B}(xy) = I_A(xy) \vee I_B(xy)$
$\leq [F_A(y) \vee F_B(y)]$
$= F_{A\cap B}(y)$ .

Hence $A \cap B \in SVNLI(R)$. Similarly, we can easily see the remaining.

(2) Let $x \in G$ and suppose $A \circ B(x) = 0_N$, then there is nothing to prove. Suppose $A \circ B(x) \neq 0_N$. Then

$$A \circ B(x) = [\bigvee_{x=yz} [T_A(y) \wedge T_B(z)], \bigvee_{x=yz} [I_A(y) \wedge I_B(z)], \bigwedge_{x=yz} [F_A(y) \vee F_B(z)]] .$$

Since $A \in FNRI(R)$ and $B \in FNLI(R)$ we have:

$$T_A(y) \leq T_A(yz) = T_A(x), I_A(y) \leq I_A(yz) = I_A(x), F_A(y) \geq F_A(yz) = F_A(x) ,$$

and

$$T_B(z) \leq T_B(yz) = T_B(x), I_B(z) \leq I_B(yz) = I_B(x), F_B(y) \geq F_B(yz) = F_B(x) .$$

Thus

$$\begin{aligned} T_{A\circ B}(x) &= \bigvee_{x=yz} [T_A(y) \wedge T_B(z)] \leq T_A(x) \wedge T_B(x) = T_{A\cap B}(x) \\ I_{A\circ B}(x) &= \bigvee_{x=yz} [I_A(y) \wedge I_B(z)] \leq I_A(x) \wedge I_B(x) = I_{A\cap B}(x) \\ F_{A\circ B}(x) &= \bigwedge_{x=yz} [F_A(y) \vee F_B(z)] \geq F_A(x) \wedge F_B(x) = F_{A\cap B}(x) . \end{aligned}$$

Hence $A \circ B \subset A \cap B$. This completes the proof. □

**Proposition 3.5.** *Let R be a ring and let* $0_N \neq A \in SVNS(R)$ . *Then A is an* $SVNI$*(resp. an SVNLI, SVNRI) in R if and only if for any* $x, y \in R$

(1) $T_A(x-y) \geq T_A(x) \wedge T_A(y)$
$I_A(x-y) \geq I_A(x) \wedge I_A(y)$
$F_A(x-y) \leq F_A(x) \vee F_A(y)$.

(2) $T_A(xy) \geq T_A(x) \vee T_A(y)$
$I_A(xy) \geq I_A(x) \vee I_A(y)$
$F_A(xy) \leq F_A(x) \wedge F_A(y)$.

*[respectively* $T_A(xy) \geq T_A(y)$ *,*$I_A(xy) \geq I_A(y)$ *and* $F_A(xy) \leq F_A(y)$

**Proposition 3.6.** *Let R be a skew field (also division ring) and let* $0_N \neq A \in SVNS(R)$ *. Then A is an SVNI(SVNLI, SVNRI) of R if and only if* $T_A(x) = T_A(e) \leq T_A(0)$ *,*$I_A(x) = I_A(e) \leq I_A(0)$ *and* $F_A(x) = F_A(e) \geq F_A(0)$ *for any* $0 \neq x \in R$*, where 0 is the unity of R for* $'+'$ *and* $e$ *is the unity of R for* $'.'$

*Proof.* Let A be neutrosophic fuzzy ideal of R and let $0 \neq x \in R$. Then

$$T_A(x) = T_A(xe) \geq T_A(e), T_A(e) = T_A(x^{-1}x) \leq T_A(x)$$
$$I_A(x) = I_A(xe) \geq I_A(e), I_A(e) = I_A(x^{-1}x) \leq I_A(x)$$
$$F_A(x) = F_A(xe) \leq F_A(e), F_A(e) = F_A(x^{-1}x) \geq F_A(x)\,.$$

Thus $T_A(x) = T_A(e), I_A(x) = I_A(e)$,$F_A(x) = F_A(e)$ , and

$$\begin{aligned} T_A(0) &= T_A(e-e) \geq T_A(e) \wedge T_A(e) = T_A(e) \\ I_A(0) &= I_A(e-e) \geq I_A(e) \wedge I_A(e) = I_A(e) \\ F_A(0) &= F_A(e-e) \leq F_A(e) \vee F_A(e) = F_A(e)\,. \end{aligned}$$

So $T_A(e) \leq T_A(0), I_A(e) \leq I_A(0), F_A(e) \geq F_A(0)$. Hence the necessary condition hold.

Suppose the necessary condition hold, let $x \in R$. Then we have the following four cases:

Case(i) Suppose $x \neq 0, y \neq 0$ and $x \neq y$. Then:

$$T_A(x-y) = T_A(e) \geq T_A(x) \wedge T_A(y)$$
$$I_A(x-y) = I_A(e) \geq I_A(x) \wedge I_A(y)$$
$$F_A(x-y) = F_A(e) \leq F_A(x) \vee F_A(y)\,.$$

$$T_A(xy) = T_A(e) \geq T_A(x) \vee T_A(y)$$
$$I_A(xy) = I_A(e) \geq I_A(x) \vee I_A(y)$$
$$F_A(xy) = F_A(e) \leq F_A(x) \wedge F_A(y)\,.$$

Case(ii) Suppose $x \neq 0, y \neq 0$ and $x = y$. Then:

$$T_A(x-y) = T_A(0) \geq T_A(x) \wedge T_A(y)$$
$$I_A(x-y) = I_A(0) \geq I_A(x) \wedge I_A(y)$$
$$F_A(x-y) = F_A(0) \leq F_A(x) \vee F_A(y)\,.$$

$$T_A(xy) = T_A(e) \geq T_A(x) \vee T_A(y)$$
$$I_A(xy) = I_A(e) \geq I_A(x) \vee I_A(y)$$
$$F_A(xy) = F_A(e) \leq F_A(x) \wedge F_A(y)\,.$$

Case(iii) Suppose $x \neq 0, y = 0$ . Then:

$$T_A(x-y) = T_A(x) = T_A(e) \geq T_A(x) \wedge T_A(y)$$
$$I_A(x-y) = I_A(x) = I_A(e) \geq I_A(x) \wedge I_A(y)$$
$$F_A(x-y) = F_A(x) = F_A(0) \leq F_A(x) \vee F_A(y)\,.$$

$$T_A(xy) = T_A(0) \geq T_A(x) \vee T_A(y)$$
$$I_A(xy) = I_A(0) \geq I_A(x) \vee I_A(y)$$
$$F_A(xy) = F_A(0) \leq F_A(x) \wedge F_A(y)\,.$$

Case(iv) Similar to case(iii).

In all the cases A is a fuzzy neutrosophic set of R. □

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